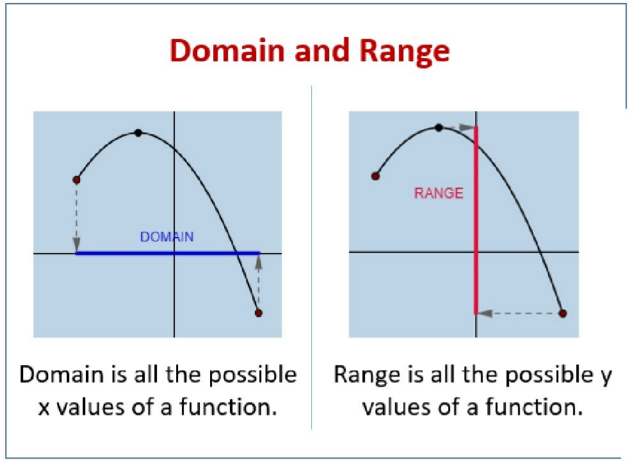
Methods of Solving Optimization Problems

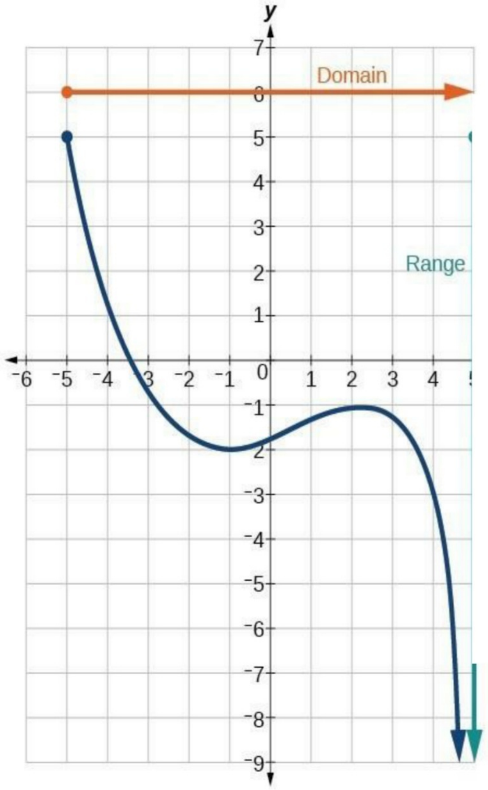
– Graphical method
– Analytical methods (or classical methods)
– Numerical Methods

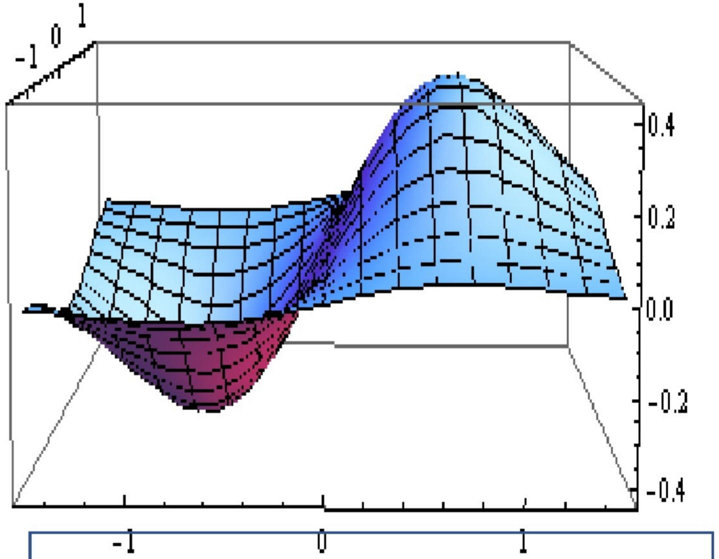
Graphical method

- Unconstrained or Constrained
- Optimization problems with one and two variables only

