Amrita School of Engineering, Bengaluru-35
23MAT206 – Optimization Techniques
Lab Practice Sheet-1
Identifying definiteness of matrices using eigenvalues, and use of Hessian matrix to identify concavity of the surfaces using MATLAB

Surface Plots

The parametric representation of any surface is given using two parameters. If a surface is given by the equation, Z=f(X,Y), the parametric representation of this surface can be taken as r(u,v)=(u,v,f(u,v)). For example, parametric representation of the plane z=x+y is r(u,v)=[u,v,u+v].

```
% surface plot of the paraboloid z=x^2+y^2 using command 'surf'
[X,Y] = meshgrid(-10:0.5:10,-10:0.5:10); % returns 2-D grid coordinates between -10<x,y<10
surf(X,Y,X.^2+Y.^2)
xlabel('x')
ylabel('y')
zlabel('z')
title('Surface z=x^2+y^2')


% surface plot of the cone z=Root(x^2+y^2) using command 'mesh'

[X,Y] = meshgrid(-2:0.01:2);% returns 2-D grid coordinates for x and y between -2 and 2

Z=(X.^2+Y.^2).^(0.5);

mesh(X,Y,Z)

xlabel('x')

ylabel('y')

zlabel('z')

title('Cone z=Root(x^2+y^2)')
```

Positive Definite Matrix:

A symmetric matrix S is defined to be a positive definite matrix if for any point x in the search space the quantity(energy) $x^T S x > 0$ for all x $\neq$ 0. Geometrically we can understand this if graph of $f(\boldsymbol{x}) = \boldsymbol{x}^T\boldsymbol{S}\boldsymbol{x}$ curves up. If $f(\boldsymbol{x}) = \boldsymbol{x}^T\boldsymbol{S}\boldsymbol{x}$ curves down, the symmetric matrix S is negative definite.

Example: S= $\begin{bmatrix} 3 & 0 \\ 0 & 5 \end{bmatrix}$ is positive definite as $f(x) = x^T S x = 3x^2 + 5y^2$ curves up. It can be verified by plotting the surface $z = 3x^2 + 5y^2$ in MATLAB.

Contour curves/ level curves

```
x = linspace(-2*pi,2*pi); y = linspace(0,4*pi);
[X,Y] = meshgrid(x,y);
Z = sin(X)+cos(Y);
contour(X,Y,Z)     % contour curves for the function f(x,y)=sinx+cosy
```

Contour curves/ level curves with value of the function written on the curves

```
x = linspace(-2*pi,2*pi); y = linspace(0,4*pi);
[X,Y] = meshgrid(x,y);
Z = sin(X)+cos(Y);
contour(X, Y, Z, 'ShowText' ,'on')
```

contour(x,y,z,50) can be used to get fixed number of contour curves/ level curves

- contour(X,Y,Z,'--') can be used to get contour curves in -- style

Surface plots with contour curves

```
x = linspace(-2*pi,2*pi); y = linspace(0,4*pi); [X,Y] = meshgrid(x,y);
Z = sin(X)+cos(Y); surfc(X,Y,Z);
shading interp
```

Gradient using MATLAB

```
syms x y z; gradient(x^2 + 3*y^2-5*z, [x, y, z])
```

Plot of surface, level curves and gradient vectors of a scalar function f at different points on the level curves

```
[x,y] = meshgrid(-2:.2:2,-2:.2:2); z = x.^2 + y.^2;
figure
surf(x,y,z);
figure
[px,py] = gradient(z); contour(x,y,z,'ShowText','on');
hold on
quiver(x,y,px,py);
hold off;
axis image
% Here surface plot will appear in one figure window and the gradient vectors in
another
```

Hessian matrix using MATLAB

```
syms x y z; hessian(x*y + 2*z*x, [x, y, z])
```

Practice Questions

1. Plot the surface given by $z = sin(x + y), -1 \leq x, y \leq 1$.

2. Plot the surface of the cone given by $z = \sqrt{6x^2 + 9y^2 - 5}$, $-5 \leq x, y \leq 5$.

3. Plot the surface $z = sin(x + y) - x^2 - xy + y^2, -2\pi \leq x, y \leq 2\pi$.

4. By plotting the surface $f(x) = x^T S x$ identify if the given matrices are positive definite, negative definite or indefinite. Also verify by finding the eigenvalues of each of these matrices.

(i) A= $\begin{bmatrix} 3 & 2 \\ 2 & 5 \end{bmatrix}$ (ii) B= $\begin{bmatrix} 9 & 4 \\ 4 & 6 \end{bmatrix}$ (iii) C= $\begin{bmatrix} -5 & 3 \\ 3 & -8 \end{bmatrix}$ (iv) D= $\begin{bmatrix} -7 & -1 \\ -1 & -4 \end{bmatrix}$

5. Generate a 6 by 2 matrix A. Find S=A^TA and verify that S=A^TA is a positive definite matrix by:
   a. by finding the eigenvalues of S.
   b. by plotting the surface $f(x) = x^T S x$.

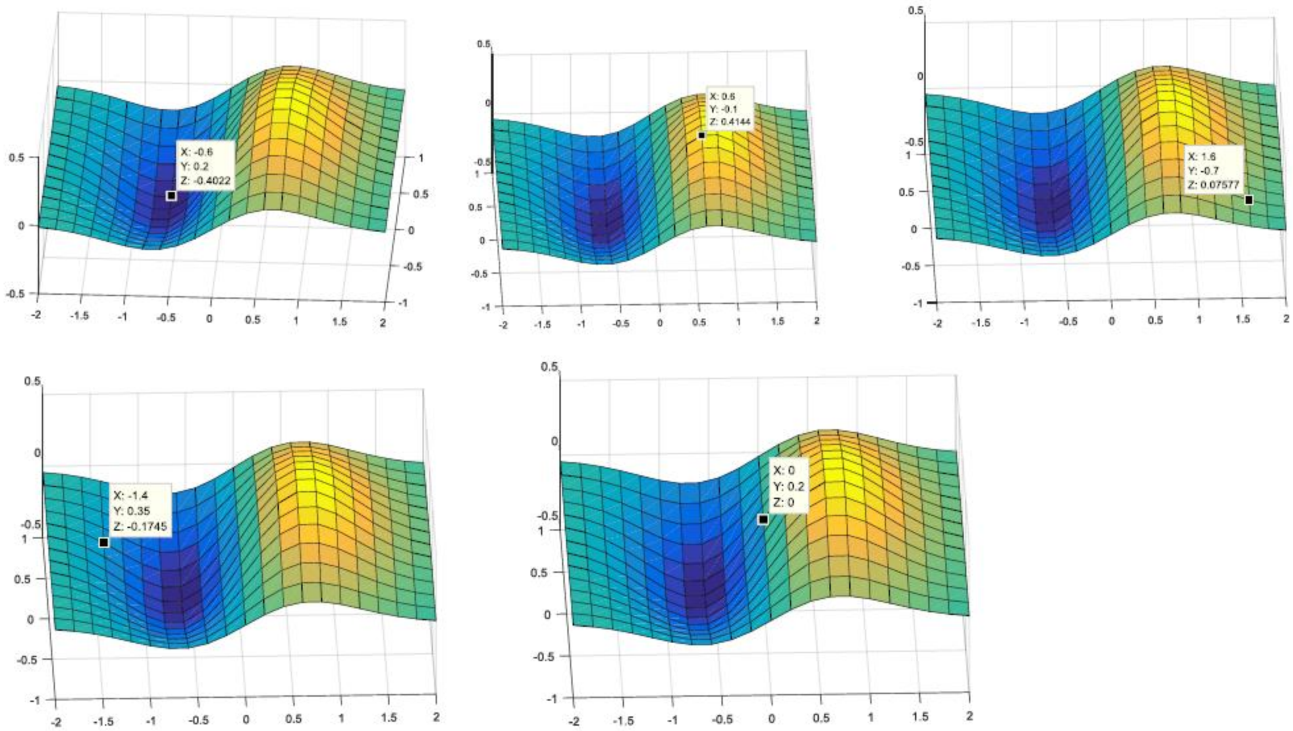
6. Find the gradient and Hessian of the following scalar function using the MATLAB command
   a. $f(x, y, z) = 4x^3 e^{xy} - y^3 z$
   b. $g(x, y) = e^{xy}(x + y)$ at $P: (-1, 1)$
   c. $h(x, y, z) = (x^2 + y^2)/(x^2 - y^2)$

