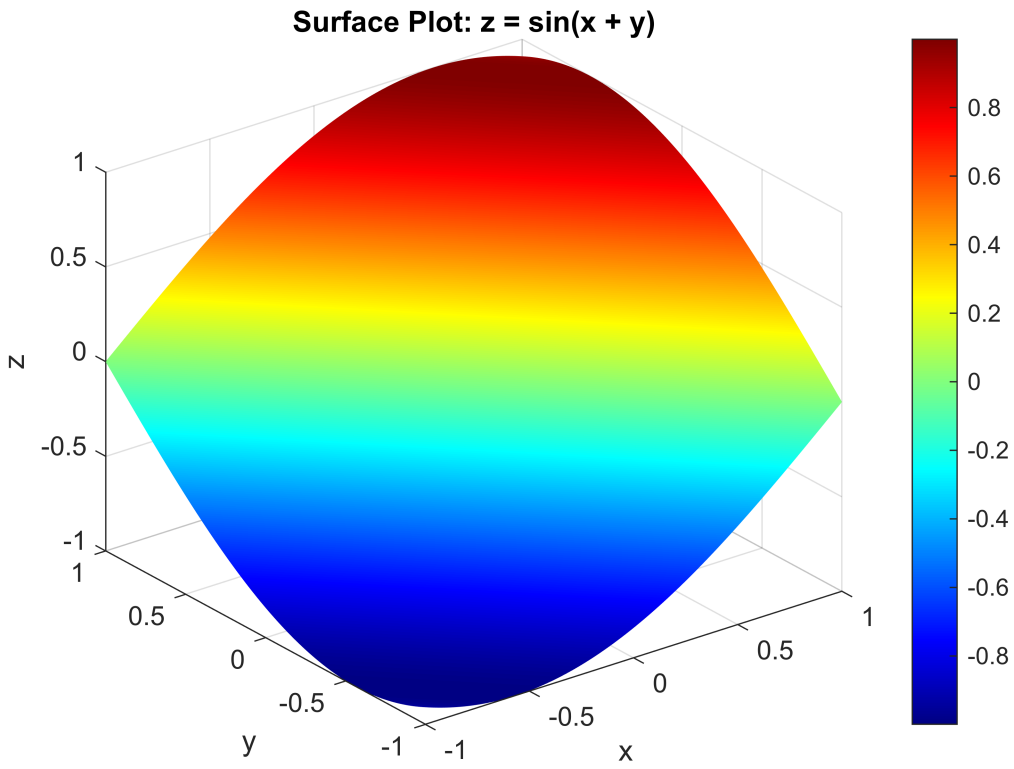
QUESTION1

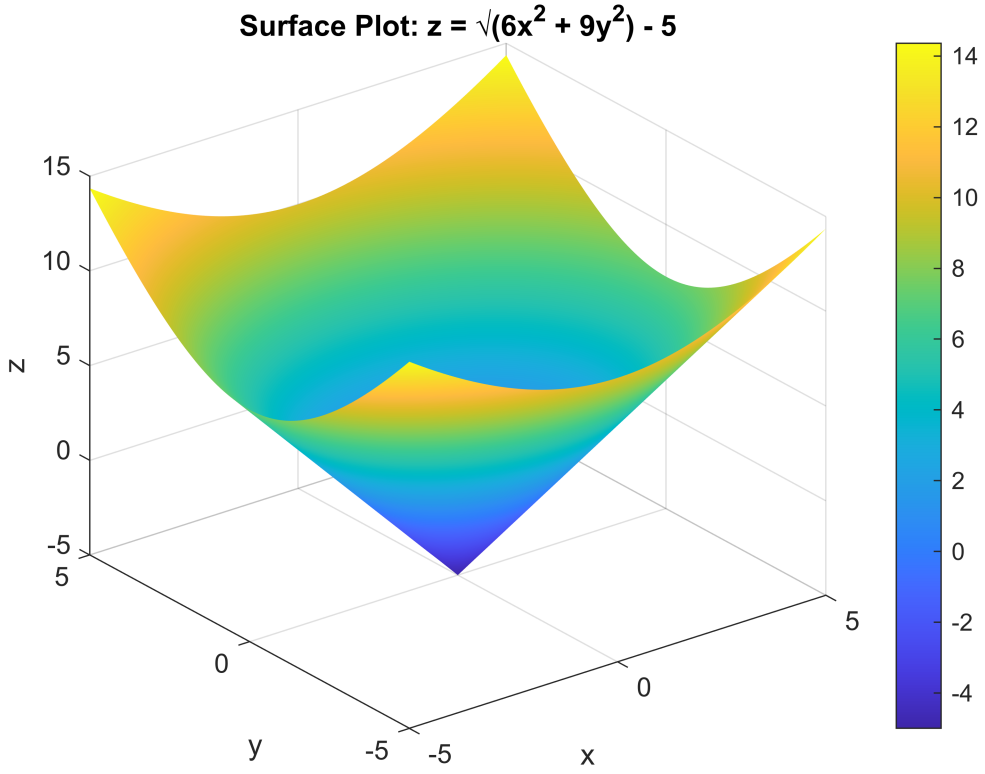
```
[x, y] = meshgrid(-1:0.01:1, -1:0.01:1);


z = sin(x + y);


surf(x, y, z)

xlabel('x')
ylabel('y')
zlabel('z')
title('Surface Plot: z = sin(x + y)')

shading interp
colormap jet
colorbar
```



