Opportunism and ordering strategies in derivative-free optimization

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> > **GERAD**





Presentation Outline

- 1 What are Derivative-Free and Blackbox Optimization?
- 2 Direct-search Methods
- Opportunism and ordering
- Mumerical results

- 1 What are Derivative-Free and Blackbox Optimization?
- ② Direct-search Methods
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Derivative-Free and Blackbox Optimization

Optimization problem :

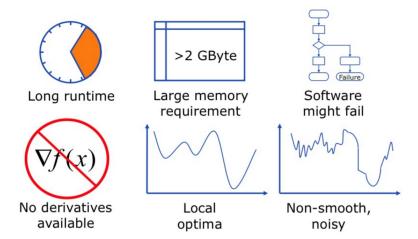
$$\begin{cases} \min_{\mathbf{x} \in \mathbb{R}^n} & f(\mathbf{x}) \\ \text{s.à.} & c_j(\mathbf{x}) \le 0 \ \forall j \in \{1, \dots, m\} \\ & l_i \le x_i \le u_i \ \forall i \in \{1, \dots, n\} \end{cases}$$

Optimization problem:

$$\begin{cases} \min_{x \in \mathbb{R}^n} & f(x) \\ \text{s.à.} & c_j(x) \le 0 \quad \forall j \in \{1, \dots, m\} \\ & l_i \le x_i \le u_i \quad \forall i \in \{1, \dots, n\} \end{cases}$$

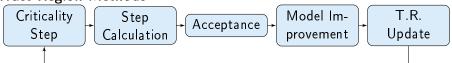
• f(x) and $c_i(x)$ are treated as blackboxes.

Blackbox



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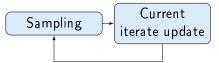
Trust Region Methods



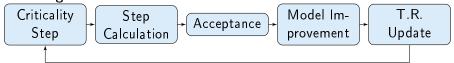
Trust Region Methods



Direct-search Methods



Trust Region Methods



Direct-search Methods



Trust Region Methods



Direct-search Methods



Trust Region Methods



Direct-search Methods



Other methods

Trust Region Methods



Direct-search Methods



Other methods

Heuristics (Particle Swarm, Simulated Annealing)

Trust Region Methods



Direct-search Methods



Other methods

Heuristics (Particle Swarm, Simulated Annealing)

Hybrids (Implicit Filtering)

Our goal: reduce the amount of calls to the blackbox

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To do so, we consider the **Opportunistic Strategy**.

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The opportunistic strategy designates the premature termination of an algorithmic step as soon as the necessary conditions to proceed to the next step are met.

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To do so, we consider the **Opportunistic Strategy**.

Opportunistic Strategy

The opportunistic strategy designates the premature termination of an algorithmic step as soon as the necessary conditions to proceed to the next step are met.

Often mentionned but never studied per se.

- What are Derivative-Free and Blackbox Optimization?
- ② Direct-search Methods
- Opportunism and ordering
- 4 Numerical results

Question 1.

For which methods is the opportunistic strategy applicable?

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- Already and inevitably opportunistic
- Directional direct-search methods \checkmark
- Allows opportunistic termination.
- Hybrid directional direct-search methods
- Same as direct-search methods, but might impact performance.
- Heuristics?
- Highly dependent on the heuristic.

Direcitonal direct-search framework

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• Samples f(x) and c(x) on \mathcal{L}^k , a list of point specific to the k^{th} iteration.

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- Generates the next list of candidates \mathcal{L}^{k+1} based on these values.

Algorithm 1 Directional direct-search framework

```
for k = 1, 2, ... do
   Search: Evaluate f(x) at a finite set of point S^k.
   If successful, update x^k
```

```
Poll: Evaluate f(x) at a finite set of point
   P^k := \{x^k + \delta^k d : d \in D\}, where D is a positive spanning
   set of directions.
   If successful, update x^k and mesh parameters.
end for
```

Remark: this work only encompasses the study of the impact of the opportunistic strategy on poll steps.

Coordinate Search (CS)

Algorithm 2 Coordinate Search

for
$$k=1,2,\ldots$$
 do Poll: Evaluate $f(x)$ at $P^k:=\{x^k+\delta^kd:d\in D_{\oplus}\}$, where $D_{\oplus}:=\{\pm e_1,\pm e_2,\ldots,\pm e_n\}$. If \exists t for which $f(t)< f(x^k)$, $t\in P^k$ Successful step update $x^{k+1}\leftarrow t$ et $\delta^{k+1}\leftarrow \delta^k$. Else \nexists t for which $f(t)< f(x^k)$, $t\in P^k$ Unsuccessful step update $x^{k+1}\leftarrow x^k$ et $\delta^{k+1}\leftarrow \frac{\delta^k}{2}$.

