

# Forecasting Transportation Services Index (TSI) Values: A Time Series Analysis

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04-29-2024

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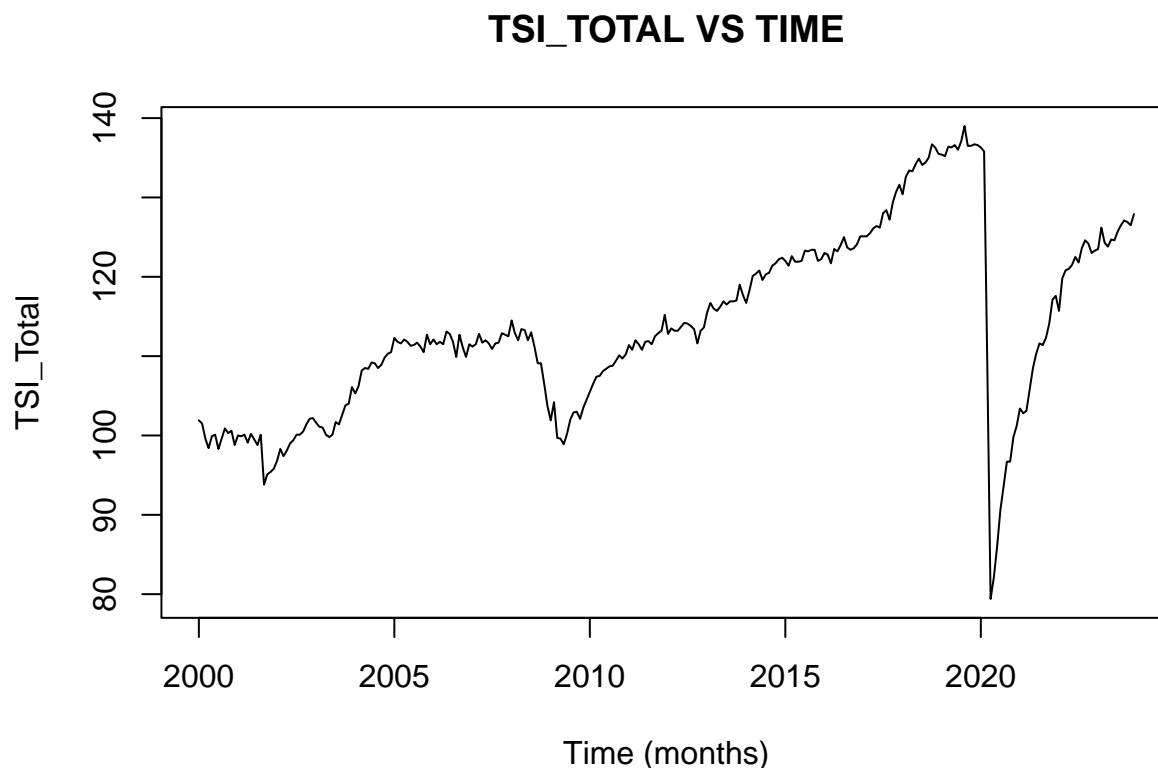
## 1 Introduction

The aim of this project is to develop a forecasting model for the Transportation Services Index (TSI) based on historical data. The TSI, produced by the Bureau of Transportation Statistics (BTS), serves as a measure of the volume of freight and passenger transportation services moved monthly by the for-hire transportation sector in the United States. This index is composed of three components: a freight index, a passenger index, and a combined index, each incorporating data from various for-hire transportation modes.

## 2 Data Description

Data Source : <https://data.bts.gov/Research-and-Statistics/Freight-Transportation-Service-Index/n68x-u7m7>

Following is the data set for my analysis where we have around 288 data monthly data points with no nulls in the data set from the year 01/2000 till 12/2023.



From the above plot we can observe there is an upward trend in the data over the period of time along with some external shocks also there is a sharp drop in the data set in the year 2020 due to the covid pandemic and later the trend normalise after the covid effect.

### 3 Objective

The primary focus of this project is to forecast the TSI values exclusively, while disregarding the other components of the index. Changes in the TSI reflect fluctuations excluding the external shocks in the demand for transportation services, which are indicative of broader economic trends. For instance, during periods of economic expansion, there is typically an increase in the demand for goods and services, leading to a corresponding rise in the TSI.

To achieve this objective, we will utilize time series analysis techniques to model the historical TSI data and generate forecasts for future TSI values. Time series analysis allows us to identify patterns, trends, and seasonal variations in the data, which are crucial for developing accurate forecasting models. We will explore various time series forecasting methods, such as autoregressive integrated moving average (ARIMA), exponential smoothing methods.

The forecasted TSI values will provide valuable insights for stakeholders in the transportation industry, policymakers, and economic analysts. By anticipating changes in the demand for transportation services, informed decisions can be made regarding infrastructure investments, capacity planning, and economic policy formulation. Additionally, understanding future trends in transportation demand can help businesses optimize their supply chain management strategies and logistics operations.

Overall, this project aims to contribute to a better understanding of the dynamics of transportation services

demand and provide actionable forecasts to support decision-making processes in both the public and private sectors.

## 4 Methodology

In this analysis, a comprehensive suite of time series models was employed to capture the effects of both short-term shocks, such as those induced by the COVID-19 pandemic, and long-term trends. The models utilized encompassed a range of complexities, from simple linear, quadratic, and cubic regressions to more sophisticated ARIMA, ARMAX, and GARCH models.

For the ARIMA models, a systematic approach was undertaken to identify the optimal model specifications. This involved fitting multiple models with varying orders and evaluating their predictive power and goodness-of-fit metrics, such as AIC and BIC. Additionally, diagnostic checks were performed on the residuals to ensure the adequacy of the chosen model in capturing the underlying patterns of the data.

The GARCH models were implemented to assess the presence of volatility clustering and to select the most appropriate specification for the data. Various model configurations were tested, and the model exhibiting minimal autocorrelation in the squared residuals, indicating the absence of ARCH effects, was selected.

To account for the exogenous impact of the COVID-19 pandemic, a binary “covid\_indicator” variable was manually constructed. This variable served as a proxy to isolate and quantify the specific effects of COVID-19 on the time series data.

A rigorous validation process was conducted for each model, encompassing both in-sample and out-of-sample evaluations. This included assessing forecasting performance metrics, such as RMSE and MAE, to gauge the accuracy of the models in predicting future observations.

Furthermore, the analysis delved into the interpretation of the model coefficients and their significance, shedding light on the underlying relationships between the variables and the dynamics of the time series data.

By employing this multifaceted approach, the analysis aimed to provide a robust understanding of the impact of both short-term shocks and long-term trends on the data, facilitating informed decision-making and forecasting in a dynamic environment.

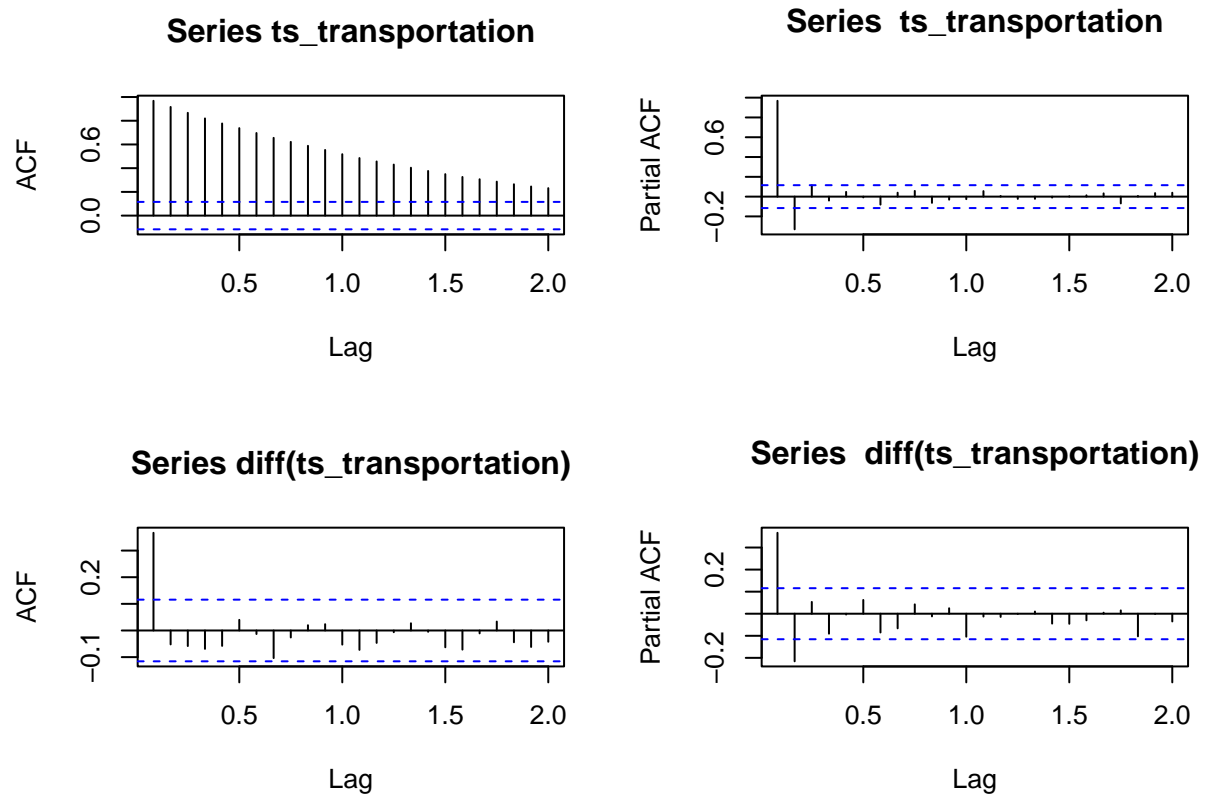
## 5 Exploratory Data Analysis

From the initial examination of the time series plot, it is evident that the data exhibits a non-stationary behavior characterized by a gradual upward trend, interrupted by a decline during the COVID-19 period. Additionally, both the autocorrelation function (ACF) and partial autocorrelation function (PACF) plots display non-stationary characteristics, with noticeable significant lags persisting throughout the observation period.