QUESTION 2

```
[x, y] = meshgrid(-5:0.1:5, -5:0.1:5);
```

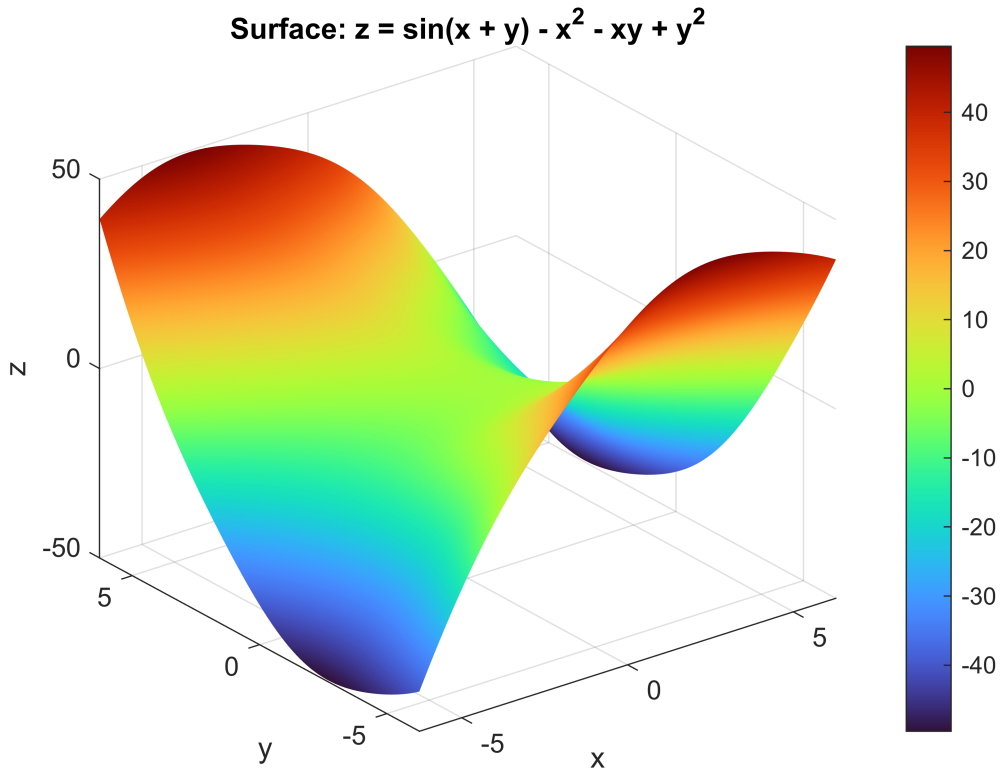
```
% Compute z values
z = sqrt(6*x.^2 + 9*y.^2) - 5;

% Plot the surface
surf(x, y, z)
xlabel('x')
ylabel('y')
zlabel('z')
title('Surface Plot: z = √(6x^2 + 9y^2) - 5')
shading interp
colormap parula
colorbar
```



QUESTION 3

```
% Define grid
x = linspace(-2*pi, 2*pi, 100);
y = linspace(-2*pi, 2*pi, 100);
[X, Y] = meshgrid(x, y);

% Define function
Z = sin(X + Y) - X.^2 - X.*Y + Y.^2;

% Plot surface
```

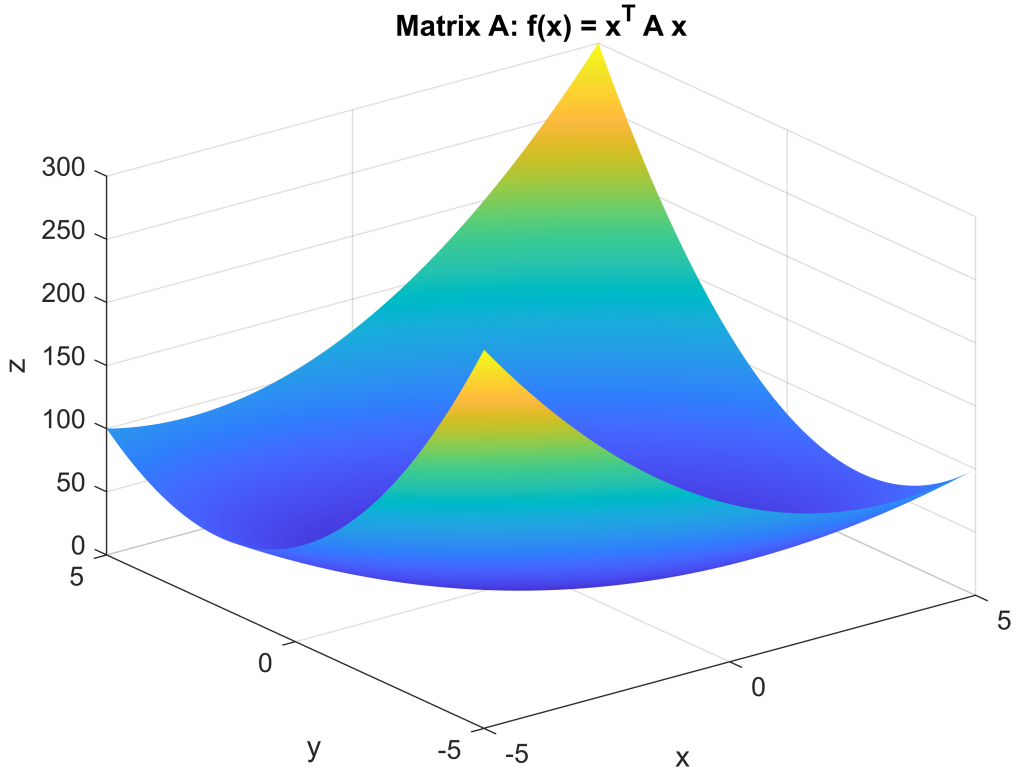
```
surf(X, Y, Z)
xlabel('x')
ylabel('y')
zlabel('z')
title('Surface: z = sin(x + y) - x^2 - xy + y^2')
shading interp
colormap turbo
colorbar
```



question4i)

```
A = [3 2; 2 5];

% Grid setup
[x, y] = meshgrid(-5:0.1:5, -5:0.1:5);

% Quadratic form f(x) = x' * A * x
z = A(1,1)*x.^2 + (A(1,2)+A(2,1))*x.*y + A(2,2)*y.^2;

% Plot
figure
surf(x, y, z)
xlabel('x'), ylabel('y'), zlabel('z')
title('Matrix A: f(x) = x^T A x')
```

```
shading interp
```




```
% Eigenvalues
eigA = eig(A)
```

```
eigA = 2×1
    1.7639
    6.2361
```

```
disp('Eigenvalues of A:')
```

```
Eigenvalues of A:
```

