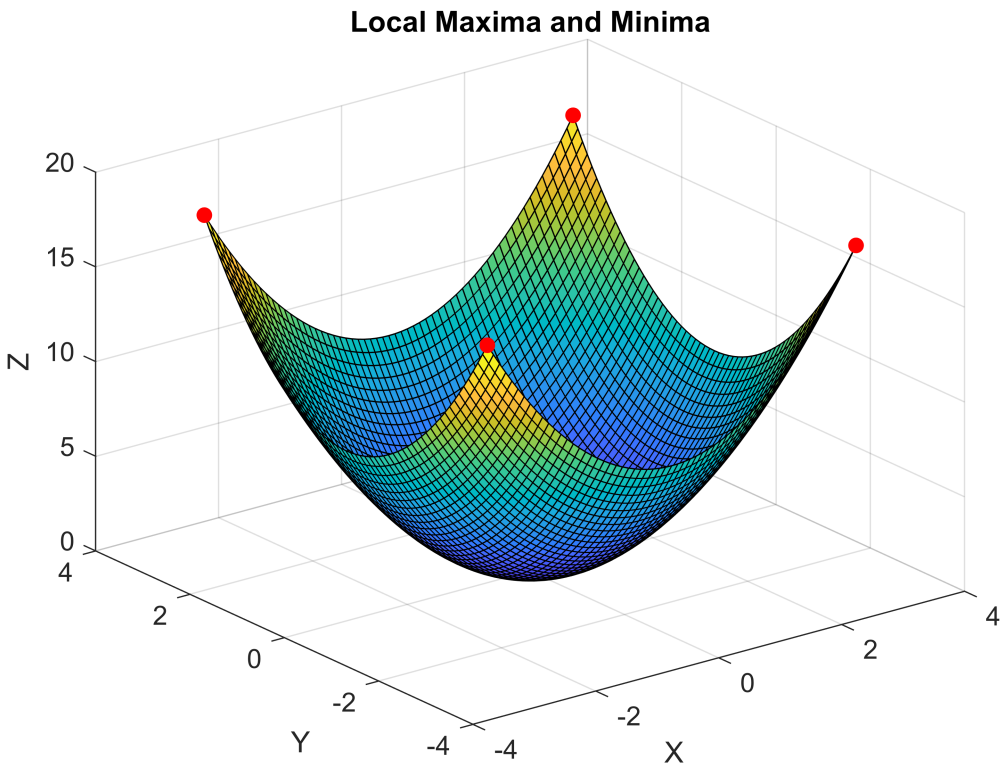
Examples

1)

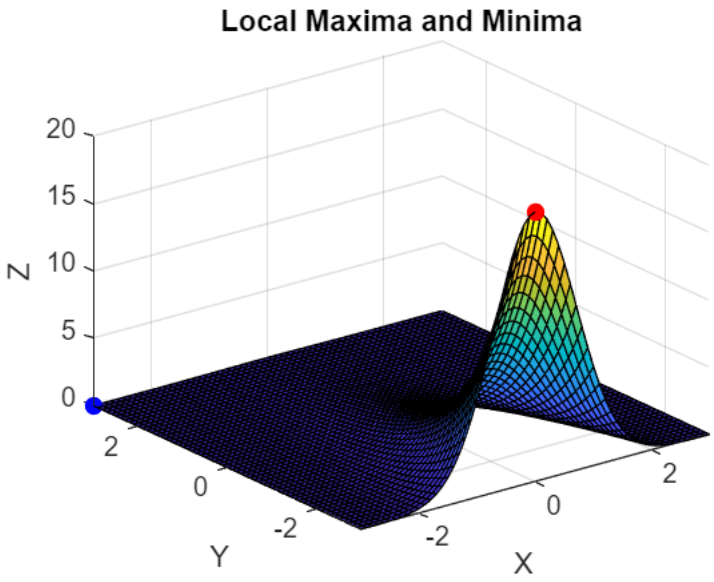
```
x =- 3:0.1:3;
y=x;
[x1,y1]=meshgrid(x,y);
Z=x1.^2+y1.^2;
surf(x1,y1,Z)
maximaMask=imregionalmax(Z);
minimaMask=imregionalmin(Z);
hold on;
plot3(x1(maximaMask),y1(maximaMask),Z(maximaMask),'r.','MarkerSize',20);
plot3(x1(minimaMask),y1(minimaMask),Z(minimaMask),'r.','MarkerSize',20);
hold off;
title("Local Maxima and Minima");
xlabel("X")
ylabel("Y")
zlabel("Z")
```