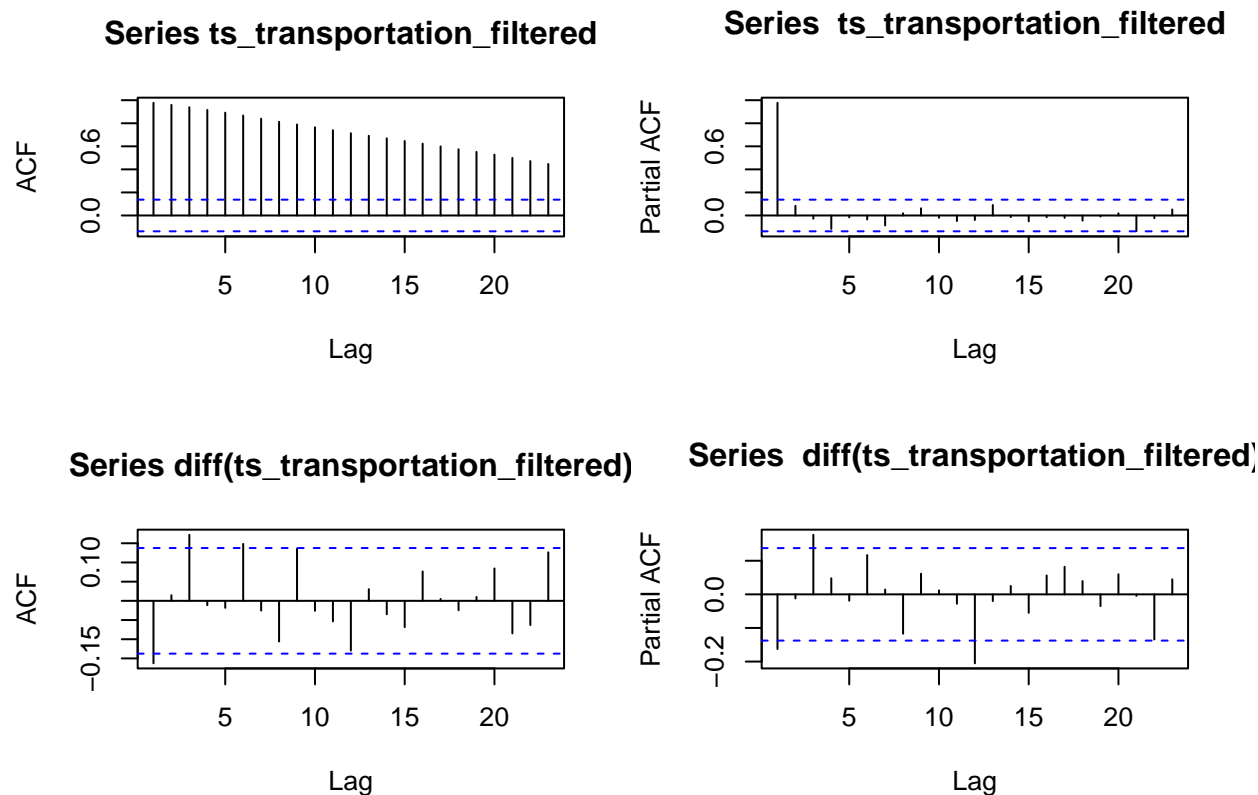
Upon differencing the data, a transformation commonly employed to induce stationarity, the non-stationary trend is effectively mitigated, rendering the series stationary except for the COVID-19 period. Furthermore, the ACF and PACF plots of the differenced series exhibit desirable stationary properties, indicating a lack of significant autocorrelation apart from at lag 1, as observed particularly in the PACF plot.

This statistical analysis underscores the efficacy of differencing in achieving stationarity and elucidates the persistence of autocorrelation dynamics, notably at lag 1, which warrants consideration in subsequent modeling endeavors.

Applied log on the data however it did not binded the sharp dip in the data closer to 0.

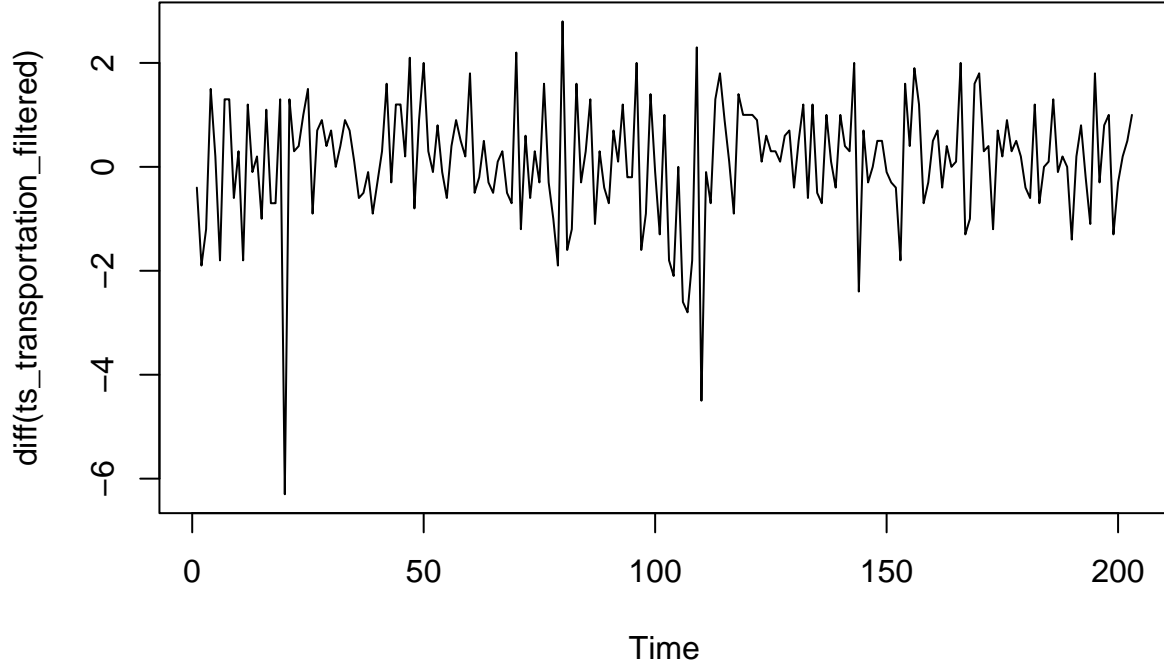


Following are the acf and pacf plots of the filtered data till the year 2020.



The provided plots showcase the time series data alongside the corresponding autocorrelation function (ACF) and partial autocorrelation function (PACF) plots of the filtered series. Notably, the ACF plot indicates stationarity, with significance observed at the first lag. Similarly, the PACF plot displays significance at the first lag. These observations collectively suggest that the data exhibits stationary behavior, with autocorrelation dynamics predominantly manifesting at the initial lag.

Following difference time series plot is stationary which was also the application of the Augmented Dickey-Fuller (ADF) test reveals that the non-differenced data fails to meet the criteria for stationarity, evidenced by the non-significant result and a p-value exceeding 0.05. Conversely, the differenced data demonstrates stationarity, with a significant result leading to the rejection of the null hypothesis of non-stationarity.



## 6 Modeling Procedure and Analysis

Split the data into train and test where the train contains the points from 1 to 180 where as the test set contains the data points from 180 to 203 of the filtered data set.

### 6.0.1 Linear Model

The linear regression model indicates a strong positive relationship between time and train transportation, with each unit increase in time associated with a 0.102 unit increase in train transportation, holding other variables constant. The model explains approximately 63.34% of the variability in train transportation, with both the intercept and time coefficient being statistically significant predictors ( $p < 0.001$ ). However, the residual plots does not looks good as there is a huge spikes in the residual plot from center.

### 6.0.2 Quadratic Model

The multiple linear regression model shows that for each unit increase in time\_train\_transportation, train transportation increases by approximately 0.118 units, holding other variables constant. However, the coefficient for time\_sqft\_train\_transportation is not statistically significant ( $p = 0.488$ ), suggesting it does not significantly impact train transportation. The overall model, including both predictors, explains approximately 63.03% of the variability in train transportation, with a statistically significant F-statistic ( $< 2.2e-16$ ). However, the residual plots does not looks good as there is a huge spikes in the residual plot from the center and is not normal.

### 6.0.3 Cubic Model

The multiple linear regression model with `time_train_transportation`, `time_sqft_train_transportation`, and `time_cubic_train_transportation` as predictors indicates that, for each unit increase in `time_train_transportation`, train transportation increases by approximately 0.466 units, holding other variables constant. Additionally, both `time_sqft_train_transportation` and `time_cubic_train_transportation` have statistically significant negative coefficients, suggesting that increases in these variables are associated with decreases in train transportation. The overall model explains approximately 71.54% of the variability in train transportation, with a significant F-statistic ( $p < 2.2e-16$ ). However, the residual plots does not looks good as there is a huge spikes in the residual plot from the center and is not normal.

### 6.0.4 Poly Quadratic Model

The multiple linear regression model with `time_train_transportation`, `time_sqft_train_transportation`, `time_cubic_train_transportation`, and `time_four_train_transportation` as predictors indicates that only `time_sqft_train_transportation` and `time_cubic_train_transportation` have statistically significant coefficients. Specifically, for each unit increase in `time_sqft_train_transportation` and `time_four_train_transportation`, train transportation increases by approximately 0.01708 and decreases by approximately 0.0005851, respectively. However, the residual plots does not looks good as there is a huge spikes in the residual plot from the center and is not normal.

### 6.0.5 Model Summaries

##	Model	Multiple_R_squared	Adjusted_R_squared	F_statistic	Residual_DF
## 1	Linear	0.6334	0.6313	307.5	178
## 2	Quadratic	0.6344	0.6303	153.6	177
## 3	Cubic	0.7202	0.7154	151.0	176
## 4	4th Degree	0.7773	0.7722	152.7	175
##	Residual_standard_error				
## 1		4.052			
## 2		4.057			
## 3		3.560			
## 4		3.185			

This table summarizes the performance of different regression models based on their multiple R-squared, adjusted R-squared, F-statistic, residual degrees of freedom, and residual standard error.

- The linear model has a multiple R-squared of 0.6334 and an adjusted R-squared of 0.6313, indicating that it explains about 63.13% of the variability in the response variable with 307.5 F-statistic.
- The quadratic model shows similar performance with a slightly lower adjusted R-squared of 0.6303 and a residual standard error of 4.057.
- The cubic model performs better with a higher adjusted R-squared of 0.7154 and a lower residual standard error of 3.560.
- The 4th Degree model outperforms the others with the highest adjusted R-squared of 0.7722 and the lowest residual standard error of 3.185.

##	Model	AIC	BIC	ME	MPE	MSE	MAE
## 1	Linear	1018.4830	1019.9930	NA	NA	NA	NA
## 2	Quadratic	1028.0620	1032.7650	NA	NA	NA	NA
## 3	Cubic	973.8577	989.8224	NA	NA	NA	NA
## 4	Linear	NA	NA	-1.540922	-1.258430	31.23893	2.455101
## 5	Quadratic	NA	NA	19.081480	15.530510	1304.15300	19.081480
## 6	Cubic	NA	NA	10.727620	8.733014	835.27390	12.641230
##	MAPE						
## 1	NA						
## 2	NA						
## 3	NA						

```
## 4 2.000836
## 5 15.530510
## 6 10.294510
```

Based on the above table, the cubic model appears to be the best choice among the three models (linear, quadratic, and cubic). It has the lowest AIC and BIC values, indicating better fit compared to the other models. Additionally, it has lower error metrics such as mean absolute error (MAE) and mean squared error (MSE), suggesting better predictive performance.

Incorporating trigonometric functions such as cosine (cos) and sine (sin) into the model to assess seasonality is a common approach in time series analysis. However, in this analysis, despite implementing models with trigonometric functions, no significant relationship was observed between these functions and the time series index (TSI) data.

Statistically, the absence of a significant relationship between the trigonometric functions and the TSI data may be attributed to several factors:

1. **Data Characteristics:** The TSI data may not exhibit clear seasonal patterns that can be effectively captured by trigonometric functions. Seasonality in time series data often manifests as periodic fluctuations at fixed intervals, and if such patterns are not prominent or consistent in the TSI data, the models incorporating trigonometric functions may fail to capture meaningful relationships.
2. **Model Specification:** The chosen model specifications, including the selection of trigonometric functions and their parameters, may not adequately capture the underlying seasonality present in the TSI data. Different time series may require different functional forms or transformations to properly model seasonality, and the selected approach may not align well with the characteristics of the TSI data.
3. **Statistical Significance:** The absence of a statistically significant relationship does not necessarily imply the absence of seasonality in the data. It is possible that the sample size or the variability of the TSI data is insufficient to detect subtle seasonal patterns, leading to non-significant results in the analysis.
4. **Complexity vs. Parsimony:** Introducing additional complexity to the model, such as trigonometric functions, should be justified by improvements in model fit or explanatory power. If the inclusion of these functions does not result in meaningful enhancements in model performance or interpretability, simpler models may be preferred to avoid overfitting and improve model generalization.