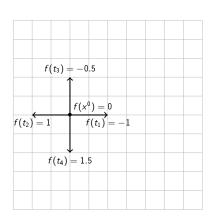


Figure: CS

end for

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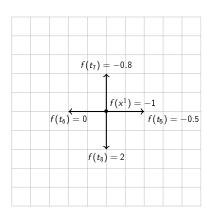


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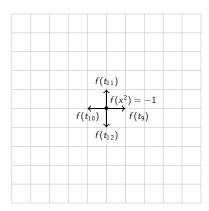


Figure: CS

end for

Generalized Pattern Search (GPS)

Algorithm 3 Generalized Pattern Search

```
\begin{array}{l} \text{for } k=1,2,\dots \text{ do} \\ \text{ with } \tau \in \{0,1\}. \\ \text{Poll} : \text{ Evaluate } f(x) \text{ at } \\ P^k := \{x^k + \delta^k d : d \in D\}, \text{ where } \\ D \text{ is a positive spanning set.} \end{array}
```

If \exists t for which $f(t) < f(x^k)$, $t \in P^k$ Successful step update $x^{k+1} \leftarrow t$ et $\delta^{k+1} \leftarrow \tau^{-1} \delta^k$.

Else \nexists t for which $f(t) < f(x^k)$, $t \in P^k$

Unsuccessful step update $x^{k+1} \leftarrow x^k$ and $\delta^{k+1} \leftarrow \tau \delta^k$. end for

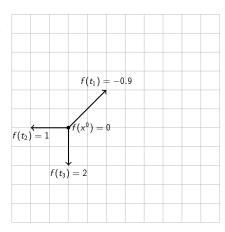


Figure: GPS

Generalized Pattern Search (GPS)

Algorithm 4 Generalized Pattern Search

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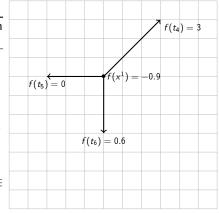


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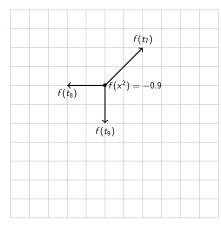


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Generating Set Search (GSS)

Algorithm 4 Generating Set Search

for k = 1, 2, ... do

```
with \tau \in \{0, 1\}.

Poll: Evaluate f(x) at P^k := \{x^k + \delta^k d : d \in D\}, where D is a positive spanning set respecting multiple conditions.

If \exists t for which f(t) < f(x^k), t \in P^k
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Successful step update
$$x^{k+1} \leftarrow t$$
 et $\delta^{k+1} \leftarrow \phi \delta^k$. Else $\nexists t$ for which $f(t) < f(x^k)$, $t \in P^k$ Unsuccessful step update $x^{k+1} \leftarrow x^k$ and $\delta^{k+1} \leftarrow \tau \delta^k$. end for

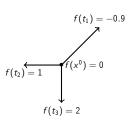


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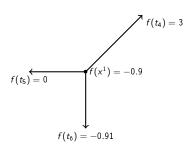


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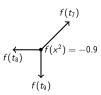


Figure: GSS

end for

Mesh Adaptive Direct Search (MADS)

Algorithm 5 Mesh Adaptive Direct Search

```
for k = 1, 2, ... do
    with \tau \in \{0, 1\}.
    Update: \delta^k \leftarrow \min(\Delta^k, (\Delta^k)^2)
    Poll: Evaluate f(x) at
    P^k := \{x^k + \delta^k d : d \in D\}, where
    D \subset F^k, with F^k frame of size \Delta^k.
    If \exists t for which f(t) < f(x^k), t \in P^k
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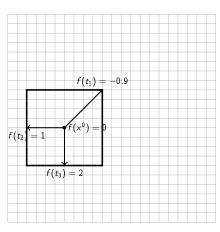


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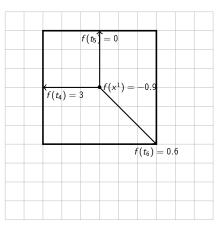


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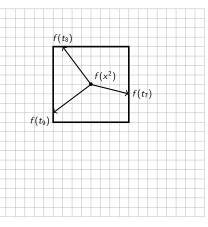


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Implicit Filtering (IMFIL)

Algorithm 6 Implicit Filtering

for
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 do Poll: Evaluate $f(x)$ at $P^k:=\{x^k+\delta^kd:d\in D_{\oplus}\}$, where $D_{\oplus}:=\{\pm e_1,\pm e_2,\ldots,\pm e_n\}$. If \exists t for which $f(t)< f(x^k)$, $t\in P^k$ Successful step Line search following $-\nabla_s f(x^k)$ update $x^{k+1}\leftarrow t$ et $\delta^{k+1}\leftarrow \delta^k$. Else \nexists t for which $f(t)< f(x^k)$, $t\in P^k$ Unsuccessful step update $x^{k+1}\leftarrow x^k$ et $\delta^{k+1}\leftarrow \delta^k$. end for

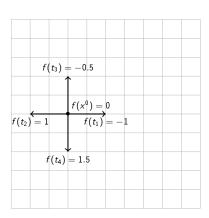


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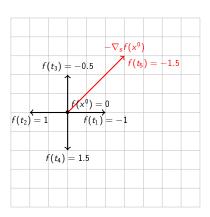


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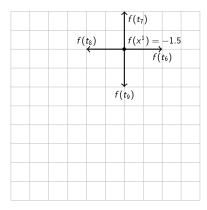


Figure: IMFIL

- Opportunism and ordering

Complete polling

Designates the evaluation of f(x) and c(x) at every point generated in the poll step.

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Simple opportunistic strategy

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Opportunistic strategy after p success

Opportunistic termination of the poll following the evaluation of p successful points.

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Opportunistic strategy after q evaluations

Opportunistic termination of the poll following the evaluations of q points.

Ordering strategy

Rule guiding the ordering of points in a set \mathcal{L} .

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Lexicographic

Ordered as in a dictionary, i.e. $(0,0,1) \prec (0,0,3) \prec (0,1,0)$.

Ordering strategy

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 if $\tilde{f}(A) < \tilde{f}(B)$

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where $\tilde{f}(x)$ is a dynamic quadratic surrogate of f(x).

Use f(x) as a surrogate for f(x) to simulate the best ordering possible.

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Omniscient

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6 Omniscient

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Omniscient

$$A \prec B$$
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Use -f(x) as a surrogate for f(x) to simulate the worst ordering possible.

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Reverse Omniscient

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 if $f(A) > f(B)$

No practical use, for comparaison only.

- What are Derivative-Free and Blackbox Optimization?
- ② Direct-search Methods
- Opportunism and ordering
- Mumerical results

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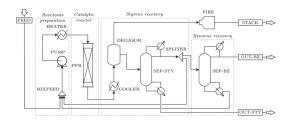


Figure: Styrene production chart [?]

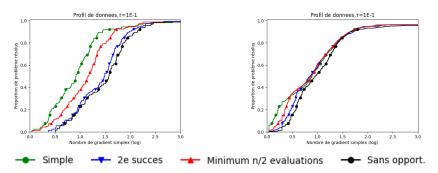


Figure: Left: CS on Moré-Wild, Right MADS on Moré-Wild

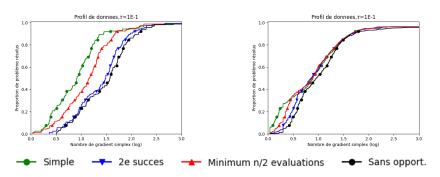


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Model ordering used.

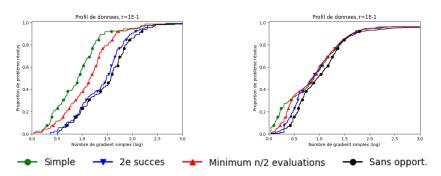


Figure: Left: CS on Moré-Wild, Right MADS on Moré-Wild

- Model ordering used.
- Most efficient opprtunistic strategy: simple

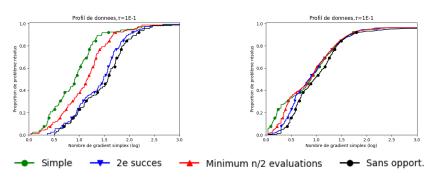


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Impact less important on MADS.

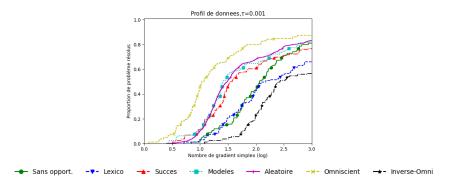


Figure: CS on Moré-Wild

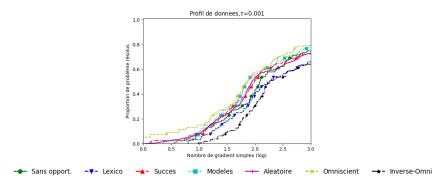


Figure: GPS on Moré-Wild

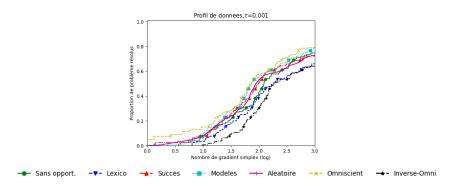


Figure: GPS on Moré-Wild

 Omniscient strategy less imactful.

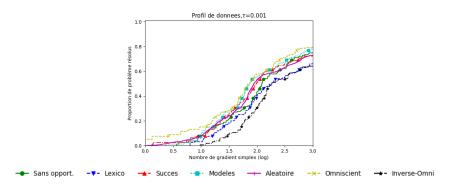


Figure: GPS on Moré-Wild

1 Omniscient strategy less imactful.

2 Model ordering less dominant