Report Lab 3 MARY Loïc

In [1]:
```
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
import pandas as pd
from IPython.display import display
from sklearn.model_selection import train_test_split
from sklearn.preprocessing import StandardScaler
from sklearn.discriminant_analysis import LinearDiscriminantAnalysis
from sklearn.metrics import roc_auc_score
from sklearn.metrics import accuracy_score
from sklearn.model_selection import GridSearchCV
from sklearn.discriminant_analysis import QuadraticDiscriminantAnalysis
from sklearn.naive_bayes import GaussianNB
from sklearn.linear_model import LogisticRegression
from sklearn.neural_network import MLPClassifier
from sklearn.ensemble import RandomForestClassifier
from sklearn.svm import SVC
from sklearn.ensemble import GradientBoostingClassifier
from sklearn.metrics import confusion_matrix, ConfusionMatrixDisplay,classification_report
from sklearn.inspection import permutation_importance
```

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1. Datasets

Load digit dataset

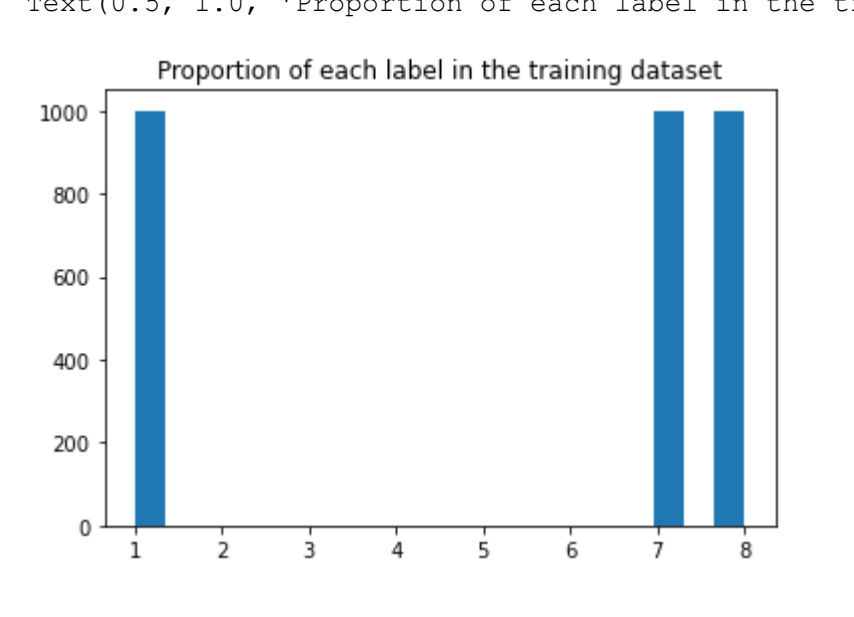
In [2]:
```
digit = np.load("digits.npz")
```

Because digit['x'].shape[0] > digit['xt'].shape[0] , I consider that x and y are the training dataset and xt,yt the test dataset

In [3]:
```
x2_train = (1/255)*digit["x"]
y2_train = np.ravel(digit["y"])

x2_test = (1/255)*digit["xt"]
y2_test = np.ravel(digit["yt"])
```

In [5]:
```
plt.hist(y2_train,bins=20)
plt.title("Proportion of each label in the training dataset")
```

Out[5]: `Text(0.5, 1.0, 'Proportion of each label in the training dataset')`



In the training dataset , all the classes are balanced. Indeed, for each class, there is the same number of samples (1000). There are 3 classes and each class represents 1/3 of the dataset

Load pima dataset

In [7]:
```
pima = np.load("pima.npz")
```

In [8]:
```
pima.files
```

Out[8]: `['xall', 'yall', 'varnames']`

In [9]:
```
list(pima['varnames'])
```

Out[9]:
```
['Pregnancies',
 'Glucose',
 'BloodPressure',
 'SkinThickness',
 'Insulin',
 'BMI',
 'DiabetesPedigreeFunction',
 'Age']
```

In [10]:
```
Xall = pima["xall"]
Yall = pima["yall"]
```

In [11]:
```
Xall.shape
```

Out[11]: `(709, 8)`

In [12]:
```
df = pd.DataFrame(Xall, columns = list(pima["varnames"]))
df["diabete"] = Yall
```

In [13]:
```
df.head()
```

Out[13]:

	Pregnancies	Glucose	BloodPressure	SkinThickness	Insulin	BMI	DiabetesPedigreeFunction	Age	diabete
0	7.0	159.0	64.0	0.0	0.0	27.4	0.294	40.0	-1
1	0.0	180.0	66.0	39.0	0.0	42.0	1.893	25.0	1
2	1.0	146.0	56.0	0.0	0.0	29.7	0.564	29.0	-1
3	2.0	71.0	70.0	27.0	0.0	28.0	0.586	22.0	-1
4	7.0	103.0	66.0	32.0	0.0	39.1	0.344	31.0	1

Discussion about the metrics used in the lab

This lab is a classification lab. Thus all samples belong to an unique class.
For the pima dataset , the classes are -1 (not diabetic) and 1 (diabetic).
For the digit dataset , the classes are 1, 7 and 8.

The first metric is the Accuracy (ACC) , it gives the proportion of good classification on the test dataset. More ACC is close to 1 , and more the model predicts well the class.

The other metric is the Area Under the Receiver Operating Curve (AUC), it measures the capacity of a model to separe well the classes. We plot the True Positive Rate depending on the False Positive rate with different threshold. The threshold allows to assign a class to a sample (by default if's 0.5 , if the propability to belong to class 1 is higher than 0.5 than this element is classified in class 1). Then, we measure the area under this curve. More the AUC is close to 1 and more the model has a very good capacity to separe the class and thus to have a model that predicts very well.

An important thing is that all those metrics are computed on the test dataset and the best parameters are estimated in a validation set (part of the training set) through GridSearch CV !

2.Predicting Diabetes on the Pima dataset

2.1 Know the data

In [14]:
```
sns.pairplot(df, hue = 'diabete' , palette = ["blue","red"])
```

Out[14]: `<seaborn.axisgrid.PairGrid at 0x24f6d27d550>`

To see the variables that seem to help predict the class , the histograms are better tool than scatter plot because we can directly see if the distributions created by 2 variables are similar or not. There are always for pairplot , 2 distributions that represent each class. So we can see the repartition of a feature depending on the class.

We remark in the diagonal of the plot above that for the features Glucose, BMI and Age , the two distributions are not the same. They dont' have the same mean and variance.
What is interesting is that thoses features are linked to the diabete desease from a medical point of view :

- The older you get, the more likely you are to have diabetes because the pancreas is not working as well
- the higher the blood glucose threshold, the more likely you are to have diabetes
- The BMI (Body Mass Index) , if it is high can mean that the patient is obese and therefore he has more chance to have diabete

An other interesting representation can be the correlation matrix to see the correlation between all the variables and more precisely the correlation between all the features and the target variable "

In [15]:
```
plt.figure(figsize=(8, 6))
sns.heatmap(round(df.corr(),2), annot=True)
plt.title("Correlation Matrix of the features and the target Variable")
plt.show()
```

We should focus on the last colums which represents the correlation between the target variable "diabete" (Y) and all the others features.
We remark that the variables most (positively) correlated to Y are "Glucose","BMI" and "Age". Thoses are the features that I highlight the importance to predict Y with the histograms.

