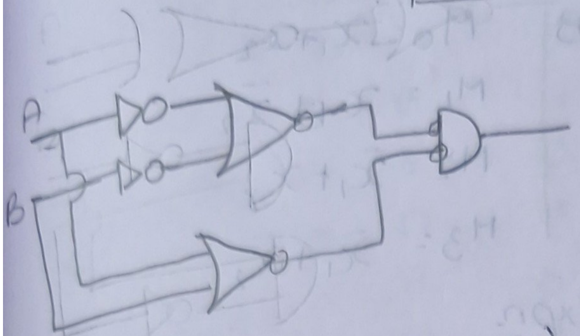
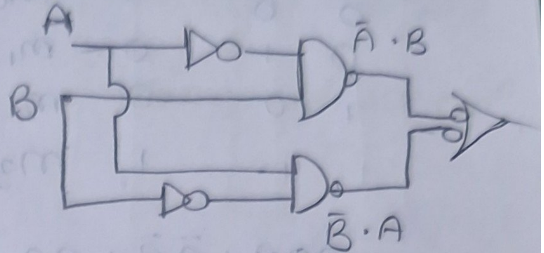
XOR using NOR gate

$\boxed{XOR = \bar{A}B + A\bar{B}}$



XOR using NAND gate



(minterm & maxterm) synthesis :-

ROW NO	x_1	x_2	minterm	maxterm
0	0	0	$m_0 = \bar{x}_1 \cdot \bar{x}_2$	$M_0 = x_1 + x_2$
1	0	1	$m_1 = \bar{x}_1 x_2$	$M_1 = x_1 + \bar{x}_2$
2	1	0	$m_2 = x_1 \bar{x}_2$	$M_2 = \bar{x}_1 + x_2$
3	1	1	$m_3 = x_1 \cdot x_2$	$M_3 = \bar{x}_1 + \bar{x}_2$

Y	A	B	A⊕B	
1	0	0	0	
1	0	1	1	$\rightarrow m_1 = \bar{A}B$
0	1	0	1	$\rightarrow m_2 = A\bar{B}$
1	1	1	0	

$= (m_1 + m_2)$

$\boxed{A \oplus B = \bar{A}B + A\bar{B} \Rightarrow \text{SOP expn.}}$ (sum of product)

$Y = m_0 + m_1 + m_3 \cdot \; \sum m(0,1,3) \rightarrow$ (minterm list)

$= \bar{A}\bar{B} + \bar{A}B + AB.$

$\boxed{\text{canonical SOP exp}^n}$

$= \bar{A}+\bar{B} + B(A + \bar{A})$

$= \bar{A} \cdot \bar{B} + B. \Rightarrow \bar{A} + B.$

[canonical → each product term is a minterm]

XNOR gate:

A	B	$A \odot B = \overline{A \oplus B}$		maxterm
0	0	1	$m_0 = \bar{A}\bar{B} + \cancel{AB}$	$M_0 = x_1 + x_2$
0	1	0	$m_1 = \bar{A}B$	$M_1 = x_1 + \bar{x}_2$
1	0	0	$m_1 - A\bar{B}$	$M_2 = \bar{x}_1 + x_2$
1	1	1	$m_3 = AB$	$M_3 = \bar{x}_1 + \bar{x}_2$

$A \odot B = \bar{A}\bar{B} + AB \rightarrow$ SOP expn

$Y = m_0 + m_3$.

ex:

R.No.	x_1	x_2	x_3	minterm	O/P Z	Z' (compliment)	maxterm
0	0	0	0	$m_0 = \bar{A}\bar{B}\bar{C}$	0	1	$M_0 = \bar{m}_0 = x_1 + x_2 + x_3$
1	0	0	1	$m_1 = \bar{A}\bar{B}C$	1	0	$M_1 = x_1 + x_2 + \bar{x}_3$
2	0	1	0	$m_2 = \bar{A}B\bar{C}$	0	1	$M_2 = x_1 + \bar{x}_2 + x_3$
3	0	1	1	$m_3 = \bar{A}BC$	1	0	$M_3 = x_1 + \bar{x}_2 + \bar{x}_3$
4	1	0	0	$m_4 = A\bar{B}\bar{C}$	1	0	$M_4 = \bar{x}_1 + x_2 + x_3$
5	1	0	1	$m_5 = A\bar{B}C$	0	1	$M_5 = \bar{x}_1 + x_2 + \bar{x}_3$
6	1	1	0	$m_6 = AB\bar{C}$	0	1	$M_6 = \bar{x}_1 + \bar{x}_2 + x_3$
7	1	1	1	$m_7 = ABC$	1	0	$M_7 = \bar{x}_1 + \bar{x}_2 + \bar{x}_3$

$Z(x_1, x_2, x_3) = m_1 + m_3 + m_4 + m_7$

MSB (most significant bit) LSB

$= \bar{x}_1\bar{x}_2 x_3 + \bar{x}_1 x_2 x_3 + x_1 \bar{x}_2 \bar{x}_3 + x_1 x_2 x_3$

(canonical SOP exprn)

$= \Sigma m(1, 3, 4, 7)$.

