EECS 740 DIGITAL IMAGE PROCESSING

Assignment #3

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11/29/2015

Abstract:

In this assignment we are going to discusses basic principles for filtering in the frequency domain and image restoration. The usual convolution operation in spatial domain transforms to array multiplication in frequency domain, so any transformation can be performed in frequency domain by just multiplying with suitable filter matrix. Techniques like Gaussian low pass filter, Butterworth band filter, Median filter, Adaptive median filter, Full inverse filtering and Wiener filtering are discussed and implemented using two ideal images. We will add noise to the images, apply the filtering techniques and observe the results. We will check this results with theoretical results and draw conclusions.

Technical discussion:

Filtering in Frequency Domain:

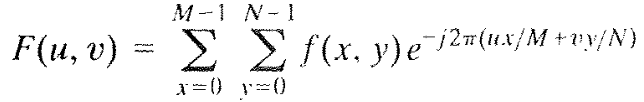
Filtering in frequency domain is much easier because of convolution.

*f(x,y)\*h(x,y)* ⬄ *F(u,v)H(u,v)*

*f(x,y)h(x,y)* ⬄ *F(u,v)\*H(u,v)*

If the two functions *f(x),* *h(x)* are of different lengths, then when we do the convolution there could be wraparound error. To avoid this, we append them with zeros.

The Discrete Fourier Transform (DFT) of an image *f(x,y)* is given by the expression



Where M, N are the dimensions of the image.

Edges and other sharp transitions (such as noise) in the gray levels of an image contribute significantly to the high-frequency content of its Fourier transform. Hence blurring (smoothing) is achieved in the frequency domain by attenuating a specified range of high frequency components in the transform of a given image.

The general procedure for performing filtering in frequency domain can be listed as follows,

1. Given an input image *f(x,y)* of size M*x*N, obtain the padding parameter P and Q. (typically we select P=2\*M and Q=2\*N).
2. Form a padded image, *fp(x,y)* of size P*x*Q by appending the necessary # of zeros to *f(x,y).*
3. Multiply *fp(x,y)* by *(-1)x+y* to center its transform.
4. Compute the DFT *F(u,v)* of the image obtained above.
5. Generate a real symmetric filter function *H(u,v)* of size P*x*Q with center at coordinates (P/2,Q/2). Form the product *G(u,v)* = *H(u,v)F(u,v)* using array multiplication; that is *Gp(i,k)*=*H(i,k)F(i,k).*
6. Obtain the processed image



Where the real part is selected in order to ignore parasitic complex components resulting from computational inaccuracies, *p* indicates we are working with the padded image.

1. Obtain the processed result, *g(x,y)* by extracting the MxN region from the top, left quadrant of *gp*.

Gaussian Low pass Filter:

* Gaussian low pass filter is used to smooth the image.
* Gaussian low pass filter in 2 dimensions is given by



We can replace sigma by D0

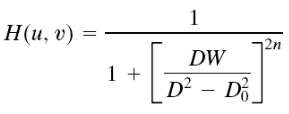


* D0 is a positive constant and D(u,v) is the distance between a point (u,v) in the frequency domain and the center of the frequency rectangle.



Butterworth Band Filter:

* Band filters work on specific band instead of the entire spectrum.
* The Butterworth filter is the best compromise between attenuation and phase response. It has no ripple in the pass band or the stop band.
* The transformation function for Butterworth band pass filter is



Where W is the width of the band, *D* is the distance *D(u,v)* from the center of the filter.

Median Filter:

Adaptive Median Filter:

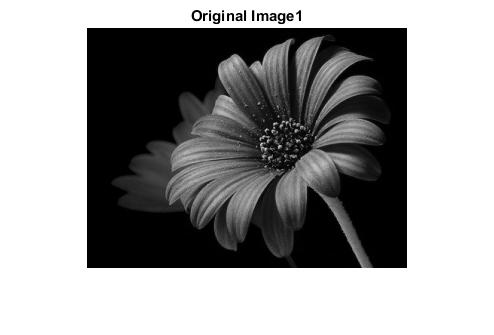
\*atmospheric turbulence

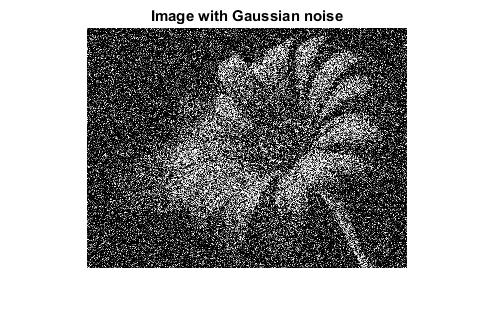
Full Inverse Filter:

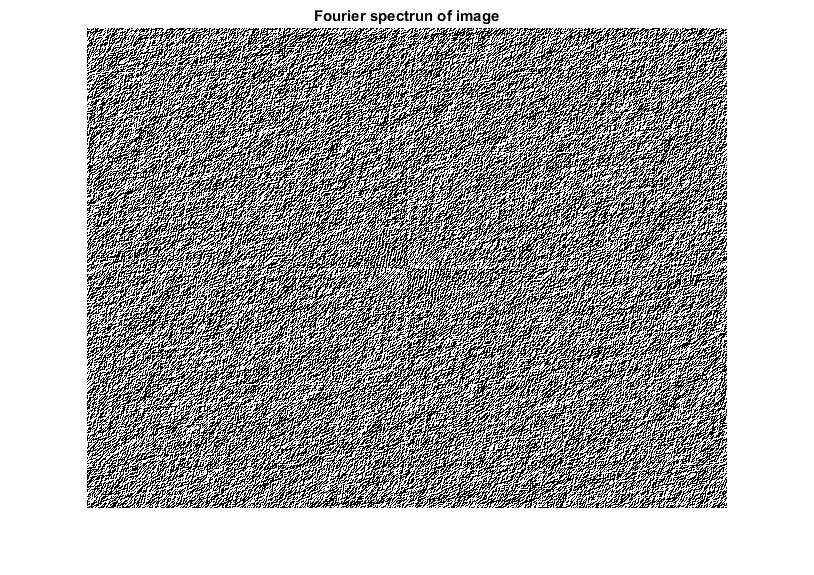
Weiner Filter:

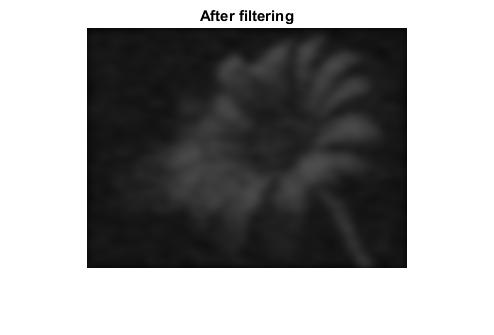
**Results:**

**Problem 1,**

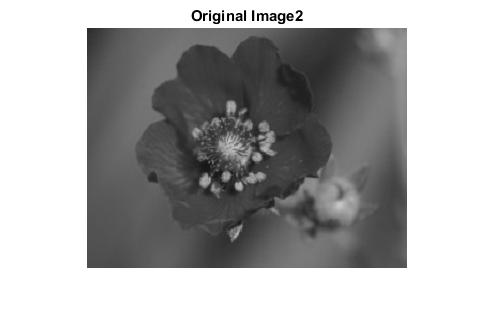
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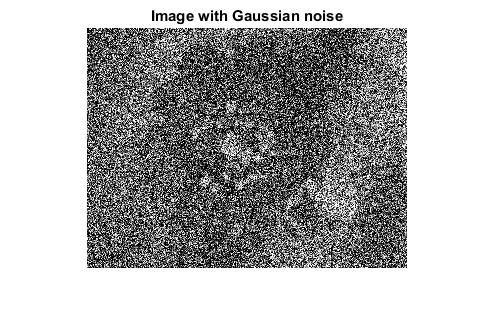
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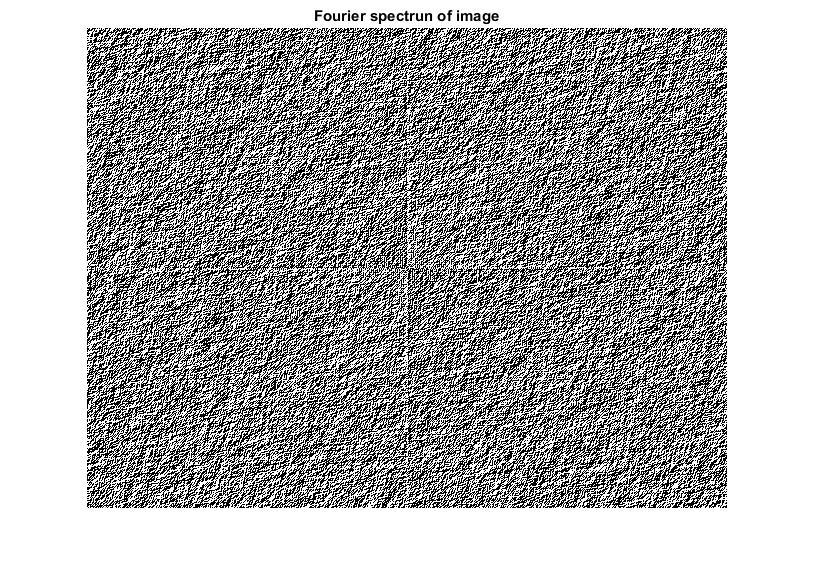
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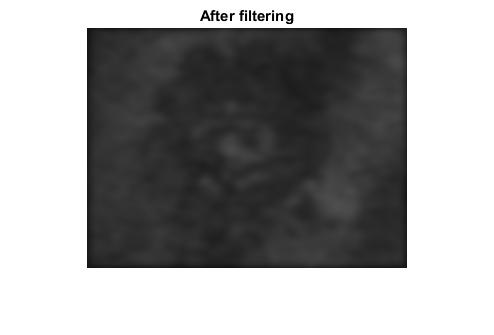
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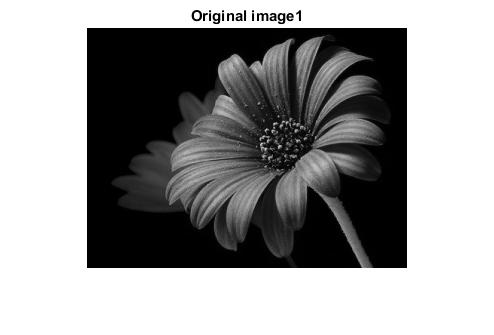
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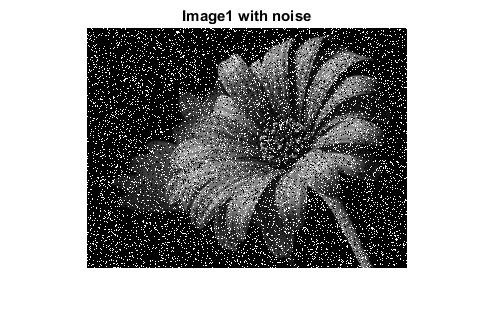
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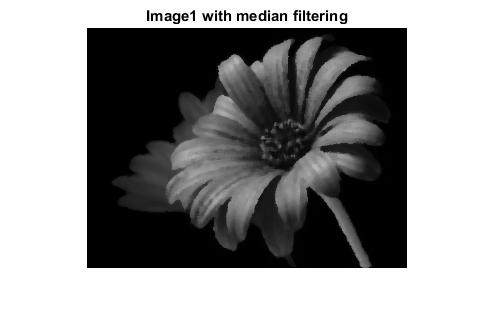
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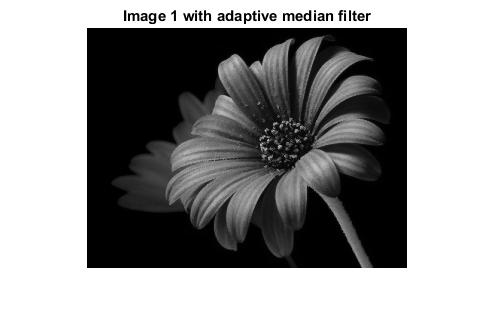
**Problem 2:**