7. Consider the function: $f(x, y) = 1/(|x| + |y| + 0.5)$. Plot 15 level curves for the function $f(x, y)$ in the rectangular domain $-5 \le x, y \le 5$.

8. Consider the function: $f(x, y) = 4x^2 + 9y^2 - 72$.
   a) Plot the level curves for $f(x, y)$ with function values written in each curve in $-10 \le x, y \le 10$
   b) Find the gradient and hessian of the function using ‘syms’.
   c) Plot the gradient vectors at some of the points on the level curves.

9. Consider the function $f(x, y) = \sin x + \cos y + 3e^{x^2 - y^2}$.
   d) Show the surface plot of the function in one figure and the gradient vectors at some points on level curves in another figure.
   e) Looking at the surface plot and from the figure with function values on level curves, try to find a minimum and a maximum for the function.

10. Consider the function: $f(x, y) = xe^{x^2 - y^2}$.
    a) Plot the surface given for x and y varying from -2 to 2
    b) Write the General expression for gradient and Hessian for the function
    c) Evaluate Hessian at the points given in the following figures
    d) Find eigenvalues of those hessian matrices and decide the nature of curvature at the points where it is evaluated. (curved upward, downward, neither)


X: -0.6
Y: 0.2
Z: -0.4022
X: 0.6
Y: -0.1
Z: 0.4144
X: 1.6
Y: -0.7
Z: 0.07577
X: -1.4
Y: 0.35
Z: -0.1745
X: 0
Y: 0.2
Z: 0

Amrita School of Engineering, Bengaluru-35
23MAT206 – Optimization Techniques
Lab Practice Sheet-2
Graphical solution of optimization problems up to 2 variables using MATLAB and computing numerical derivatives using Excel

MATLAB sample code (Hard copy) provided for the following problems:

1. Find the local maxima and local minima of the following.
   a. $x^2 + y^2 \quad x, y \,\epsilon[-3,3]$
   b. $-x^2 + y \quad x, y \,\epsilon[-3,3]$
   c. $e^{-(x^2+y^2)} \quad x, y \,\epsilon[-3,3]$
2. Minimize z=$x^2 + y^2$ subject to $x + y \geq 1; x, y \geq 0$
3. Minimize z= $x + y$ subject to $x^2 + y^2 \quad \leq 1; x, y \geq 0$
4. Maximize z=$x + y$ subject to $x^2 + y^2 \quad \leq 25\,; x + y \geq 5;\ x, y \geq 0$
5. Minimize z= $x + y$ subject to $x^2 + y^2 \quad \leq 25\,; x + y \geq 5;\ x, y \geq 0$
6. Minimize z=$x^2 + y^2$ subject to $x^2 + y^2 \quad \leq 25\,; x - y \geq 1;\ x \leq 3;\ x, y \geq 0$

Practice Questions:

1. *Maximize x+y subject to y ≤ 5, x+y ≤10; x,y ≥0*

2. *Maximize 3y subject to x ≤3; -x+y ≤ 4; x+y ≤6 and x, y≥0*

3. *Maximize x+y subject to x ≤ 3, y ≤ 7 and x, y ≥0*

4. *Maximize and minimize* $x^2 + y^2$ *in the first quadrant, subject to x ≤ 3, y ≤ 7 and x, y ≥0*

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23MAT206 – Optimization Techniques

Lab Sheet-3

Implementation of Exhaustive Search Method and Bounding Phase Method for Single variable optimization problem

- **Exhaustive Search Method**

Bracket the minimum of the function $f(x) = x^2 + \frac{54}{x}$ using an exhaustive search method in the interval (0,5) by considering only 10 intermediate points.

```
% Define the function
 f = @(x) x.^2 + 54./ x;
 % Define the interval and discretization
 a = 0; % Start just above 0 to avoid division by zero
 b = 5;
 n = 10;
 dx = (b - a) / n;
 % Initialize variables
 x1 = a;
 x2 = x1 + dx;
 x3 = x2 + dx;
 % Initialize best value
 fmin = Inf;
 lower = 0;
 upper= 0;
 % Perform the grid search
 while x3 <= b
    f1 = f(x1);
    f2 = f(x2);
    f3 = f(x3);
 % Check if x2 is a local minimum
 if f1 >= f2 && f2 <= f3
 if f2 < fmin
            fmin = f2;
            lower= x1;
            upper = x3;
 end
```

```
end
% Update x1, x2, and x3
   x1 = x2;
   x2 = x3;
   x3 = x2 + dx;
end
fprintf('Minimum lies in (%.3f, %.3f)\n', lower, upper);
x=(lower+upper)/2;
fprintf('Optimized values:\n x=%.3f \n minimum f=%.3f', x,f(x))
```

Output:

Minimum lies in (2.500, 3.500)

Optimized values: x=3.000

minimum f=27.000

Alternative code for the Exhaustive Search Method

```
% Function definition
f = @(x) x.^2 + 54./x;
% Interval and number of points
a = 0;        % Start of interval
b = 5;        % End of interval
n = 10;       % Number of intermediate points
% Step size
delta_x = (b - a) / (n);
% Initialize variables
x_lower = a;
f_lower = f(a);
min_found = false;
% Exhaustive Search Loop
for i = 1: n
   x_current = a + i * delta_x;
   f_current = f(x_current);
   % Compare current function value with previous one
   if f_current < f_lower
      x_lower = x_current;
      f_lower = f_current;
```

```
        min_found = true;
    else
        % Stop if we find the function value starts increasing
        if min_found
            x_upper = x_current;
            break;
        end
    end
end
% Compute function values at the interval bounds
f_lower_bound = f(x_lower - delta_x);
f_upper_bound = f(x_upper);
% Display the final result
fprintf('\nThe minimum lies in the range [%.3f, %.3f]\n', x_lower - delta_x, x_upper);
fprintf('Function value at lower bound: %.3f\n', f_lower_bound);
fprintf('Function value at upper bound: %.3f\n', f_upper_bound);
fprintf('Function value at the minimum: %.3f\n', f_lower);
```

