23MAT206
OPTIMIZATION TECHNIQUES

L-T-P-C: 3-0-2-4

SEMESTER III

23MAT206	OPTIMIZATION TECHNIQUES	L-T-P-C: 3-0-2-4

Course Objectives

- To understand the concept of search space and optimality for solutions of engineering problems.
- To understand some computation techniques for optimizing single variable functions.
- To understand various computational techniques for optimizing severable variable functions.

Course Outcomes

CO1: Understand different types of Optimization Techniques in engineering problems. Learn Optimization methods such as Bracketing methods, Region elimination methods, Point estimation methods.
CO2: Understand Optimizations Techniques in single variable functions.
CO3: Understand the optimality criteria for the multivariable optimizations.
CO4: Understand Optimizations Techniques in multi variable functions.
CO5: Understand constrained optimization techniques and Kuhn-Tucker conditions.

Syllabus

Unit 1

Introduction to optimization: classical optimization, Optimality criteria – Necessary and sufficient conditions for existence of extreme point.

Direct search methods: unidirectional search, evolutionary search method, simplex search method, Introduction, Conditions for local minimization. One dimensional Search methods: Golden search method, Fibonacci method, Newton's Method, Secant Method, Remarks on Line Search Sections. Hook-Jeeves pattern search method.

Unit 2

Gradient-based methods- introduction, the method of steepest descent, analysis of Gradient Methods, Convergence, Convergence Rate. Analysis of Newton's Method, Levenberg-Marquardt Modification, Newton's Method for Nonlinear Least-Squares.

Conjugate direction method, Introduction, The Conjugate Direction Algorithm, The Conjugate Gradient Algorithm for Non-Quadratic Quasi Newton method.

Unit 3

Nonlinear Equality Constrained Optimization- Introduction, Problems with equality constraints Problem Formulation, Tangent and Normal Spaces, Lagrange Condition

Nonlinear Inequality Constrained Optimization -Introduction - Problems with inequality constraints: Kuhn-Tucker conditions.

Lab Practice Problems: Single and multivariable optimizations. Implementation of iterative methods.

Case studies

Textbook(s)

Edwin K.P. Chong, Stanislaw H. Zak, "An introduction to Optimization", 2nd edition, Wiley, 2013.

Reference(s)

S.S. Rao, "Optimization Theory and Applications", Second Edition, New Age International (P) Limited Publishers, 1995.

Kalyanmoy Deb, "Optimization for Engineering Design: Algorithms and Examples, Prentice Hall, 2002.

Lab Experiments:

1. Identifying definiteness of matrices using eigenvalues and use of Hessian matrix to identify concavity of the surfaces (revision from Calculus and Linear Algebra)
2. Implementation of Golden Section Search, Fibonacci search for single variable optimization problems
3. Evaluation of ordinary and partial derivatives numerically (in excel/MATLAB)
4. Implementation of Secant method and Newton's method for single variable optimization problems
5. Implementation of evolutionary search method for multivariable optimization problems
6. Implementation of Simplex search method for multivariable optimization problems
7. Implementation of Hooke-Jeeve's Pattern Search method for multivariable optimization problems
8. Implementation of Newton's method for solving system of non-linear equations
9. Implementation of Newton's method for solving multivariable optimization problems
10. Identifying whether a constrained optimization problem is convex or not and solutions using 'cvx'

Evaluation Pattern: 70:30

Assessment	Internal	External
Midterm	20	
*Continuous Assessments (CA)	50	
**End Semester		30 (50 Marks; 2 hours exam)

*CA – Can be Quizzes, Assignment, Lab Practice, Projects, and Reports

**End Semester can be theory examination/ lab-based examination

Introduction

What is Optimization?

- Optimization is the act of obtaining the best result under a given circumstances.
- Optimization is the mathematical discipline which is concerned with finding the maxima and minima of functions, possibly subject to constraints.

OPTIMIZATION THEORY

Dictionary Meaning

- Optimization : an act, process, methodology or procedure of making something (as a design, system, or decision) as fully perfect, functional, or effective as possible (Minimization or Maximization)

Theory: an idea or set of ideas that is intended to explain facts or events

Historical development

- ***Isaac Newton*** *(1642-1727) :The development of differential calculus methods of optimization.*
- ***Joseph-Louis Lagrange*** *(1736-1813) :Calculus of variations, minimization of functionals, method of optimization for constrained problems.*
- ***Augustin-Louis Cauchy*** *(1789-1857) :Solution by direct substitution, steepest descent method for unconstrained optimization.*
- ***George Bernard Dantzig*** *(1914-2005):Linear programming and Simplex method (1947).*
- ***Albert William Tucker*** *(1905-1995):Necessary and sufficient conditions for the optimal solution of programming problems, nonlinear programming.*

Why Optimization?