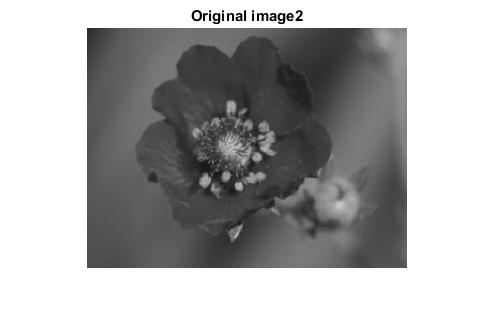
**Problem 3:**

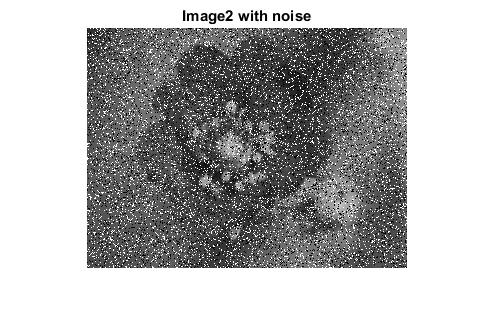
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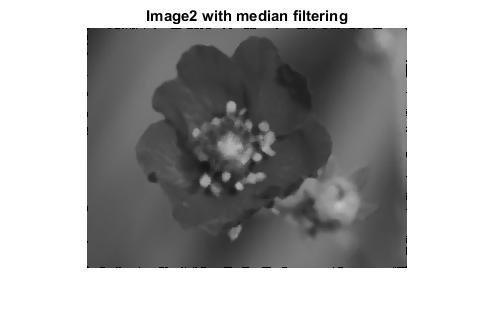
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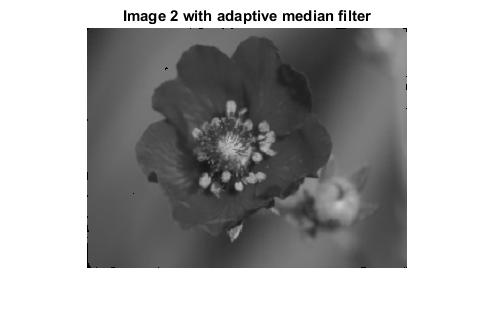
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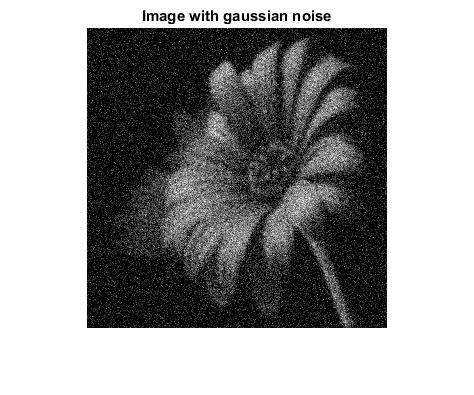
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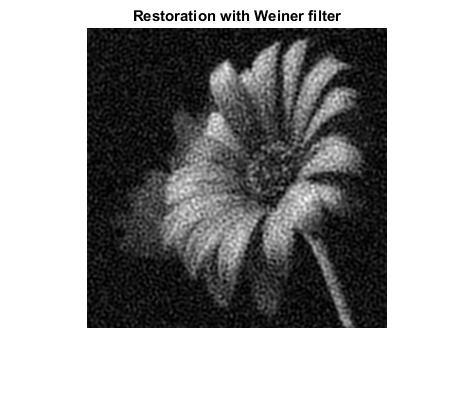
**Problem 4:**