Question 2

```
clc;
clear all;
x =- 3:0.1:3;
y=x;
```

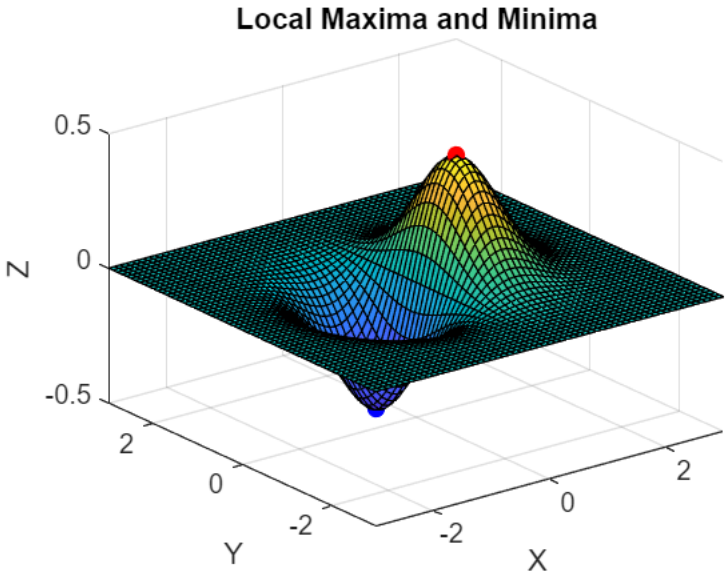
```
[x1,y1]=meshgrid(x,y);
z1=exp(-(x1.^2+y1));
[maxZ,maxIndex]=max(z1(:));
[minZ,minIndex]=min(z1(:));
[maxRow,maxCol]=ind2sub(size(z1),maxIndex);
[minRow,minCol]=ind2sub(size(z1),minIndex);
figure;
surf(x1,y1,z1);
hold on;
plot3(x1(maxRow,maxCol),y1(maxRow,maxCol),maxZ,'r.','MarkerSize',20);
plot3(x1(minRow,minCol),y1(minRow,minCol),minZ,'b.','MarkerSize',20);
hold off;
title("Local Maxima and Minima");
xlabel("X")
ylabel("Y")
zlabel("Z")
```



Question 3

```
clc;
clear all;
x =- 3:0.1:3;
y=x;
[x1,y1]=meshgrid(x,y);
z2=x .* exp(-(x1.^2+y1.^2));
[maxZ,maxIndex]=max(z2(:));
[minZ,minIndex]=min(z2(:));
[maxRow,maxCol]=ind2sub(size(z2),maxIndex);
[minRow,minCol]=ind2sub(size(z2),minIndex);
figure;
surf(x1,y1,z2);
hold on;
plot3(x1(maxRow,maxCol),y1(maxRow,maxCol),maxZ,'r.','MarkerSize',20);
```