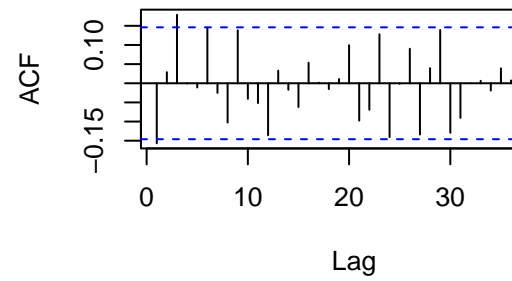
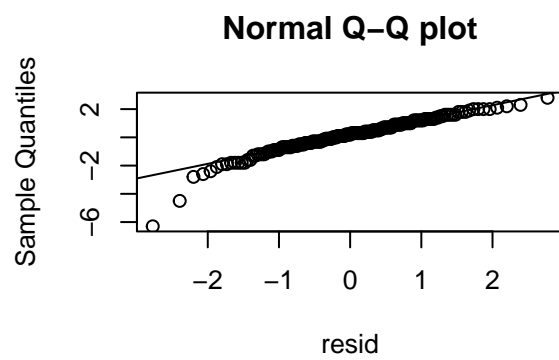
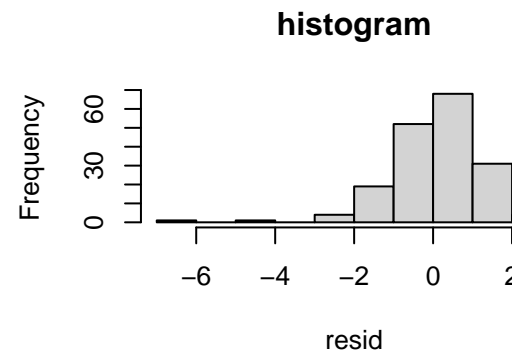
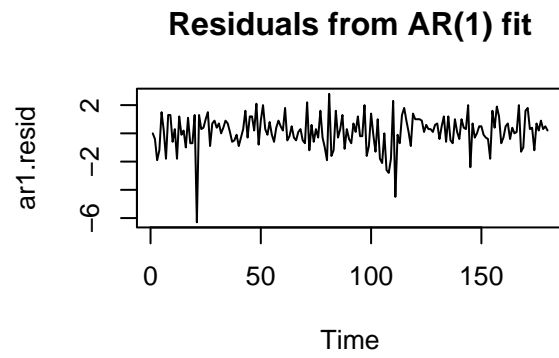
In conclusion, the lack of a discernible relationship between trigonometric functions and the TSI data suggests that seasonality may not be a prominent feature of the dataset or that the chosen modeling approach may not be appropriate for capturing seasonal patterns effectively. Further investigation and refinement of the modeling strategy may be warranted to better understand the underlying dynamics of the TSI data.

### 6.0.6 ARIMA

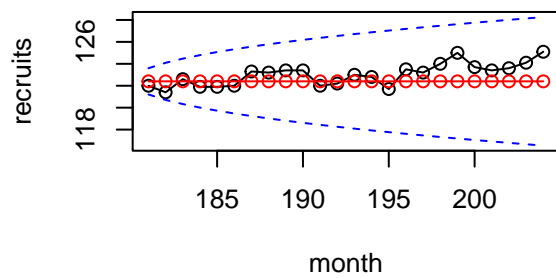
The Auto ARIMA function identified candidate model orders, including (3, 1, 2), (0, 1, 0), and (0, 1, 1). These orders were determined based on the examination of the autocorrelation function (ACF) and partial autocorrelation function (PACF) plots. Additionally, iterative model selection was performed, considering the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) values to refine the choice of the optimal model.







#### 6.0.6.1 Order-(0,1,0)

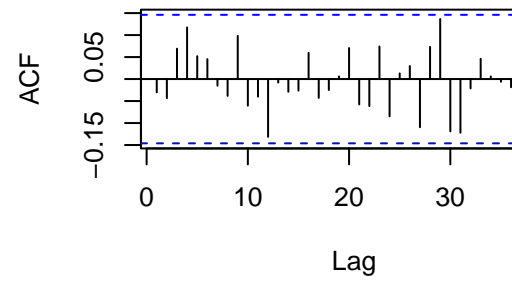
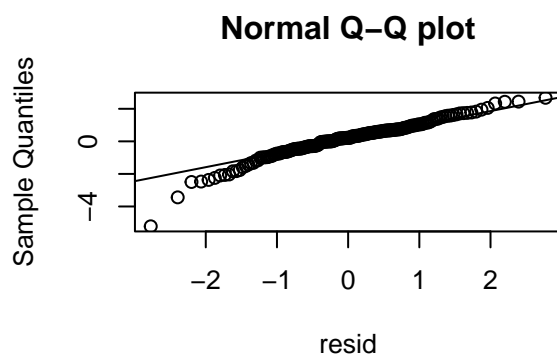
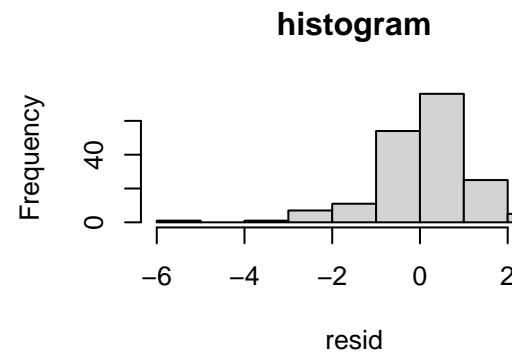
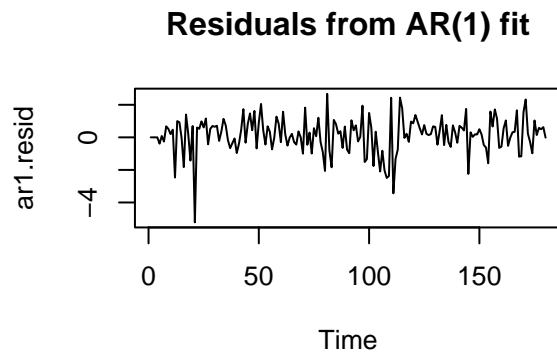


The obtained p-value of  $2.256 \times 10^{-7}$  suggests a significant deviation from normality in the residuals, indicating non-normality.

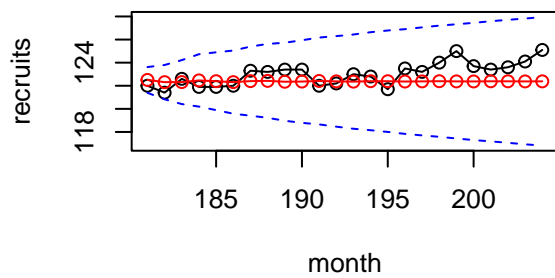
The Box-Pierce and Box-Ljung tests were conducted to assess the autocorrelation of the residuals. Both tests resulted in p-values below conventional significance levels (0.05), indicating significant autocorrelation in the residuals. This implies that the residuals lack independence, violating one of the assumptions of the ARIMA model.

Upon examining the predictions for the two-year data, it is observed that the data points are near some points but not entirely aligned.





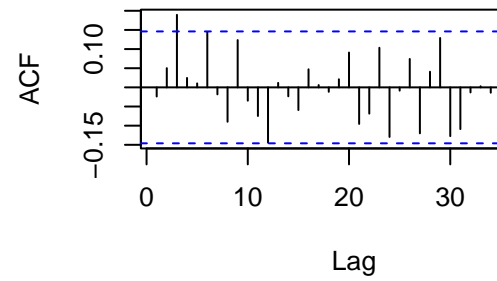
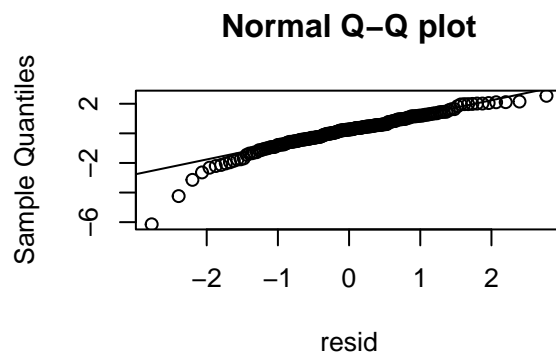
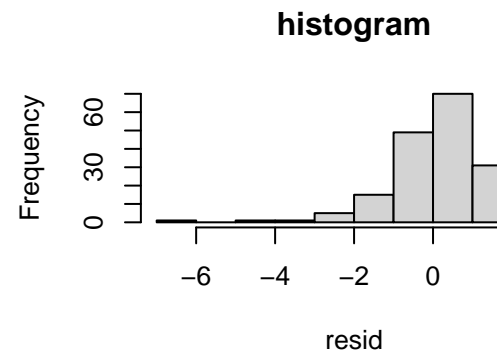
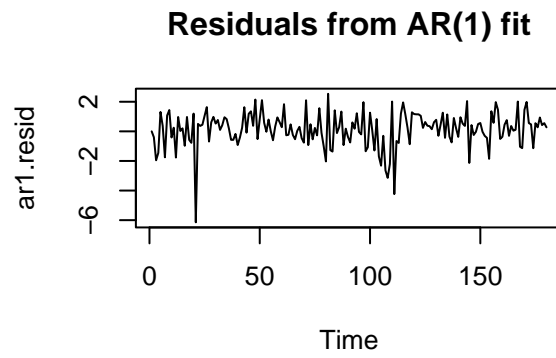
#### 6.0.6.2 Order-(3,1,2)



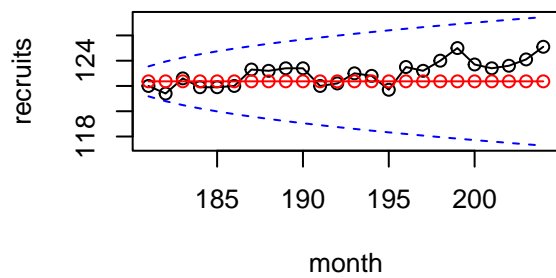
The Shapiro-Wilk test indicates that the residuals deviate from a normal distribution.

Examining the predictions for the next two years, we notice that while the data points approximate certain observed values, they are not entirely congruent. However, this alignment is slightly improved compared to the previous two models.





#### 6.0.6.3 Order-(0, 1, 1)





The Shapiro test reveals that the residuals do not adhere to a normal distribution.