Graphical method – Domain and Range

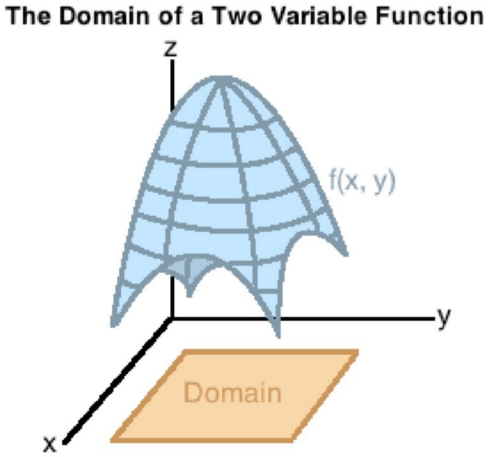






Domain: $-1 \leq x, y \leq 1$

Range: $-0.4 \leq f(x, y) \leq 0.4$



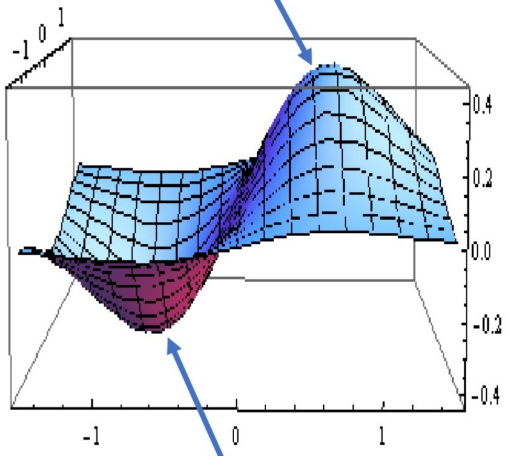
Solutions for an Optimization problem

- **Local Optimum Solution**
  - A function f(**X**) is said to have
    - A local minimum at $\mathbf{X}^*$ if $f(\mathbf{X}^*) \leq f(\mathbf{X})$
    - A local maximum at $\mathbf{X}^*$ if $f(\mathbf{X}^*) \geq f(\mathbf{X})$

    for all X in an open neighbourhood of **X**.
- **Global Optimum Solution**
  - A function f(**X**) is said to have
    - A global minimum at $\mathbf{X}^*$ if $f(\mathbf{X}^*) \leq f(\mathbf{X})$
    - A global maximum at $\mathbf{X}^*$ if $f(\mathbf{X}^*) \geq f(\mathbf{X})$

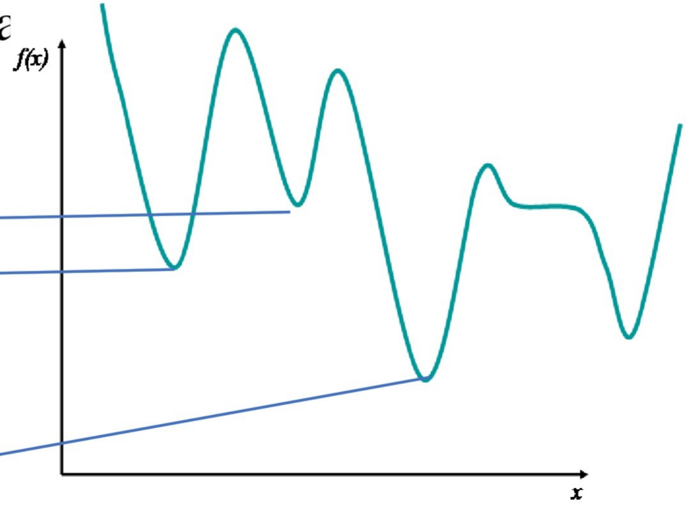
    for all X in the doma

Local and global Maximum



Local and global Minimum

Local Minimum



Global Minimum

Shifting a Graph of a Function

Shift Formulas

Vertical Shifts

$y = f(x) + k$	Shifts the graph of f *up* k units if $k > 0$ Shifts it *down* $\lvert k \rvert$ units if $k < 0$

Horizontal Shifts

$y = f(x + h)$	Shifts the graph of f *left* h units if $h > 0$ Shifts it *right* $\lvert h \rvert$ units if $h < 0$

