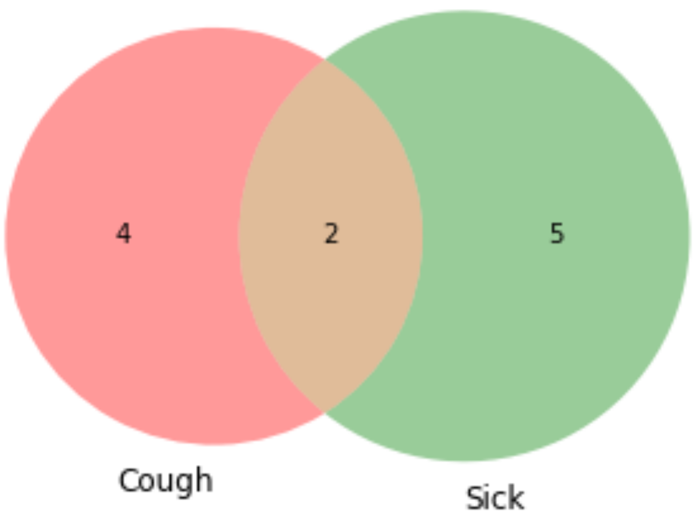
BAYES THEOREM

Assume the example we are in a place where only 14 people are living and conducted a survey if the people having cough or people are sick or both? and after all the survey we are ended up with



Also

	Have cough	Doesn't have cough	Row total
Are sick	2 2/14	5 5/14	7 7/14
Aren't sick	4 4/14	3 3/14	7 7/14
Column total	6 6/14	8 8/14	

Applying **CONDITIONAL PROBABILITY**

probability of someone doesn't have cough but are sick knowing that they are sick can be calculated as

P(Dosen't have cough and Are sick | Are sick) = $\frac{5/14}{7/14}$ = 0.71

If we cleanly observer

The fraction we are doing can be rewritten as

$$P(Dosen'thavecoughandAresick|Aresick) = \frac{P(Dosen'thavecoughandAresick)}{P(Aresick)} = \frac{5}{7} = 0.71$$

probability of someone doesn't have cough but are sick knowing that they aren't sick can be calculated as

P(Dosen't have cough and Are sick | Dosen't have cough) = $\frac{5/14}{8/14}$ = 0.63

If we cleanly observer

The fraction we are doing can be rewritten as

$$P(Dosen'thavecoughandAresick|Dosen'thavecough) = \frac{P(Dosen'thavecoughandAresick)}{P(Dosen'thavecough)} = \frac{5}{8} = 0.63$$

In the above two cases we want to know the probability that someone doesn't have cough but are sick soo the numerator is same for both cases but since we have different knowledge in each case the denominator will be different then we get a different probability

QUESTION

Can we solve the conditional probability with out knowing the P(Dosen't have cough and Are sick) ?

$$P(Dosen'thavecoughandAresick|Aresick) = \frac{P(Dosen'thavecoughandAresick)}{P(Aresick)}$$
$$P(Dosen'thavecoughandAresick|Dosen'thavecough) = \frac{P(Dosen'thavecoughandAresick)}{P(Dosen'thavecough)}$$

Rewritting we get

$P(Dosen'thavecoughandAresick|Aresick)$ X $P(Aresick)$=$P(Dosen'thavecoughandAresick)$

$P(Dosen'thavecoughandAresick|Dosen'thavecough)$ X $P(Dosen'thavecough)$=$P(Dosen'thavecoughandAresick)$

If we look carefully we could see that the RHS side is same for both the cases for lets substitue it and rewrite it again

$P(Dosen'thavecoughandAresick|Aresick)$ X $P(Aresick)$ = $P(Dosen'thavecoughandAresick|Dosen'thavecough)$ X $P(Dosen'thavecough)$

$P(Dosen'thavecoughandAresick|Aresick)$ Solving this term first we get

$$P(Dosen'thavecoughandAresick|Aresick) = \frac{P(Dosen'thavecoughandAresick|Dosen'thavecough)XP(Dosen'thavecough)}{P(Aresick)}$$

Similarly we sovlve for this $P(Dosen'thavecoughandAresick|Dosen'thavecough)$ we get

$$P(Dosen'thavecoughandAresick|Dosen'thavecough) = \frac{P(Dosen'thavecoughandAresick|Aresick)XP(Aresick)}{P(Dosen'thavecough)}$$

$$------$$
$$THIS$$
$$IS$$
$$THE$$
$$BAYES$$
$$THEOREM$$
$$------$$

Making it simpler

A = Dosen't have cough and Are sick

B = Are sick

$$P(AandB|B) = \frac{P(AandB|A)XP(A)}{P(B)}$$
$$P(AandB|A) = \frac{P(AandB|B)XP(B)}{P(A)}$$

NOTE : When we have all the data like the venn diagram or contingency table then BAYES THEOREM

isn't that usefull we can do it directly however when we have only less data like

P(Dosen't have cough and are sick | are sick) = 0.71

P(Are sick) ~ 0.6 #approximate value

P(Dosen't have cough) = 0.57

$$P(Dosen'thavecoughandaresick|Dosen'thavecough) = \frac{0.71X0.6}{0.57} = 0.75$$

BAYESIAN STATISTICS : Is about understanding what it means to make a guess like P(Are sick) ~ 0.6