In [16]:
```
round(df.describe(),2)
```

Out[16]:

	Pregnancies	Glucose	BloodPressure	SkinThickness	Insulin	BMI	DiabetesPedigreeFunction	Age	diabete
count	709.00	709.00	709.00	709.00	709.00	709.00	709.00	709.00	709.00
mean	3.75	121.61	69.64	20.77	80.53	32.45	0.47	33.02	-0.32
std	3.34	30.49	18.14	15.90	112.68	6.95	0.33	11.67	0.95
min	0.00	44.00	0.00	0.00	0.00	18.20	0.08	21.00	-1.00
25%	1.00	99.00	64.00	0.00	0.00	27.50	0.24	24.00	-1.00
50%	3.00	117.00	72.00	23.00	40.00	32.30	0.37	29.00	-1.00
75%	6.00	141.00	80.00	32.00	130.00	36.60	0.63	40.00	1.00
max	17.00	199.00	122.00	99.00	744.00	67.10	2.42	81.00	1.00

The features have not the same variance and the same scaling. It is important for all the features in a model to have the same scaling, so we should normalize the data. Indeed, it equalizes the weight of each feature and it allows some algorithms to converge faster or to work simply.

In the following parts, we will split our dataset in a train and test set to evaluate the performance of several models to determine if a patient given the features, is diabetic or not.

In [17]:
```
Xall_train, Xall_test, Yall_train, Yall_test = train_test_split(Xall, Yall, train_size=300, random_stat
e=42)
```

In [18]:
```
print("Number of elements in class 1 : ",np.sum(Yall_train[Yall_train == 1]))
print("Number of elements in class -1 : ",np.abs(np.sum(Yall_train[Yall_train == -1])))
```

```
Number of elements in class 1 :  112
Number of elements in class -1 :  188
```

In the training set, there are a bit more of examples for the class -1 but there is not a class that is predominant. So, the model can really train effectively and its capacity to generalize could be good.

In [19]:
```
#Standardization of the data
scaler = StandardScaler()
scaler.fit(Xall_train)
X_train_norm = scaler.transform(Xall_train)
X_test_norm = scaler.transform(Xall_test)
```

In [20]:
```
def performances_model_default(model,X_train_norm,Yall_train,X_test_norm,Yall_test):
    model.fit(X_train_norm , Yall_train)
    auc = roc_auc_score(Yall_test, model.predict_proba(X_test_norm)[:, 1])
    acc = model.score(X_test_norm,Yall_test)

    name_model = str(model).replace('()','')

    dic={}
    dic["model"] = name_model
    dic["AUC"] = round(auc,3)
    dic["ACC"] = round(acc,3)


    return dic
```

In [21]:
```
def performances_grid_model(model,grid_parameters,X_train_norm,Yall_train,X_test_norm,Yall_test):
    model_grid = GridSearchCV(model,grid_parameters,cv=5,scoring="accuracy")
    model_grid.fit(X_train_norm,Yall_train)

    auc = roc_auc_score(Yall_test, model_grid.predict_proba(X_test_norm)[:, 1])
    acc = model_grid.score(X_test_norm,Yall_test)

    name_model = str(model).replace('()','')

    dic={}
    dic["model"] = name_model
    dic["AUC"] = round(auc,3)
    dic["ACC"] = round(acc,3)
    dic['Best Parameters'] = model_grid.best_params_

    return dic
```

2.2 Bayesian decision and linear classification

2.2.1 Linear Discriminant Analysis (LDA)

2.2.1.1 Default Model

The default parameter for LDA is :

- shrinkage = None (no shrinkage)

In [22]:
```
performances_model_default(LinearDiscriminantAnalysis(),X_train_norm,Yall_train,X_test_norm,Yall_test)
```

Out[22]: `{'model': 'LinearDiscriminantAnalysis', 'AUC': 0.831, 'ACC': 0.787}`

2.2.1.2 Best Parameters

In [23]:
```
parameters_lda = {'shrinkage': np.linspace(0,1,100)}
```

In [24]:
```
performances_grid_model(LinearDiscriminantAnalysis(solver='eigen'),parameters_lda,X_train_norm,Yall_tra
in,X_test_norm,Yall_test)
```

Out[24]:
```
{'model': "LinearDiscriminantAnalysis(solver='eigen')",
 'AUC': 0.853,
 'ACC': 0.797,
 'Best Parameters': {'shrinkage': 0.37373737373737376}}
```

It seems that LDA with a shrinkage of 0.38 gives best result than LDA with default parameter. Shrinkage is a regularization parameter that reduce overfitting. Thus, the capacity of this model to generalize is better.

The metrics are quite close to 1 , which means that this is a good model. But it is difficult to judge its performance without comparing it with others models.

2.2.2 Quadratic Discriminant Analysis (QDA)

2.2.2.1 Default Mode

The default parameter for QDA is :

- reg_param = 0

In [25]:
```
performances_model_default(QuadraticDiscriminantAnalysis(),X_train_norm,Yall_train,X_test_norm,Yall_tes
t)
```

Out[25]: `{'model': 'QuadraticDiscriminantAnalysis', 'AUC': 0.814, 'ACC': 0.768}`

2.2.2.2 Best Parameters

In [26]:
```
parameters_qda = {"reg_param" : np.linspace(0,1,100)}
```

In [27]:
```
performances_grid_model(QuadraticDiscriminantAnalysis(),parameters_qda,X_train_norm,Yall_train,X_test_n
orm,Yall_test)
```

Out[27]:
```
{'model': 'QuadraticDiscriminantAnalysis',
 'AUC': 0.828,
 'ACC': 0.775,
 'Best Parameters': {'reg_param': 0.1919191919191919}}
```

It seems that QDA with reg_param = 0.20 is better than QDA with default parameter. The reason can be that reg_param is a regularization parameter and most of the time regularization increase the capacity of a model to generalize.

But this model is not better than LDA even if its metrics are good.

2.2.3 Gaussian Naive Bayes

In [28]:
```
gnb = GaussianNB()
gnb.fit(X_train_norm,Yall_train)
```

Out[28]: `GaussianNB()`

In [29]:
```
performances_model_default(GaussianNB(),X_train_norm,Yall_train,X_test_norm,Yall_test)
```

Out[29]: `{'model': 'GaussianNB', 'AUC': 0.842, 'ACC': 0.804}`

In terms of AUC , Gaussian Naive Bayes is better than QDA but less efficient than LDA. But , regarding ACC , this model is better than the previous one.

2.2.4 Logistic Regression

2.2.4.1 Default Model

The default parameter for Logistic Regression is :

- C = 1.0

In [30]:
```
performances_model_default(LogisticRegression(random_state=0),X_train_norm,Yall_train,X_test_norm,Yall_
test)
```

Out[30]: `{'model': 'LogisticRegression(random_state=0)', 'AUC': 0.832, 'ACC': 0.775}`

2.2.4.2 Best Parameters

In [31]:
```
parameters_lr = {"C" : np.linspace(0.1,10,100) , "solver" : ['liblinear' , 'saga']}
```

In [32]:
```
performances_grid_model(LogisticRegression(penalty="l1",random_state=0),parameters_lr,X_train_norm,Yall
_train,X_test_norm,Yall_test)
```

Out[32]:
```
{'model': "LogisticRegression(penalty='l1', random_state=0)",
 'AUC': 0.83,
 'ACC': 0.775,
 'Best Parameters': {'C': 3.5000000000000004, 'solver': 'saga'}}
```

The model with the best parameter C is better than the default one because of the regularization C. The model is better with an important regularization.

In terms of ACC and AUC, Logistic Regression is the worst model so far.

Let's see the value of the weighs of each feature to interpret them.