Output:

Output:

The minimum lies in the range [2.500, 3.500]

Function value at lower bound: 27.850

Function value at upper bound: 27.679

Function value at the minimum: 27.000

- **Bounding Phase Method**

Bracket the minimum of the function $f(x) = x^2 + \frac{54}{x}$ using bounding phase method, with an increment parameter of 0.6 and initial guess of $x = 0.5$.

```
% Define the function
f = @(x) x.^2 + 54./x;
% Initialization
x0 = 0.6;     %Initial Guess
d = 0.5;      %Increment Parameter
k = 0;
```

```
% Evaluate Neighbouring points
f0 = f(x0);
f1= f(x0 + d);
f_1=f(x0-d);
% Determine the direction of search
if  (f_1>=f0 && f0>=f1)
    disp("d is positive")
elseif (f_1<=f0 && f0<=f1)
        disp("d is negative")
    d = -d;     % Change direction if the function value increases
else
    d=d/2;
end
%Initial Update
xk = x0;
xk_next = xk + (2^k) * d;
% Bounding Phase Method Loop (Iterative Search)
while f(xk_next) < f(xk)
    % Update xk and increment k
    xk = xk_next;
    k = k + 1;
    xk_next = xk + (2^k) * d;
end
% Termination and Bracketed Range
x_lower = xk - (2^(k-1)) * d;
x_upper = xk_next;
% Compute function values at bounds and minimum
f_lower_bound = f(x_lower);
f_upper_bound = f(x_upper);
f_minimum = f(xk);
% Display the final result
fprintf('\nThe minimum lies in the range [%.3f, %.3f]\n', min(x_lower,
x_upper), max(x_lower, x_upper));
fprintf('Function value at lower bound: %.3f\n', f_lower_bound);
fprintf('Function value at upper bound: %.3f\n', f_upper_bound);
fprintf('Function value at the minimum: %.3f\n', f_minimum);
```

Output: d is positive.

The minimum lies in the range [2.100, 8.100]

Function value at lower bound: 30.124

Function value at upper bound: 72.277

Function value at the minimum: 29.981

Practice Questions:

1) Bracket the minimum for the function $f(x) = x^3 - 4x^2 + 6x - \exp(0.1x)$ in the interval (1,4) using the exhaustive search method with 20 intermediate points.
2) Determine the bracket for the minimum of the function $f(x) = \sqrt{x^2 + 1} - 2x$ within the interval (-2.5, 1) by dividing the interval into 15 equal parts using exhaustive search method.
3) Consider the function $f(x) = x^2 \log(x + 1) - \frac{1}{x+1}$. Bracket a minimum of function by applying exhaustive search method within an interval of size 10 in the range (0.5, 2.5) where the minimum of the function lies.
4) Bracket the minimum for the function $f(x) = (1 - x)^4 - (2x + 10)^2$ using the bounding phase method, with an increment parameter of 0.6 and an initial guess of $x = 0.5$.
5) Determine the bracket for the minimum of the function $f(x) = \exp\left((x - 0.2)^2/2\right)$ using the bounding phase method, with an initial guess of -10 and an increment parameter of 0.8.
6) Consider the function $f(x) = \frac{(x+2)^2}{sin(x+1)+2}$. Bracket a minimum of function by applying bounding phase method with an initial guess of –1.5 and an increment value of 0.05.
7) Bracket the minimum of the function $f(x) = \sqrt{x} - \sin(x^2)$.
   a) Using the exhaustive search method in the interval (-1,2), taking 15 intermediate points.
   b) Using the bounding phase method with an initial guess of 1 and an increment value of 0.4.
8) Consider the function $f(x) = \frac{cos(x)^2+x}{(x-2)}$.
   a) Using the exhaustive search method, determine the interval where the minimum lies within (-3,3) by dividing the interval into 14 equal parts.
   b) Applying the bounding phase method, find the minimum with an initial guess of $x = 0.08$ and increment parameter of -0.5.

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Lab Sheet-4

Implementation of Fibonacci Search Methods and Golden Section Search for Single Variable Optimization Problem

- **Fibonacci Search Method**

  Find the interval of a minimum of the function $f(x) = x^2 + \frac{54}{x}$ using the Fibonacci search method in the interval (0, 5) with 4 function evaluations.

```
% Define the function using a function handle
clear all;
clc; clf;
f=@(x)x.^2+54./x;
% Defining interval
a=0;     % Start of interval
b=5;      % End of interval
L=b-a;    % Length of the interval
% Fibonacci search loop
n=input('Enter the number of functional evaluations:');
k=2;
for i=k:n
Lk=fibonacci(n-i+1)/fibonacci(n+1)*L;
x1=a+Lk;
x2=b-Lk;
f1=f(x1);
f2=f(x2);
plot(x1,f1,'rx')      % plotting x
hold on
plot(x2,f2,'rx')
if f1<f2
 b=x2;
else
 a=x1;
end
fprintf('\n The New interval is (%.3f,%.3f) after %d iterations',a,b,i-1)
end
NewL=b-a;
fprintf('\n The required approximation to the optimum point lies in the interval
(%.3f,%.3f) of length %d',a,b, NewL)
min_x=(a+b)/2;
min_f=f(min_x);
plot(min_x,min_f,'ro')
hold off
%fprintf('\n The minimum lies in the interval (%.3f,%.3f)',a,b)
fprintf('\n The approximation to the minimum value of the function is %.3f, for
x=%.3f',min_f,min_x)
% Fibonacci function
function f=fibonacci(n)
if n==0
f=1;
elseif n==1
 f=1;
else
 f=fibonacci(n-1)+fibonacci(n-2);
```

```
end
end
```

Output

```
Enter the number of functional evaluations:4
Lk =    1.8750
 The New interval is (1.875,5.000) after 1 iterations
Lk =    1.2500
 The New interval is (1.875,3.750) after 2 iterations
Lk =    0.6250
 The New interval is (2.500,3.750) after 3 iterations
The minimum value lies in the interval (2.500,3.750)
```

Practice Questions

Write MATLAB code for solving the following problems using the Fibonacci Search Method and Golden Section Search method till the length of the final interval is less than 0.25.

1. Bracket the minimum for the function $f(x) = -4x^3 + 100 + e^x$ in the interval (3,8)
2. For the function $f(x) = e^x - 2x$, determine the minimum value within the interval [0,3].
3. Minimize the function f(x)=$\frac{1}{x+1} + x^2$ within the interval [0,2].
4. Find the minimum value of the piecewise function

$$f(x) = \begin{cases} 2x + 3 & 0 \le x < 1 \\ (x-2)^2 + 1 & 1 \le x \le 3 \end{cases}$$

   within the interval [0,3].