Problem stat: ~~Th~~ whenever ~~the~~ press a even number then the bulb should glow (with 3 inputs) write logical expn.

minterm list for compliment; canonical SOP exp

$z' = \Sigma m(\underset{000}{0}, \underset{010}{2}, \underset{101}{5}, \underset{110}{6})$

$= \bar{x}_1\bar{x}_2\bar{x}_3 + \bar{x}_1 x_2 \bar{x}_3 + x_1 \bar{x}_2 x_3 + x_1 x_2 \bar{x}_3$

$= \bar{x}_1 \bar{x}_3 (x_2 + \bar{x}_2) + x_1 \bar{x}_2 x_3 + x_1 x_2 \bar{x}_3$

(P.T.O)

R.NO	A	B	C	Z
0	0	0	0	1
1	0	0	1	0
2	0	1	0	1
3	0	1	1	0
4	1	0	0	1
5	1	0	1	0
6	1	1	0	1
7	1	1	1	0

$= (\bar{x}_1 + x_2)\bar{x}_3 + x_1\bar{x}_2 x_3$

$= \bar{x}_1\bar{x}_3 + x_2\bar{x}_3 + x_1\bar{x}_2 x_3$

↓

minimized SOP expⁿ

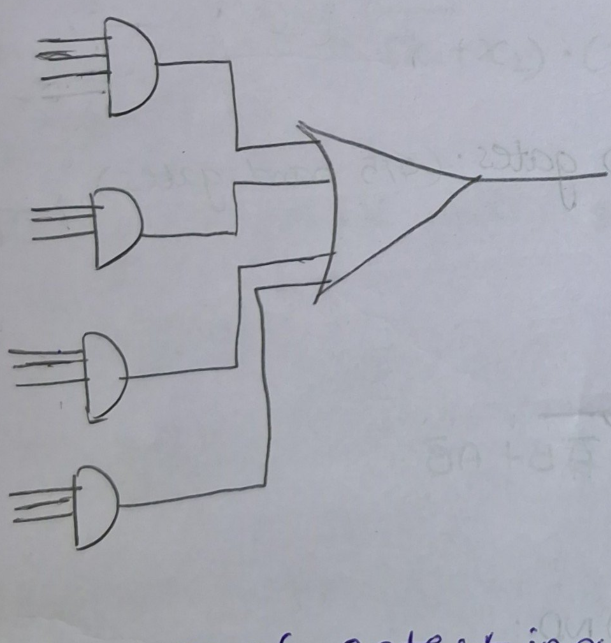
$Z(A,B,C) = m_0 + m_2 + m_4 + m_6.$

$= \sum m(0,2,4,6).$

$= \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + A\bar{B}\bar{C} + AB\bar{C}.$

$= \bar{A}\bar{C}(B+\bar{B}) + A\bar{C}(B+\bar{B})$

$= \bar{A}\bar{C} + A\bar{C}. = \bar{C}.$

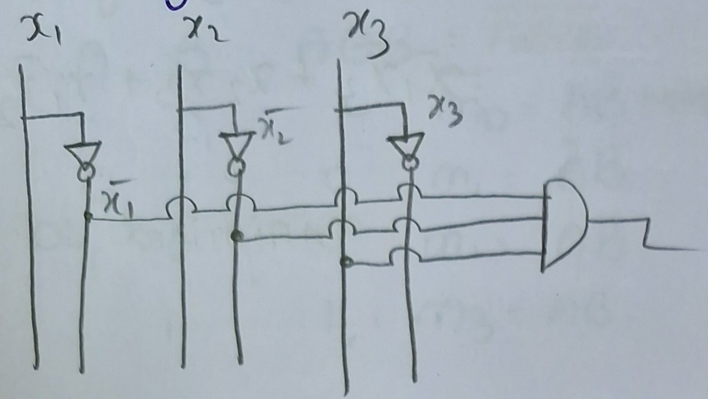


cost = no.of gatest input

$= 16 + 5 \Rightarrow 21$

cost of reduced one $\Rightarrow 0$

neat way:



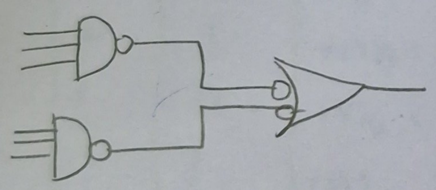
* when we implement SOP expⁿ using minterms, we get two level circuit.

first level → AND gates

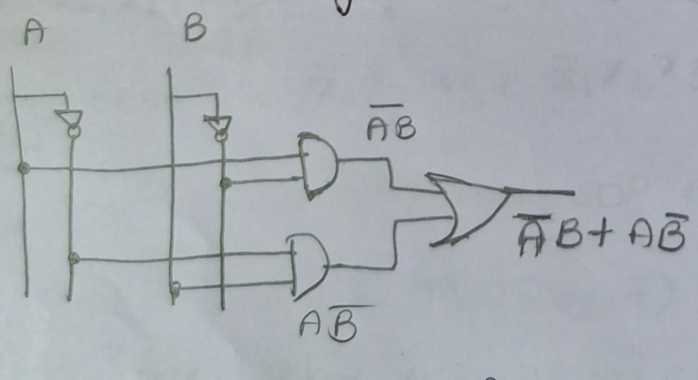
and second level → one OR gate

[This is called AND-OR combination]

This combination can be easily converted to NAND-NAND gates.



Draw XOR using NAND gates:-(4/5 nand gates)



Draw XNOR using NAND:-

minterm:

It is a product term in which all the input variables are there in true or complimented form.

maxterm:

It is a sum term in which all the input variables are there in true/complimented form.

$\rightarrow M_0 = \bar{m}_0$

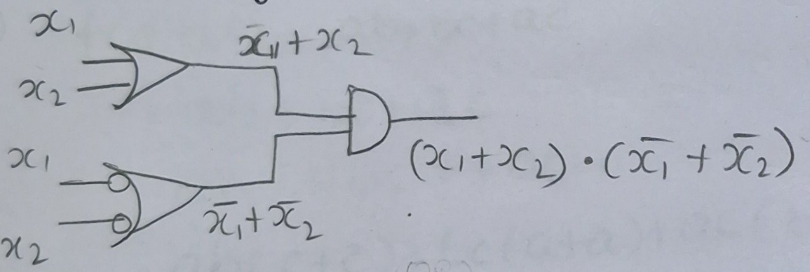
(whereever the o/p is zero).

$A \oplus B = \Sigma_m(1,2) = \boxed{M_0 \cdot M_3 = (x_1 + x_2) \cdot (\bar{x}_1 + \bar{x}_2)}$

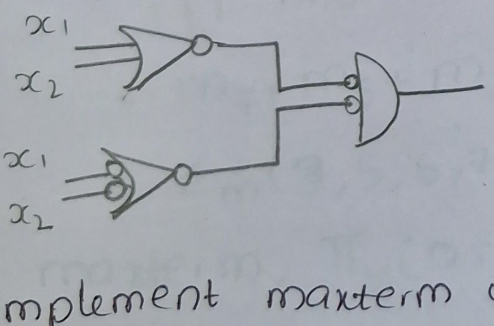
(product of sum terms)

$\Rightarrow \boxed{\Pi_M(0,3)}$

$\rightarrow$ logic diagram for POS:-



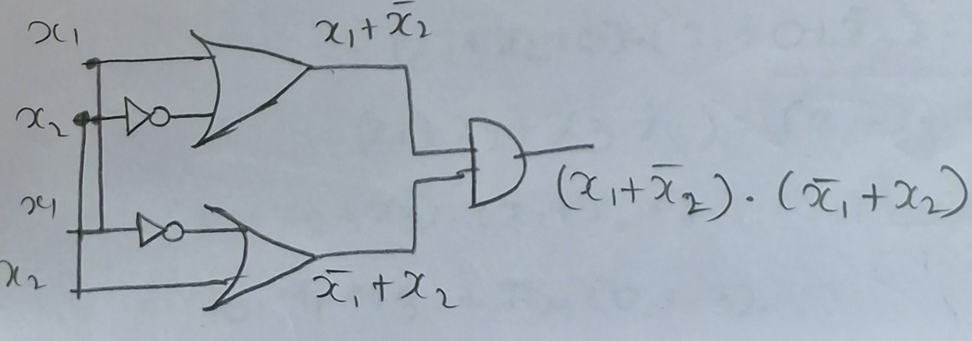
using universal gates (NOR)