In [33]:
```
lr_best = LogisticRegression(penalty='l1',random_state=0,C=3.5000000000000004,solver='saga')
lr_best.fit(X_train_norm,Yall_train)
```

Out[33]:
```
LogisticRegression(C=3.5000000000000004, penalty='l1', random_state=0,
                   solver='saga')
```

In [110]:
```
coef_lr = {}
coef_lr["weight"] = lr_best.coef_[0]
coef_lr["Correlation with diabete"] = df.corr()['diabete']
coef_lr_dataframe = pd.DataFrame(coef_lr,index=list(pima['varnames']))
display(coef_lr_dataframe)
```

	weight	Correlation with diabete
Pregnancies	0.594613	0.220255
Glucose	1.435167	0.499845
BloodPressure	-0.366214	0.056029
SkinThickness	0.012138	0.055354
Insulin	-0.109993	0.113659
BMI	0.442325	0.311803
DiabetesPedigreeFunction	0.319019	0.174826
Age	-0.233396	0.229811

In [117]:
```
plt.figure(figsize=(20,5))
plt.stem(list(pima['varnames']), np.ravel(lr_best.coef_[0]),use_line_collection=True)
plt.title("Weights for features for Logistic Regression")
plt.show()
```

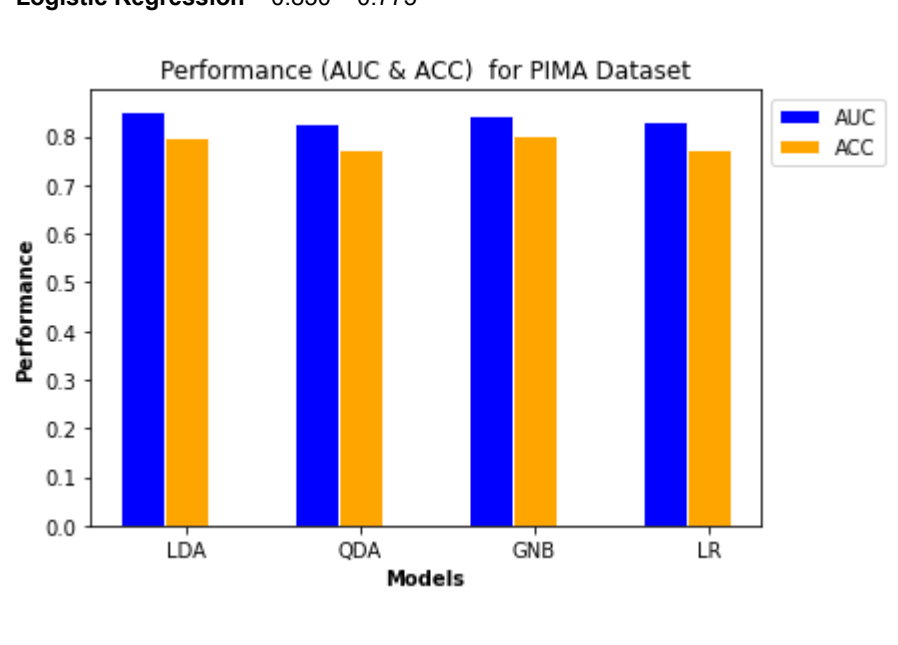
Even with a L1 penalization, any weight is equal to 0. But we remark that for the feature "SkinThickness", its weight has not the same order of magnitude as the other features. Indeed, for the majority of the features, their weights are around 1 or 0.1 (in absolute value) but for "SkinThickness" it is around 0.01.
So it seems that this feature is less important in the model than the others. And in fact, if we see the correlation between "SkinThickness" and "diabete", it is equal to 0.06 (pretty close to 0), so it makes sense !

2.2.4 Review of Bayesian decision & Linear Classification

Let's recap the performance of all models used so far

In [35]:
```
dic_linear_model = {}
Models = ["LDA","QDA","Gaussian NB","Logistic Regression"]
dic_linear_model["AUC"] = [0.853,0.828,0.842,0.830]
dic_linear_model["ACC"] = [0.797,0.775,0.804,0.775]
perf_linear_model_dataframe = pd.DataFrame(dic_linear_model,index = Models)
display(perf_linear_model_dataframe)


# set width of bars
barWidth = 0.25
Models_bis = ["LDA","QDA","GNB","LR"]
# set heights of bars
bars1 = dic_linear_model["AUC"]
bars2 = dic_linear_model["ACC"]

# Set position of bar on X axis
r1 = np.arange(len(bars1))
r2 = [x + barWidth for x in r1]

# Make the plot
plt.bar(r1, bars1, color='blue', width=barWidth, edgecolor='white', label='AUC')
plt.bar(r2, bars2, color='orange', width=barWidth, edgecolor='white', label='ACC')


# Add xticks on the middle of the group bars
plt.xlabel('Models', fontweight='bold')
plt.ylabel('Performance', fontweight='bold')
plt.title("Performance (AUC & ACC)  for PIMA Dataset")
plt.xticks([r + barWidth for r in range(len(bars1))], Models_bis)

# Create legend & Show graphic
plt.legend(bbox_to_anchor=(1, 1))
plt.show()
```

	AUC	ACC
LDA	0.853	0.797
QDA	0.828	0.775
Gaussian NB	0.842	0.804
Logistic Regression	0.830	0.775



Generally, LDA is the best model so far. But in terms of ACC, Gaussian Naive Bayes is is a little better than LDA. As AUC measures the capacity of separability of classes, I consider that this metric is more interesting than ACC to estimate the capacity of generalization of a model (even if a good model should have also an ACC close to 1).

But it is important to note that the metrics for all those models are quite close . They are all superior than 0.80 (for AUC) and 0.78 (for ACC) which mean that those models are quite perfomant to separate well the class and thus to predict well the class for new samples.

Let's compare those performances with non linear methods

2.3 Nonlinear methods

Remark : For those models I don't use linspace or logspace for SearchGrid CV , because my computer made too long time to give a result! So I use several values that fill the space of integers that I wanted to explore.

2.3.1 Random Forest Classifier

2.3.1.1 Default Model

The default parameters for Random Forest Classifier are :

- n_estimators = 100
- criterion = 'gini'
- max_dept = None

In [36]:
```
performances_model_default(RandomForestClassifier(random_state = 0),X_train_norm,Yall_train,X_test_norm
```

Out[36]: `{'model': 'RandomForestClassifier(random_state=0)', 'AUC': 0.844, 'ACC': 0.78}`

2.3.1.2 Best Parameters

In [37]:
```
parameters_rfc = {'n_estimators' : [5,10,30,50,100,150,200,250,300 ], 'criterion' : ['gini','entropy'],
                  'max_depth' : [5,10,30,50,100,150,200,250,300]+[None]}
```

In [38]:
```
performances_grid_model(RandomForestClassifier(random_state = 0),parameters_rfc,X_train_norm,Yall_train
,X_test_norm,Yall_test)
```

Out[38]:
```
{'model': 'RandomForestClassifier(random_state=0)',
 'AUC': 0.838,
 'ACC': 0.782,
 'Best Parameters': {'criterion': 'entropy',
  'max_depth': 30,
  'n_estimators': 150}}
```

It seems that the model with the best parameter is slighty better than the default one for ACC but less better for ACC.