Graphical method
-Unconstrained, Single Variable Optimization Problems

$$f(x) = 6 - x^2$$

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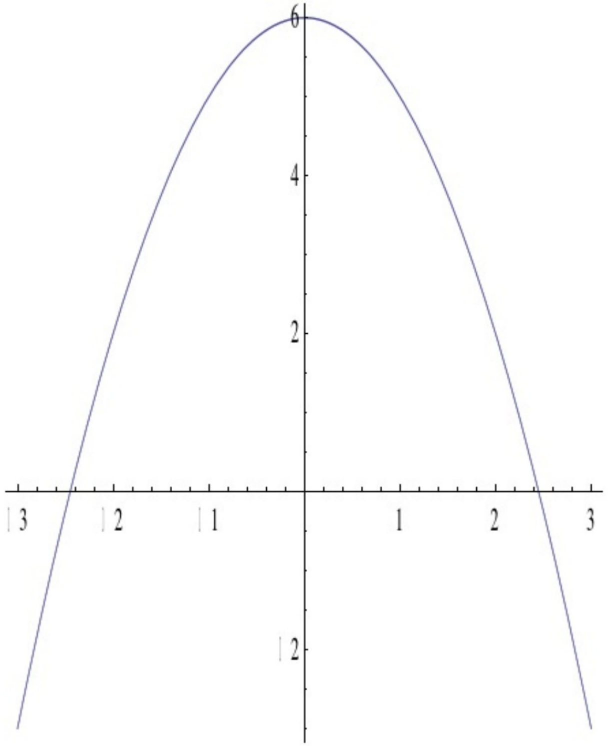


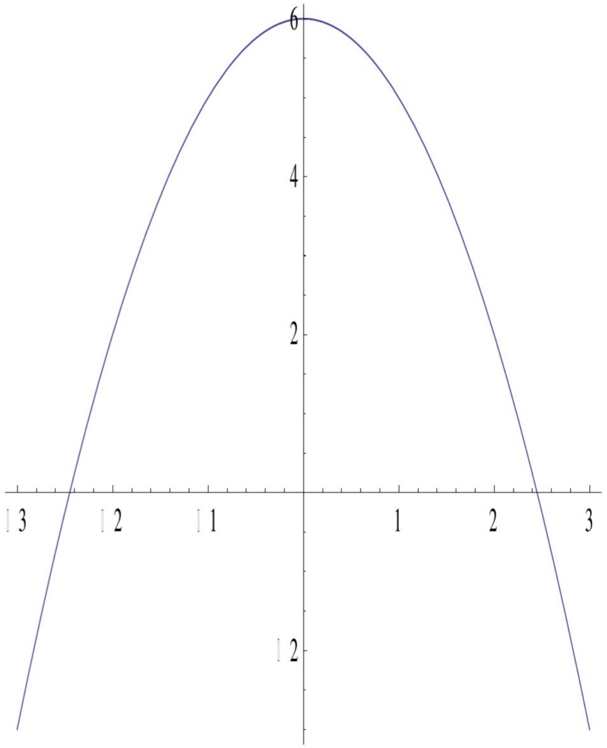
Graphical method
-Unconstrained, Single Variable Optimization Problems

$f(x) = 6 - x^2$

Local maximum and Global maximum

$x^* = 0$

$f(x^*) = 6$



Graphical method

-Unconstrained, Single Variable Optimization Problems

$$f(x) = (x-3)^2 + 1$$

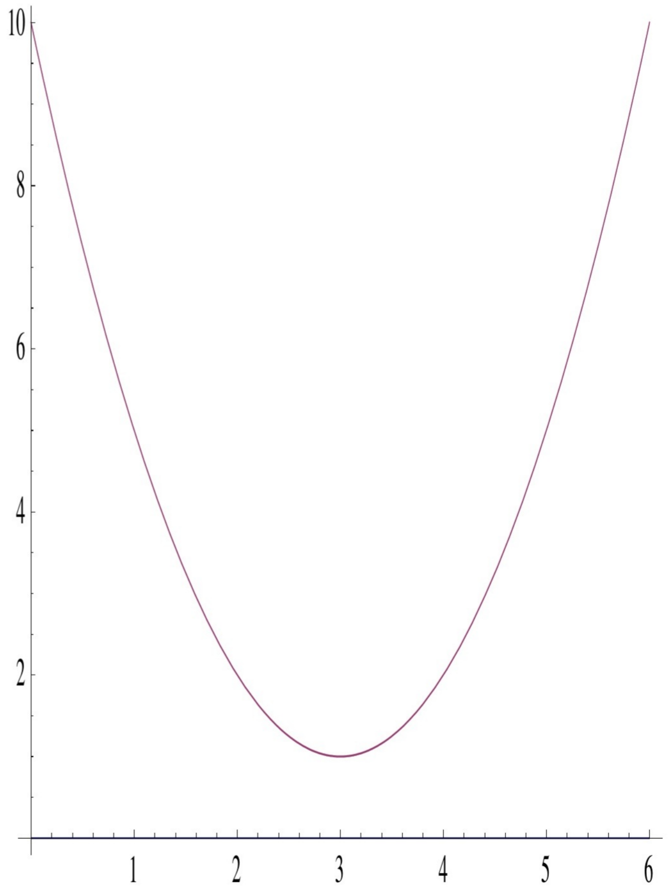
Graphical method
-Unconstrained, Single Variable Optimization Problems