5. Compute the minimum value of the function f(x)=$e^{-x^2} + 0.1x^2$ within the interval [-2,2].
6. Find the minimum value of f(x)= x^2log(x+1)+ $\frac{1}{x+1}$ within the interval [0,4].
7. Find the minimum value of the piecewise function

$$f(x) = \begin{cases} (x-1)^2 + 2 & 0 \le x < 2 \\ (x-3)^2 - 1 & 2 \le x \le 4 \end{cases}$$

   within the interval [0,4].

Amrita School of Engineering, Bengaluru-35

23MAT206 – Optimization Techniques

Lab Sheet-5

Implementation of Golden Section Search, Powell's algorithm and Newton Raphson method for Single Variable Optimization Problem

- Find the interval of a minimum of the function $f(x) = x^2 + \frac{54}{x}$ in the interval (0, 5).

Golden Section Search

```
clc;
clear all;
syms x w;
g= @(x) x^2+54/x;
a=0;b=5;
w = (x - a)/(b-a);
x_w= finverse(w,x);
f = g(x_w);
disp("Function after the transformation:");
disp(f);
aw = 0;
bw = 1;
Lw = bw - aw;
Lx=subs(x_w,bw)-subs(x_w,aw);
k = 1;
e_val = input("Enter the termination paramenter: ");
if(e_val == 0)
n = input("Enter the number of function evaluations");
e_val = (0.618).^n*(b - a);
end
while Lx>0.25
 w1 = aw + ((0.618)*(Lw));
 w2 = bw - ((0.618)*(Lw));
 f_w1=subs(f,x,w1);
 f_w2=subs(f,x,w2);
 if ((w1>w2)&(f_w1>f_w2))
 bw = w1 ;
 elseif ((w1>w2)&(f_w1<f_w2))
 aw = w2;
 elseif((w1<w2)&(f_w1>f_w2))
 aw=w1;
 elseif((w1<w2)&(f_w1<f_w2))
 bw=w2;
 end
fprintf("New interval of the function (%.3f , %.3f) after iteration 
%d\n",aw,bw,k);
Lw = bw - aw;
k = k + 1;
Lx=subs(x_w,bw)-subs(x_w,aw);
end
fprintf("Final interval of the function in w (%.3f , %.3f) \n",aw, bw);
fprintf("Final interval of the function in x (%.3f , %.3f) 
\n",subs(x_w,aw),subs(x_w,bw));
```

Successive quadratic estimation method (Powell's algorithm)

```
clc;
clear all;
a = 1; % Lower bound
b = 5; % Upper bound
dx = 1;%1e-5; % Step size
fcp = 0.01; %1e-8; % Convergence parameter for function evaluation

% Equation to be minimized
f = @(x) x^2 + 54/x;
x = linspace(a, b, 1000);
% Initialization
x1 = a;
x2 = x1 + dx;
f1 = f(x1);
f2 = f(x2);

if f1 > f2
    x3 = x1 + 2*dx;
elseif f1 <= f2
    x3 = x1 - dx;
end
f3 = f(x3);

% Determine minimum function value for the initial step
f_min = min([f1, f2, f3])

% Determine x-value corresponding to minimum function value
if f_min == f1
    x_min = x1;
elseif f_min == f2
    x_min = x2;
elseif f_min == f3
    x_min = x3;
end

% Determine x_bar
a1 = (f2 - f1) / (x2 - x1);
a2 = (1 / (x3 - x2)) * (((f3 - f1) / (x3 - x1)) - ((f2 - f1) / (x2 - x1)));
x_bar = 0.5 * ((x1 + x2) - (a1 / a2))
f_bar = f(x_bar)

 while abs(f_min - f_bar) > fcp
    x_s=sort([f1,f2,f3,f_bar])
    fn1 = x_s(1,1)
    fn2 = x_s(1,2)
    fn3 =x_s(1,3)
if fn1==f1
    xn1=x1
elseif fn1==f2
    xn1=x2
elseif fn1==f3
    xn1=x3
elseif fn1==f_bar
    xn1=x_bar
end
if fn2==f1
    xn2=x1
elseif fn2==f2
    xn2=x2
```

```
elseif fn2==f3
    xn2=x3
elseif fn2==f_bar
    xn2=x_bar
end
if fn3==f1
    xn3=x1
elseif fn3==f2
    xn3=x2
elseif fn3==f3
    xn3=x3
elseif fn3==f_bar
    xn3=x_bar
end
x_s=sort([xn1,xn2,xn3])
    x1 = x_s(1,1)
    x2 = x_s(1,2)
    x3 =x_s(1,3)
    f1 = f(x1)
    f2 = f(x2)
    f3 = f(x3)
   f_min = min([f1, f2, f3]);
    if f_min == f1
        x_min = x1;
    elseif f_min == f2
        x_min = x2;
    elseif f_min == f3
        x_min = x3;
    end
    a1 = (f2 - f1) / (x2 - x1);
    a2 = (1 / (x3 - x2)) * (((f3 - f1) / (x3 - x1)) - ((f2 - f1) / (x2 - x1)));
    x_bar = 0.5 * ((x1 + x2) - (a1 / a2));
    f_bar = f(x_bar);
end

fprintf('The approximate minimum function value is %.8f, at %.8f\n', f_min,
x_min);

% Plot the function
x_values = linspace(a, b, 1000);
y_values = arrayfun(f, x_values);

figure;
plot(x_values, y_values,'DisplayName', func2str (f));
%plot(x_values, y_values) %'DisplayName', func2str (f));
xlabel('x', 'FontWeight', 'bold');
ylabel('f(x)', 'FontWeight', 'bold');
grid on;
title('Successive Quadratic Estimation Method', 'FontWeight', 'bold');
hold on;
scatter(x_min, f_min, 'red', 'filled', 'DisplayName', 'Approximate Minimum
Point');
legend('Location', 'Best');
saveas(gcf, 'Successive_Quadratic_Estimation_Method.png');
```

Newton Raphson method

```
syms x
x0=1; % Initial condition
f=@(x)x^2+54/x;  % Defining function
fdx=inf;
tol=10^-3;   % Setting the tolerance value
while abs(fdx)>tol
 fdx=subs(diff(f,x),x0);    % First derivative of the function at x= x0
 fd2x=subs(diff(f,x,2),x0);    % Second derivative of the function at x= x0
 xn=x0-fdx/fd2x;            % Newton's Method formula
 x0=xn;
end
fx=f(xn);
fprintf("The minimum value of the function is %.3f at x=%.3f",fx,xn)
```

Practice Questions

Write MATLAB code for solving the following problems using the Golden Section Search till the length of the final interval is less than 0.25, using the Successive quadratic estimation method and by using Newton Raphson method with the specified initial value and termination parameter.