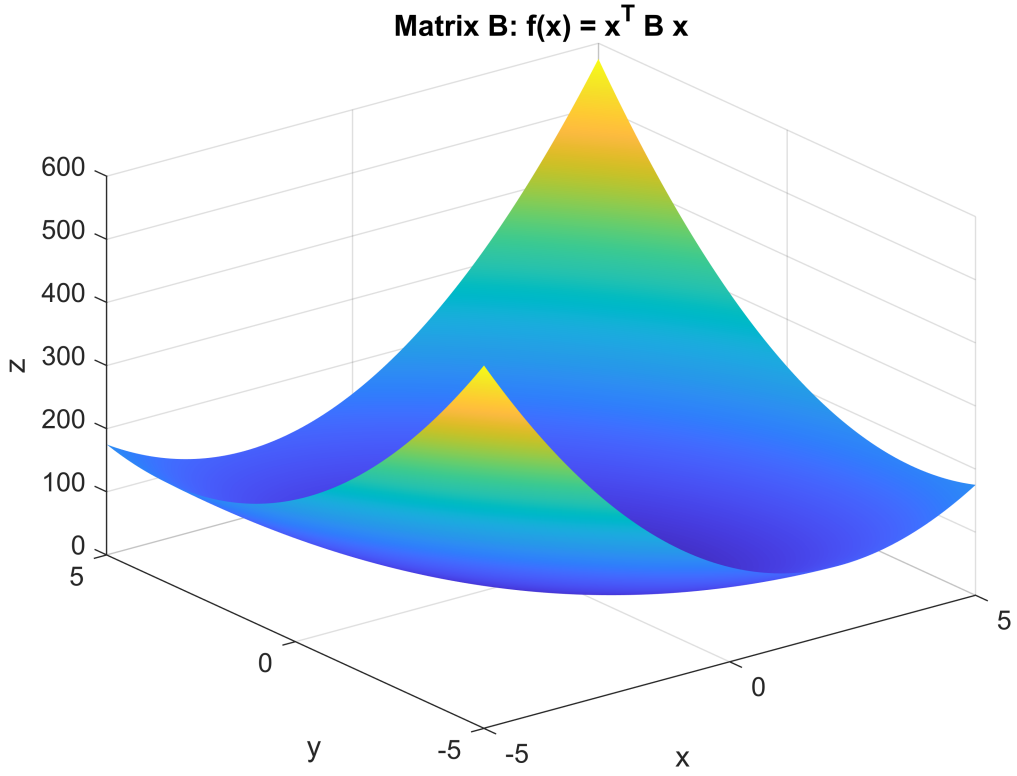
```
disp(eigA)
```

```
    1.7639
    6.2361
```

ii)

```
B = [9 4; 4 6];
z = B(1,1)*x.^2 + (B(1,2)+B(2,1))*x.*y + B(2,2)*y.^2;

figure
surf(x, y, z)
xlabel('x'), ylabel('y'), zlabel('z')
title('Matrix B: f(x) = x^T B x')
```

```
shading interp
```




```
eigB = eig(B)
```

```
eigB = 2×1
    3.2280
   11.7720
```

```
disp('Eigenvalues of B:')
```

```
Eigenvalues of B:
```

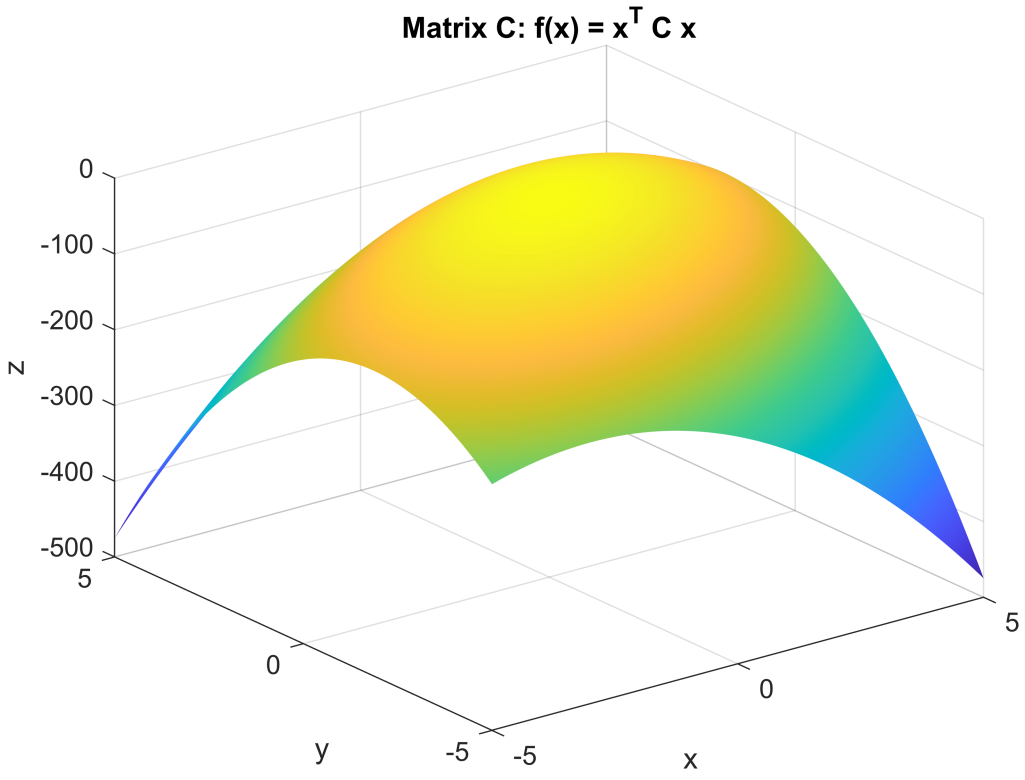
```
disp(eigB)
```

```
    3.2280
   11.7720
```

iii)

```
C = [-5 3; 3 -8];
z = C(1,1)*x.^2 + (C(1,2)+C(2,1))*x.*y + C(2,2)*y.^2;

figure
surf(x, y, z)
xlabel('x'), ylabel('y'), zlabel('z')
title('Matrix C: f(x) = x^T C x')
```

```
shading interp
```




```
eigC = eig(C)
```

```
eigC = 2×1
   -9.8541
   -3.1459
```

```
disp('Eigenvalues of C:')
```

```
Eigenvalues of C:
```