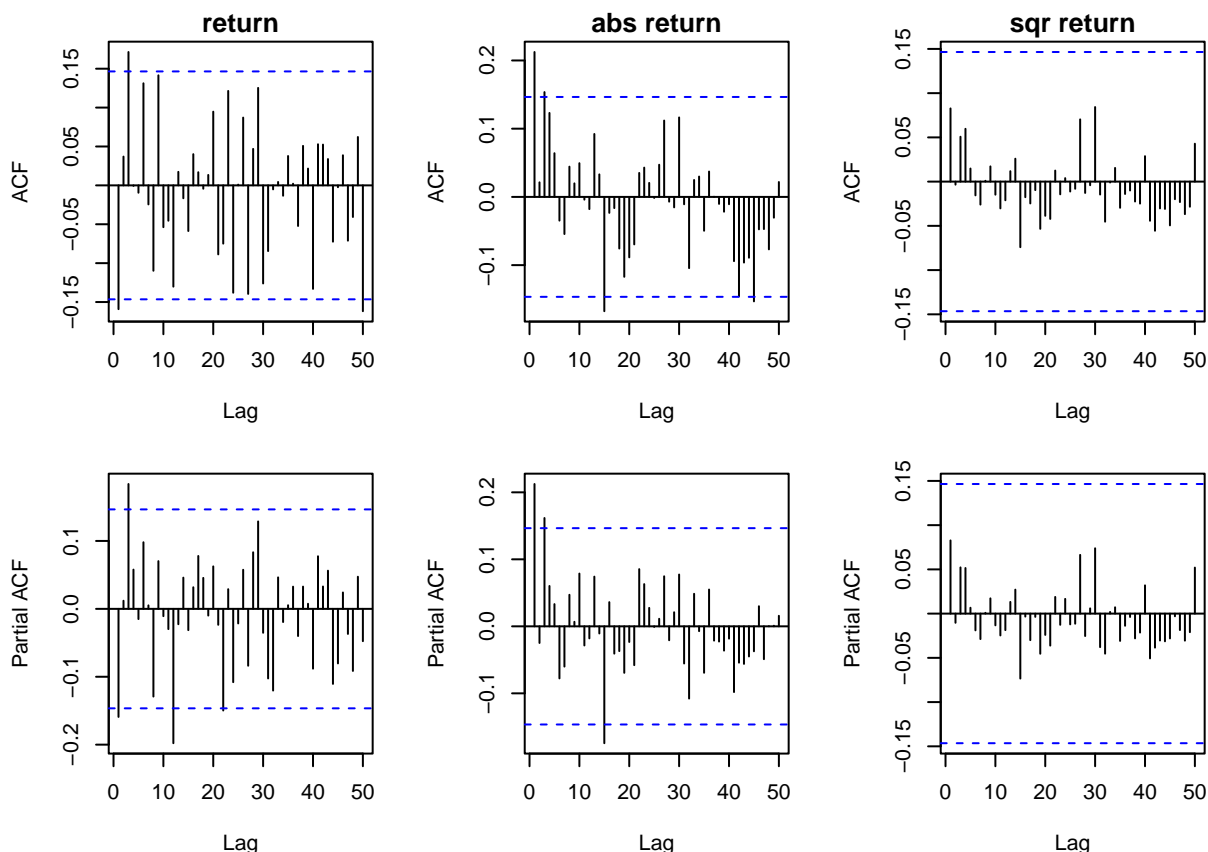
The Box-Pierce and Box-Ljung tests evaluate the autocorrelation of the residuals derived from the ARIMA(0,1,1) model. The resulting p-values are 0.06788, 0.08795, and 0.06216, respectively. These values surpass the conventional significance threshold of 0.05, indicating our failure to reject the null hypothesis of no autocorrelation in the residuals. Consequently, we infer that the residuals exhibit relatively independent behavior, signifying that the ARIMA(0,1,1) model adequately captures the temporal dependence in the data.

Upon reviewing the predictions for the next two years, it's evident that while the data points closely approximate some observed values, they don't precisely align.

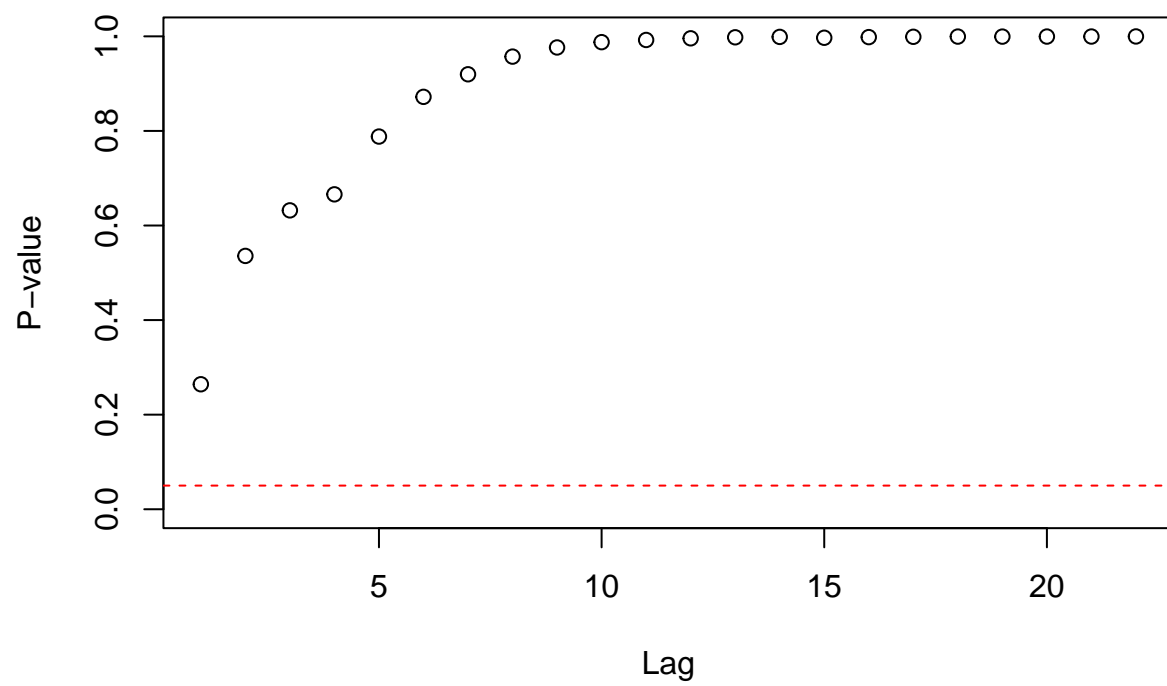
##	Model	ME	MPE	MSE	MAE	MAPE
## 1	freight_3_1_2	0.5212157	0.4190192	1.036990	0.8416413	0.6820952
## 2	freight_0_1_1	0.5393900	0.4337677	1.066341	0.8575744	0.6950459
## 3	freight_0_1_0	0.5041667	0.4051070	1.029583	0.8458333	0.6856508

Based on the provided metrics, the model “freight\_0\_1\_0” has the lowest values for Mean Error (ME), Mean Percent Error (MPE), Mean Squared Error (MSE), Mean Absolute Error (MAE), and Mean Absolute Percent Error (MAPE). Therefore, the “freight\_0\_1\_0” model performs better compared to the other models listed.

Based on the coefficients and their standard errors, the ARIMA model with order (3, 1, 2) appears to be better as it includes multiple autoregressive (AR) and moving average (MA) terms, indicating a more complex and potentially better capturing of the underlying patterns in the data. Additionally, the standard errors of the coefficients in the (3, 1, 2) model are relatively smaller compared to the (0, 1, 0) model, suggesting more precise estimates. However, model selection should also consider other factors such as model fit diagnostics and forecast performance.



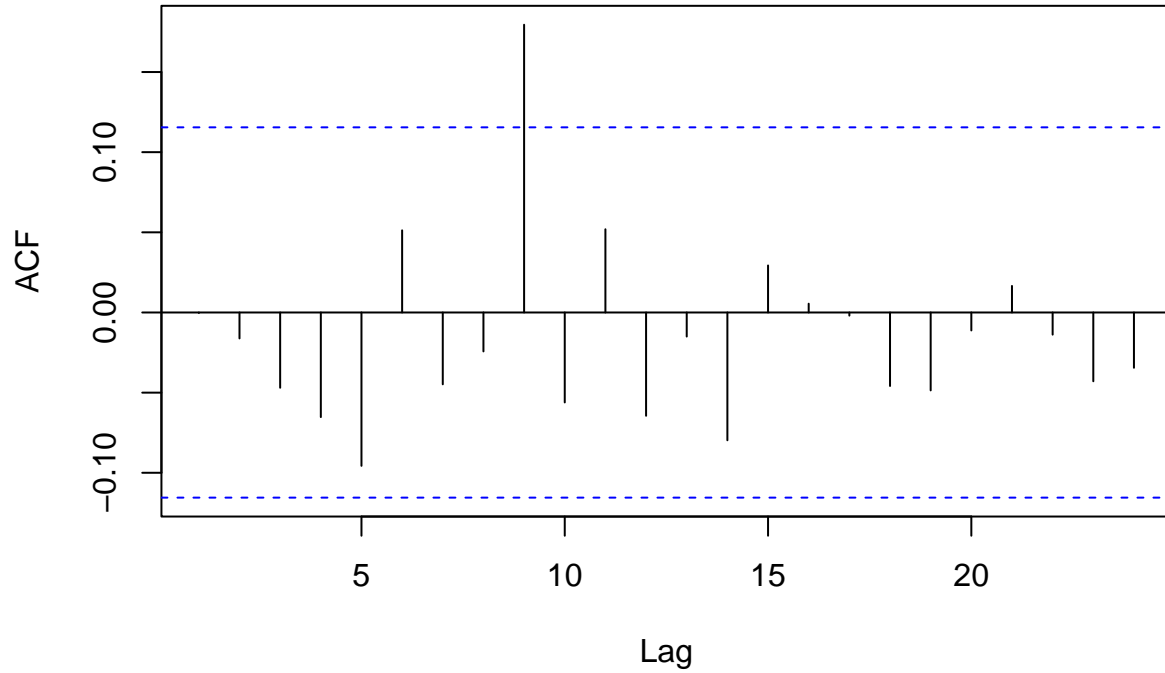
Based on the Chi-square return plots and the McLeod-Li test results, it's evident that there is no ARCH effect present in the data. However, it's worth noting that the data does not exhibit normality as indicated by the Q-Q plot and the Shapiro-Wilk test.



Above McLeod.Li.test indicates that there is no arch effect on the non covid data.