To understand and get solutions to many questions like:

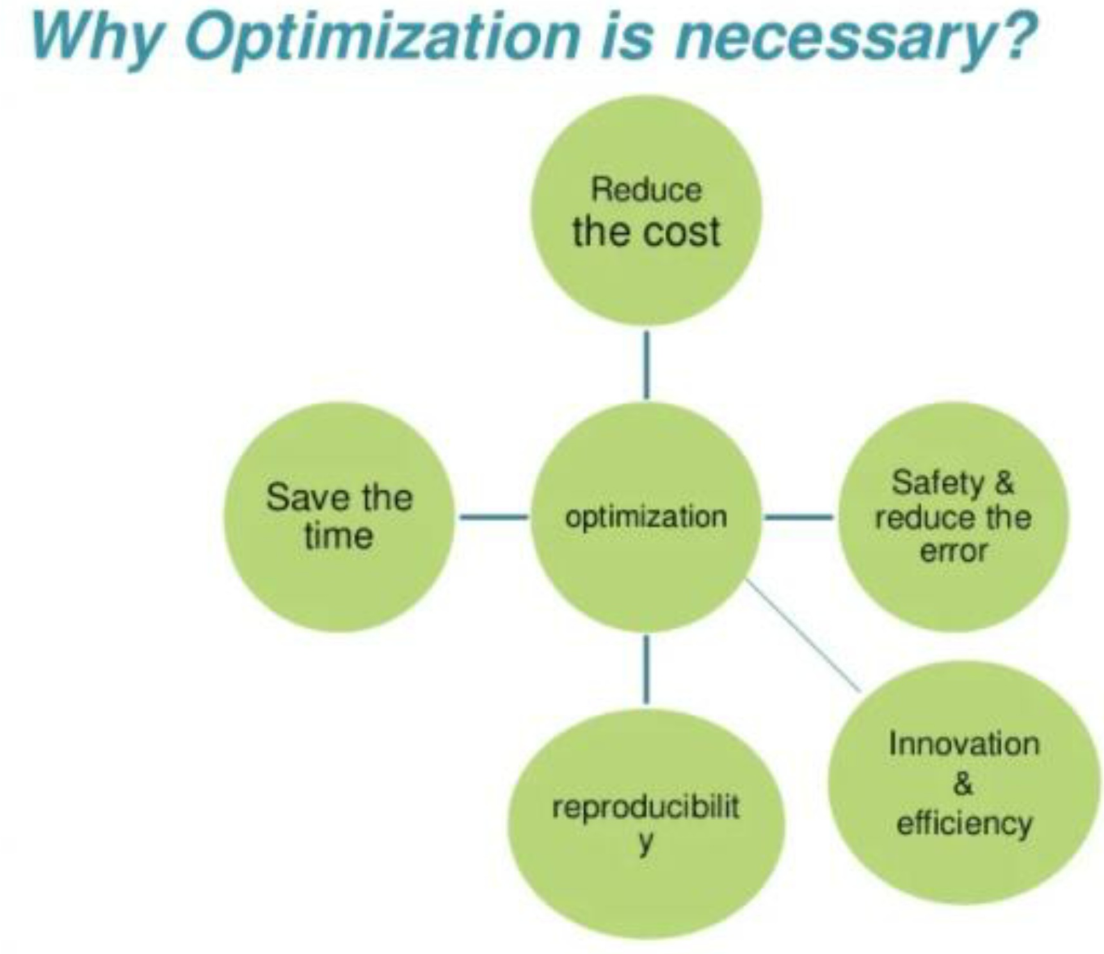
- ➢ How can a car manufacturer get the most parts out of a piece of sheet metal?
- ➢ How can a household moving company fit the most furniture into a truck of a certain size?
- ➢ How can the phone company route calls to get the best use of its lines and connections?
- ➢ How can a university schedule its classes to make the best use of classrooms without conflicts?

Similarly, engineers have to take many technological and managerial decisions at several stages. The main and important goal of all such decisions is either to minimize the required effort (time / cost / power consumption) or maximize the desired benefit (profit / efficiency of the engine / processer speed) .

Why Optimization?

Helps to improve the quality of decision making in every field

Engineering
Sciences
Business
Economics
Military Planning



Applications

- Design of Aircrafts, racing cars
- Maximize the efficiency of a power plant
- Maximize the efficiency of a IC engine
- Optimized Scheduling of a machine
- Minimization of the manufacturing cost
- Optimum product mixing in a fractionating problem (petroleum refinery)
- Minimizing transmission loss
- Faster Communication between nodes in a communication network
- Maximizing the processor speed
- Robot path planning
- Fitting of a data
- …

Questions Should be in mind

- ✓ How we can make Formulation perfect ?
- ✓ What should be characteristics?
- ✓ What should be the conditions?