$f(x) = (x-3)^2 + 1$

Local minimum and Global minimum

$x^* = 3$

$f(x^*) = 1$



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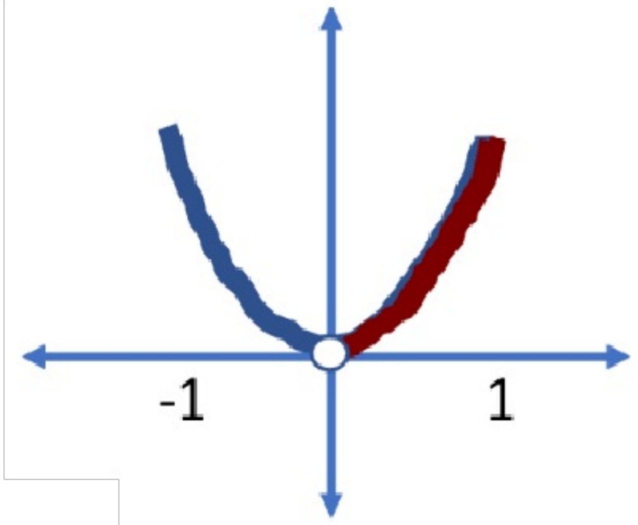
Graphical method
- Single Variable Optimization Problems

Find the points of minimum and maximum (local and global) for the following functions

(a) $f(x) = x^2$ in $[-1,1]$
(b) $f(x) = x^2$ in $[0,1]$
(c) $f(x) = x^2$ in $(0,1]$
(d) $f(x) = x^2$ in $[-1,0)$
(e) $f(x) = x^2$ in $(-1,0)$
(f) $f(x) = 5x + 3$
(g) $f(x) = 5x + 3$ in $[0,5]$



Graphical method
- Single Variable Optimization Problems

Find the points of minimum and maximum (local and global) for the following functions

(a) $f(x) = x^2$ in $[-1,1]$

(b) $f(x) = x^2$ in $[0,1]$

(c) $f(x) = x^2$ in $(0,1]$

(d) $f(x) = x^2$ in $[-1,0)$

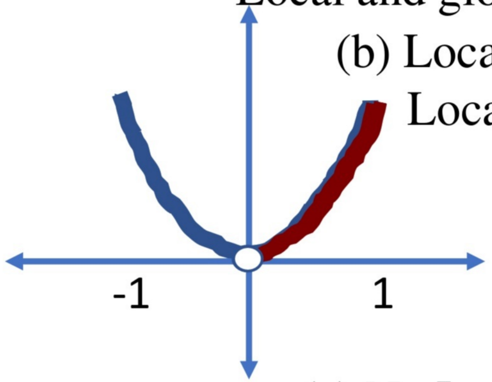
(e) $f(x) = x^2$ in $(-1,0)$

(f) $f(x) = 5x + 3$

(g) $f(x) = 5x + 3$ in $[0,5]$

(a) Local and global minimum at $x=0$
Local and global maximum at $x= -1$ and $x=1$

(b) Local and global minimum at $x=0$
Local and global maximum at $x=1$



(c) No Local and global minimum
Local and global maximum at $x=1$

(e) and (f) No Local/global minimum/maximum

The minimum / maximum depends on not only the function but also on the region in which the optimum has to be found.