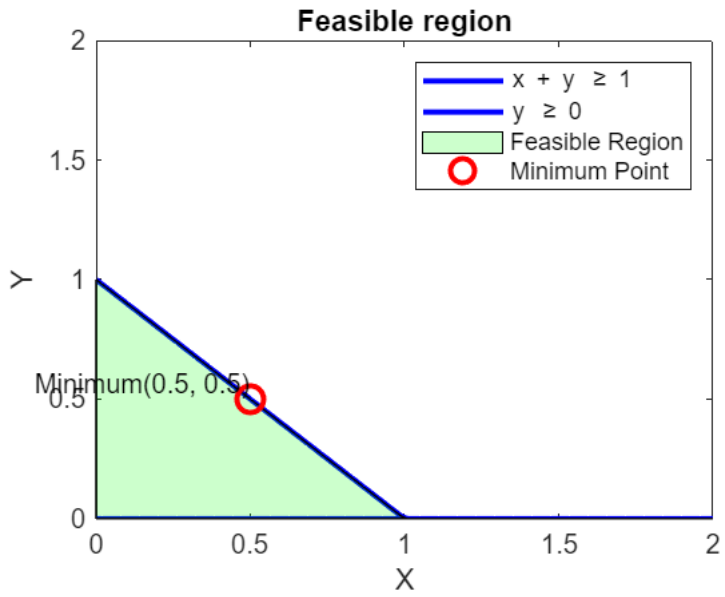
```
plot3(x1(minRow,minCol),y1(minRow,minCol),minZ,'b.','MarkerSize',20);
hold off;
title("Local Maxima and Minima");
xlabel("X")
ylabel("Y")
zlabel("Z")
```



Question 4

```
clc;
clear all;
x = linspace(0,2,100);
y1 = max(0,1 - x);
y2 = zeros(size(x));
figure;
plot(x, y1, 'b', 'LineWidth', 2);
hold on;
plot(x, y2, 'b', 'LineWidth', 2);
xlabel("X");
ylabel("Y");
title("Feasible region");
ylim([0,2]);
fill([x, fliplr(x)], [y1, fliplr(y2)], 'g', 'FaceAlpha', 0.2);
objective = @(x, y) x.^2 + y.^2;
[X, Y] = meshgrid(0:0.1:2, 0:0.1:2);
Z = objective(X, Y);
Z((X + Y) < 1) = NaN;
[minZ, minIndex] = min(Z(:));
[minRow, minCol] = ind2sub(size(Z), minIndex);
minX = X(minRow, minCol);
minY = Y(minRow, minCol);
plot(minX, minY, 'ro', 'MarkerSize', 10, 'LineWidth', 2);
```

```
text(minX, minY, ['Minimum(', num2str(minX), ', ', num2str(minY), ')'],
'VerticalAlignment', 'bottom', 'HorizontalAlignment', 'right');
xlabel("X");
ylabel("Y");
zlabel("Z");
legend('x + y \geq 1', 'y \geq 0', 'Feasible Region', 'Minimum Point');
hold off;
```



Question 5

```
clc;
fun = @(x) x(1)^2 + x(2)^2;
x0 = [0, 0];
A = [-1, -1];
B = -1;
lb = [0, 0];
[x_opt, fval] = fmincon(fun, x0, A, B, [], [], lb, []);
```

```
Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the value of the optimality tolerance,
and constraints are satisfied to within the value of the constraint tolerance.

<stopping criteria details>
```

```
disp('Optimal Solution : ');
```

```
Optimal Solution :
```

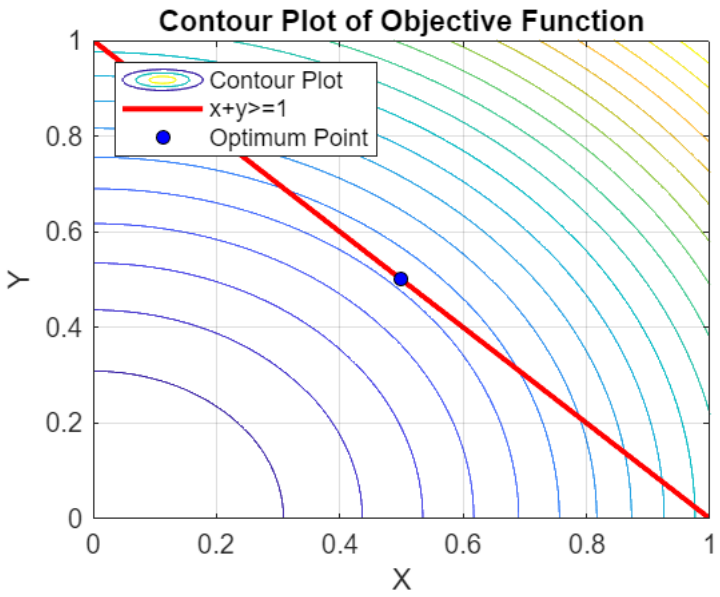
```
disp(['x=', num2str(x_opt(1)), ',y=', num2str(x_opt(2))]);
```

```
x=0.5,y=0.5
```

```
disp(['Function value at optimum : ', num2str(fval)]);
```

```
Function value at optimum : 0.5
```

```
x = 0:0.001:1;
y = 0:0.001:1;
[X, Y] = meshgrid(x, y);
Z = X.^2 + Y.^2;
contour(X, Y, Z, 20);
hold on;
plot(x, 1 - x, 'r', 'LineWidth', 2);
plot(x_opt(1), x_opt(2), 'ko', 'MarkerSize', 5, 'MarkerFaceColor', 'b');
xlabel("X");
ylabel("Y");
title("Contour Plot of Objective Function");
legend('Contour Plot', 'x+y>=1', 'Optimum Point', 'Location', 'northwest');
grid on;
hold off;
```