2.3.2 Support Vector Classification

2.3.2.1 Default Model

The default parameters are :

- C : 1.0
- kernel : 'rbf'
- degree = 3 (for polynomial kernel)

In [39]:
```
performances_model_default(SVC(probability=True),X_train_norm,Yall_train,X_test_norm,Yall_test)
```

Out[39]: `{'model': 'SVC(probability=True)', 'AUC': 0.833, 'ACC': 0.785}`

2.3.2.2 Best parameters

In [40]:
```
parameters_svc = {'kernel': ['poly','rbf']}
```

In [41]:
```
performances_grid_model(SVC(probability=True),parameters_svc,X_train_norm,Yall_train,X_test_norm,Yall_t
est)
```

Out[41]:
```
{'model': 'SVC(probability=True)',
 'AUC': 0.799,
 'ACC': 0.765,
 'Best Parameters': {'kernel': 'poly'}}
```

In [42]:
```
parameters_svc = {'C': np.linspace(1,10,10),'kernel': ['poly'],'degree':[1,2,3,4,5],}
```

In [43]:
```
performances_grid_model(SVC(probability=True),parameters_svc,X_train_norm,Yall_train,X_test_norm,Yall_t
est)
```

Out[43]:
```
{'model': 'SVC(probability=True)',
 'AUC': 0.833,
 'ACC': 0.782,
 'Best Parameters': {'C': 7.0, 'degree': 1, 'kernel': 'poly'}}
```

The default model is better than any SVC models with customized parameters.

2.3.3 Multi Layers Perceptron Classifier

2.3.3.1 Default Model

The default parameter for MLP is :

- hidden_layer_sizes = (100,)

In [44]:
```
performances_model_default(MLPClassifier(random_state=1 ),X_train_norm,Yall_train,X_test_norm,Yall_test
)
```

```
C:\Users\Public\anaconda3\lib\site-packages\sklearn\neural_network\_multilayer_perceptron.py:692: Con
vergenceWarning: Stochastic Optimizer: Maximum iterations (200) reached and the optimization hasn't c
onverged yet.
  warnings.warn(
```

Out[44]: `{'model': 'MLPClassifier(random_state=1)', 'AUC': 0.826, 'ACC': 0.748}`

2.3.3.2 Best Parameters

In [45]:
```
parameters_mlp = {'hidden_layer_sizes':[(50,),(100,),(200,),(500,)]}
```

In [80]:
```
digit_lr = LogisticRegression()
digit_lr.fit(x2_train,y2_train)
print(classification_report(y2_test,digit_lr.predict(x2_test)))
```

```
              precision    recall  f1-score   support

           1       0.94      0.99      0.96       500
           7       0.98      0.96      0.97       500
           8       0.97      0.95      0.96       500

    accuracy                           0.97      1500
   macro avg       0.97      0.97      0.97      1500
weighted avg       0.97      0.97      0.97      1500
```

```
C:\Users\Public\anaconda3\lib\site-packages\sklearn\linear_model\_logistic.py:814: ConvergenceWarnin
g: lbfgs failed to converge (status=1):
STOP: TOTAL NO. of ITERATIONS REACHED LIMIT.

Increase the number of iterations (max_iter) or scale the data as shown in:
    https://scikit-learn.org/stable/modules/preprocessing.html
Please also refer to the documentation for alternative solver options:
    https://scikit-learn.org/stable/modules/linear_model.html#logistic-regression
  n_iter_i = _check_optimize_result(
```

SVC

In [81]:
```
digit_svc = SVC()
digit_svc.fit(x2_train,y2_train)
print(classification_report(y2_test,digit_svc.predict(x2_test)))
```

```
              precision    recall  f1-score   support

           1       0.98      0.99      0.98       500
           7       0.99      0.97      0.98       500
           8       0.99      0.99      0.99       500

    accuracy                           0.98      1500
   macro avg       0.98      0.98      0.98      1500
weighted avg       0.98      0.98      0.98      1500
```

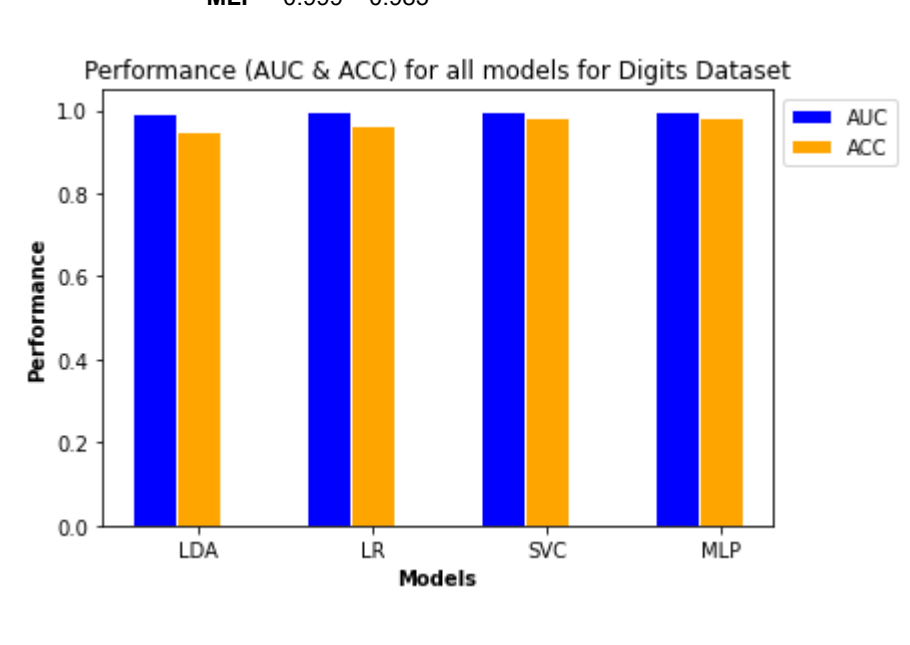
MLP Classifier

In [82]:
```
digit_mlp = MLPClassifier(hidden_layer_sizes=(200,500))
digit_mlp.fit(x2_train,y2_train)
print(classification_report(y2_test,digit_mlp.predict(x2_test)))
```

```
              precision    recall  f1-score   support

           1       0.98      0.99      0.99       500
           7       0.98      0.98      0.98       500
           8       0.99      0.98      0.99       500

    accuracy                           0.98      1500
   macro avg       0.98      0.98      0.98      1500
weighted avg       0.98      0.98      0.98      1500
```

In [83]:
```
dic_models_digit = {}
Models = ["LDA","Logistic Regression","SVC","MLP"]
dic_models_digit["AUC"] = [0.995,0.997,0.998,0.999]
dic_models_digit["ACC"] = [0.947,0.965,0.983,0.985]
perf_dic_models_digit_dataframe = pd.DataFrame(dic_models_digit,index = Models)
display(perf_dic_models_digit_dataframe)

# set width of bars
barWidth = 0.25
Models_bis = ["LDA","LR","SVC","MLP"]
# set heights of bars
bars1 = dic_models_digit["AUC"]
bars2 = dic_models_digit["ACC"]

# Set position of bar on X axis
r1 = np.arange(len(bars1))
r2 = [x + barWidth for x in r1]

# Make the plot
plt.bar(r1, bars1, color='blue', width=barWidth, edgecolor='white', label='AUC')
plt.bar(r2, bars2, color='orange', width=barWidth, edgecolor='white', label='ACC')


# Add xticks on the middle of the group bars
plt.xlabel('Models', fontweight='bold')
plt.ylabel('Performance', fontweight='bold')
plt.title("Performance (AUC & ACC) for all models for Digits Dataset")
plt.xticks([r + barWidth for r in range(len(bars1))], Models_bis)

# Create legend & Show graphic
plt.legend(bbox_to_anchor=(1, 1))
plt.show()
```