### 6.0.7 ARIMAX

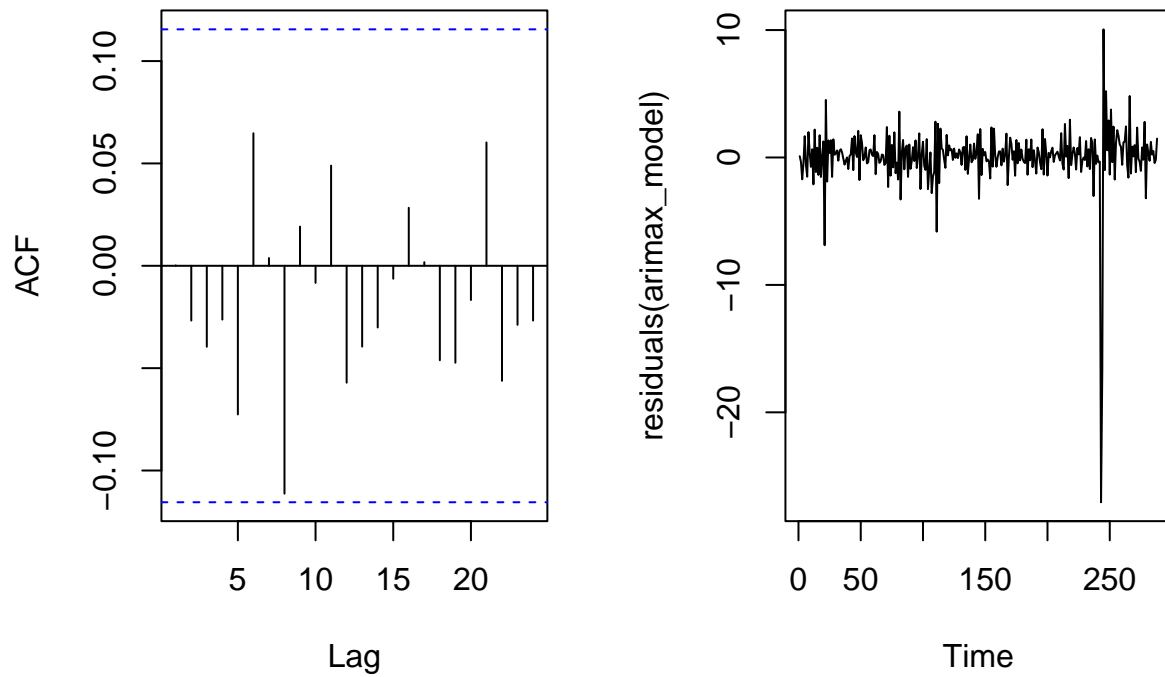
**ACF of ARIMAX Model Residuals**



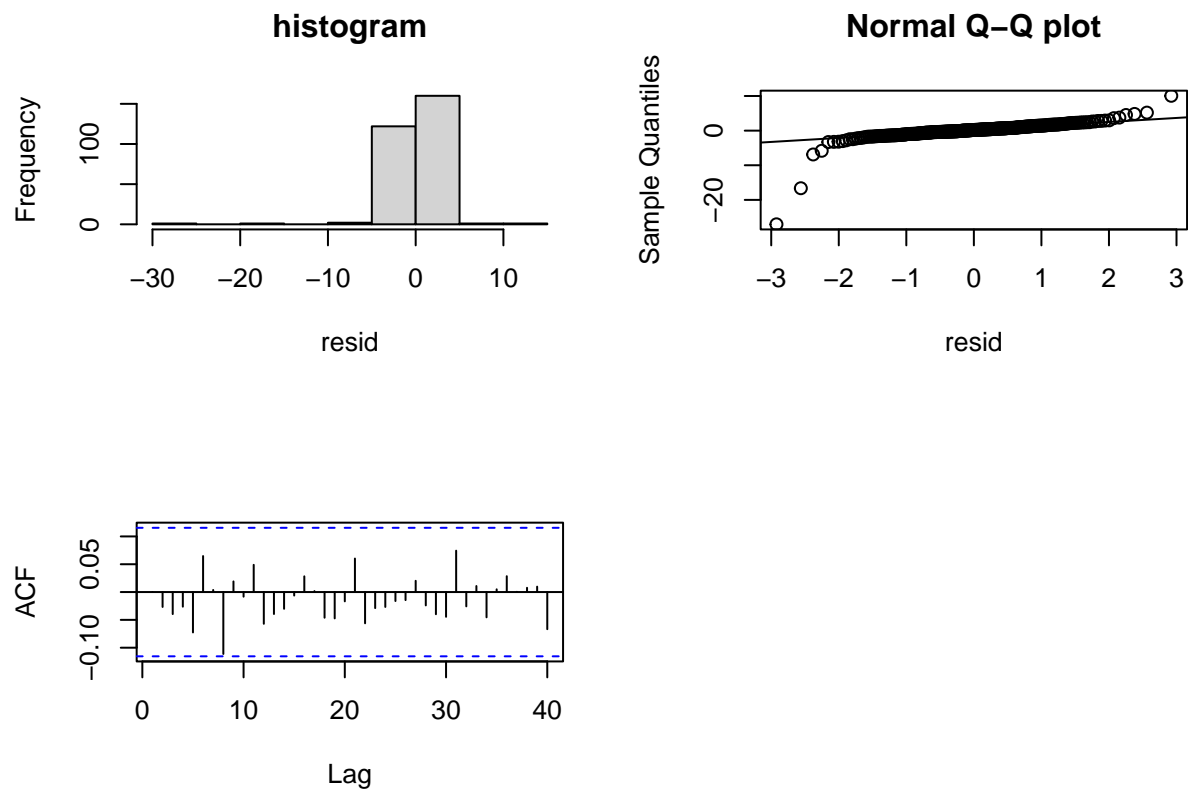
Based on the ARIMAX model fitted using the exogenous variable “covid\_indicator,” it appears that this variable does not have a discernible effect on the model. Additionally, examination of the residuals suggests the presence of significant lags, indicating potential autocorrelation issues.

Based on the autocorrelation function (ACF) and partial autocorrelation function (PACF) plots of the entire dataset, an ARIMA model with order (1, 1, 1) was selected for implementation.

### ACF of ARIMAX Model Residual

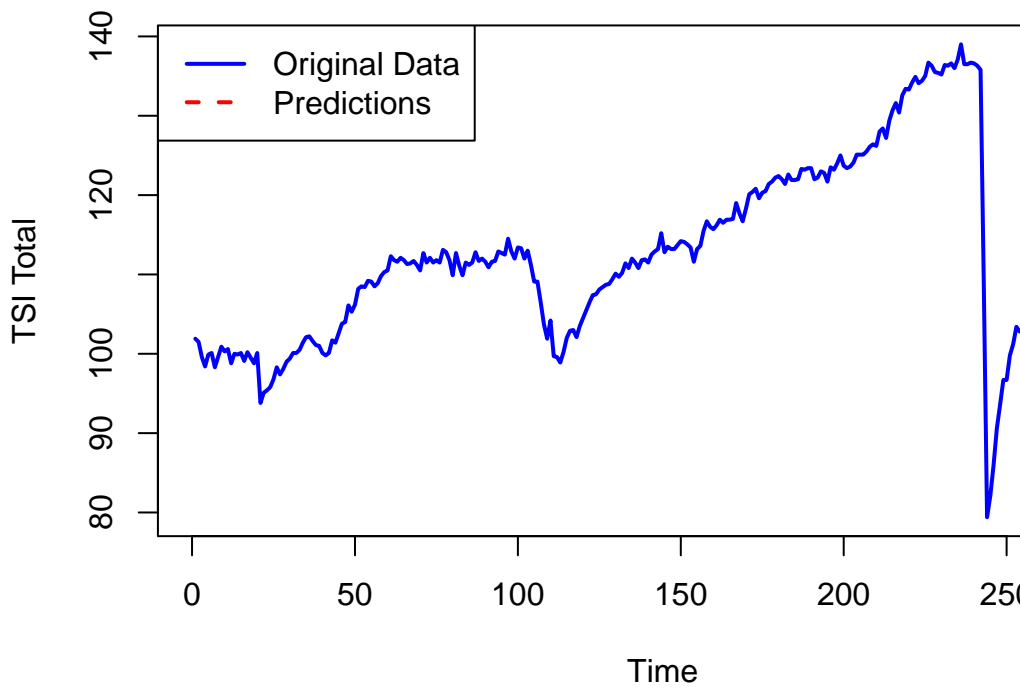


From the visual inspection of the autocorrelation function (ACF) plot of the residuals, it can be determined that the residuals exhibit desirable characteristics. Specifically, the ACF plot demonstrates a lack of significant autocorrelation at various lags, indicating that the residuals are relatively uncorrelated and exhibit no systematic patterns. This suggests that the ARIMA model adequately captures the temporal dependence in the data, and the residuals display randomness, meeting one of the key assumptions of the model.



Despite the apparent adequacy of the residuals observed in the autocorrelation function (ACF) plot, further examination using the Shapiro-Wilk test, histograms, and Q-Q plots reveals that the residuals do not conform to a normal distribution. These diagnostic tests suggest departures from normality, indicating potential limitations in the model's ability to fully capture the underlying variability in the data.

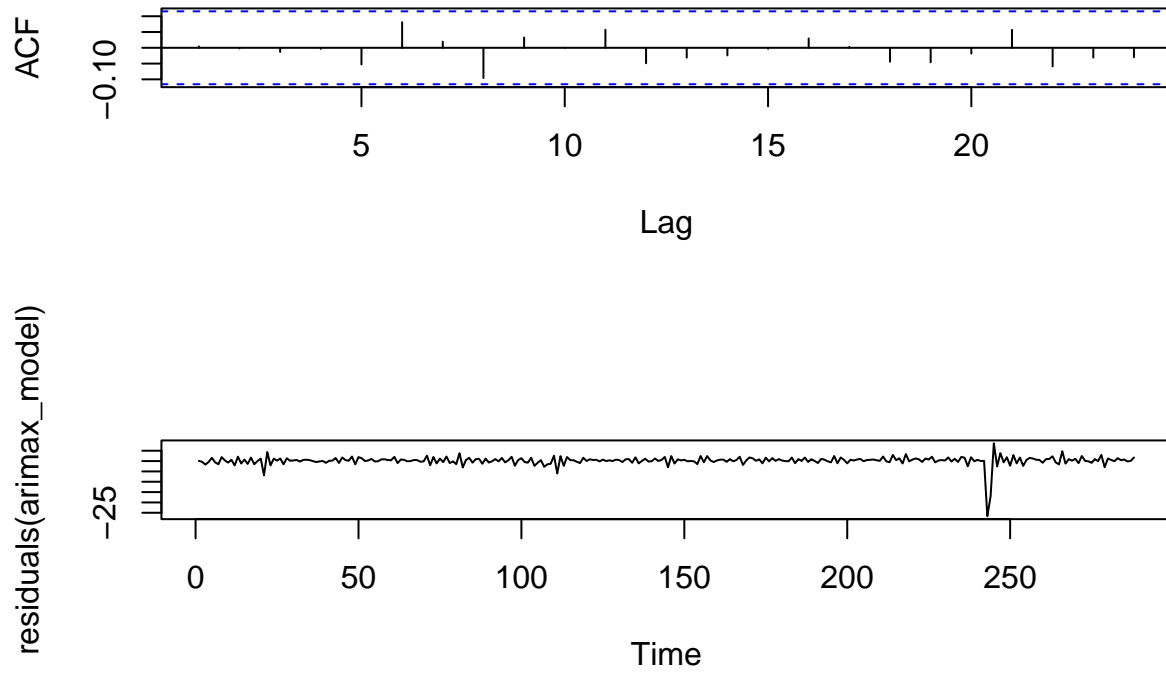
## ARIMAX Model Predictions with Intervals



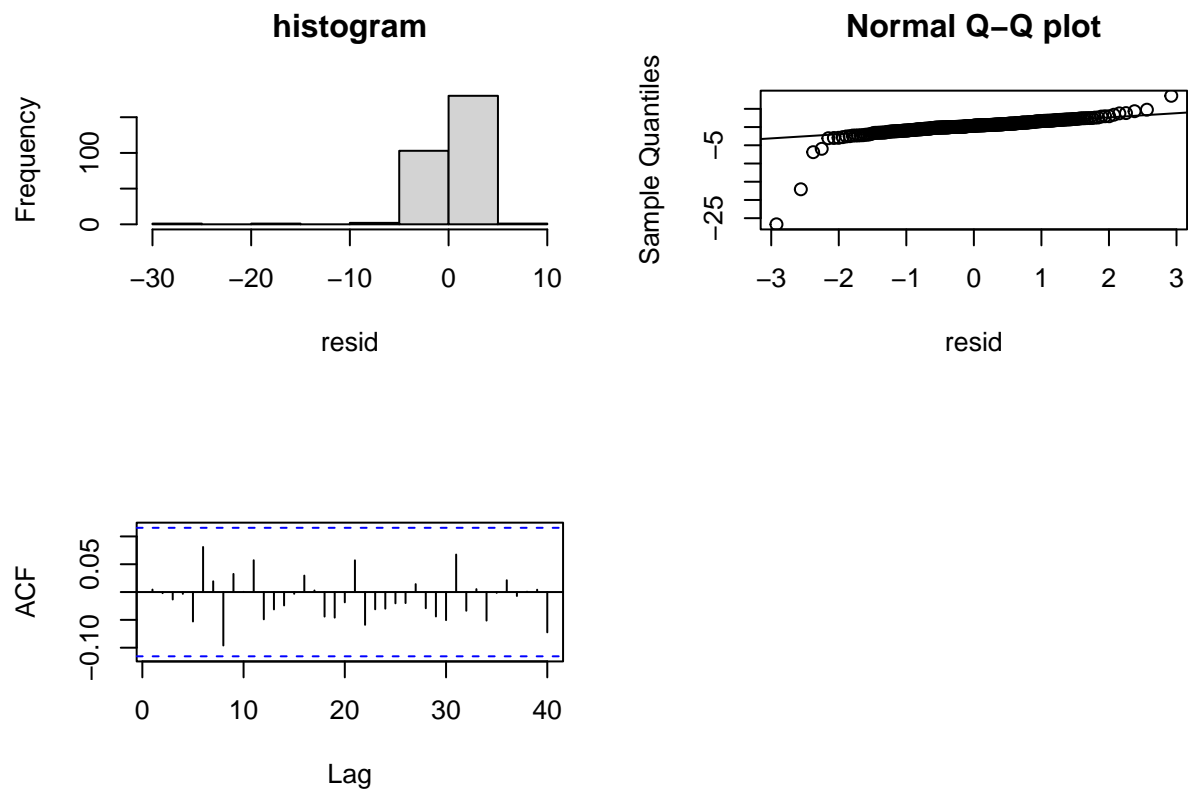
Following is the forecast for two years

I applied an ARIMA model with an order of (1, 1, 2), determined using the `auto.arima` function, to the entire dataset, encompassing both the COVID and non-COVID periods.

