HOW TO CALCULATE P-VALUE

There are 2 types of P-Values

- One sided P-Values
- Two sided P-Values

Two sided P-Values are the most common and one sided P-Values are rarely used

Lets take an example to demonstrate how to calculate P-Value

Assuem we had a coin and flipped it once and got HEADs, flipped it again and got HEADs again and at this point we might think that the coin is super special

'we might think that the coin is super special' ----> This is HYPOTHESIS

however in statistics we consider hypotesis as ----> 'Even though we got 2 HEADs in a row, the coin is no different a normal coin'

NOTE: Although we want to know if our coin is special the statistics language version considers it as opposite which is called as NULL HYPOTHESIS and we can reject NULL HYPOTHESIS using P-Value

if we reject the Null Hypothesis we will know that our coin is special

Lets calculate the P-Value for flipping a coin

```
       __C__
      /     \
     H       T      ---> First Flip
    / \     / \
   H   T   H   T    ----> Second Flip
```

Outcomes would be

HH HT TH TT

we can calculate the probability of getting 2 HEADs = $\frac{nooftimewegot2HEADs}{Totalnoofoutcomes} = \frac{1}{4} = 0.25$

same way we can calculate the probability of getting 2 TAIL = $\frac{nooftimewegot2Tails}{Totalnoofoutcomes} = \frac{1}{4} = 0.25$

Now to calculate the probability of getting 1 HEAD 1 TAIL = $\frac{nooftimewegotHEADsandTails}{Totalnoofoutcomes} = \frac{2}{4} = 0.5$

Outcomes - Probability

HH - 0.25

HT - 0.5

TH - 0.5

TT - 0.25

Now lets calculate the P-Value for getting 2 HEADs

NOTE: P-Value consists of 3 parts

- Probability random chance would result in the observation
- Probability of something else that is equally rare
- Probability of observing something rarer or most extreme

P-Value for 2 Heads = $(Probabilityrandomchancewouldresultintheobservation) + (Probabilityofsomethingelsethatisequallyrare) + (Probabilityofobservingsomethingrarerormostextreme)$P-Value for 2 Heads = $(Probabilitythatnormalcoinget2HEADs = 0.25) + (2TAILsisasrareas2HEADs = 0.25) + (Nootheroutcomeareasrareas2TAILS/2HEADs = 0)$

P-Value for 2 Heads =0.5

P-Value is used to reject the NULL HYPOTHESIS which is

`Even though we got 2 HEADs in a row, the coin is no different a normal coin`

NOTE : we reject this HYPOTHESIS only if our P-Value is less than 0.05

Since we have P-Value = 0.5 > 0.05 we failed to reject the HYPOTHESIS

In other words data getting 2 HEADs in a row failed to convince that our coin is special

Now that we know getting 2 HEADs in a row doesn't make our coin special lets take a new example

$$HHHTH$$

four HEADs and one TAIL

HYPOTHESIS : Even though we get 4 HEADs and 1 TAIL , out coin is no different from a normal coin

First we know that we can flip coin 5 times and get 5 HEADS

'H', 'H', 'H', 'H', 'H'

5 ways to get 4 HEADs and 1 TAIL

'H', 'H', 'H', 'H', 'T'

'H', 'T', 'H', 'H', 'H'

'H', 'H', 'H', 'T', 'H'

'T', 'H', 'H', 'H', 'H'

'H', 'H', 'T', 'H', 'H'

10 ways to get 3 HEADs and 2 TAILs

'H', 'H', 'T', 'H', 'T'

'H', 'T', 'T', 'H', 'H'

'H', 'H', 'T', 'T', 'H'

'H', 'T', 'H', 'H', 'T'

'H', 'T', 'H', 'T', 'H'

'H', 'H', 'H', 'T', 'T'

'T', 'H', 'T', 'H', 'H'

'T', 'H', 'H', 'T', 'H'

'T', 'H', 'H', 'H', 'T'

'T', 'T', 'H', 'H', 'H'

10 ways to get 2 HEADs and 3 TAILs

'T', 'T', 'H', 'T', 'H'

'T', 'T', 'T', 'H', 'H'

'T', 'T', 'H', 'H', 'T'

'T', 'H', 'T', 'T', 'H'

'T', 'H', 'T', 'H', 'T'

'T', 'H', 'H', 'T', 'T'

'H', 'T', 'T', 'T', 'H'

'H', 'T', 'T', 'H', 'T'

'H', 'T', 'H', 'T', 'T'

'H', 'H', 'T', 'T', 'T'

5 ways to get 4 TAILs and 1 HEAD

'T', 'T', 'T', 'H', 'T'

'T', 'T', 'T', 'T', 'H'

'T', 'H', 'T', 'T', 'T'

'H', 'T', 'T', 'T', 'T'

'T', 'T', 'H', 'T', 'T'

1 way to get all TAILs

'T', 'T', 'T', 'T', 'T'

overall when we flip a coin 5 times we get 32 possible outcomes

P-Value for 4 HEADs and 1 TAIL is

P-Value for 4 HEADs and 1 TAIL = $(Probabilityrandomchancewouldresultintheobservation) + (Probabilityofsomethingelsethatisequallyrare) + (Probabilityofobservingsomethingrarerormostextreme)$

1. Probability we randomly get 4 HEADs and 1 TAIL = $\frac{5}{32}$
2. Probability of something else that is equally rare [1 H and 4 T] = $\frac{5}{32}$
3. Probability of observing something rarer or most extreme [5 TAILs / 5 HEADs] = $\frac{2}{32}$

P-Value for 4 HEADs and 1 TAIL=$\frac{5}{32} + \frac{5}{32} + \frac{2}{32} = 0.375$

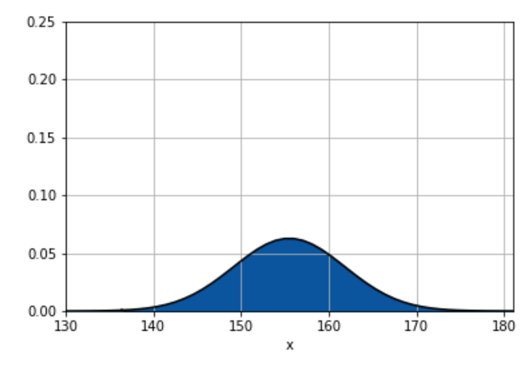
0.375 > 0.05 --> fail to reject NULL HYPOTHESIS

We are good till here with the example of the flipping the coin but what if we want to calculate the probability and p-value for heights of the people?