1. Minimize the function $f(x) = x^2 + \frac{54}{x}$ in the interval (1, 5) with increment 0.5, the initial value $x_0 = 1$ and termination parameter 0.0001.
2. Minimize the function $f(x) = -3x^3 sin(2x) + 4xcos(3x) + 2x + 4$ in the interval (3.5, 5) with increment 0.5, the initial value $x_0 = 4$ and termination parameter 0.00001.
3. Minimize the function $f(x) = x^3 cos(2x) + e^x$ in the interval (4, 6) with increment 0.5, initial value $x_0 = 5$ and termination parameter 0.00001.
4. Minimize the function $f(x) = 2x^2 + \frac{16}{x}$ in the interval (1, 3) with increment 0.5, the initial value $x_0 = 1$ and termination parameter 0.000001.
5. Minimize the function $f(x) = x^5 - 5x^3 - 20x + 5$ in the interval (1, 4) with increment 0.5, the initial value $x_0 = 2$ and termination parameter 0.00001.
6. Find the minimum for the function $f(x) = -4x^3 + 100 + e^x$ in the interval (5,7) with increment 1, the initial value $x_0 = 6$ and termination parameter 0.01.
7. For the function $f(x) = e^x - 2x$, determine the minimum value within the interval [-1, 1] with increment 0.6, the initial value $x_0 = 0$ and termination parameter 0.0001.
8. Minimize the function f(x) = $\frac{1}{x+1} + x^2$ within the interval [0, 2] with increment 0.6, the initial value $x_0 = 1$ and termination parameter 0.0001.

Amrita School of Engineering, Bengaluru-35

23MAT206 – Optimization Techniques

Lab Sheet-6

Implementation of the Bisection and Secant method for single variable optimization problem

- **Bisection method**

Minimize the function $f(x) = x^2 + \frac{54}{x}$ using the Bisection method in the interval (1,5) with termination tolerance of 0.001.

```
syms x x1 x2
f=@(x) x^2+54/x;
a=1; % Lower limit of the interval
b=5; % Upper limit of the interval
fda=double(subs(diff(f,x),x,a)); % first derivative of the function at x=a
fdb=double(subs(diff(f,x),x,b)); % first derivative of the function at x=b
%Check if the interval is valid
if fda<0 && fdb>0
x1=a;
x2=b;
elseif fda>0 && fdb<0
x1=b;
x2=a;
else
fprintf("Invalid interval")
end
tol=0.001; % Set termination parameter
fdz=inf; % Set the value of the first derivative of the function at x=z to a maximum to ensure that the while loop runs at least once.
while abs(fdz)>tol
z=(x1+x2)/2; % Evaluate z
fdz=double(subs(diff(f,x),x,z)); % First derivative of the function at x=z
```

```
if fdz<0
x1=z;
else
x2=z;
end
end
%Calculate the function minimum
f_minimum=f(z);
fprintf("Minimum value of the function is %.3f at x= %.3f",f_minimum,z)
```

Output

Minimum value of the function is 27.000 at x= 3.000

- ## Secant method

Minimize the function $f(x) = x^2 + \frac{54}{x}$ using the Secant method in the interval (1,5) with termination tolerance of 0.001.

```
syms x x1 x2
f=@(x) x.^2+54./x;
a=1; % Lower limit of the interval
b=5; % Upper limit of the interval
fda=double(subs(diff(f,x),x,a)); % first derivative of the function at x=a
fdb=double(subs(diff(f,x),x,b)); % first derivative of the function at x=b
if fda<0 && fdb>0
   x1=a;
   x2=b;
elseif fda>0 && fdb<0
   x1=b;
   x2=a;
else
   fprintf("Invalied interval")
end
fdz=inf;
while abs(fdz)>0.001
```

```
        fdx1=double(subs(diff(f,x),x,x1)); % first derivative of the function at x=x1
        fdx2=double(subs(diff(f,x),x,x2)); % first derivative of the function at x=x2
        z=x2-fdx2/((fdx2-fdx1)/(x2-x1)); % Evaluating the z
       fdz=double(subs(diff(f,x),x,z)); % first derivative of the function at x=z
       if fdz<0
           x1=z;
       else
           x2=z;
      end
end
f_minimum=f(z);
fprintf("Minimum value of the function is %.3f at x= %.3f",f_minimum,z)

% Plot the function
x_values = linspace(a, b, 1000);
y_values = f(x_values);
figure;
plot(x_values, y_values, 'DisplayName', func2str (f));
xlabel('x', 'FontWeight', 'bold');
ylabel('f(x)', 'FontWeight', 'bold');
grid on;
title('Secant Method', 'FontWeight', 'bold');
hold on;
scatter(z,f_minimum, 'red', 'filled', 'DisplayName', 'Approximate Minimum
Point');
legend('Location', 'Best');
saveas(gcf, 'Secant_Method.png');
```

Output

Minimum value of the function is 27.000 at x=3.000.

Practice Questions:

Write MATLAB code for solving the following problems using the Bisection method and the Secant method with the specified intervals and termination parameters.

1) Minimize the function $f(x) = x^2 + \frac{54}{x}$ in the interval (2,5) and termination parameter 0.0001.
2) Minimize the function $f(x) = -3x^3 sin(2x) + 4xcos(3x) + 2x + 4$ in the interval (3.5, 5) and termination parameter 0.00001.

3) Minimize the function $f(x) = x^3cos(2x) + e^x$ in the interval (4, 6) and termination parameter 0.00001.

4) Minimize the function $f(x) = 2x^2 + \frac{16}{x}$ in the interval (1,3) and termination parameter 0.000001.

5) Minimize the function $f(x) = x^5 - 5x^3 - 20x + 5$ in the interval (1,4) and termination parameter 0.00001.

6) Find the minimum for the function $f(x) = -4x^3 + 100 + e^x$ in the interval (5,7) and termination parameter 0.01.

7) For the function$f(x) = e^x - 2x$, determine the minimum value within the interval [-1,1] and termination parameter 0.0001.

8) Minimize the function f(x) = $\frac{1}{x+1} + x^2$ within the interval [0,2] and termination parameter 0.0001.