### ACF of ARIMAX Model Residuals



Based on the autocorrelation function (ACF) plots of the ARIMAX models, the lags appear to be within acceptable limits, consistently falling below the significance level.

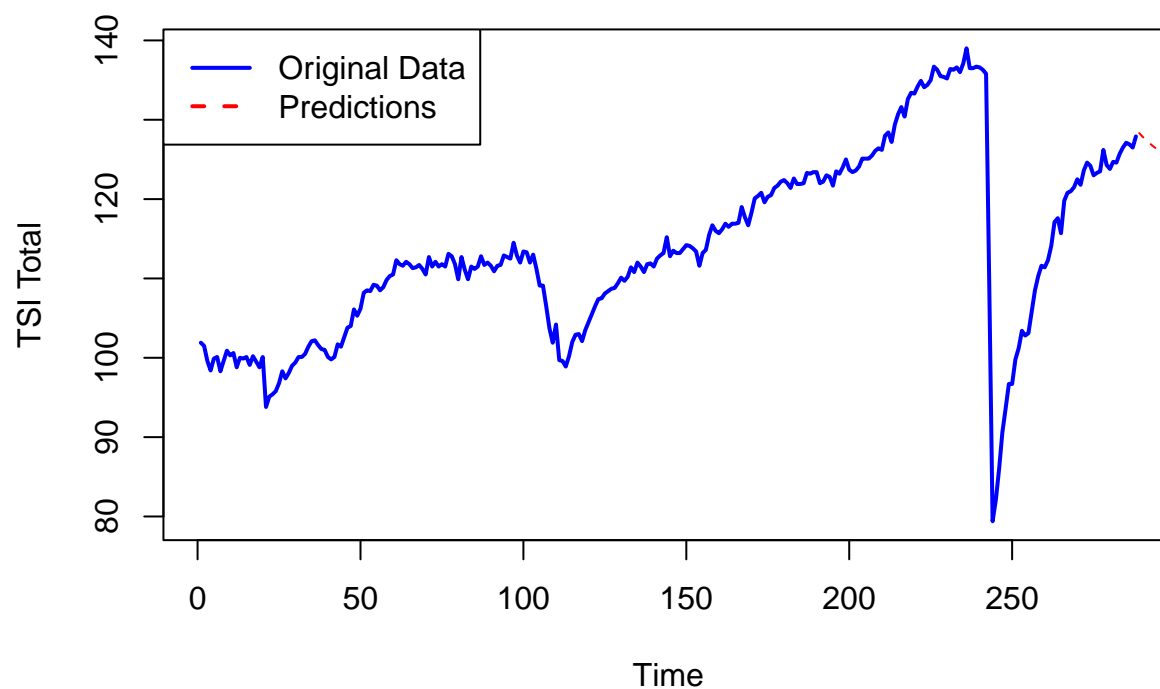


Following an analysis of the histograms and normal quantile-quantile (Q-Q) plots, it's evident that the data deviates from normality. Additionally, a notable spike in the residuals is observed, largely attributable to the impact of the COVID-19 pandemic.

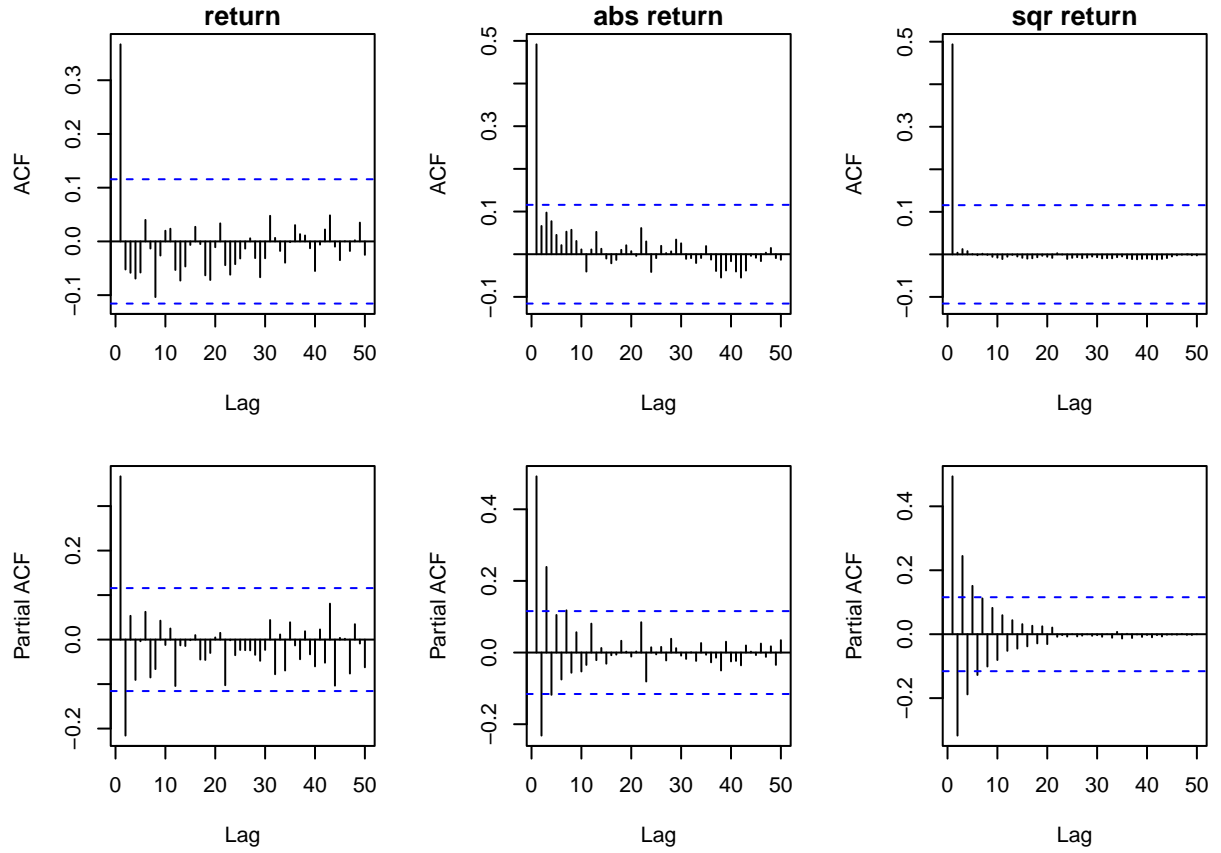
Following is the forecast for next 2 years



## ARIMAX Model Predictions with Intervals

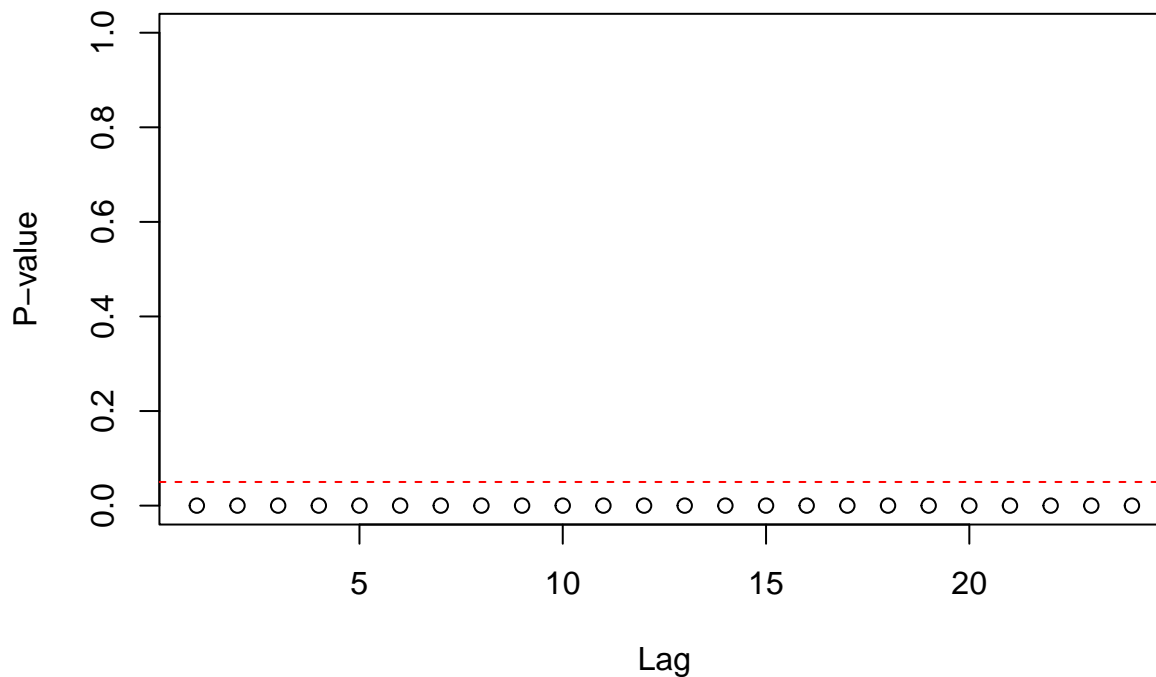


The model fitted with order  $(1, 1, 2)$  exhibits non-normal residuals, primarily attributed to the pronounced decline during the COVID period. This deviation from normality is likely a consequence of the extraordinary impact of the pandemic on the data dynamics.



Upon examination of the absolute returns and square returns of the differenced TSI total data, it becomes apparent that an ARCH effect is present. This observation is further validated by the McLeod-Li test, as depicted in the plot below.

## 6.0.8 ARCH



I applied a GARCH model with parameters (1, 0) to the data. This model aims to capture volatility clustering, a phenomenon where periods of high volatility tend to cluster together. By specifying the GARCH(1,0) model, we focus on modeling the persistence of volatility in the data without considering any additional effects from past squared innovations.