Typical steps for Solving Mathematical Optimization Problems

Problem formulation

Understanding and Defining the Variables

Defining the objective function using variables

Writing the constraints based on the restrictions using variables

Checking the existence of a solution

Solving the optimization problem, if a solution exists

- **Calculus methods/ Analytical methods**
- **Search methods/ Numerical methods**

Solution analysis

Mathematical model of an Optimization Problem

Example 1:

- Use a string of 100 m to form a rectangle of maximum area.

Mathematical model of an Optimization Problem

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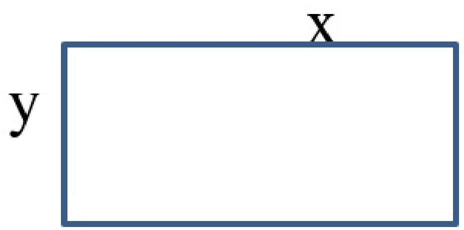


Variable definitions

x = length in meters

y = breadth in meters

Maximize xy

Objective function

subject to $2x + 2y \leq 100$, $x > 0$, $y > 0$.

constraints

Example 2:

Form a rectangle of area $1000 m^2$, whose perimeter is as small as possible.

Form a rectangle of area $1000m^2$, whose perimeter is as small as possible.

Variable: x = length, y = breadth

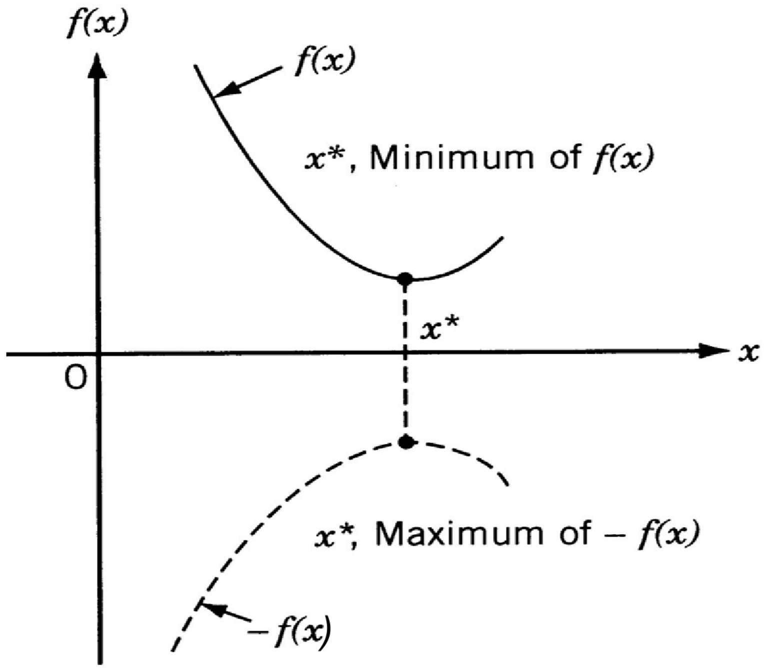
Minimize $2x + 2y$

subject to $xy = 1000$, $x > 0$, $y > 0$.

- A mathematical formulation of an optimization problem:

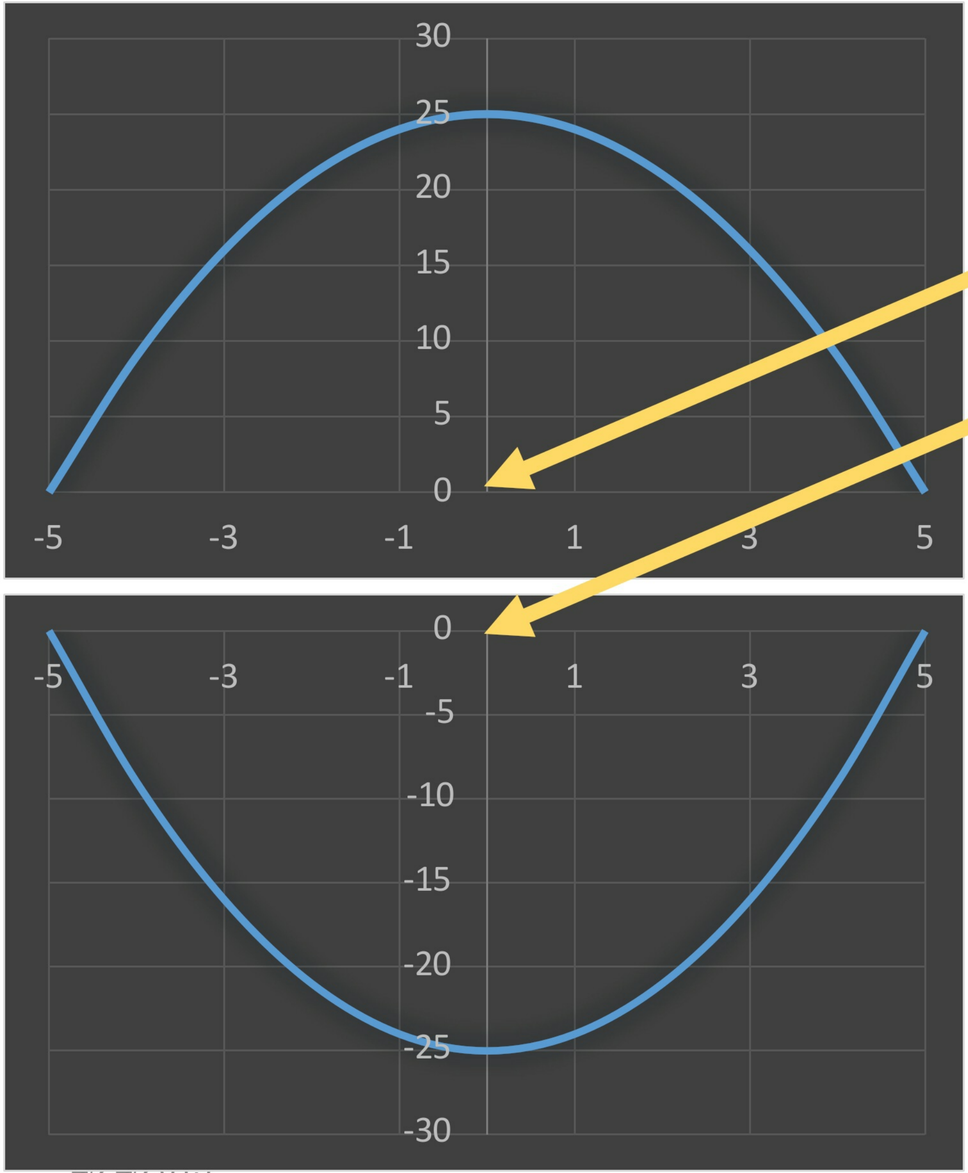
$$\text{Minimize } f(\mathbf{x}) \text{ subject to } \mathbf{x} \in S$$

- If x^* has a minimum of f(x), then x^* will also have the maximum of -f(x) and Minimum value of $f(\mathbf{x}) = -$ Maximum value of $-f(\mathbf{x})$



Minimum of $f(x)$ is same as maximum of $-f(x)$.

Unimodal and duality principle



Optimal solution $x^* = 0$

Optimal solution $x^* = 0$

Minimization $f(x)$ = Maximization $(-f(x))$

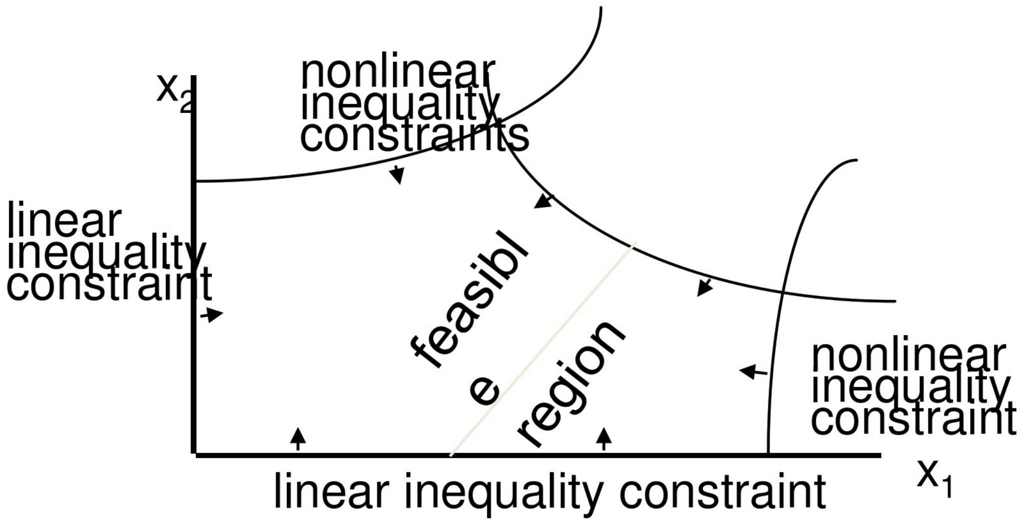
- Essential Components in an Optimization problem

  Minimize f(**x**) subject to **x** ϵ S

  x: set of variables

  f : objective function

  S: feasible region (set of all points that satisfy all the constraints)



- Solution of an optimization problem: Minimize f(**x**) subject to **x** ϵ S

  x* ϵ S such that f (**x***) ≤ f (**x**) for every **x** ϵ S

  x* : solution, f (**x***) : optimal objective function value

  x* may not be unique and may not even exist.