Amrita School of Engineering, Bengaluru-35

23MAT206 – Optimization Techniques

Lab Sheet-7

Implementation of the numerical algorithms for unconstrained multivariable optimization problems

- **Unidirectional Search**

➢ Find the minimum of the given function $f(x, y, z, w) = x^2 + y^2 + z^2 + w^2 - 2x - 4y + 6z + 8w + 15$ from (1,1,1,1) along the direction (-1,0,1,1) using unidirectional search.

```
syms x y z w t
f=x^2+y^2+z^2+w^2-2*x-4*y+6*z+8*w+15;
a=[1 1 1 1];
b=[-1 0 1 1];
st=a+t*b;
fst=subs(f,[x y z w],st);
ft=diff(fst,t);
t1=solve(ft==0,t)
ftt=diff(fst,t,2);
if subs(ftt,t,t1)>0
X=subs(st,t1);
disp("The minimum value of the function is at :")
disp(X);
disp(" Minimum value =")
disp(subs(f,[x,y,z,w],X))
end
```

OUTPUT:
t1 =-3
The minimum value of the function is at: (4 1 -2 -2)
Minimum value = 0

- **Evolutionary Optimization Method**

➢ Find a minimum for the function, $f(x, y) = (x - 2)^2 + (y - 3)^2$ starting from the point (1,0), with a reduction parameter $\Delta = (2,2)$ and a termination parameter $\varepsilon = 0.8$.

```
%% Define function
```

```matlab
f = @(X) (X(1) - 2).^2 + (X(2) - 3).^2;
X0 = [1,0]; %Initial point
D = [2, 2]; % Reduction parameter
e = 0.8; % Termination parameter
X_bar = X0;
NormD = sqrt(D(1)^2 + D(2)^2);
k=0;
while NormD > e
 % Finding corner points
 X1 = X_bar + [-D(1)/2, -D(2)/2];
 X2 = X_bar + [ D(1)/2, -D(2)/2];
 X3 = X_bar + [ D(1)/2, D(2)/2];
 X4 = X_bar + [-D(1)/2, D(2)/2];

 % Evaluate the function at these points
 f0 = f(X_bar);
 f1 = f(X1);
 f2 = f(X2);
 f3 = f(X3);
 f4 = f(X4);

 % Store the function values in an array
 F = [f0, f1, f2, f3, f4];

 % Finding minimum value and corresponding index
 [minimum, idx] = min(F);

 % Update X_bar and D based on the index of the minimum
 switch idx
 case 1
 D = D / 2; % Reduce the size of D
 NormD = sqrt(D(1)^2 + D(2)^2); % Update NormD
 case 2
 X_bar = X1; % Update X_bar to the point with the minimum value
 case 3
 X_bar = X2;
 case 4
 X_bar = X3;
 otherwise
 X_bar = X4;
 end
 k=k+1;
 fprintf('\n The minimum value after %d iteration is %.3f at
X=(%.3f,%.3f)',k,f(X_bar),X_bar(1),X_bar(2));
end
% Display results
disp('Optimal point:');
disp(X_bar);
disp('Function value at optimal point:');
disp(f(X_bar));
```

OUTPUT:

The minimum value after 1 iteration is 4.000 at X= (2.000,1.000)
The minimum value after 2 iteration is 2.000 at X= (3.000,2.000)
The minimum value after 3 iteration is 0.000 at X= (2.000,3.000)
The minimum value after 4 iteration is 0.000 at X= (2.000,3.000)
The minimum value after 5 iteration is 0.000 at X= (2.000,3.000)
Optimal point: 2, 3
Function value at optimal point: 0

- <u>**Hooke-Jeeve's Pattern Search Method**</u>

➢ Find a minimum for the function, $f(x,y) = (x-1)^2 + (y-2)^2$ using Hook-Jeeve's pattern search method starting from the initial point (0,0), with an increment vector (0.5, 0.5), a termination parameter $\varepsilon = 0.5$, and a reduction factor $\alpha = 2$.

```
% Define the objective function
f = @(X) (X(1)-1)^2 + (X(2)-2)^2;

% Initial parameters
X0 = [0, 0]; % Starting point
D = [0.5, 0.5]; % Step size for each direction
a = 2; % Reduction factor for the step size
E = 0.5; % Termination criterion (norm of D)
k = 1; % Iteration counter


% Initial exploratory move from X0
X1 = exploratory(X0, D, f); % First exploratory move


% Display the result for the first iteration
fprintf('Iteration %d: X = [%f, %f], f(X) = %f\n', k, X1(1), X1(2), f(X1));
k = k + 1; % Increment iteration counter


% Main optimization loop
while norm(D) > E
    % Perform pattern move
    X1p = 2 * X1 - X0; % Pattern move

    % Exploratory move from the pattern point X1p
    X2 = exploratory(X1p, D, f);

    % Check if the exploratory move was successful
    if any(X1p ~= X2) || f(X2) < f(X1)
        % Successful move
        X0 = X1; % Update base point
```

```
        X1 = X2; % Move to the new minimum
    else
        % Unsuccessful move, reduce step size
        D = D / a;
    end

    % Display current iteration
    fprintf(' X = [%f, %f], f(X) = %f\n', X1(1), X1(2), f(X1));
    k = k + 1; % Increment iteration counter
end


% Output the final result
fprintf('Final Solution: X = [%f, %f], f(X) = %f\n', X1(1), X1(2), f(X1));
```

```
function [xmin] = exploratory(X0, D, f)
    % f: function handle (objective function)

    % Step size for each direction (assumed equal step sizes in x and y)
    d = D(1);

    % Explore in the x direction
    f0 = f(X0); % Current function value at X0
    f_plus = f(X0 + [d, 0]); % Step in positive x direction
    f_minus = f(X0 - [d, 0]); % Step in negative x direction
    [fmin1, idx] = min([f0, f_plus, f_minus]); %  minimum value in x direction
    switch idx
        case 1
            xmin1 = X0; % No change
        case 2
            xmin1 = X0 + [d, 0]; % Move in positive x direction
        otherwise
            xmin1 = X0 - [d, 0]; % Move in negative x direction
    end

    % Explore in the y direction (using the best x from above)
    f0 = f(xmin1); % Function value at the new xmin1
    f_plus = f(xmin1 + [0, d]); % Step in positive y direction
    f_minus = f(xmin1 - [0, d]); % Step in negative y direction
    [fmin, idx] = min([f0, f_plus, f_minus]); % minimum value in y direction
    switch idx
        case 1
            xmin = xmin1; % No change
        case 2
            xmin = xmin1 + [0, d]; % Move in positive y direction
        otherwise
            xmin = xmin1 - [0, d]; % Move in negative y direction
    end
end
```

OUTPUT:

Iteration 1: X = [0.500000, 0.500000], f(X) = 2.500000
Iteration 2: X = [1.000000, 1.500000], f(X) = 0.250000
Iteration 3: X = [1.000000, 2.000000], f(X) = 0.000000
Iteration 4: X = [1.000000, 2.000000], f(X) = 0.000000
Iteration 5: X = [1.000000, 2.000000], f(X) = 0.000000
Final Solution: X = [1.000000, 2.000000], f(X) = 0.000000

Practice Questions

1) Find the minimum of the following functions using unidirectional search method.

a) $f(x,y,z,w) = x^3 + y^2 + 2z^2 + w^2 - 2xy + 6z + 15w$ from (0, 1, -2, 1) along the direction (0, 2, -1, 0).

b) $f(x,y,z,w) = x^2 + y^2 + z^2 + (x+y)(z-w) + 5$ from (2, -1, 0, 3) along the direction (1, -1, 2, 1).

2) Find the minimum of the following functions using Evolutionary optimization search.

a) $f(x,y) = \sin(x) + \cos(y) + (x-2)^2 + (y-3)^2$ starting from the point (3,0) with a reduction parameter $\Delta = (0.5, 1)$ and a termination parameter $\varepsilon = 0.2$.

b) $f(x,y) = e^x + e^y + (x-1)^2 + (y+3)^2$ starting from the point (1, -2) with a reduction parameter $\Delta = (1, 1)$ and a termination parameter $\varepsilon = 0.9$.