Question 6

```
clc;
x = 0:0.1:1;
y = 0:0.1:1;
fun = @(x) x(1) + x(2);
nonlcon = @(x) deal([], x(1)^2 + x(2)^2 - 1);
x0 = [0.5, 0.5];
lb = [0, 0];
[x_opt, fval] = fmincon(fun, x0, [], [], [], [], lb, [], nonlcon);
```

```
Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the value of the optimality tolerance,
and constraints are satisfied to within the value of the constraint tolerance.
```

```
<stopping criteria details>
```

```
disp('Optimal Solution');
```

```
Optimal Solution
```

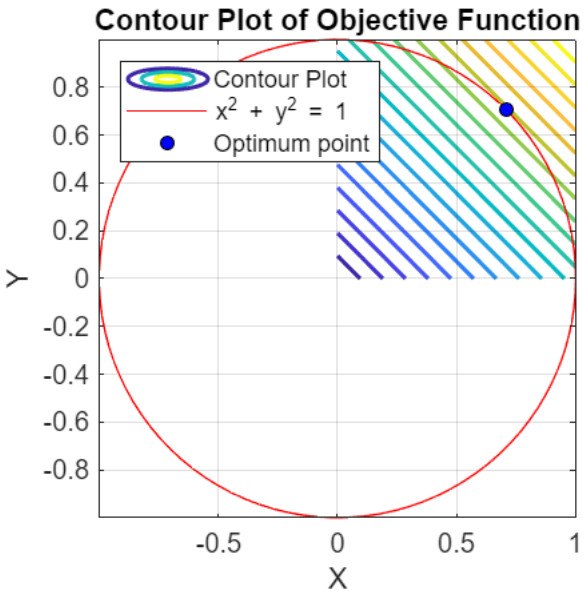
```
disp(['x = ', num2str(x_opt(1)), ', y = ', num2str(x_opt(2))]);
```

```
x = 0.70711, y = 0.70711
```

```
disp(['Minimum value of x + y at optimum: ', num2str(fval)]);
```

```
Minimum value of x + y at optimum: 1.4142
```

```
[X, Y] = meshgrid(x, y);
Z = X + Y;
contour(X, Y, Z, 20, 'LineWidth', 1.5);
hold on;
theta = linspace(0, 2*pi, 100);
r = 1;
xc = r * cos(theta);
yc = r * sin(theta);
plot(xc, yc, 'r');
plot(x_opt(1), x_opt(2), 'ko', 'MarkerSize', 5, 'MarkerFaceColor', 'b');
xlabel("X");
ylabel("Y");
title("Contour Plot of Objective Function");
legend('Contour Plot', 'x^2 + y^2 = 1', 'Optimum point', 'Location', 'Northwest');
axis equal;
grid on;
hold off;
```



Question 7

```
clc;
fun=@(x)-(x(1)+x(2));
nonlcon=@(x) deal(x(1)^2+x(2)^2-25,[]);
x0=[0,0];
A=[-1,-1];
b =- 5;
b=[0,0];
[x_opt, fval]=fmincon(fun,x0,A,B,[],[],1*B,[],nonlcon);
```

```
Warning: Length of lower bounds is < length(x); filling in missing lower bounds with -Inf.
Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the value of the optimality tolerance,
and constraints are satisfied to within the value of the constraint tolerance.

<stopping criteria details>
```

```
fval =- fval;
disp("Optimal solution : ");
```

```
Optimal solution :
```

```
disp(['x=',num2str(x_opt(1)),',y=',num2str(x_opt(2))]);
```