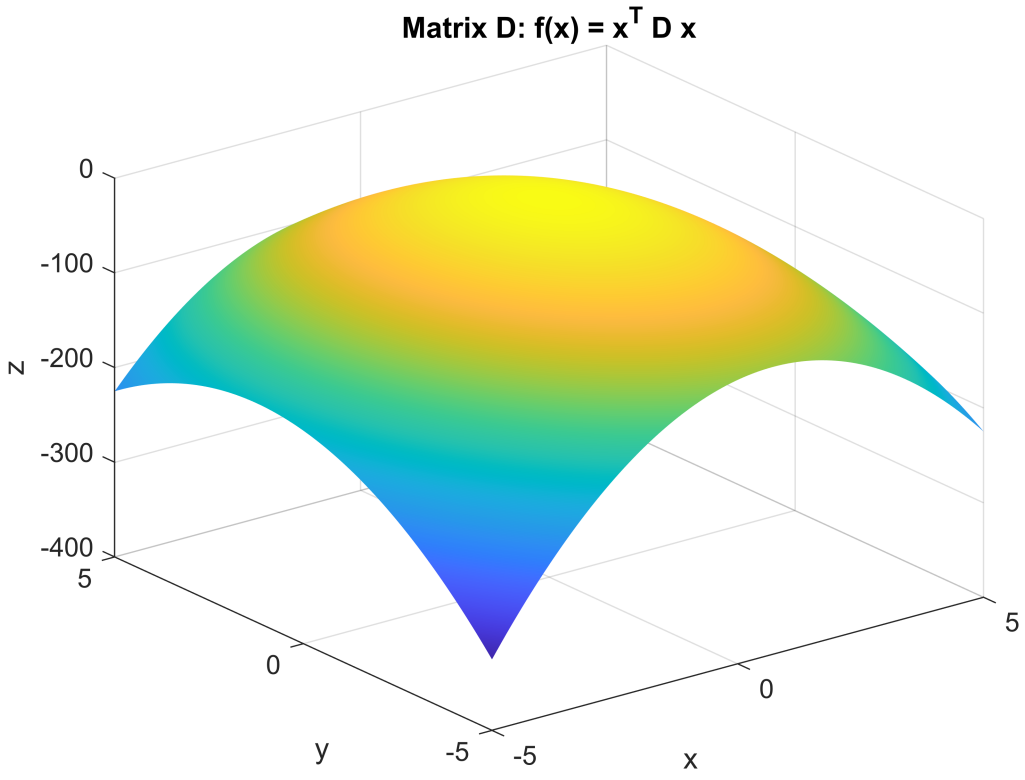
```
disp(eigC)
```

```
   -9.8541
   -3.1459
```

iv)

```
D = [-7 -1; -1 -4];
z = D(1,1)*x.^2 + (D(1,2)+D(2,1))*x.*y + D(2,2)*y.^2;

figure
surf(x, y, z)
xlabel('x'), ylabel('y'), zlabel('z')
title('Matrix D: f(x) = x^T D x')
```

```
shading interp
```




```
eigD = eig(D)
```

```
eigD = 2×1
   -7.3028
   -3.6972
```

```
disp('Eigenvalues of D:')
```

```
Eigenvalues of D:
```

```
disp(eigD)
```

```
   -7.3028
   -3.6972
```

question 5

```
% Generate random 6x2 matrix A
A = rand(6, 2);

% Compute S = A' * A
S = A' * A;

% a) Eigenvalues of S
```

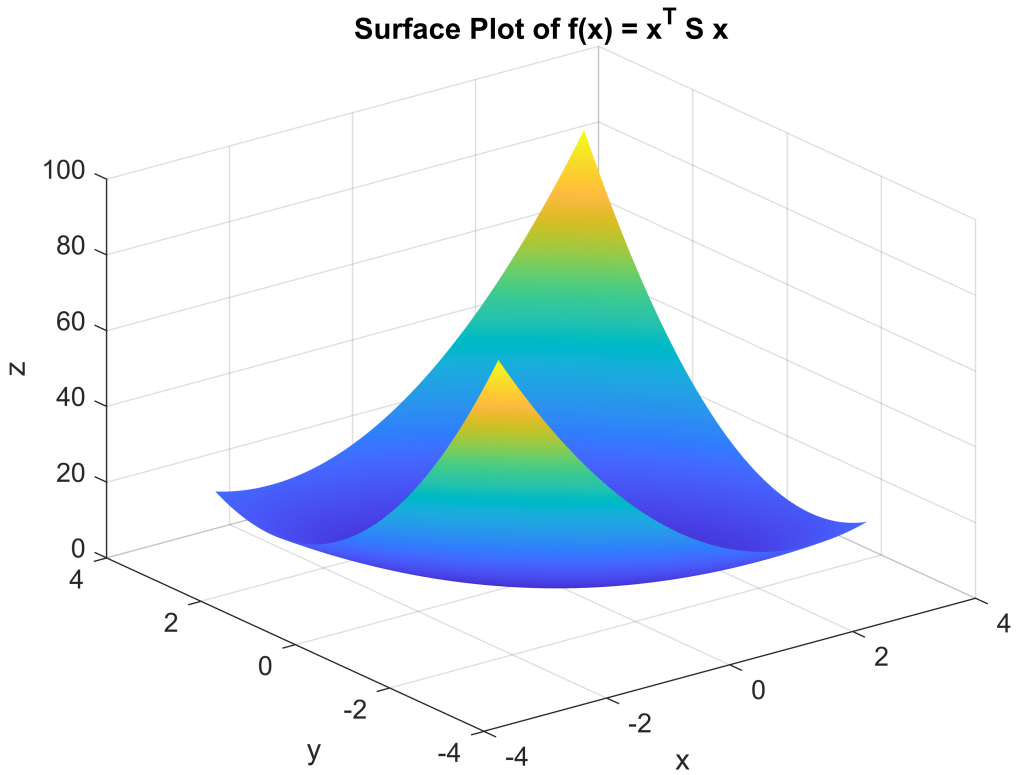
```
eigS = eig(S);
disp('Eigenvalues of S:')
```

```
Eigenvalues of S:
```

```
disp(eigS)
```

```
    1.0352
    4.9002
```

```
% b) Plot f(x) = x' * S * x
[x, y] = meshgrid(-3:0.1:3, -3:0.1:3);
z = S(1,1)*x.^2 + (S(1,2)+S(2,1))*x.*y + S(2,2)*y.^2;

figure
surf(x, y, z)
xlabel('x'), ylabel('y'), zlabel('z')
title('Surface Plot of f(x) = x^T S x')
shading interp
```




```
```

question 6

```
    syms x y z

% a) f(x,y,z) = 4*x^3*exp(x*y) - y^3*z
```

```
f = 4*x^3*exp(x*y) - y^3*z;
grad_f = gradient(f, [x y z]);
hess_f = hessian(f, [x y z]);
disp('Gradient of f:')
```

```
Gradient of f:
```

```
disp(grad_f)
```

$$\begin{pmatrix} 12\,x^2\,\mathrm{e}^{x\,y} + 4\,x^3\,y\,\mathrm{e}^{x\,y} \\ 4\,x^4\,\mathrm{e}^{x\,y} - 3\,y^2\,z \\ -y^3 \end{pmatrix}$$

```
disp('Hessian of f:')
```

```
Hessian of f:
```

```
disp(hess_f)
```

$$\begin{pmatrix} 24\,x\,\mathrm{e}^{x\,y} + 24\,x^2\,y\,\mathrm{e}^{x\,y} + 4\,x^3\,y^2\,\mathrm{e}^{x\,y} & \sigma_1 & 0 \\ \sigma_1 & 4\,x^5\,\mathrm{e}^{x\,y} - 6\,y\,z & -3\,y^2 \\ 0 & -3\,y^2 & 0 \end{pmatrix}$$

where

$$\sigma_1 = 16\,x^3\,\mathrm{e}^{x\,y} + 4\,x^4\,y\,\mathrm{e}^{x\,y}$$

```
% b) g(x,y) = exp(x*y)*(x + y) at point P = (-1, 1)
g = exp(x*y)*(x + y);
grad_g = gradient(g, [x y]);
grad_g_atP = subs(grad_g, [x y], [-1 1]);
disp('Gradient of g at P = (-1,1):')
```

```
Gradient of g at P = (-1,1):
```

```
disp(grad_g_atP)
```

$$\begin{pmatrix} \mathrm{e}^{-1} \\ \mathrm{e}^{-1} \end{pmatrix}$$

```
% c) h(x,y,z) = (x^2 + y^2) / (x^2 - y^2)
% Note: h depends only on x,y; z is irrelevant here.
syms x y
h = (x^2 + y^2)/(x^2 - y^2);
grad_h = gradient(h, [x y]);
hess_h = hessian(h, [x y]);
disp('Gradient of h:')
```

Gradient of h:

```
disp(grad_h)
```

$$\begin{pmatrix} \frac{2\,x}{x^2-y^2}-\frac{2\,x\,(x^2+y^2)}{(x^2-y^2)^2} \\ \frac{2\,y}{x^2-y^2}+\frac{2\,y\,(x^2+y^2)}{(x^2-y^2)^2} \end{pmatrix}$$

```
disp('Hessian of h:')
```

Hessian of h:

```
disp(hess_h)
```

$$\begin{pmatrix} \frac{2}{x^2-y^2}-\frac{2\,\sigma_4}{\sigma_1}-\frac{8\,x^2}{\sigma_1}+\frac{8\,x^2\,\sigma_4}{\sigma_3} & \sigma_2 \\ \sigma_2 & \frac{8\,y^2}{\sigma_1}+\frac{2\,\sigma_4}{\sigma_1}+\frac{2}{x^2-y^2}+\frac{8\,y^2\,\sigma_4}{\sigma_3} \end{pmatrix}$$

where

$$\sigma_1 = (x^2-y^2)^2$$

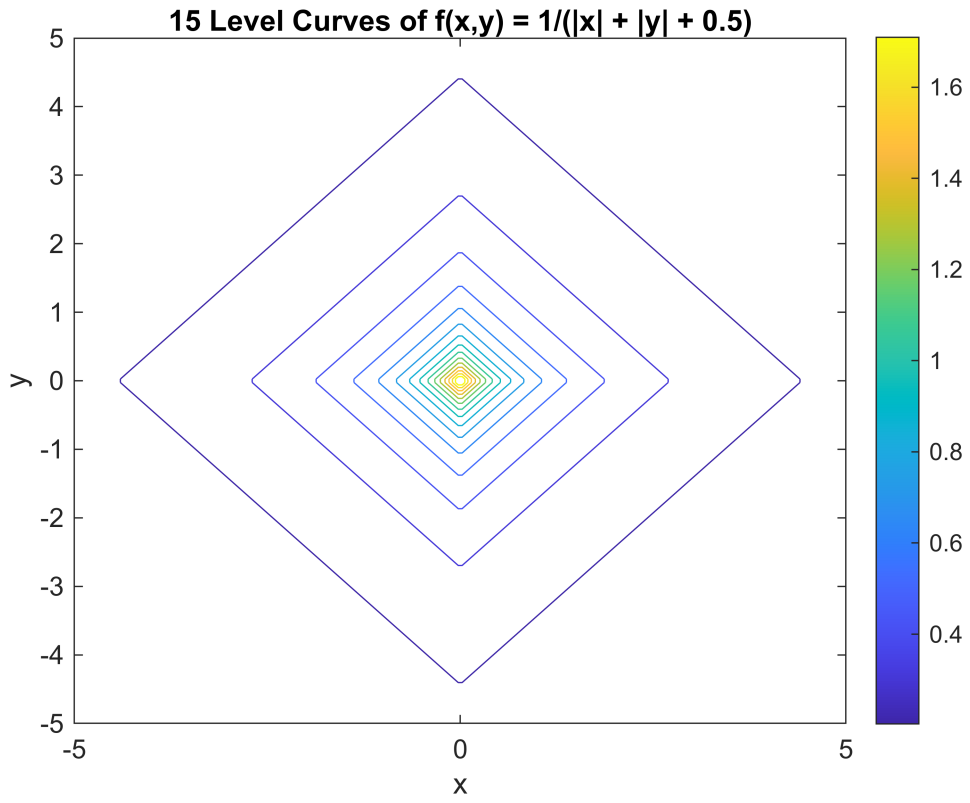
$$\sigma_2 = -\frac{8\,x\,y\,\sigma_4}{\sigma_3}$$