3) Find the minimum of the following functions using Hooke Jeeve's search method.

a) $f(x,y) = (x^2 + y^2 - 1)^2 + (x^2 - y)^2$ starting from the initial point (1,2), with an increment vector (0.5, 0.5), a termination parameter $\varepsilon = 0.01$, and a reduction factor $\alpha = 2$.

b) $f(x,y) = (x+3)^2 + (y-4)^2 + \frac{1}{2}(xy-6)^2$ starting from the initial point (2, -2), with an increment vector (0.3, 0.5), a termination parameter $\varepsilon = 0.4$, and a reduction factor $\alpha = 2$.

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23MAT206 – Optimization Techniques

Lab Sheet-8

Implementation of the numerical algorithms for unconstrained multivariable optimization problems

Simplex Search Method

Practice Questions

Find the minimum of the following functions using simplex search method.

1) $f(x, y) = x^2 - 2xy - 19x + 7y^2$ starting from the point (0.5, 1.5), (3, 5.5), (9, 12.5). Perform three iterations taking $\gamma = 2$ and $\beta = 0.5$.
2) $f(x, y) = (x-1)^2 + (y-2)^2$ choosing the initial simplex as (2, 0), (0,1) and (1,1) with termination parameter 0.01.
3) $f(x, y) = \sin(x) + \cos(y) + (x-2)^2 + (y-3)^2$ starting from the point (0.5, 1), (3, 5) and (7, 9) with termination parameter 0.8.
4) Perform three iterations to minimize $f(x, y) = x^4 + y^4 + 2xy - 5x^2 + 3y$ starting from the initial simplex points (1, -1), (2, 0), and (0,2) with reflection parameter 1.8 and contraction parameter 0.6.
5) $f(x, y) = e^{x+y} + \sin(x)$ starting from the initial simplex points (0, 0.5), (0.5, 1), and (1,1) with termination parameter 10^{-4}.

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23MAT206 – Optimization Techniques

Lab Sheet-9

Implementation of the numerical algorithms for unconstrained multivariable optimization problems

- **Steepest Descent Method**

Find a minimum for the function, $f(x, y) = 2x^2 - 2xy + +y^2$ using steepest descent method. Perform three iterations starting from the point (1, 2).

```
clear all;
clc;
syms x y t
f=2*x^2-2*x*y+y^2;
x0=[1 2];
n=3; % Number of iterations
gradf=gradient(f,[x,y]);
for i=1:n
df=subs(gradf,[x,y], x0);
% Unidirectional search from x0 along df
xt=x0-t*df';
f_xt=subs(f,[x,y], xt);
fdt=diff(f_xt, t);
t1=solve(fdt==0, t);
x0=subs(xt,t, t1);
f0=subs(f,[x,y], x0);
fprintf("X value after %d iteration is:\n", i)
disp(x0);
fprintf(" The corresponding function value is: %.3f \n", f0)
end
```

OUTPUT:

```
X value after 1 iteration is:
```

$(1 \quad 1)$

```
 The corresponding function value is: 1.000
X value after 2 iteration is:
```

$(1 \quad 1)$

```
 The corresponding function value is: 0.500
X value after 3 iteration is:
```

$(1 \quad 1)$

```
 The corresponding function value is: 0.250
Iteration 1: X = [0.500000, 1.000000, 1.000000], f(X) = -0.500000
Converged point:
    0.5000
    1.0000
    1.0000
Function value at the minimum:
   -0.5000
```

- <u>**Newton's Method**</u>

Find a minimum for the function, $f(x, y, z) = 2x^2 - 2xy + y^2 - z - yz + z^2$ using Newton's method with starting point as (1, 2, 3) and termination parameter as 0.1.

```
clear all;
clc;
syms x y z
k=0;
f=2*x^2-2*x*y+y^2-z-y*z+z^2;
X=[1;2;3];
% f=100*(y-x^2)^2+(1-x)^2
% X=[-1;1]
% f = (x - 2)^2 + (y - 3)^2; % Define the function
% X = [-1; 4]; % Initial guess (as a column vector)
Df = gradient(f, [x, y, z]); % Compute the gradient
E = 0.1; % Tolerance for convergence
% Newton's Method Loop
while true
 df = double(subs(Df, {x, y, z}, {X(1), X(2), X(3)})); % Evaluate the gradient
at X


 if norm(df) <= E
 break; % Exit the loop if converged
 end
% Evaluate Hessian
 H = double(subs(hessian(f, [x, y, z]), {x, y, z}, {X(1), X(2), X(3)}));
```

```
% Compute the Newton step (Hessian inverse times the gradient)
d = -H \ df; % Use the backslash operator for matrix division  d=-inv(H)*df
% Update X using the Newton step
X = X + d; % Update X disp('Converged point:');
k=k+1;
fprintf('Iteration %d: X = [%f, %f, %f], f(X) = %f\n', k, X(1), X(2), X(3),
double(subs(f, {x, y, z}, {X(1), X(2), X(3)})));
end
disp('Converged point:');
disp(X);
disp('Function value at the minimum:');
disp(double(subs(f, {x, y, z}, {X(1), X(2), X(3)})));
```

OUTPUT:

```
Iteration 1: X = [0.500000, 1.000000, 1.000000], f(X) = -0.500000
Converged point:
    0.5000
    1.0000
    1.0000
Function value at the minimum:
   -0.5000
```

Practice Questions

I. Find the minimum of the following functions using Steepest Descent Method.

1) $f(x,y) = (x-1)^2 + (y-2)^2$ starting from the point (10, -1) with termination parameter 0.1.

2) Perform three iterations to minimize $f(x,y) = y^2 + 4x^2 - 10y + 25 - xy$ starting from the point (0, 5).

3) $f(x,y) = x^2 + y^2 + x + y + 2$ starting from the point (1, 1) with termination parameter 0.01.

4) Perform five iterations to minimize $f(x,y) = (x-y+2)^2 + (y+x-7)^2$ starting from the point (-1, 3).

II. Find the minimum of the following functions using Newton's Method.

1) $f(x,y) = \sin(x) + y^2 + 4x^2 - 10y + 50$ with starting point as (-3,5) and termination parameter 0.001.

2) $f(x,y) = x^4 + y^4 + 2xy - 5x^2 + 3y$ with starting point as (1, 3.5) and termination parameter 0.1.

3) $f(x, y) = x^2 - 2xy - 19x + 7y^2$ with starting point as (1.5, 5) and termination parameter 0.8.

4) $f(x, y) = e^{x+y} + \sin(x)$ with starting point as (-2,3) and termination parameter 0.001.