Graphical Method – SVO

- An open rectangular box need to be made using a sheet of length 12cm and breadth 8cm. What would be the height of the box if the volume of it should be maximum?

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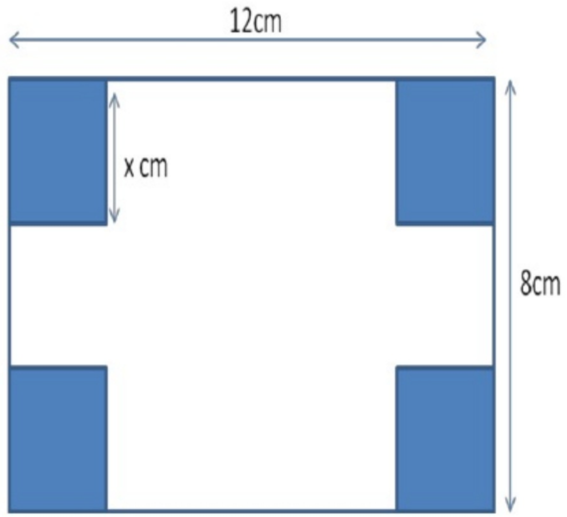

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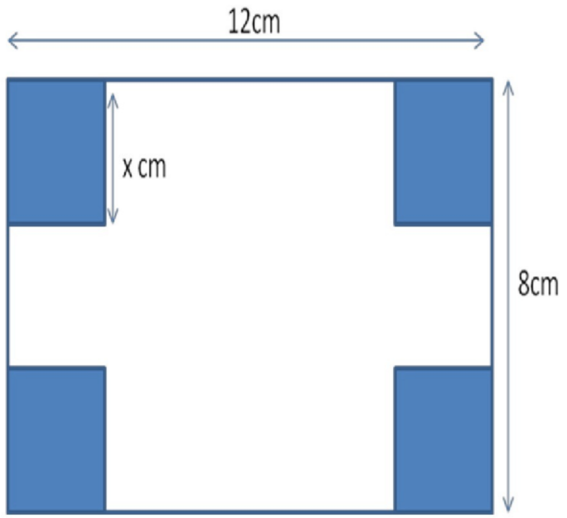
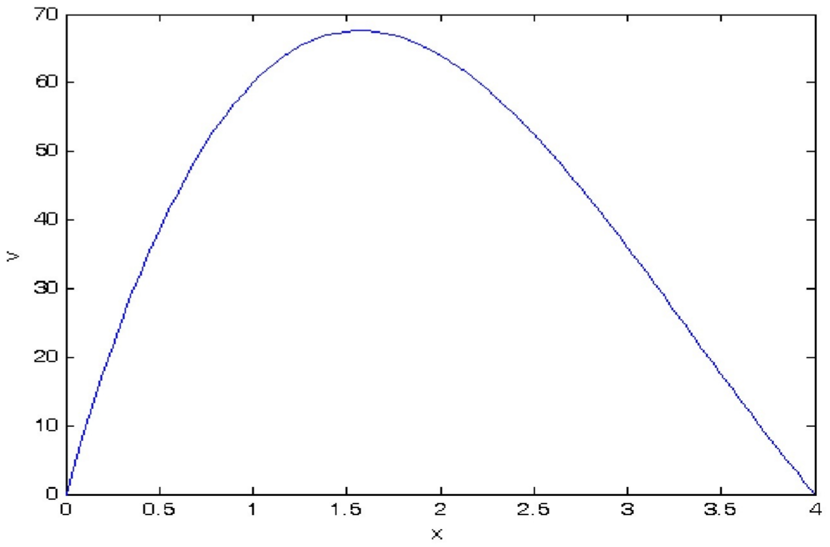


Problem: Maximize V

$$V = (12 - 2x)(8 - 2x)x = 4x^3 - 40x^2 + 96x$$



Graphical Method – S.V.O.P. – Using Shifting

- If x^* is the minimum of $f(x)$, then x^* will also have the minimum of $f(x)+a$ and Minimum value of $[f(\mathbf{x})+a]$ = Minimum value of $f(\mathbf{x}) + a$.
- If x^* is the minimum of $f(x)$, then minimum of $f(x+a)$ will be at $x^*- a$. The minimum values of $f(x)$ and $f(x+a)$ is the same.