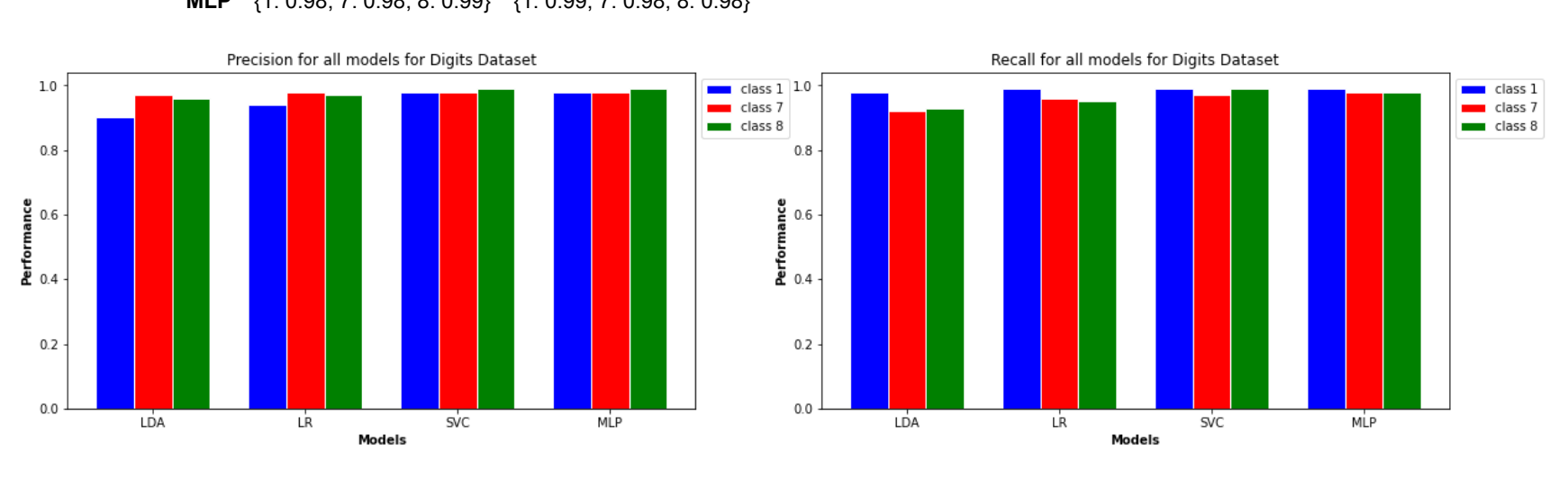
	AUC	ACC
LDA	0.995	0.947
Logistic Regression	0.997	0.965
SVC	0.998	0.983
MLP	0.999	0.985



In terms of AUC and ACC, MLP Classifier is the best although the others models have excellent score too (the metrics are always superior to 0.95).

In [84]:
```
dic_models_digit = {}
Models = ["LDA","Logistic Regression","SVC","MLP"]
dic_models_digit["Precision"] = [{1:0.90,7:0.97,8:0.96},{1:0.94,7:0.98,8:0.97},{1:0.98,7:0.98,8:0.99},{
1:0.98,7:0.98,8:0.99}]
dic_models_digit["Recall"] = [{1:0.98,7:0.92,8:0.93},{1:0.99,7:0.96,8:0.95},{1:0.99,7:0.97,8:0.99},{1:
0.99,7:0.98,8:0.98}]
perf_dic_models_digit_dataframe = pd.DataFrame(dic_models_digit,index = Models)
display(perf_dic_models_digit_dataframe)



plt.figure(figsize=(20,5))
plt.subplot(1,2,1)
# set width of bars
barWidth = 0.25
Models_bis = ["LDA","LR","SVC","MLP"]
# set heights of bars
bars1 = [x[1] for x in dic_models_digit["Precision"]]
bars2 = [x[7] for x in dic_models_digit["Precision"]]
bars3 = [x[8] for x in dic_models_digit["Precision"]]

# Set position of bar on X axis
r1 = np.arange(len(bars1))
r2 = [x + barWidth for x in r1]
r3 = [x + barWidth for x in r2]

# Make the plot
plt.bar(r1, bars1, color='blue', width=barWidth, edgecolor='white', label='class 1')
plt.bar(r2, bars2, color='red', width=barWidth, edgecolor='white', label='class 7')
plt.bar(r3, bars3, color='green', width=barWidth, edgecolor='white', label='class 8')


# Add xticks on the middle of the group bars
plt.xlabel('Models', fontweight='bold')
plt.ylabel('Performance', fontweight='bold')
plt.title("Precision for all models for Digits Dataset")
plt.xticks([r + barWidth for r in range(len(bars1))], Models_bis)

# Create legend & Show graphic
plt.legend(bbox_to_anchor=(1, 1))



plt.subplot(1,2,2)
# set width of bars
barWidth = 0.25
Models_bis = ["LDA","LR","SVC","MLP"]
# set heights of bars
bars1 = [x[1] for x in dic_models_digit["Recall"]]
bars2 = [x[7] for x in dic_models_digit["Recall"]]
bars3 = [x[8] for x in dic_models_digit["Recall"]]

# Set position of bar on X axis
r1 = np.arange(len(bars1))
r2 = [x + barWidth for x in r1]
r3 = [x + barWidth for x in r2]

# Make the plot
plt.bar(r1, bars1, color='blue', width=barWidth, edgecolor='white', label='class 1')
plt.bar(r2, bars2, color='red', width=barWidth, edgecolor='white', label='class 7')
plt.bar(r3, bars3, color='green', width=barWidth, edgecolor='white', label='class 8')


# Add xticks on the middle of the group bars
plt.xlabel('Models', fontweight='bold')
plt.ylabel('Performance', fontweight='bold')
plt.title("Recall for all models for Digits Dataset")
plt.xticks([r + barWidth for r in range(len(bars1))], Models_bis)

# Create legend & Show graphic
plt.legend(bbox_to_anchor=(1, 1))
plt.show()
```

	Precision	Recall
LDA	{1: 0.9, 7: 0.97, 8: 0.96}	{1: 0.98, 7: 0.92, 8: 0.93}
Logistic Regression	{1: 0.94, 7: 0.98, 8: 0.97}	{1: 0.99, 7: 0.96, 8: 0.95}
SVC	{1: 0.98, 7: 0.98, 8: 0.99}	{1: 0.99, 7: 0.97, 8: 0.99}
MLP	{1: 0.98, 7: 0.98, 8: 0.99}	{1: 0.99, 7: 0.98, 8: 0.98}



The precision and recall for each class and for each model are pretty close and more important they are close to 1 . But, in general, MLP still has the best result.
Regarding the recall, digit 1 is the class that is well classified (the recall for class 1 is higher than classes 7 and 8), for all the models. So, it seems that 1 is not hard to classify . For MLP, the recall for class 7 and 8 is almost the same , and therefore they are the classes that the model has difficulties to separate. Maybe , it is because the "white" pixel of a 8 and 7 are almost the same. But the recall for thoses classes is 0.98 which is very good !

For the precision for the MLP , they are almost the same and very close to 1.

In conclusion, all the model separates well the classes and MLP is the best one regarding all the metrics (AUC,ACC,Precision & Recall).