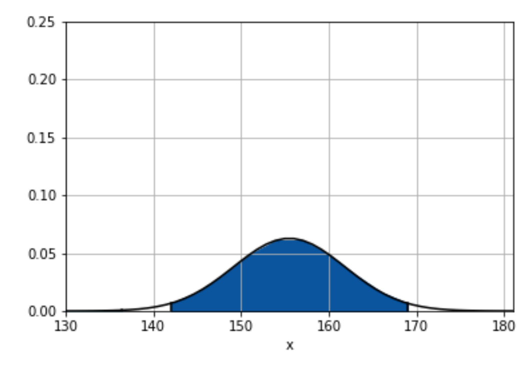
theoretically we give a try to list all the values for height however in pratical we calculate the probability and p-value for height by using **Statistical distribution**

Assume we have a distribution of height measurements from INDIAN WOMEN between 15-49 years old taken in 1996.

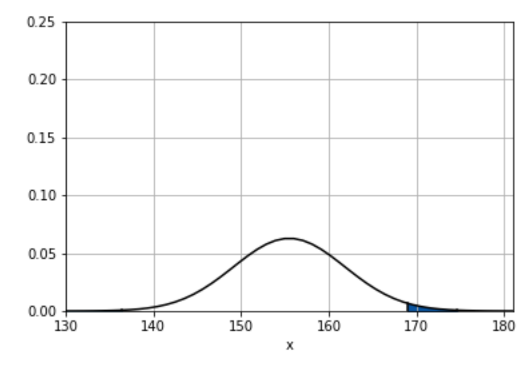
The blue region tells us the probability that a person's height will be within a range of possible values



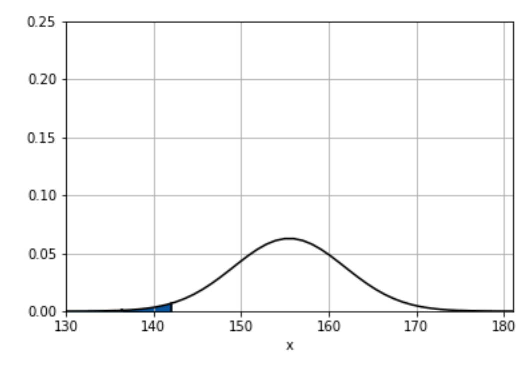
Assume 95% of area is under curve between 142cm and 169cm in other words 95% probability that each time we measure height it will be between 142cm - 169cm



2.5% of total area under the curve is greater than 169cm that means there is 2.5% probability that each time we measure the height it will be greater than 169cm



similarly there is 2.5% of total area under curve less that 142cm which means there is 2.5% probability that each time we measure the height it will be less than 142cm



In order to calculate the P-value with a distribution we must add up the percentage of area under the curve

for example imagine we measure someone's height around 142cm we might wonder if it came from distribution which has average value of 155.7

$$or$$

of it came from another distribution which has average value of 142

so the question is,

`is this measurement 142cm so far away from the mean of distribution with 155.7cm avg value that can reject the idea that it came from it?`

if so then that would suggest that the another distribution with 142cm avg value might do better in explaining data

P-Value for the hypotheses " measurement comes from curve with 155.5cm avg value "

P-Value for 142cm with 155.5 avg value curve = (2.5% area less or equal to 142cm)+(2.5% area more or equal to 169cm) = 0.025+0.025 = 0.05

So the P-Value for the hypothesis

`Some one 142cm tall could be from the curve with 155.7 avg value is 0.05`

however our threshold value is 0.05 and actual p-value we got is also 0.05. Maybe it could come from distribution with avg value 155.7cm, maybe not .Its hard to say since the p-value is on the threshold.

If we measure someone who is 141cm tall the p-value would be 0.016

P-Value for 141cm with 155.5 avg value curve = 0.016+0.016 = 0.03

which is less than 0.05 we can reject the hypothesis that given the distribution with 155.7cm avg value its normal to measure someone 141cm tall

and that suggest that different distribution of height explains better.

ONE SIDED P-VALUE

Imagine we created a new drug and wanted to see if it helped people recover in less days

if we gave it to some people and avg recovery was 4.5 days

For a one sided p-value we decide which direction we want to see change in. In this case we want the drug to shorten the time it takes to recover from the illness.

because we want to see change in the direction less 5 days the only more extreme vaues are < 4.5 days

vaues are > 4.5 days are less extreme

so when we calculate a one sided p-value we only calculate that area in the desired direction we want to see the change 0.016

One-Sided p-value for 4.5 days = 0.016 so some other distribution might give better explination

Now imagine the drug wasn't that effective and took almost 15.5 days on average to recover

if we want to see the shorter recovery times then it is 0.98

since 0.98 > 0.05 one sided p-value would not detect that drug was doing anything unusual.

One sided p-value is only looking to see if a distribution to left of the original mean makes more sense and since the observation is on the right side of the mean we fail to reject the hypothesis that original distribution explians better

This is why one-sided p-value is some what dangerous