$$\sigma_3 = (x^2-y^2)^3$$

$$\sigma_4 = x^2+y^2$$

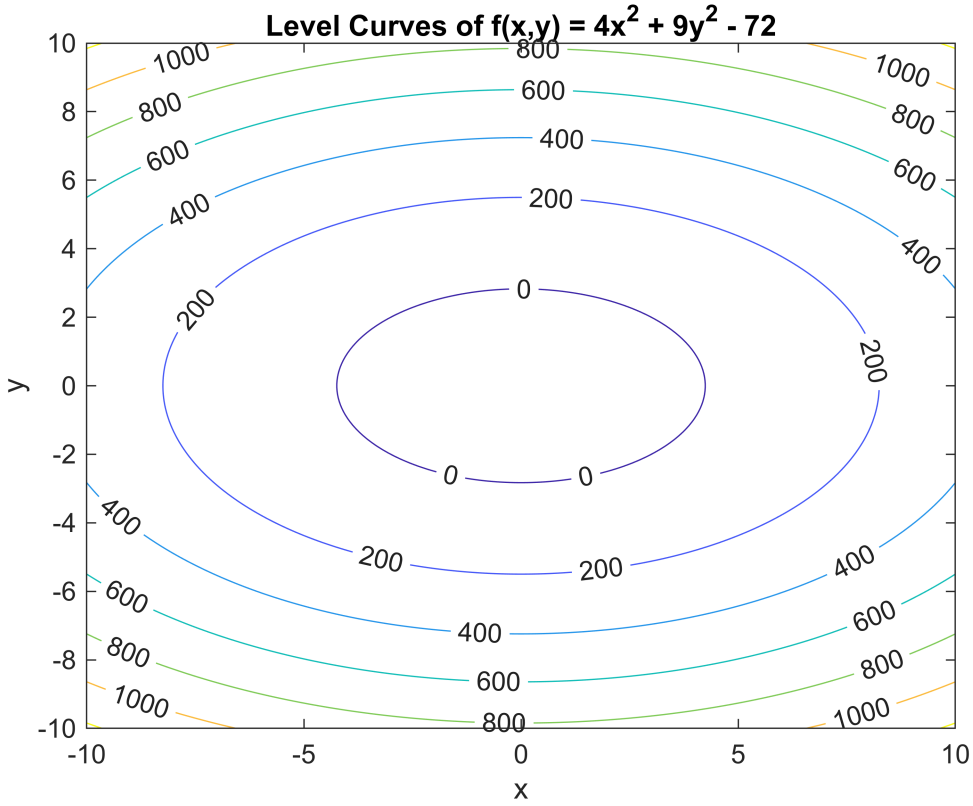
question 7

```
x = linspace(-5,5,200);
y = linspace(-5,5,200);
[X, Y] = meshgrid(x,y);
Z = 1 ./ (abs(X) + abs(Y) + 0.5);

contour(X, Y, Z, 15) % 15 level curves
xlabel('x'), ylabel('y')
title('15 Level Curves of f(x,y) = 1/(|x| + |y| + 0.5)')
colorbar
```



question 8

```
% a) Plot level curves with values labeled
x = linspace(-10,10,200);
y = linspace(-10,10,200);
[X, Y] = meshgrid(x,y);
Z = 4*X.^2 + 9*Y.^2 - 72;

contour(X, Y, Z, 'ShowText', 'on')
xlabel('x'), ylabel('y')
title('Level Curves of f(x,y) = 4x^2 + 9y^2 - 72')
```




```
% b) Gradient and Hessian
syms x y
f = 4*x^2 + 9*y^2 - 72;
grad_f = gradient(f, [x y]);
hess_f = hessian(f, [x y]);
disp('Gradient of f:')
```

```
Gradient of f:
```

```
disp(grad_f)
```

$$\begin{pmatrix} 8\,x \\ 18\,y \end{pmatrix}$$

```
disp('Hessian of f:')
```

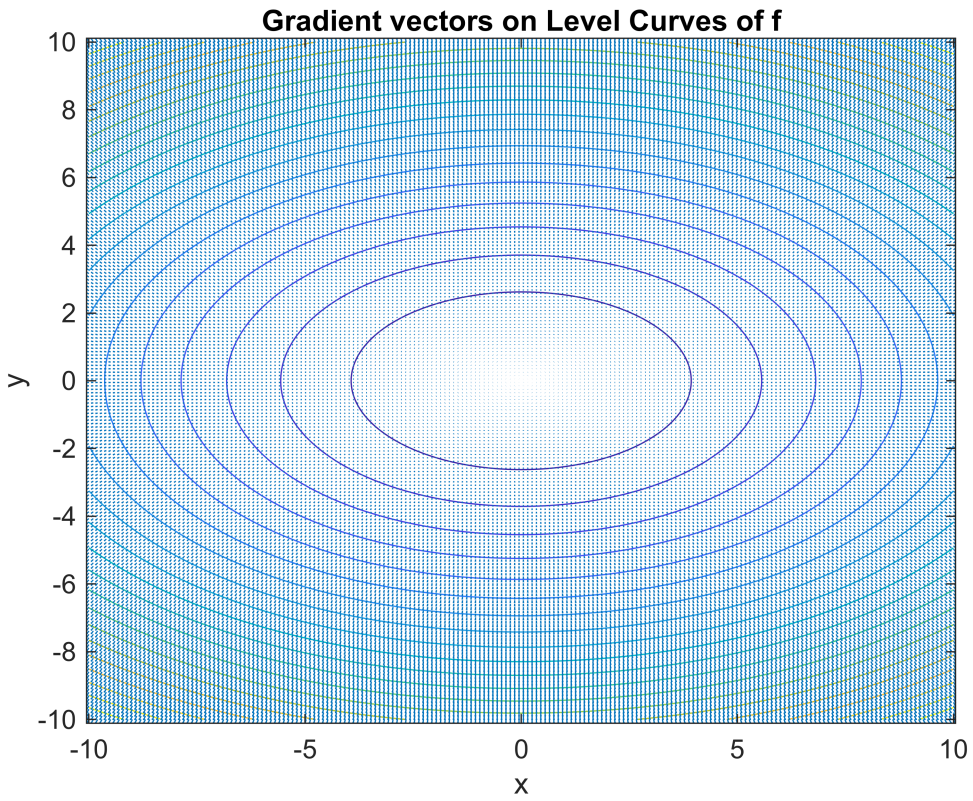
```
Hessian of f:
```

```
disp(hess_f)
```