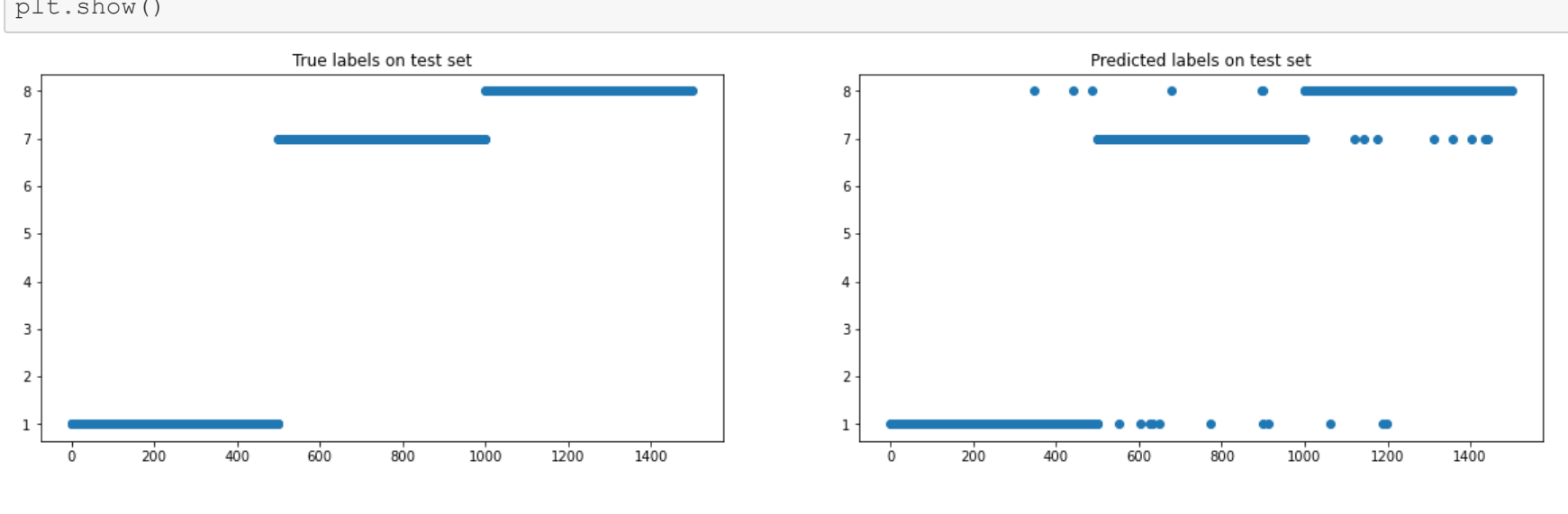
3.2.2 Study with the best model

3.2.2.1 Performances

In [85]:
```
best_model = MLPClassifier(random_state=1)
best_model.fit(x2_train,y2_train)
```

Out[85]:
```
MLPClassifier(random_state=1)
```

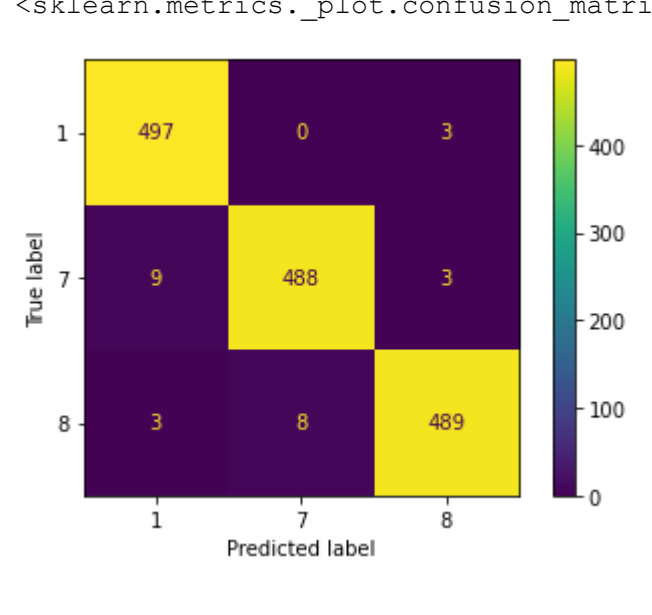
In [86]:
```
plt.figure(figsize=(20,5))
plt.subplot(1,2,1)
plt.plot(y2_test,"o")
plt.title("True labels on test set")
plt.subplot(1,2,2)
plt.plot(best_model.predict(x2_test),"o")
plt.title("Predicted labels on test set")
plt.show()
```



If we compare the two graphs, they are almost the same, which means that the classification is very good. There are some points that are not classified but the overwhelming majority is well classified. The model recognizes very well the digits !

In [87]:
```
cm_digit = confusion_matrix(y2_test , best_model.predict(x2_test))
cm_disp_digit = ConfusionMatrixDisplay(confusion_matrix=cm_digit , display_labels=best_model.classes_)
cm_disp_digit.plot()
```

Out[87]:
```
<sklearn.metrics._plot.confusion_matrix.ConfusionMatrixDisplay at 0x24f71275340>
```



Clearly, 7 and 8 are the most difficult classes to recognize even if there is a small amount of misclassification for 7 and 8. This remark is to connect with the recall of this model.

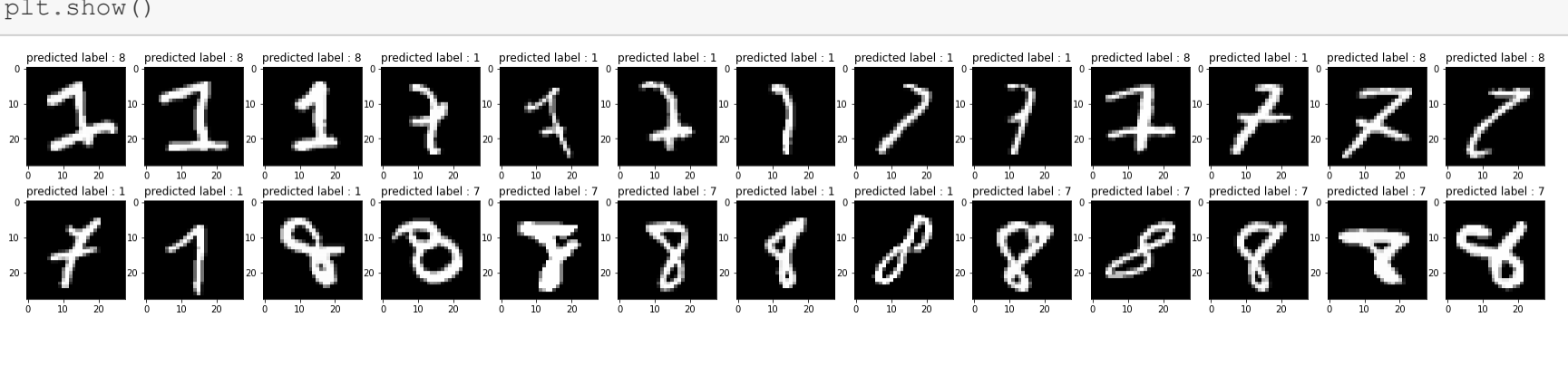
In [88]:
```
def classified_digit(y2_test,y2_predicted):
    bad_ind = [] #index of misclassified digit
    good_ind = [] #index of well classified digit
    n = len(y2_test)
    for i in range(n):
        if y2_test[i] != y2_predicted[i] :
            bad_ind.append(i)
        else :
            good_ind.append(i)
    return good_ind,bad_ind
```

In [89]:
```
y2_predicted = best_model.predict(x2_test)
```

In [90]:
```
good_predictions,bad_predictions = classified_digit(y2_test,y2_predicted)
```

In [92]:
```
def show_digit(x2,y2,index,title="label"):
    plt.imshow(x2[index].reshape(28,28),interpolation='none', cmap=plt.gray())
    plt.title(""+title+ f" : {y2[index]}")
```

In [93]:
```
plt.figure(figsize=(30,5))
row=2
col=13
ind=1
for indice in bad_predictions:
    plt.subplot(row,col,ind)
    show_digit(x2_test,y2_predicted,indice,"predicted label")
    ind+=1
plt.show()
```



For the misclassified data, the behaviour is the following :

- 8 is classified as 7
- 7 is classified as 1 (most of th time) and 8
- 1 is classified as 8 (in minority)
  The misclassification between 7 and 8 can be explained by the fact that 7 and 8 have many "white" pixels in common and therefore the model confuses the two digits.
  But for the fact that 1 is classified as 8 , I don't expected that. I would've said that 1 can be classified as 7 because depending to the handwritting of 1 , it could look like a 7.
  Finnaly, the fact that a 7 can be classified as 1 is predictable because the english handwritting of a 7 looks like a french handwritting of a 1 !