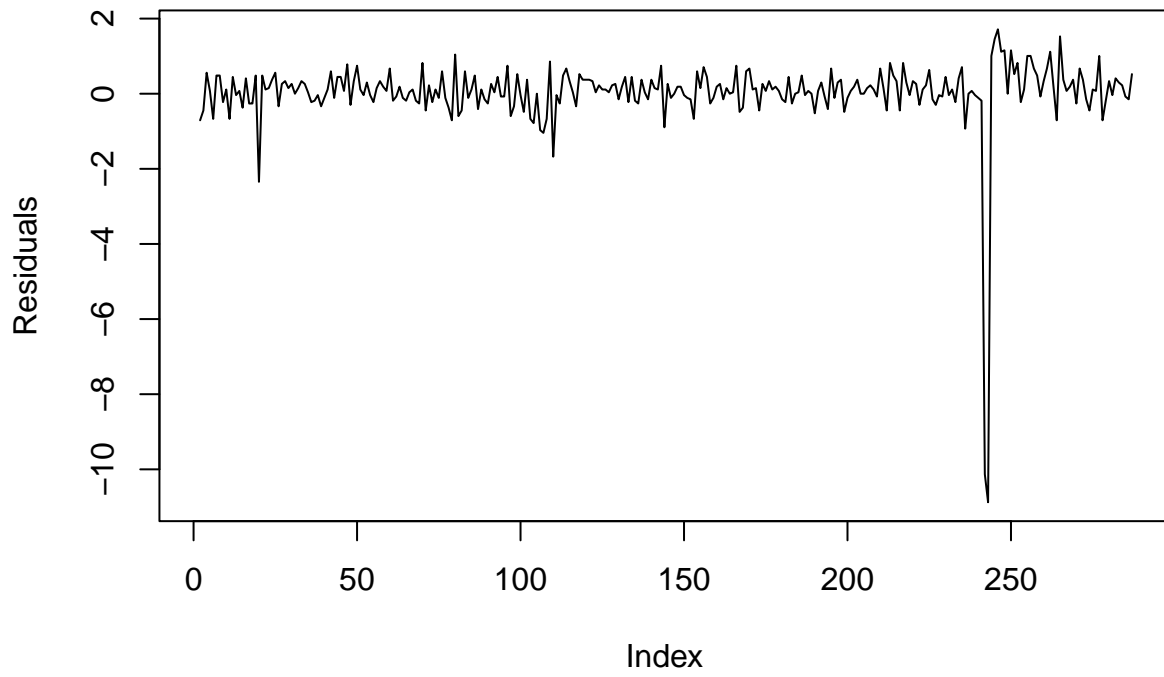
```
##
## ***** ESTIMATION WITH ANALYTICAL GRADIENT *****
##
##
##      I      INITIAL X(I)      D(I)
##
##      1      6.839683e+00      1.000e+00
##      2      5.000000e-02      1.000e+00
##
##      IT      NF      F      RELDF      PRELDF      RELDX      STPPAR      D*STEP      NPRELDF
##
##      0      1      4.254e+02
##      1      4      4.254e+02      1.44e-07      7.31e-07      1.3e-04      1.0e+02      1.8e-03      3.67e-05
##      2      5      4.254e+02      8.52e-08      8.98e-08      1.3e-04      1.2e+00      1.8e-03      9.01e-08
##      3      6      4.254e+02      6.36e-10      5.77e-09      1.7e-04      0.0e+00      2.3e-03      5.77e-09
##      4      7      4.254e+02      4.85e-10      2.04e-08      3.3e-04      1.5e+00      4.6e-03      1.01e-07
##      5      8      4.254e+02      9.55e-10      1.10e-08      1.7e-04      2.0e+00      2.3e-03      6.56e-07
##      6      9      4.254e+02      1.18e-09      6.12e-09      8.2e-05      2.0e+00      1.1e-03      4.10e-06
##      7      10      4.254e+02      1.03e-09      1.16e-08      1.7e-04      2.0e+00      2.3e-03      4.98e-06
##      8      11      4.254e+02      2.96e-10      5.58e-09      8.3e-05      2.0e+00      1.1e-03      5.61e-06
##      9      12      4.254e+02      2.78e-10      2.91e-09      4.1e-05      2.0e+00      5.7e-04      7.85e-06
##      10     16      4.254e+02      4.03e-12      1.17e-10      1.8e-06      4.3e+00      2.5e-05      8.84e-06
```

```

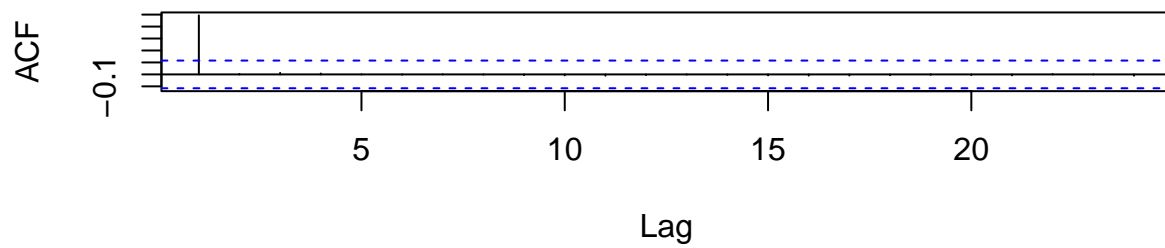
##      11      17  4.254e+02  1.02e-11  6.34e-11  8.2e-07  9.0e-01  1.3e-05 -4.31e-06
##      12      18  4.254e+02  8.94e-12  1.26e-10  1.8e-06  5.3e+02  2.5e-05  8.89e-06
##      13      19  4.254e+02  4.00e-12  6.30e-11  9.1e-07  9.2e-01  1.3e-05 -4.36e-06
##      14      20  4.254e+02  2.10e-12  3.15e-11  4.6e-07  5.4e+00  6.3e-06 -4.44e-06
##      15      21  4.254e+02  1.05e-12  1.57e-11  2.3e-07  3.1e+01  3.1e-06 -4.93e-06
##      16      22  4.254e+02  5.27e-13  7.87e-12  1.1e-07  2.7e+01  1.6e-06 -4.92e-06
##      17      23  4.254e+02  2.63e-13  3.94e-12  5.7e-08  2.1e+01  7.8e-07 -4.89e-06
##      18      24  4.254e+02  1.32e-13  1.97e-12  2.8e-08  1.2e+01  3.9e-07 -4.80e-06
##      19      25  4.254e+02  6.41e-14  9.84e-13  1.4e-08  6.8e+00  2.0e-07 -4.58e-06
##      20      26  4.254e+02  3.47e-14  4.92e-13  7.1e-09  3.8e+00  9.8e-08 -4.11e-06
##      21      27  4.254e+02  1.38e-14  2.46e-13  3.6e-09  2.4e+00  4.9e-08 -3.25e-06
##      22      28  4.254e+02  1.07e-14  1.23e-13  1.8e-09  2.0e+00  2.5e-08 -2.91e-06
##      23      29  4.254e+02  3.47e-15  6.15e-14  8.9e-10  2.0e+00  1.2e-08 -3.49e-06
##      24      30  4.254e+02  2.27e-15  3.08e-14  4.4e-10  2.0e+00  6.1e-09 -3.68e-06
##      25      31  4.254e+02  1.34e-15  1.54e-14  2.2e-10  2.0e+00  3.1e-09 -3.71e-06
##      26      32  4.254e+02  1.34e-16  7.69e-15  1.1e-10  2.0e+00  1.5e-09 -3.69e-06
##      27      33  4.254e+02  9.35e-16  3.84e-15  5.6e-11  2.0e+00  7.7e-10 -3.65e-06
##      28      38  4.254e+02  4.01e-16  2.65e-16  3.8e-12  2.0e+00  5.3e-11 -3.62e-06
##      29      43  4.254e+02 -1.34e-16  1.16e-18  1.7e-14  4.2e+00  2.3e-13 -4.24e-06
##
## ***** FALSE CONVERGENCE *****
##
## FUNCTION      4.254393e+02  RELDX      1.686e-14
## FUNC. EVALS    43          GRAD. EVALS    29
## PRELDF        1.164e-18    NPRELDF   -4.244e-06
##
##      I      FINAL X(I)      D(I)      G(I)
##
##      1      6.823964e+00      1.000e+00      2.114e-03
##      2      5.329347e-02      1.000e+00      -2.930e-04

```

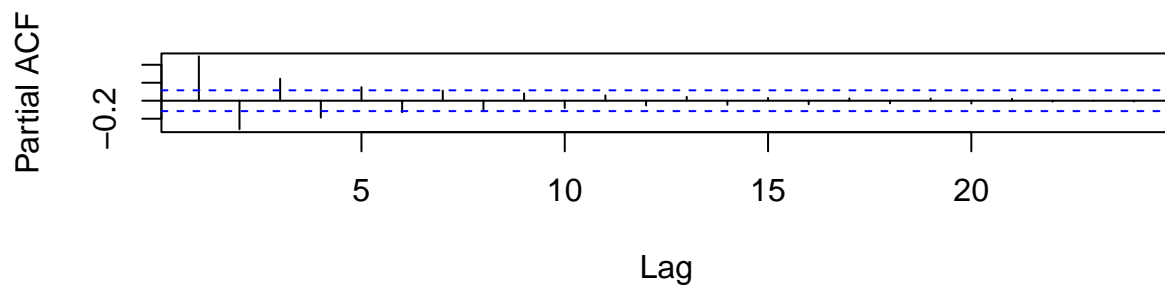
**Residuals Plot**

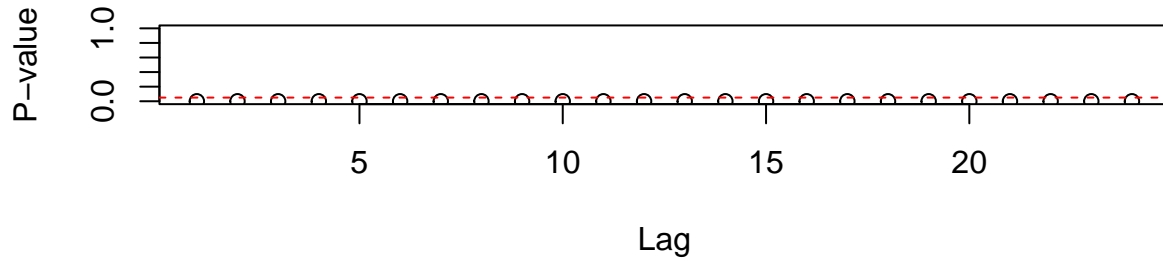


**ACF of Squared Residuals**



**PACF of Squared Residuals**





Upon careful examination of the residual plots, including the autocorrelation function (ACF) and partial autocorrelation function (PACF), the data points exhibit characteristics indicative of normality. This suggests that the residuals follow a normal distribution. However, additional verification of the ARCH effect in the residuals using the McLeod-Li test reveals its presence, highlighting the need for further consideration of volatility clustering in the data.

I implemented a GARCH model with parameters (1, 1) on the dataset. This particular model specification aims to capture both the short-term and long-term persistence of volatility in the data. The first parameter (1) represents the autoregressive component, which captures the short-term volatility dynamics, while the second parameter (1) represents the moving average component, which accounts for the long-term memory of volatility shocks. This comprehensive model seeks to provide a nuanced understanding of the volatility dynamics inherent in the dataset.