$$\begin{pmatrix} 8 & 0 \\ 0 & 18 \end{pmatrix}$$

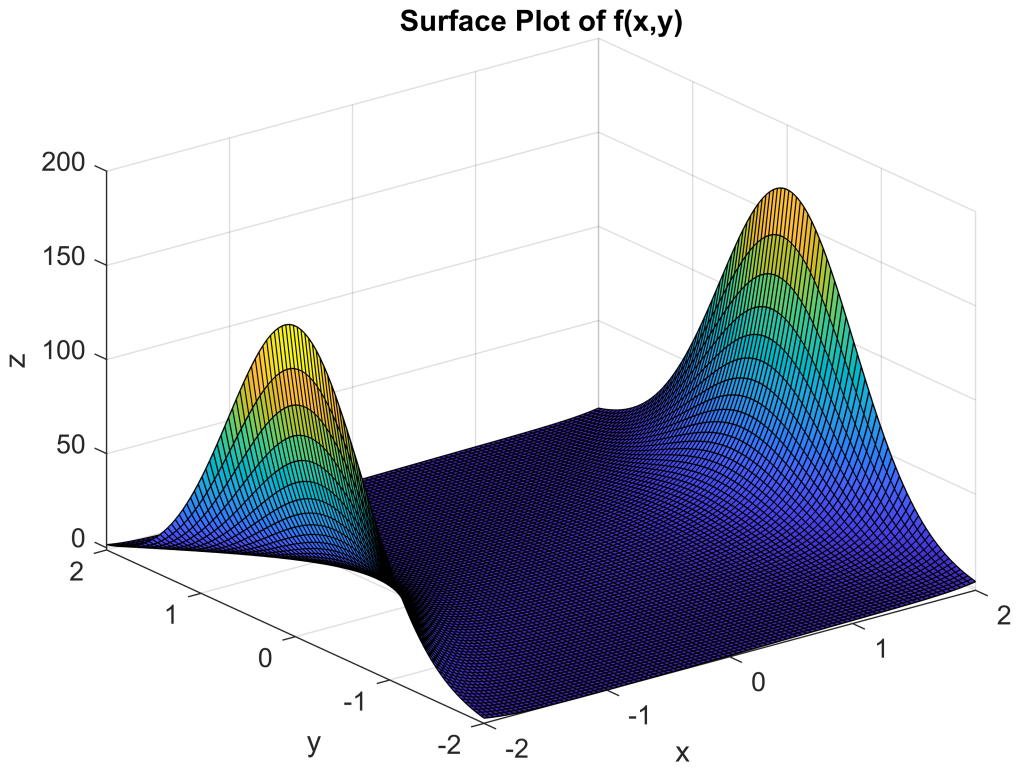
```
% c) Plot gradient vectors on level curves
[px, py] = gradient(4*X.^2 + 9*Y.^2 - 72);
```

```
figure
contour(X, Y, Z, 20)
hold on
quiver(X, Y, px, py)
xlabel('x'), ylabel('y')
title('Gradient vectors on Level Curves of f')
hold off
```

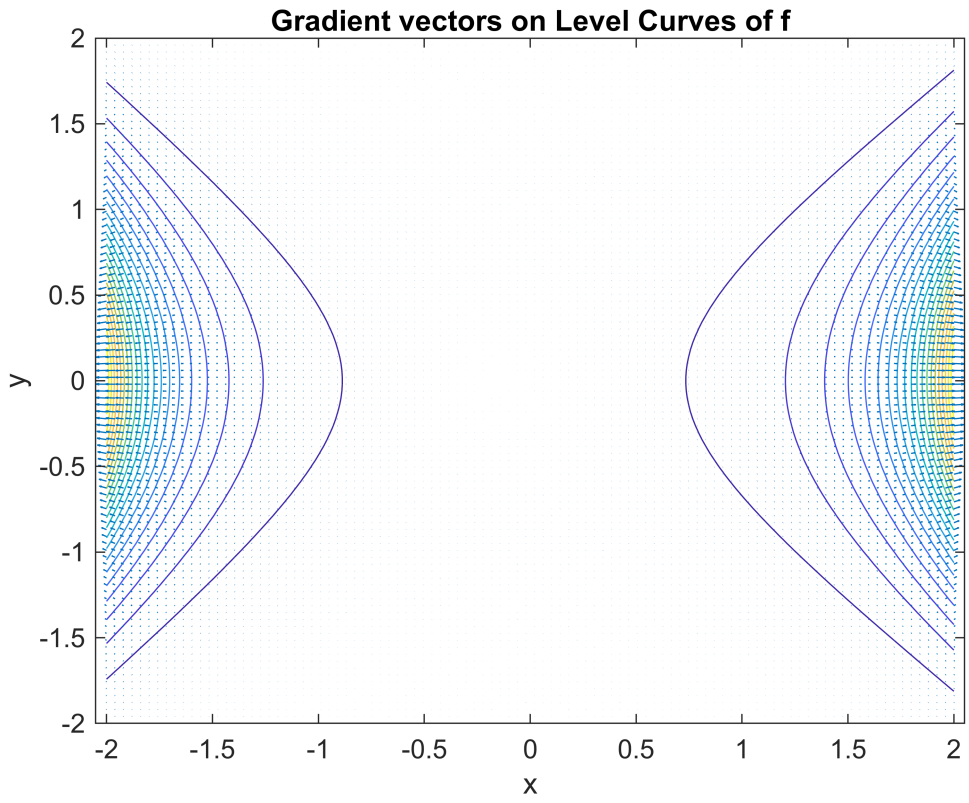


question 9

```
x = linspace(-2, 2, 100);
y = linspace(-2, 2, 100);
[X, Y] = meshgrid(x,y);

Z = sin(X) + cos(Y) + 3*exp(X.^2 - Y.^2);

% d) Surface plot
figure
surf(X, Y, Z)
xlabel('x'), ylabel('y'), zlabel('z')
title('Surface Plot of f(x,y)')
```