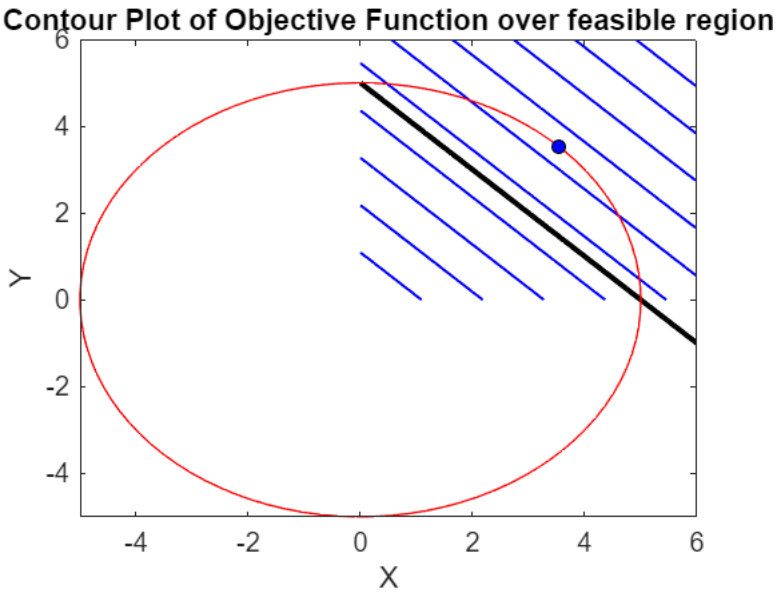
```
x=3.5355,y=3.5355
```

```


disp(['Maximum value of x+y at optimum : ',num2str(fval)]);
```

```
Maximum value of x+y at optimum : 7.0711
```

```
x=linspace(0,6,100);
y=linspace(0,6,100);
[X,Y]=meshgrid(x,y);
Z=X+Y;
contour(X,Y,Z,10,'b','LineWidth',1);
hold on;
plot(x,5-x,'black','LineWidth',2);
theta=linspace(0,2*pi,100);
r=5;
x=r*cos(theta);
y=r*sin(theta);
plot(x,y,'r');
plot(x_opt(1),x_opt(2),'ko','MarkerSize',5,'MarkerFaceColor','b');
xlabel("X");
ylabel("Y");
title('Contour Plot of Objective Function over feasible region');
```



Question 8

```
clc;
clf;
clear all;
fun=@(x)x(1)+x(2);
nonlcon=@(x) deal(x(1)^2+x(2)^2-25,[]);
x0=[0,0];
A=[-1,-1];
b =- 5;
lb=[0,0];
[x_opt,fval]=fmincon(fun,x0,A,b,[],[],lb,[],nonlcon);
```

```
Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the value of the optimality tolerance,
and constraints are satisfied to within the value of the constraint tolerance.

<stopping criteria details>
```

```
fval =- fval;
disp("Optimal solution : ");
```

```
Optimal solution :
```

```
disp(['x=',num2str(x_opt(1)),',y=',num2str(x_opt(2))]);
```