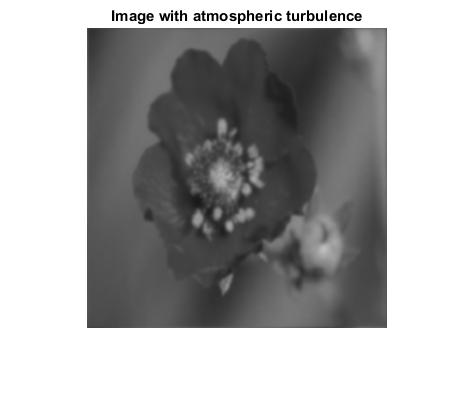
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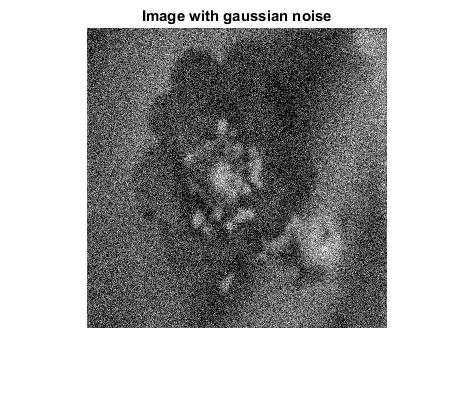
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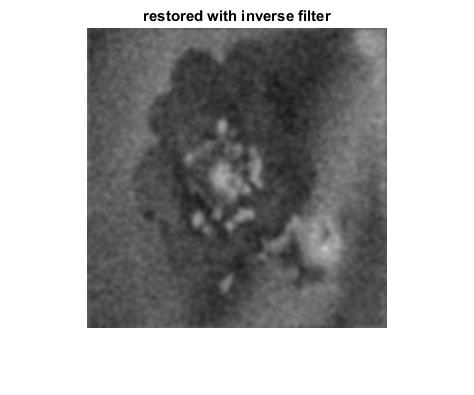
****

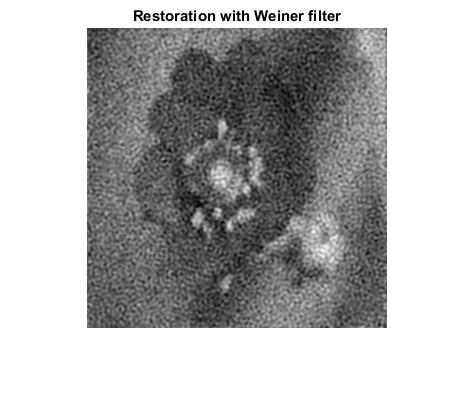
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**Analysis:**

**Appendix:**

**Problem 1:**

Problem.m

% Program for Problem 1

clear;

close all;

I = imread('flower1.jpg');

I = im2double(I);

I = rgb2gray(I);

figure, imshow(I); title('Original Image1');

Process(I);

J = imread('flower2.jpg');

J = im2double(J);

J = rgb2gray(J);

figure, imshow(J); title('Original Image2');

Process(J);

% end

Process.m

function [ Iout ] = Process( In )

% This does all the processing for the input image

Inoise = imnoise(In,'gaussian',0,0.2);

figure, imshow(Inoise); title('Image with Gaussian noise');

% Filtering in frequency domain begins here Lecture #22

[M, N] = size(Inoise);

% padding parameters

P = 2\*M;

Q = 2\*N;

% padded image

Fp = zeros(P,Q);

Fp(1:M,1:N) = Inoise;

% shifting image to center

for u = 1:P

for v = 1:Q

b = power(-1, (u+v));

Fp(u,v) = b\*Fp(u,v);

end

end

Itr = real(fft2(Fp));

figure, imshow(real(Itr)); title('Fourier spectrun of image');

% filtering using gaussian lowpass filter

I2 = gauss\_lowpass(Itr);

I3 = real(ifft2(I2));

for u = 1:P

for v = 1:Q

I3(u,v) = power(-1,(u+v))\*I3(u,v);

end

end

% applying the inverse fourier transform

Iout = I3(1:M,1:N);

figure, imshow(Iout); title('After filtering');

Ifinal = Inoise - Iout;

Imask = Ifinal - In;

Ifinal2 = Inoise - Imask;

figure, imshow(Ifinal2); title('Restored image with post processing');

end

gauss\_lowpass.m

function [out] = gauss\_lowpass(In)

% function to implement the gaussian low pass filter.

[P , Q] = size(In);

% D0 = sum(sum(In))/(P\*Q);

D0 = 20;

% out = In;

H = zeros(P,Q);

a = P/2;

b = Q/2;

K = power(D0,2);

K = 2\*K;

D = 0;

for i = 1:P

for j = 1:Q

D = power((i-a),2) + power((j-b),2);

D = sqrt(D);

k = power(D,2);

H(i,j) = exp((-1\*k)/ K);

k = 0;

end

end

out = H.\*In;

end

**Problem 2:**

**Problem 3:**

% this program is for impulse noise(salt & pepper) addition and removal

% using median and adaptive median filters

clear;

close all;

I = imread('flower1.jpg');

I = rgb2gray(I);

J = imread('flower2.jpg');

J = rgb2gray(J);

figure, imshow(I); title('Original image1');

figure, imshow(J); title('Original image2');

% adding noise

In = imnoise(I,'salt & pepper',0.2);

figure, imshow(In); title('Image1 with noise');

Jn = imnoise(J,'salt & pepper',0.2);

figure, imshow(Jn); title('Image2 with noise');

% median filter

Iout = medfilt2(In, [5 5]);

figure, imshow(Iout); title('Image1 with median filtering');

Jout = medfilt2(Jn, [5 5]);

figure, imshow(Jout); title('Image2 with median filtering');

% adaptive median filtering

Iad = ad\_medianfilter(I,7);

figure, imshow(Iad); title('Image 1 with adaptive median filter');

Jad = ad\_medianfilter(J,15);

figure, imshow(Jad); title('Image 2 with adaptive median filter');

% end

**Problem 4:**

Problem4.m

% This program reads the input files and calls the process function

clear;

close all;

I = imread('flower1.jpg');

Process4(I);

J = imread('flower2.jpg');

Process4(J);

% end

Process.m

function [ ] = Process4( In )

% Process4 does all the processing

In = rgb2gray(In);

In = im2double(In);

[M,N]=size(In);

figure, imshow(In); title('Original image');

Ft = fftshift(fft2(In));

% Atmospheric turbulence with k = 0.002

k = 0.002;

u0 = M/2;

v0 = N/2;

u = (1:M)-u0;

v = (1:N)-v0;

[U,V] = meshgrid(u,v);

D = (U.^2+V.^2);

H = exp(-k\*(D.^(5/6)));

Ftr = Ft .\* H;

Fblurred = real(ifft2(ifftshift(Ftr)));

figure, imshow(Fblurred); title('Image with atmospheric turbulence');

% lets add gaussian noise with mean = 0 and variance 0.02

Gn = imnoise(Fblurred,'gaussian',0,0.02);

figure, imshow(Gn); title('Image with gaussian noise');

% designing filter with width 70

Fblurrnoisy = fftshift(fft2(Gn));

Fblurr = gauss\_lowpass(Fblurrnoisy);

figure, imshow(real(ifft2(ifftshift(Fblurr)))); title('Gaussian noise removed');

radius = 70;

a = floor(M/2)+1;

b = floor(N/2)+1;

for i = 1:M

for j = 1:N

dist = ((i-a)^2 + (j-b)^2)^0.5;

if dist < radius

Fblurr(i,j) = Fblurr(i,j)/H(i,j);

else

Fblurr(i,j) = 0;

end

end

end

% restoring

Frreal = abs(ifft2(ifftshift(Fblurr)));

figure, imshow(Frreal); title('restored with inverse filter');

% weiner filter

% W(i,j) = (1/H(i,j))\*((abs(H(i,j))^2)/(abs(H(i,j))^2+1000));

L = conj(H)./(H .\* (conj(H) + 1000));

% H1 = abs(H).^2;

% H2 = H1 + 1000;

% H3 = H1./H2;

% L = conj(H) \* H3;

Fweiner = L .\* Fblurr;

figure, imshow(abs(ifft2(ifftshift(Fweiner)))); title('Restored with Weiner filter');

end

gauss\_lowpass.m

function [out] = gauss\_lowpass(In)

% function to implement the gaussian low pass filter.

[P , Q] = size(In);

% D0 = sum(sum(In))/(P\*Q);

D0 = 20;

% out = In;

H = zeros(P,Q);

a = P/2;

b = Q/2;

K = power(D0,2);

K = 2\*K;

D = 0;

for i = 1:P

for j = 1:Q

D = power((i-a),2) + power((j-b),2);

D = sqrt(D);

k = power(D,2);

H(i,j) = exp((-1\*k)/ K);

k = 0;

end

end

out = H.\*In;

end