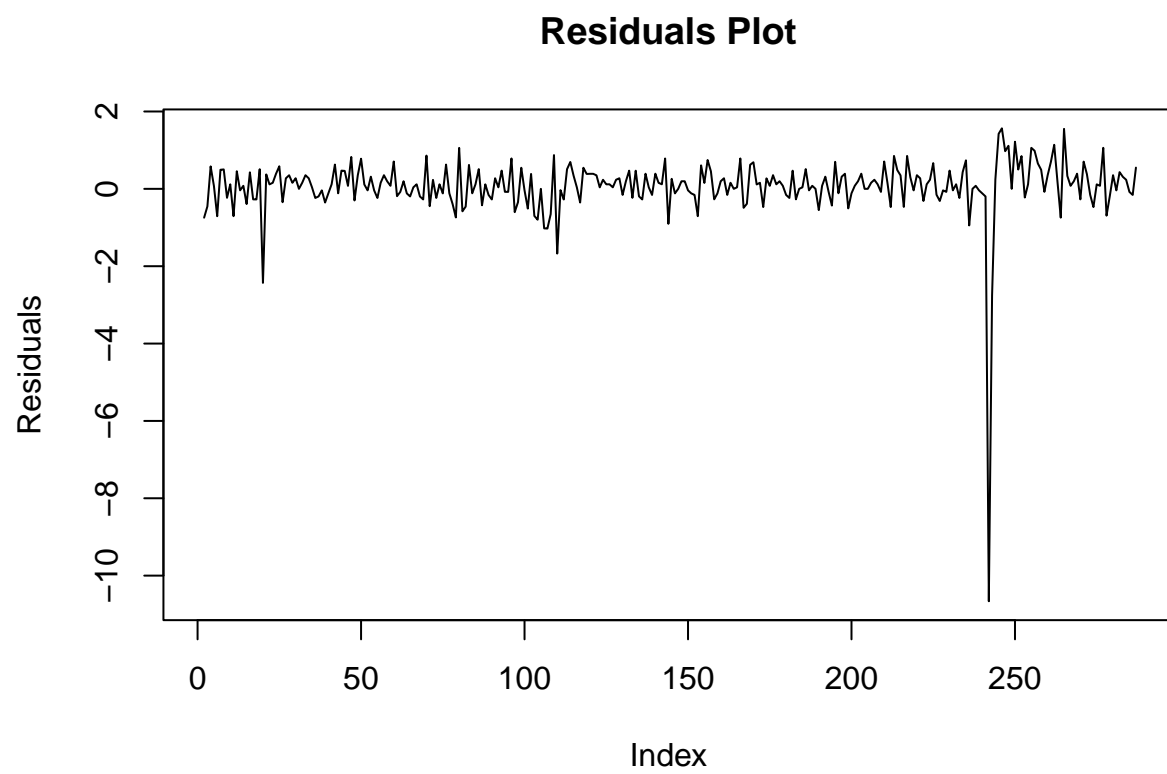
```
##
## ***** ESTIMATION WITH ANALYTICAL GRADIENT *****
##
##
##      I      INITIAL X(I)      D(I)
##
##      1      6.479700e+00      1.000e+00
##      2      5.000000e-02      1.000e+00
##      3      5.000000e-02      1.000e+00
##
##      IT  NF      F      RELDF      PRELDF      RELDX      STPPAR      D*STEP      NPRELDF
##      0   1  3.750e+02
##      1   3  3.693e+02  1.52e-02  3.85e-02  7.1e-03  1.4e+03  1.0e-01  2.79e+01
##      2   5  3.689e+02  1.32e-03  1.37e-03  7.6e-04  3.0e+02  1.0e-02  3.92e+00
##      3   8  3.689e+02  2.52e-05  2.52e-05  1.5e-05  5.6e+02  2.0e-04  5.68e-02
```

```

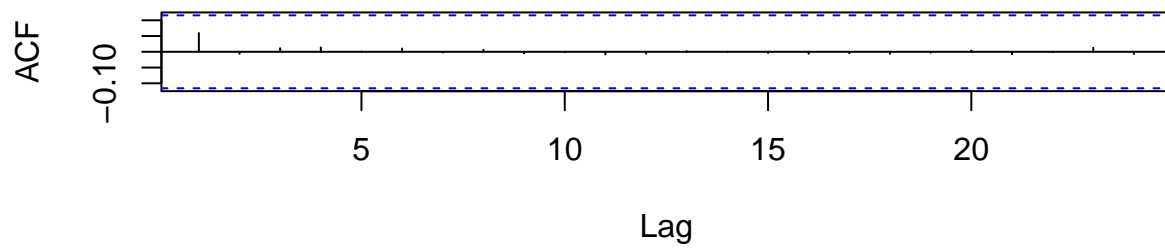
##      4      11  3.688e+02  2.00e-04  2.00e-04  1.2e-04  1.6e+01  1.6e-03  5.90e-02
##      5      14  3.688e+02  3.96e-06  3.96e-06  2.4e-06  2.8e+03  3.2e-05  5.92e-02
##      6      16  3.688e+02  7.91e-06  7.91e-06  4.9e-06  3.3e+02  6.4e-05  5.96e-02
##      7      18  3.688e+02  1.58e-06  1.58e-06  9.7e-07  6.6e+03  1.3e-05  5.96e-02
##      8      20  3.688e+02  3.16e-06  3.16e-06  1.9e-06  8.3e+02  2.6e-05  5.96e-02
##      9      22  3.688e+02  6.32e-07  6.32e-07  3.9e-07  1.7e+04  5.1e-06  5.97e-02
##     10      24  3.688e+02  1.26e-06  1.26e-06  7.8e-07  2.1e+03  1.0e-05  5.97e-02
##     11      27  3.688e+02  2.53e-08  2.53e-08  1.6e-08  4.1e+05  2.0e-07  5.97e-02
##     12      29  3.688e+02  5.06e-08  5.06e-08  3.1e-08  5.2e+04  4.1e-07  5.97e-02
##     13      31  3.688e+02  1.01e-08  1.01e-08  6.2e-09  1.0e+06  8.2e-08  5.97e-02
##     14      33  3.688e+02  2.02e-09  2.02e-09  1.2e-09  5.2e+06  1.6e-08  5.97e-02
##     15      35  3.688e+02  4.05e-10  4.05e-10  2.5e-10  2.6e+07  3.3e-09  5.97e-02
##     16      37  3.688e+02  8.09e-10  8.09e-10  5.0e-10  3.2e+06  6.6e-09  5.97e-02
##     17      39  3.688e+02  1.62e-10  1.62e-10  1.0e-10  6.5e+07  1.3e-09  5.97e-02
##     18      42  3.688e+02  1.29e-09  1.29e-09  8.0e-10  2.0e+06  1.0e-08  5.97e-02
##     19      46  3.688e+02  2.59e-12  2.59e-12  1.6e-12  1.4e+00  2.1e-11 -1.99e-02
##     20      48  3.688e+02  5.18e-12  5.18e-12  3.2e-12  1.4e+00  4.2e-11 -1.99e-02
##     21      50  3.688e+02  1.04e-12  1.04e-12  6.4e-13  1.4e+00  8.4e-12 -1.99e-02
##     22      52  3.688e+02  2.07e-12  2.07e-12  1.3e-12  1.4e+00  1.7e-11 -1.99e-02
##     23      55  3.688e+02  4.16e-14  4.14e-14  2.5e-14  1.4e+00  3.4e-13 -1.99e-02
##     24      57  3.688e+02  8.39e-14  8.29e-14  5.1e-14  1.4e+00  6.7e-13 -1.99e-02
##     25      59  3.688e+02  1.65e-14  1.66e-14  1.0e-14  1.4e+00  1.3e-13 -1.99e-02
##     26      61  3.688e+02  3.33e-14  3.31e-14  2.0e-14  1.4e+00  2.7e-13 -1.99e-02
##     27      63  3.688e+02  6.01e-15  6.63e-15  4.1e-15  1.4e+00  5.4e-14 -1.99e-02
##     28      65  3.688e+02  1.33e-14  1.33e-14  8.2e-15  1.4e+00  1.1e-13 -1.99e-02
##     29      66  3.688e+02 -2.71e+07  2.65e-14  1.6e-14  1.4e+00  2.1e-13 -1.99e-02
##
## ***** FALSE CONVERGENCE *****
##
## FUNCTION      3.687723e+02  RELDX      1.631e-14
## FUNC. EVALS    66          GRAD. EVALS    29
## PRELDF         2.652e-14    NPRELDF    -1.987e-02
##
##      I      FINAL X(I)      D(I)      G(I)
##
##      1      6.472179e+00      1.000e+00      7.635e+00
##      2      1.430427e-01      1.000e+00     -3.288e+00
##      3      3.184051e-15      1.000e+00      4.477e+01

```

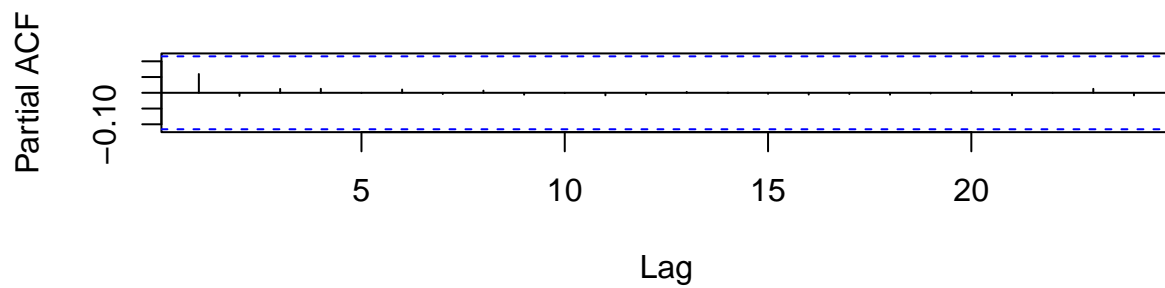


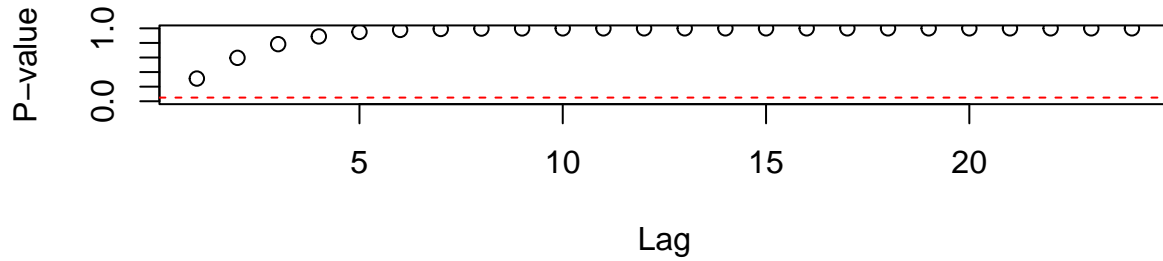


**ACF of Squared Residuals**



**PACF of Squared Residuals**





Upon implementing the GARCH(1, 1) model, it was observed that there is no ARCH effect present in the residuals. Furthermore, the autocorrelation function (ACF) and partial autocorrelation function (PACF) plots exhibited desirable characteristics, with all lags appearing insignificant. Notably, any sharp declines observed in the residuals were attributed to the impact of the COVID-19 pandemic.

## 7 Conclusions

After analyzing the various time series models applied to transportation data, it's evident that each model has its strengths and weaknesses. Here's a concise conclusion:

The ARIMA models, including (0,1,0), (3,1,2), and (0,1,1), all offer valuable insights into the data. The (0,1,0) model demonstrates the best performance based on traditional metrics such as Mean Error, Mean Squared Error, and Mean Absolute Error. However, it falls short in capturing more complex patterns compared to the (3,1,2) model, which incorporates multiple autoregressive and moving average terms. The (0,1,1) model strikes a balance between simplicity and capturing temporal dependence.

The ARIMAX model incorporating the exogenous variable “covid\_indicator” did not significantly improve performance, suggesting that the pandemic's impact may not be adequately captured by this variable alone.

On the other hand, the GARCH models, particularly the GARCH(1,1) model, effectively capture volatility clustering without exhibiting an ARCH effect in residuals. This suggests its suitability for modeling the volatility dynamics inherent in the transportation data.

## 8 Limitations

One limitation of the analysis is the presence of external shocks, particularly the significant decline in freight services due to the COVID-19 pandemic. This unprecedented event introduced a level of volatility

and disruption that traditional time series models may struggle to accurately capture. Despite attempts to incorporate the “covid\_indicator” as an exogenous variable in the ARIMAX model, the impact of the pandemic on transportation data remains challenging to fully model and predict.

The limitations stemming from the COVID-19 pandemic highlight the need for more robust modeling techniques that can effectively account for extreme events and their repercussions on time series data. Additionally, the uncertainty and unpredictability associated with such external shocks underscore the importance of supplementing time series analysis with qualitative insights and scenario planning to better anticipate and mitigate potential disruptions in the future.

## 9 References

Data Source : <https://data.bts.gov/Research-and-Statistics/Freight-Transportation-Service-Index/n68x-u7m7>