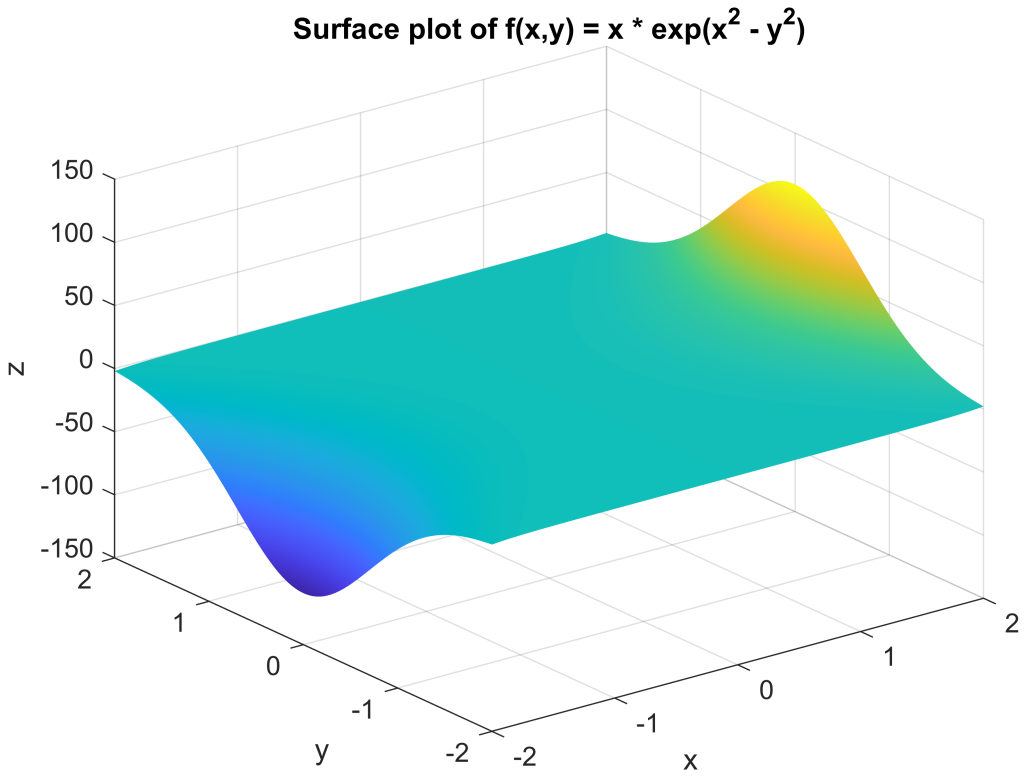



```
% Gradient calculation
[px, py] = gradient(Z, x(2)-x(1), y(2)-y(1));

% e) Gradient vectors on level curves
figure
contour(X, Y, Z, 20)
hold on
quiver(X, Y, px, py)
xlabel('x'), ylabel('y')
title('Gradient vectors on Level Curves of f')
hold off
```



question 10

```
% a) Surface plot for x,y in [-2,2]
x = linspace(-2,2,100);
y = linspace(-2,2,100);
[X, Y] = meshgrid(x,y);
Z = X .* exp(X.^2 - Y.^2);

figure
surf(X, Y, Z)
xlabel('x'), ylabel('y'), zlabel('z')
title('Surface plot of f(x,y) = x * exp(x^2 - y^2)')
shading interp
```




```
% b) Gradient and Hessian (symbolic)
syms x y
f = x * exp(x^2 - y^2);
grad_f = gradient(f, [x y]);
hess_f = hessian(f, [x y]);
disp('Gradient of f:')
```

```
Gradient of f:
```

```
disp(grad_f)
```

$$\begin{pmatrix} e^{x^2-y^2} + 2\,x^2\,e^{x^2-y^2} \\ -2\,x\,y\,e^{x^2-y^2} \end{pmatrix}$$

```
disp('Hessian of f:')
```

```
Hessian of f:
```

```
disp(hess_f)
```

$$\begin{pmatrix} 4x^3\sigma_1 + 6x\sigma_1 & -2y\sigma_1 - 4x^2y\sigma_1 \\ -2y\sigma_1 - 4x^2y\sigma_1 & 4xy^2\sigma_1 - 2x\sigma_1 \end{pmatrix}$$

where

$$\sigma_1 = e^{x^2 - y^2}$$

```
% c) Evaluate Hessian at given points (example points)
points = [0 0; 1 1; -1 1];
for i=1:size(points,1)
    val = subs(hess_f, [x y], points(i,:));
    fprintf('Hessian at (%.1f, %.1f):\n', points(i,1), points(i,2))
    disp(double(val))
end
```

```
Hessian at (0.0, 0.0):
     0     0
     0     0
Hessian at (1.0, 1.0):
    10    -6
    -6     2
Hessian at (-1.0, 1.0):
   -10    -6
    -6    -2
```

```
% d) Eigenvalues and curvature nature
for i=1:size(points,1)
    val = subs(hess_f, [x y], points(i,:));
    val_num = double(val);
    eigVals = eig(val_num);
    fprintf('Eigenvalues of Hessian at (%.1f, %.1f):\n', points(i,1), points(i,2))
    disp(eigVals)

    if all(eigVals > 0)
        disp('Curvature: curved upward (local minimum)')
    elseif all(eigVals < 0)
        disp('Curvature: curved downward (local maximum)')
    else
        disp('Curvature: saddle point or neither')
    end
    fprintf('\n')
end
```

```
Eigenvalues of Hessian at (0.0, 0.0):
     0
     0
Curvature: saddle point or neither
Eigenvalues of Hessian at (1.0, 1.0):
   -1.2111
   13.2111
```

```
Curvature: saddle point or neither
Eigenvalues of Hessian at (-1.0, 1.0):
  -13.2111
    1.2111
Curvature: saddle point or neither
```