3.2.2.2 Adversial examples

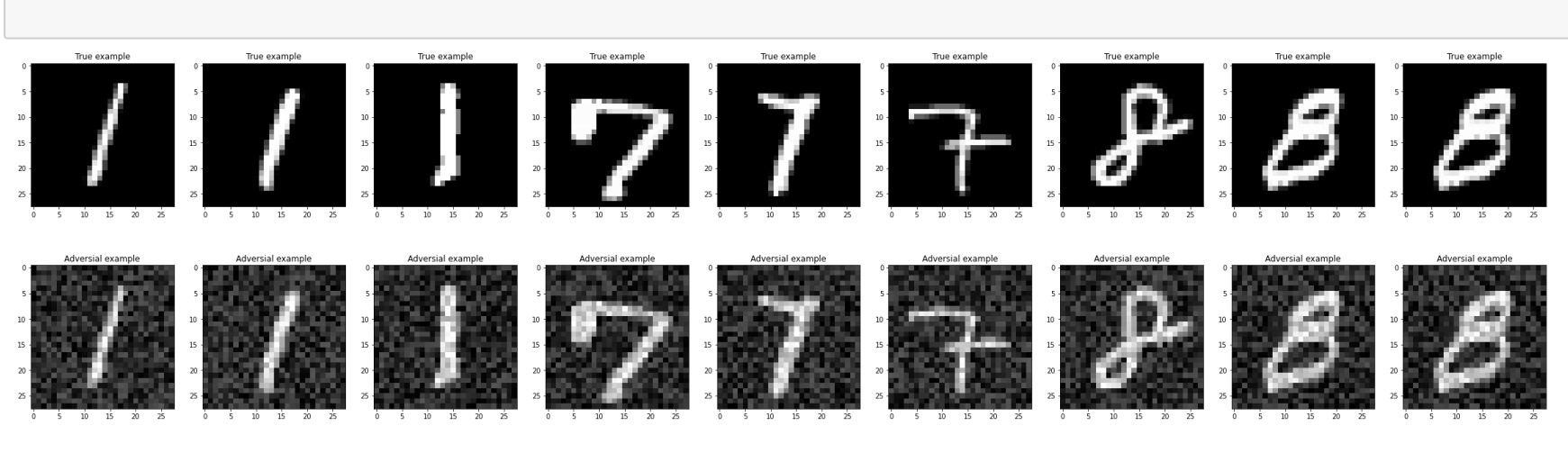
In [95]:
```
def adversial_examples_func(x2_test , ind):
    res = np.zeros((1000,784))
    for i in range(len(ind)):
        res[i] = x2_test[ind[i]]
    return res
```

In [96]:
```
indices = good_predictions[0:334]+good_predictions[500:834]+good_predictions[1000:1332]
```

In [97]:
```
adversial_examples = adversial_examples_func(x2_test,indices)
```

In [98]:
```
adversial_examples = adversial_examples + (0.5*np.random.rand(1000,784))
```

In [99]:
```
row = 2
col = 9
ind = 1
plt.figure(figsize=(40,10))
for i in [0,1,2,501,502,503,990,991,991]:
    plt.subplot(row,col,ind)
    plt.imshow(x2_test[indices[i]].reshape(28,28),interpolation='none', cmap=plt.gray())
    plt.title("True example")
    ind += 1
for i in [0,1,2,501,502,503,990,991,991]:
    plt.subplot(row,col,ind)
    plt.imshow(adversial_examples[i].reshape(28,28),interpolation='none', cmap=plt.gray())
    plt.title("Adversial example")
    ind +=1
plt.show()
```



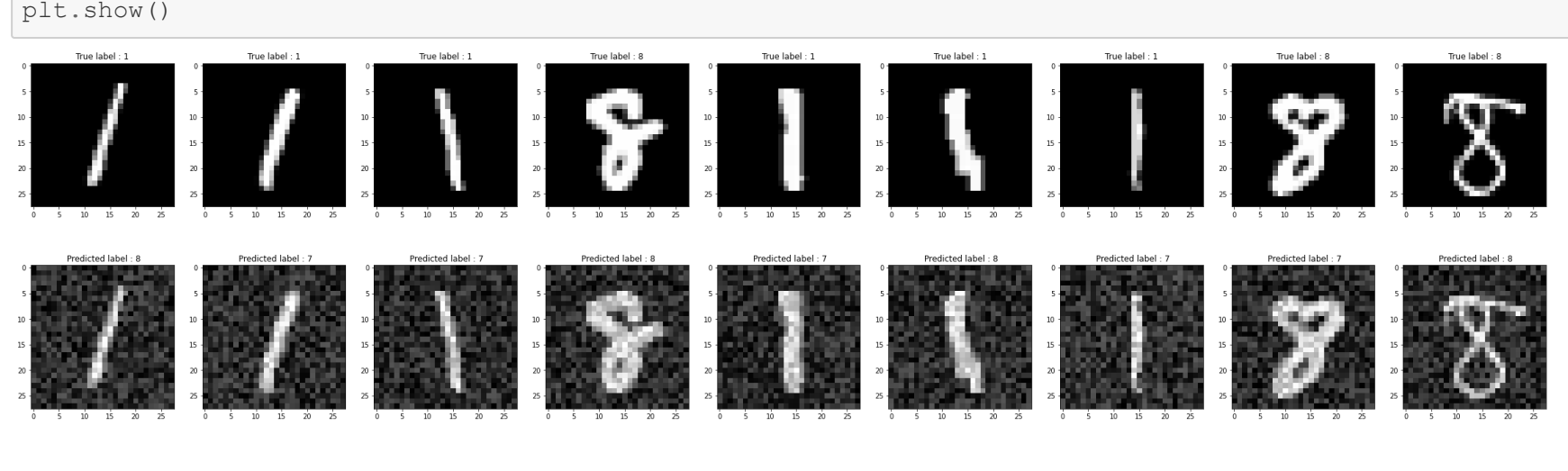
As we can see above, even with the gaussian noise, we can still distinguisg clearly the class.

Let's test our best classifier (MLP Classifier) to those adversial examples and let's see the result

In [100]:
```
true_labels_adversial_examples = [y2_test[i] for i in indices]
predictions = best_model.predict(adversial_examples)
auc_ad = roc_auc_score(true_labels_adversial_examples, best_model.predict_proba(adversial_examples) ,
multi_class = 'ovo')
acc_ad = best_model.score(adversial_examples ,true_labels_adversial_examples)
print({'AUC' : round(auc_ad,3) , 'ACC' : round(acc_ad,3)})
```

```
{'AUC': 0.989, 'ACC': 0.675}
```

In [101]:
```
row = 2
col = 9
ind = 1
plt.figure(figsize=(40,10))
for i in [0,1,4,671,321,322,323,971,996]:
    plt.subplot(row,col,ind)
    plt.imshow(x2_test[indices[i]].reshape(28,28),interpolation='none', cmap=plt.gray())
    plt.title(f"True label : {y2_test[indices[i]]}" )
    ind += 1
for i in [0,1,4,671,321,322,323,971,996]:
    plt.subplot(row,col,ind)
    plt.imshow(adversial_examples[i].reshape(28,28),interpolation='none', cmap=plt.gray())
    plt.title(f" Predicted label : {predictions[i]}")
    ind +=1
plt.show()
```



Before introducing the gaussian noise, thoses examples were well classified by the model. Now, it is not the case even if we can clearly recognize the digit witht the noise. It shows that the model is very sensitive to the value of each pixel to classify the digit. As human being, we recognize digit by seeing the picture in its globality whereas the model pays attention to each pixel to build its criteria to classify. Therefore, if all the pixels are modified, the model makes bad decision.