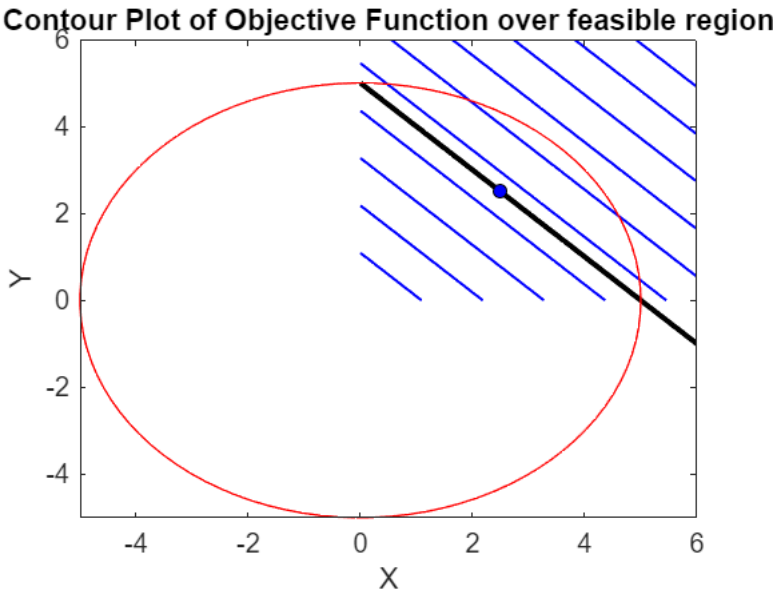
```
x=2.5,y=2.5
```

```
disp(['Maximum value of x+y at optimum : ',num2str(fval)]);
```

```
Maximum value of x+y at optimum : -5
```

```
x=linspace(0,6,100);
y=linspace(0,6,100);
```

```
[X,Y]=meshgrid(x,y);
Z=X+Y;
contour(X,Y,Z,10,'b','LineWidth',1);
hold on;
plot(x,5-x,'black','LineWidth',2);
theta=linspace(0,2*pi,100);
r=5;
x=r*cos(theta);
y=r*sin(theta);
plot(x,y,'r');
plot(x_opt(1),x_opt(2),'ko','MarkerSize',5,'MarkerFaceColor','b');
xlabel("X");
ylabel("Y");
title('Contour Plot of Objective Function over feasible region');
```



Practice questions

Question 1

```
clc;
x = 0:0.1:10;
y = 0:0.1:10;
fun = @(x) - (x(1) + x(2));
A = [0 1; 1 1];
b = [5; 10];
x0 = [1, 1];
lb = [0, 0];
[x_opt, fval] = fmincon(fun, x0, A, b, [], [], lb, []);
```

```
Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the value of the optimality tolerance,
```

```
and constraints are satisfied to within the value of the constraint tolerance.

<stopping criteria details>
```

```
disp('Optimal Solution');
```

```
Optimal Solution
```

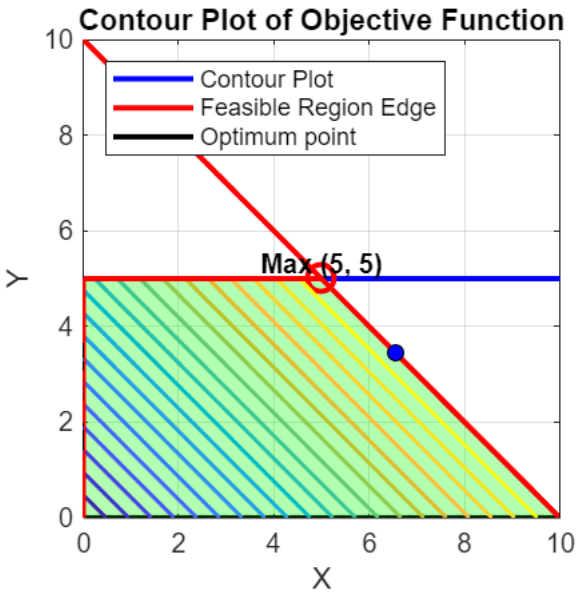
```
disp(['x = ', num2str(x_opt(1)), ', y = ', num2str(x_opt(2))]);
```

```
x = 6.5496, y = 3.4504
```

```
disp(['Maximum value of x + y at optimum: ', num2str(-fval)]);
```

```
Maximum value of x + y at optimum: 10
```

```
[X, Y] = meshgrid(x, y);
Z = X + Y;
Z(Y > 5 | (X + Y) > 10) = NaN;
contour(X, Y, Z, 20, 'LineWidth', 1.5);
hold on;
plot([0 0 5 10], [0 5 5 0], 'r', 'LineWidth', 2);
plot(x_opt(1), x_opt(2), 'ko', 'MarkerSize', 6, 'MarkerFaceColor', 'b');
xlabel("X");
ylabel("Y");
title("Contour Plot of Objective Function");
legend('Contour Plot', 'Feasible Region Edge', 'Optimum point', 'Location',
'Northwest');
axis equal;
grid on;
hold off;
```