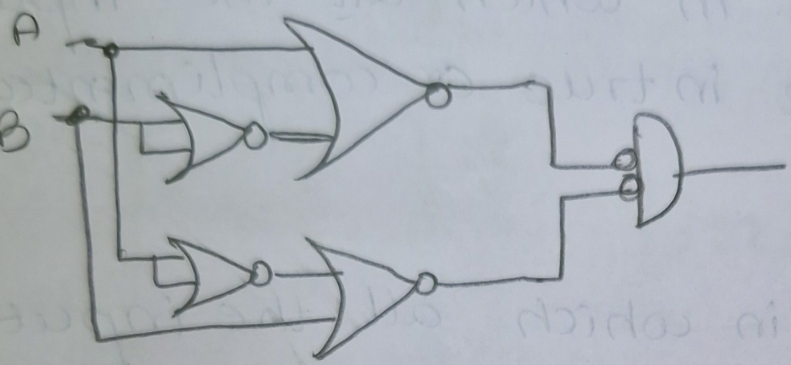
$\rightarrow$ Implement maxterm using XNOR gate:

$z(x_1, x_2) = \cancel{\Sigma}\ \Pi_M(1,2) = M_1 \cdot M_2$

$\boxed{x_1 \odot x_2 = (x_1 + \bar{x}_2) \cdot (\bar{x}_1 + x_2)}$



using NOR:



For 3 input;

same for (f')
logic diagram
simplify POS
logic diagram

$f(x_1, x_2, x_3) = \Sigma_m(1,3,4,7)$

$= \Pi_M(0,2,5,6).$

$\Rightarrow (x_1+x_2+x_3)\cdot(x_1+\bar{x}_2+x_3)\cdot(\bar{x}_1+x_2+\bar{x}_3)\cdot(\bar{x}_1+\bar{x}_2+x_3)$

write minterm & for maxterm (minterm - all 3 inputs should be there)

1) $f(x_1, x_2, x_3) = x_1 + x_2\bar{x}_3$

$$= x_1 \cdot 1 + 1 \cdot x_2\bar{x}_3$$

$$\Rightarrow x_1 \cdot (x_2 + \bar{x}_2) + (x_1 + \bar{x}_1) \cdot x_2 \cdot \bar{x}_3$$

$$= x_1x_2 \cdot 1 + x_1\bar{x}_2 \cdot 1 + \underset{110}{x_1x_2\bar{x}_3} + \underset{010}{\bar{x}_1x_2\bar{x}_3}$$

$$= x_1x_2(x_3 + \bar{x}_3) + x_1\bar{x}_2(x_3 + \bar{x}_3) + x_1x_2\bar{x}_3 + \bar{x}_1x_2\bar{x}_3$$

$$= \underset{111}{x_1x_2x_3} + \underset{110}{x_1x_2\bar{x}_3} + \underset{101}{x_1\bar{x}_2x_3} + \underset{100}{x_1\bar{x}_2\bar{x}_3} + \underset{110}{x_1x_2\bar{x}_3} + \underset{010}{\bar{x}_1x_2\bar{x}_3}$$

$$= m_7 + m_6 + m_5 + m_4 + m_2$$ canonical SOP exp^n

$$= \Sigma_m(2,4,5,6,7).$$

maxterm, $\Pi_M(0,1,3,\cancel{8})$.

2) $f(a,b,c) = ab + bc + ac.$

$$= ab + bc + ac$$

$$= ab \cdot 1 + bc \cdot 1 + ac \cdot 1$$

$$= ab(c + \bar{c}) + bc(a + \bar{a}) + ac(b + \bar{b})$$

$$= \underset{111}{abc} + \underset{110}{ab\bar{c}} + \underset{111}{abc} + \underset{011}{\bar{a}bc} + \underset{111}{abc} + \underset{101}{a\bar{b}c}$$

$$= m_7 + m_6 + m_3 + m_5$$

$$= \Sigma_m(3,5,6,7)$$

maxterm, $\Pi_M(0,1,2,4)$

(1) Do in POS form

$f(x_1, x_2, x_3) = x_1 + x_2 \cdot \bar{x}_3$

$$= (x_1 + x_2) \cdot (x_1 + \bar{x}_3)$$

$$= (x_1 + x_2 + 0) \cdot (x_1 + 0 + \bar{x}_3)$$

$$= (x_1 + x_2 + x_3 \cdot \bar{x}_3) \cdot (x_1 + x_2 \cdot \bar{x}_2 + \bar{x}_3)$$

$$= (x_1 + x_2 + x_3) \cdot (x_1 + x_2 + \bar{x}_3) \cdot (x_1 + x_2 + \bar{x}_3) \cdot (x_1 + \bar{x}_2 + \bar{x}_3)$$

$$= M_0 \cdot M_1 \cdot M_3 = \Pi_M(0,1,3).$$

4) Write func. f in which 3 input variables are there when $f=1$; majority of inputs are 1.