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23MAT206 – Optimization Techniques

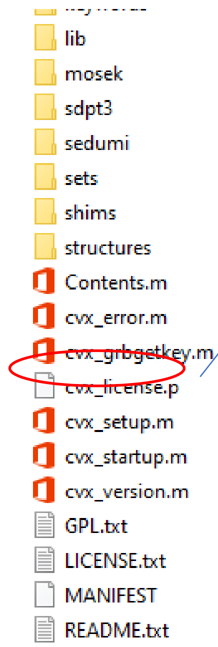
Lab Sheet-10

Implementation of the numerical algorithms for constrained multivariable optimization problems

(CVX)

- CVX – Convex Optimization Programming
  Download CVX from: http://cvxr.com/cvx/download/

OS	mexext	Download links	
Standard bundles, including Gurobi and/or MOSEK			
Linux	mexa64	cvx-a64.zip	cvx-a64.tar.gz
Mac	mexmaci64	cvx-maci64.zip	cvx-maci64.tar.gz
Windows	mexw64	cvx-w64.zip	cvx-w64.tar.gz
Redistributable: free solvers only			
All platforms		cvx-rd.zip	cvx-rd.tar.gz
All platforms (v1.22)		cvx-1.22.zip	cvx-1.22.tar.gz
Commercial solvers only			
Linux	mexa64	cvx-a64-co.zip	cvx-a64-co.tar.gz
Mac	mexmaci64	cvx-maci64-co.zip	cvx-maci64-co.tar.gz
Windows	mexw64	cvx-w64-co.zip	cvx-w64-co.tar.gz



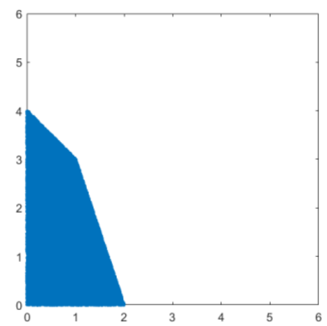
Run this file

Automatically Set the path. Matlab should be 'On'

- Plotting the feasible region of a constrained optimization problem:
  If constraints of the Optimization problem are: $x+y \leq 4$ and $3x+y \leq 6$

```
N=200000; % number (x,y) points in domain
x = rand(N,1)*6;
y = rand(N,1)*6;
f1 = x+y-4;  % An array of N fn values in domain
ind1 = (f1<0); % An N_array of logical 0s and 1s
f2 = 3*x+y-6; % An array of N fn values in domain
ind2 = (f2<0); % An N_array of logical 0s and 1s
ind3=and(ind1,ind2); % An N_array of logical 0s and 1s
a = [x(ind3),y(ind3)];% points which are in feasible region
figure
plot(a(:,1),a(:,2),'.','MarkerSize',10);
axis equal
xlim([0 6])
ylim([0 6])
```

Output:



Example1: Solve using CVX

Maximise $Z = 4x + y$

subject to the constraints:

$$x + y \le 50$$
$$3x + y \le 90$$
$$x \ge 0, y \ge 0$$

```
% Example 1
cvx_begin quiet
variables x y
maximize 4*x+y
subject to
x+y<=50
3*x+y<=90
x>=0
y>=0
cvx_end
sprintf('x=%0.2f  y=%0.2f maxvalue=%0.2f',x,y,4*x+y)
```

Output:

ans = 'x=30.00 y=0.00 maxvalue=120.00'

```
% Example 1 after introducing the slack variables and writing as matrices and vectors
A=[1 1 1 0; 3 1 0 1]; b=[50;90];
c=[4 1 0 0];
cvx_begin quiet
variables x(4)
maximize c*x
subject to
A*x==b;
x>=0
cvx_end
% display
x
Z=c*x
```

Maximize $Z = c^T x$

subject to

$Ax = b; \quad x \ge 0;$

where

$$A = \begin{pmatrix} 1 & 1 & 1 & 0 \\ 3 & 1 & 0 & 1 \end{pmatrix}; b = \begin{pmatrix} 50 \\ 90 \end{pmatrix}; x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}; c = \begin{pmatrix} 4 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

Output:

x = 30.00
0.00
20.00
0.00

Z = 120.00

Example 2: Solve using CVX

Minimise $Z = 200x + 500y$

subject to the constraints:

$$x + 2y \geq 10$$
$$3x + 4y \leq 24$$
$$x \geq 0, y \geq 0$$

```
%Example2:
cvx_begin quiet
variables x y
minimize 200*x+500*y
subject to
x+2*y>=10
3*x+4*y<=24
x>=0
y>=0
cvx_end
sprintf('x=%0.2f  y=%0.2f minvalue=%0.2f',x,y,200*x+500*y)
```

Output:
ans = 'x=4.000000 y=3.000000 minvalue=2300.000024'

```
% Example 1 using matrix and vector representations
A=[1 2 -1 0; 3 4 0 1]; b=[10;24];
c=[200 500 0 0];
cvx_begin quiet
variables x(4)
minimize c*x
subject to
A*x==b;
x>=0
cvx_end
% display
x
Z=c*x
```

Output:

x=4.00

3.00

0.00

0.00

Z=2300

Example 3: Solve using CVX

Maximize z=3x+9y

subject to the constraints: $x + 3y \leq 60$

$$x + y \geq 10$$
$$x \leq y$$

```
cvx_begin quiet
variables x y
maximize 3*x+9*y
subject to
x+3*y<=60
x+y>=10
x<=y
x>=0
y>=0
cvx_end
sprintf('x=%0.2f  y=%0.2f maxvalue=%0.2f',x,y,3*x+9*y)
```

```
'x=8.23  y=17.26 maxvalue=180.00'
```

- The solution for maximization problem is actually, ‘infinite solutions-all pts between (15,15) and (0,20) with maximum value 180. But CVX will give you one point on this line segment.
- If we change the problem to minimize we will get the answer as (5,5) with minimum value as 60.

Practice Questions:

1. Solve the following Convex optimization problems using CVX.

(a) Minimize $6x - 9y$

$$\textit{subject to } x - y \geq 2$$
$$3x + y \geq 1$$
$$2x - 3y \geq 3$$

(b) Minimize $x^2 + 2y^2$

$$\textit{subject to } x + y \geq 1; \; x, y \geq 0$$

(c) Maximize, $3 - (x-1)^2 - (y-1)^2$

$$\textit{subject to } 2x + x^2 + y^2 \leq 16$$
$$3x - 7y = 21$$

(d) Minimize $x^2 + y^2$

$$\textit{subject to } x + y \leq 4$$
$$2x + x^2 + y^2 \leq 15$$

2. Consider the problem: Minimize x+y

$$\text{subject to } (x-1)^2 + (y-1)^2 \leq 1$$
$$x \leq 1, y \leq 1$$

(a) Draw the feasible region for the problem and solve graphically.
(b) Solve the problem using CVX

3. Solve all the convex problems that was discussed in the class using CVX tool.

4. Draw the feasible region for the given problems. Also solve them using CVX.

(a) Maximize $-6x + 9y$

subject to $x - y \leq 2$

$3x + y \leq 1$

$2x - 3y \leq 3$

(b) Minimize x+y

subject to $x + y \leq 4$

$2x + x^2 + y^2 \leq 15$