Question 2

```
clc;
```

```
x = 0:0.1:10;
y = 0:0.1:10;
fun = @(v) -3 * v(2);
A = [1 0; -1 1; 1 1];
b = [3; 4; 6];
x0 = [1, 1];
lb = [0, 0];
[x_opt, fval] = fmincon(fun, x0, A, b, [], [], lb, []);
```

```
Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the value of the optimality tolerance,
and constraints are satisfied to within the value of the constraint tolerance.

<stopping criteria details>
```

```
disp('Optimal Solution');
```

```
Optimal Solution
```

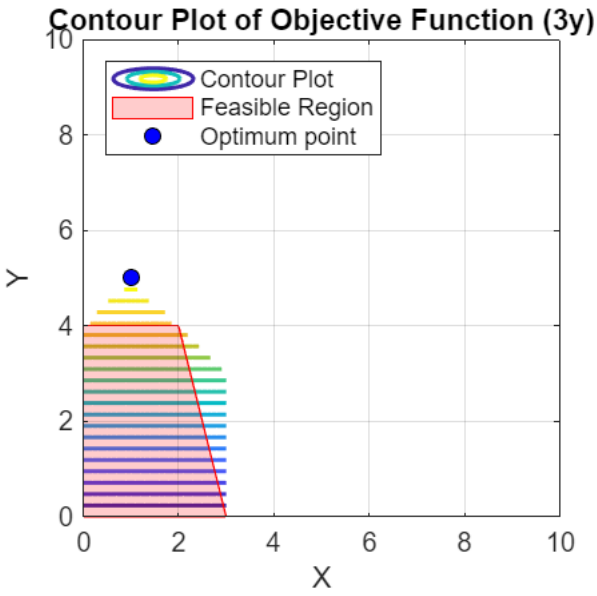
```
disp(['x = ', num2str(x_opt(1)), ', y = ', num2str(x_opt(2))]);
```

```
x = 1, y = 5
```

```
disp(['Maximum value of 3y at optimum: ', num2str(-fval)]);
```

```
Maximum value of 3y at optimum: 15
```

```
[X, Y] = meshgrid(x, y);
Z = 3 * Y;
Z(X > 3 | -X + Y > 4 | X + Y > 6) = NaN;
contour(X, Y, Z, 20, 'LineWidth', 1.5);
hold on;
x1 = [0 3 2 0];
y1 = [0 0 4 4];
fill(x1, y1, 'r', 'FaceAlpha', 0.2, 'EdgeColor', 'r');
plot(x_opt(1), x_opt(2), 'ko', 'MarkerSize', 6, 'MarkerFaceColor', 'b');
xlabel("X");
ylabel("Y");
title("Contour Plot of Objective Function (3y)");
legend('Contour Plot', 'Feasible Region', 'Optimum point', 'Location', 'Northwest');
axis equal;
grid on;
hold off;
```



Question 3

```
clc;
x = 0:0.1:10;
y = 0:0.1:10;
fun = @(v) - (v(1) + v(2));
A = [1 0; 0 1];
b = [3; 7];
x0 = [1, 1];
lb = [0, 0];
[x_opt, fval] = fmincon(fun, x0, A, b, [], [], lb, []);
```

```
Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the value of the optimality tolerance,
and constraints are satisfied to within the value of the constraint tolerance.

<stopping criteria details>
```

```
disp('Optimal Solution');
```

```
Optimal Solution
```

```
disp(['x = ', num2str(x_opt(1)), ', y = ', num2str(x_opt(2))]);
```

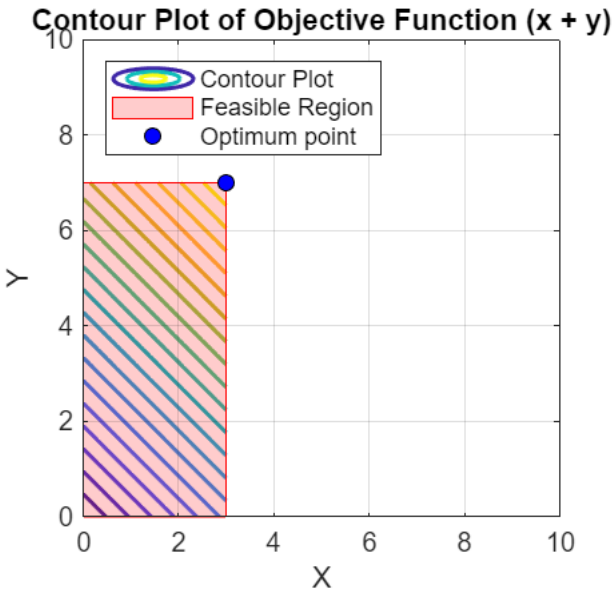
```
x = 3, y = 7
```

```
disp(['Maximum value of x + y at optimum: ', num2str(-fval)]);
```

```
Maximum value of x + y at optimum: 10
```

```
[X, Y] = meshgrid(x, y);
Z = X + Y;
Z(X > 3 | Y > 7) = NaN;
contour(X, Y, Z, 20, 'LineWidth', 1.5);
```

```
hold on;
x_rect = [0 3 3 0];
y_rect = [0 0 7 7];
fill(x_rect, y_rect, 'r', 'FaceAlpha', 0.2, 'EdgeColor', 'r');
plot(x_opt(1), x_opt(2), 'ko', 'MarkerSize', 6, 'MarkerFaceColor', 'b');
xlabel("X");
ylabel("Y");
title("Contour Plot of Objective Function (x + y)");
legend('Contour Plot', 'Feasible Region', 'Optimum point', 'Location', 'Northwest');
axis equal;
grid on;
hold off;
```



Question 4

```
clc;
x = 0:0.1:3;
y = 0:0.1:7;
fun = @(v) v(1)^2 + v(2)^2;
neg_fun = @(v) -fun(v);
x0 = [1, 1];
A = [1 0; 0 1];
b = [3; 7];
lb = [0, 0];
[x_min, fminval] = fmincon(fun, x0, A, b, [], [], lb, []);
```

```
Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the value of the optimality tolerance,
and constraints are satisfied to within the value of the constraint tolerance.

<stopping criteria details>
```

```
[x_max, fmaxval_neg] = fmincon(neg_fun, x0, A, b, [], [], lb, []);
```

```
Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the value of the optimality tolerance,
and constraints are satisfied to within the value of the constraint tolerance.

<stopping criteria details>
```

```
fmaxval = -fmaxval_neg;
disp('Minimum at:');
```

```
Minimum at:
```

```
disp(['x = ', num2str(x_min(1)), ', y = ', num2str(x_min(2)), ', f = ',
num2str(fminval)]);
```

```
x = 0.0006395, y = 0.00063953, f = 8.1796e-07
```

```
disp('Maximum at:');
```

```
Maximum at:
```

```
disp(['x = ', num2str(x_max(1)), ', y = ', num2str(x_max(2)), ', f = ',
num2str(fmaxval)]);
```

```
x = 3, y = 7, f = 58
```

```
[X, Y] = meshgrid(x, y);
Z = X.^2 + Y.^2;
contour(X, Y, Z, 30, 'LineWidth', 1.2);
hold on;
fill([0 3 3 0], [0 0 7 7], 'r', 'FaceAlpha', 0.1, 'EdgeColor', 'r');
plot(x_min(1), x_min(2), 'go', 'MarkerSize', 7, 'MarkerFaceColor', 'g');
plot(x_max(1), x_max(2), 'ro', 'MarkerSize', 7, 'MarkerFaceColor', 'r');
xlabel('X'); ylabel('Y');
title('Min and Max of f(x,y) = x^2 + y^2 in Feasible Region');
legend('Contour', 'Feasible Region', 'Minimum', 'Maximum', 'Location', 'Northwest');
axis equal;
grid on;
hold off;
```

Min and Max of f(x,y) = x^2 + y^2 in Feasible Region