	x_1	x_2	x_3	f
0	0	0	0	0
1	0	0	1	0
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	1
6	1	1	0	1
7	1	1	1	1

$\therefore f = m_3 + m_5 + m_6 + m_7$

$= \bar{x}_1 x_2 x_3 + x_1 \bar{x}_2 x_3 + x_1 x_2 \bar{x}_3 + x_1 x_2 x_3$

$= x_1 x_2 (x_3 + \bar{x}_3) + \bar{x}_1 x_2 x_3 + x_1 \bar{x}_2 x_3$

$= x_1 x_2 + \bar{x}_1 x_2 x_3 + x_1 \bar{x}_2 x_3$

$= \bar{x}_1 x_2 x_3 + x_1 (\bar{x}_2 x_3 + x_2)$ $(a + \bar{a} b = a + b)$

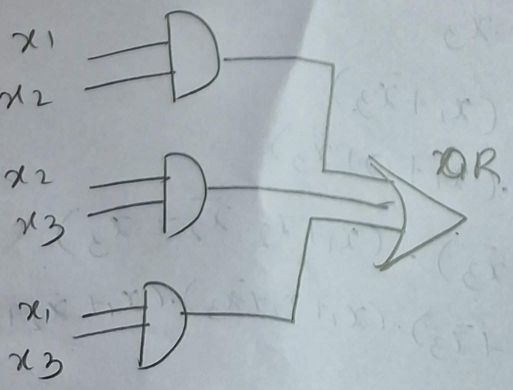
$= \bar{x}_1 x_2 x_3 + x_1 (x_2 + x_3)$

$= \bar{x}_1 x_2 x_3 + x_1 x_2 + x_1 x_3.$

$= x_2 (\bar{x}_1 x_3 + x_1) + x_1 x_3$

$= x_2 (x_1 + x_3) + x_1 x_3$

$= x_1 x_2 + x_2 x_3 + x_1 x_3$



cost:- $4 + 9$

$= 13$

5) A circuit that controls a digital system has 3 inputs (x_1, x_2, x_3). It has to recognise 3 diff conditions

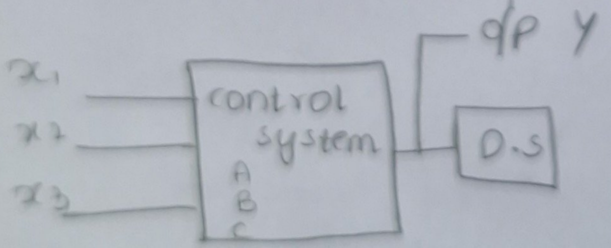
condition 'A' is true if x_3 is T & either x_1 is T or x_2 is F

cond 'B' is T if x_1 is T & either x_2 or x_3 is F.

cond 'C' is T if x_2 is T & either x_1 is T or x_3 is F

The control circuit must produce 1, if atleast two of conditions A, B, C is true. design a circuit

x_1	x_2	x_3
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1



$A = x_3 \cdot (x_1 + \bar{x}_2)$

$B = x_1 \cdot (\bar{x}_2 + \bar{x}_3)$

$C = x_2 \cdot (x_1 + \bar{x}_3)$

$Y = A \cdot B + B \cdot C + A \cdot C + A \cdot B \cdot C$

$= AB + BC + AC(1 + B)$

$= AB + BC + AC$

$= x_3 \cdot (x_1 + \bar{x}_2)\, x_1 \cdot (\bar{x}_2 + \bar{x}_3) + x_1 (\bar{x}_2 + \bar{x}_3) \cdot x_2 (x_1 + \bar{x}_3)$

$+ x_3 x_2 (x_1 + \bar{x}_2)(x_1 + \bar{x}_3)$

$= x_1 x_3 + x_1 \bar{x}_2 x_3 (\bar{x}_2 + \bar{x}_3) + x_1 (0) + x_1 x_2 \bar{x}_3 (x_1 + \bar{x}_3)$

$+ x_1 x_2 x_3 + 0 (x_1 + \bar{x}_3)$

$= x_1 x_3 + x_1 \bar{x}_2 x_3 + 0 + x_1 x_2 \bar{x}_3 + x_1 x_2 \bar{x}_3 + x_1 x_2 x_3$

$x_1 x_3 + x_1 x_2 \bar{x}_3 + x_1 x_2 x_3.$

$=$

$\Rightarrow x_1 x_2 (x_3 + \bar{x}_3) + x_1 x_3$

$= x_1 x_2 + x_1 x_3.$

$= x_1 (x_2 + x_3)$