4. BONUS CNN

In [ ]:
```
import tensorflow as tf
from tensorflow.keras.utils import to_categorical

from tensorflow.keras.models import Sequential, Model
from tensorflow.keras.layers import Dense, Dropout, Activation, Flatten, Input
from tensorflow.keras.layers import Conv2D, MaxPooling2D
from tensorflow.keras import optimizers
```

In [ ]:
```
img_rows, img_cols, nb_channels = 28, 28, 1
```

In [ ]:
```
learning_rate = 0.01
n_epochs = 10
batch_size = 64

# number of convolutional filters to use
nb_filters = 32
# convolution kernel size
kernel_size = (3, 3)
# size of pooling area for max pooling
pool_size = (2, 2)

# --- Size of the successive layers
n_h_0 = nb_channels  # number of input channels
n_h_1 = nb_filters
n_h_2 = nb_filters
```

In [ ]:
```
def category(y,num_class):



    n = y.shape[0]
    res = np.zeros((n,num_class))


    for i in range(n):

        if y[i]==1:

            res[i] = [1,0,0]
        if y[i]==7:
            res[i]= [0,1,0]
        if y[i] == 8 :
            res[i] = [0,0,1]
    return res
```

In [ ]:
```
X_train = x2_train.reshape(x2_train.shape[0], img_rows, img_cols, nb_channels)
X_test = x2_test.reshape(x2_test.shape[0], img_rows, img_cols, nb_channels)

#We transform the labels in a one hot vectors for each class
Y_train = category(np.ravel(y2_train),3)
Y_test = category(np.ravel(y2_test),3)
```

I will now define the CNN model. The input of the CNN is a set of (28,28,1) image tensors. We apply :

- a Convolutional layer of 32 filters of shape (3,3), with stride (1,1) and padding='same'
- a ReLu activation function
- a Convolutional layer of 32 filters of shape (3,3), with stride (1,1) and padding='same'
- a ReLu activation function
- a Max Pooling Layer of shape (2,2) and stride (2,2) (i.e. we reduce by two the size in each dimension)
- We then Flatten the data (reduce them to a vector in order to be able to apply a Fully-Connected layer to it)
- A softmax activation function which outputs are the $P(y_c|X)$ (multi-class problem)

In [ ]:
```
model = Sequential()
model.add(Conv2D(n_h_1 , kernel_size, input_shape=(img_rows, img_cols, n_h_0), name='Conv1' , activatio
n='relu' , padding='same' , strides=(1,1)))
model.add(Conv2D(n_h_2, kernel_size,  name='Conv2' ,input_shape=(img_rows, img_cols, n_h_1), activation
='relu' , padding='same',strides=(1,1)))
model.add(MaxPooling2D(pool_size=pool_size , strides = (2,2) , padding='same'))
model.add(Flatten())
model.add(Dense(units=3 , activation='softmax'))
```

In [ ]:
```
model.compile(optimizer=optimizers.Adam(learning_rate=learning_rate),loss='categorical_crossentropy',me
trics=['accuracy'])
model.summary()
model.fit(X_train, Y_train , batch_size = batch_size , epochs= n_epochs)
```

```
Model: "sequential_1"
_________________________________________________________________
Layer (type)                 Output Shape              Param #
=================================================================
Conv1 (Conv2D)               (None, 28, 28, 32)        320
_________________________________________________________________
Conv2 (Conv2D)               (None, 28, 28, 32)        9248
_________________________________________________________________
max_pooling2d_1 (MaxPooling2 (None, 14, 14, 32)        0
_________________________________________________________________
flatten_1 (Flatten)          (None, 6272)              0
_________________________________________________________________
dense_1 (Dense)              (None, 3)                 18819
=================================================================
Total params: 28,387
Trainable params: 28,387
Non-trainable params: 0
_________________________________________________________________
Epoch 1/10
47/47 [==============================] - 30s 10ms/step - loss: 0.2021 - accuracy: 0.9243
Epoch 2/10
47/47 [==============================] - 0s 7ms/step - loss: 0.0422 - accuracy: 0.9877
Epoch 3/10
47/47 [==============================] - 0s 7ms/step - loss: 0.0179 - accuracy: 0.9937
Epoch 4/10
47/47 [==============================] - 0s 7ms/step - loss: 0.0244 - accuracy: 0.9907
Epoch 5/10
47/47 [==============================] - 0s 7ms/step - loss: 0.0165 - accuracy: 0.9947
Epoch 6/10
47/47 [==============================] - 0s 7ms/step - loss: 0.0149 - accuracy: 0.9947
Epoch 7/10
47/47 [==============================] - 0s 7ms/step - loss: 0.0086 - accuracy: 0.9967
Epoch 8/10
47/47 [==============================] - 0s 7ms/step - loss: 0.0065 - accuracy: 0.9973
Epoch 9/10
47/47 [==============================] - 0s 7ms/step - loss: 0.0114 - accuracy: 0.9963
Epoch 10/10
47/47 [==============================] - 0s 7ms/step - loss: 0.0018 - accuracy: 1.0000
```

Out[ ]:
```
<keras.callbacks.History at 0x7fe90d169bd0>
```

In [ ]:
```
score = model.evaluate(X_test, Y_test,verbose=False)
print('Test loss:', score[0])
print('Test accuracy:', score[1])
```

```
Test loss: 0.0652097761631012
Test accuracy: 0.984000027179718
```

In terms of accuracy, the CNN is slighty less efficiant than MLP but the difference is on the second significant number. But it is still one of the best model so far !

In [ ]:
```
X_adversial = adversial_examples.reshape(adversial_examples.shape[0], img_rows, img_cols, nb_channels)
Y_adversial = category(np.ravel(true_labels_adversial_examples),3)
```

In [ ]:
```
score = model.evaluate(X_adversial, Y_adversial,verbose=False)
print('Test loss:', score[0])
print('Test accuracy:', score[1])
```

```
Test loss: 0.06105760857462883
Test accuracy: 0.9769999980926514
```

Contrary to MLP, CNN has better performance than MLP with adversial examples. The accuracy has nothing to do with the accuracy of MLP. With MLP, this metric was around 0.6 whereas with the CNN, it is about 0.98 !! One reason of that is that NLP focuses only on pixels whereas CNN focuses on features on the picture. The different convolutions extract important features to characterise each digit. Therefore, CNN has a global view of the image and focuses on different areas of the image such as us human. It can explain the robusteness of CNN face to MLP !

Discussion about the lab

This lab was very interesting because we use several methods to do classification and we can see the strengths and weaknesses of each model. I have never used Grid Search CV but I realize that this function is very useful to fit the models and have the best model possible with all its parameters configured to give good results !

The fact that we reused the digit dataset was interesting, because I could see how to use it in an unsupervised way (lab 1) and here in a supervised way.

In [ ]: