UNIVERSITY OF ORLÉANS

Advanced Financial Econometrics

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Abstract

This project, which covers the econometrics course on Advanced Financial Econometrics taught by Christophe Hurlin at the ESA Master, aims to integrate the various concepts discussed in class, in particular those relating to the implementation of Minimum-Variance (MV hereafter) and Risk Parity (RP hereafter) strategies. This project stems from one of the major concerns encountered in the study of series in Advanced Financial Econometrics, the Optimal Portfolio Construction. Once implemented, a performance comparison is done between those latter and a "naive" portfolio (where the weights of the different assets are equal). Indeed, in a very well-known article, DeMiguel, Garlappi and Uppal [2009] have shown that the performance of a naive portfolio often exceeds the performance of portfolios built on active strategies (like the MV or RP for example). In this latter, the authors have published active strategies developed on the unconditional variance-covariance matrix of asset returns, simply estimated by the empirical variance-covariance matrix from past results. This project aims to implement the MV and RP strategies based on the conditional variance-covariance matrix (Martellini, Milhau, and Tarelli [2014]). Questioning the conclusions of DeMiguel, Garlappi, and Uppal [2009], means checking whether the use of conditional moments improves the performance of these strategies. In other words, it is about assessing whether conditional moments-based MV and RP portfolios perform better than the naive one. Project codes are available on GitHub.

Contents

Chapter 1

Introduction to Market Portfolio Theory

Optimal Portfolio Construction is the process of efficiently allocating wealth among asset classes and securities.
A large body of literature during the past five decades has considered different portfolio strategies. Over the last fifty years, mean-variance optimization (Markowitz [1952, 1956]) has been widely used to manage asset portfolios and to build strategic asset allocations. Altough powerful, it faces some stability issues because of its tendency to maximize the effects of estimation errors.
Alternative methods to deal with these issues have been suggested in the literature in the late eighties, notably portfolio re-sampling (Michaud [1989]) or new shrinkage estimators of the covariance matrix (Ledoit and Wolf [2003]), which have their own disadvantages. More recently, results on ridge and lasso regressions have been considered to improve Markowitz portfolios. However, it appears that a large fraction of investors and/or portfolio managers tend to prefer less sophisticated methods by constraining directly the weights of the portfolio. Indeed, heuristic solutions are computationally simpler to implement and are presumed robust as they do not depend on expected returns.
Two well-known examples of such techniques are the **Minimum-Variance** (MV) and the **Equally-weighted Risk Contributions** (ERC) portfolios. The first one is a specific portfolio on the mean-variance efficient frontier. Equity funds applying this principle have been launched in recent years. This portfolio is easy to compute since the solution is unique. As the only mean-variance efficient portfolio not incorporating information on the expected returns as a criterion, it is also recognized as robust. However, minimum-variance portfolios generally suffer from the drawback of portfolio concentration. A simple and natural way to resolve this issue is to attribute the same weight to all the assets considered for inclusion in the portfolio. **Equally-weighted** or $1/n$ portfolios are widely used in practice (Bernartzi and Thaler [2001], Windcliff and Boyle [2004]). In addition, if all assets have the same correlation coefficient as well as identical means and variances, the equally-weighted portfolio is the unique portfolio on the efficient frontier. The drawback is that it can lead to a very limited diversification of risks if individual risks are significantly different. ERC portfolios are interesting in a sense that no asset contributes more than its peers to the total risk of the portfolios, meanwhile MV portfolios also equalizes risk contributions but only on a marginal basis.
All in all those 3 strategies are going to be benchmark versus each other in order to give a performance comparison in terms of different criterion. These portfolios are special cases of a more general allocation approach based on risk budgeting methods (called also risk parity). This approach has opened a door to develop new equity and bond benchmarks (risk-based indexation) and to propose new multi-assets allocation styles (risk-balanced allocation). More information about the market portfolio theory can be found on Thierry Roncalli 's presentation : From Portfolio Optimization to Risk Parity[1].

[1]The opinions expressed in this presentation are those of the author and are not meant to represent the opinions or official positions of Lyxor Asset Management.

1.1 Data

Let us consider $m = 5$ shares listed on EURONEXT N.V (**Eur**opean **N**ew **E**xchange Technology). Let r_{it} be the centered daily returns for $i = 1, ..., m$ from the closing prices on these 5 shares over a period of 5 years (01/21/2015 until the 01/17/2020). We end up with 1257 observations over the period. We decided to take the following 5 shares: **ABC Arbitrage** (specialized in the design of arbitrage strategies on most European and American financial markets), **Airbus** (the world's largest airliner manufacturer), **L'Oréal** (world number one company in the cosmetic industry), **BNP Paribas** (1^{st} bank in the euro zone in 2018 by its assets, and the 8^{th} international banking group) and **Sanofi** (the world's fifth-largest pharmaceutical company by prescription sales).

1.2 Evaluation strategy

In order to evaluate our different strategies, we adopt a *rolling window* scheme with an estimate window size of the variance-covariance matrix, noted h, fixed at 120 days (that is to say approximately 6 months) and a portfolio rebalancing horizon, denoted k, fixed at 20 days (approximately 1 month). The approach of this evaluation strategy is as follows:

1. Estimation of the multivariate GARCH parameters (DCC) from h assets returns $r_1, ..., r_h$ in order to construct a conditional variance-covariance matrix's prediction for the date $h+1$, denoted $\widehat{\sum}_{h+1}$.

2. From the variance-covariance matrix prediction $\widehat{\sum}_{h+1}$, construction of the optimal portfolio w^*_{h+1} for each strategy.

3. It is assumed that this optimal portfolio is kept unchanged for a period of k days (frequency of portfolio rebalancing). We then construct and stock the return of the optimal portfolio R^*_{pt} for the following periods $t = h+1, ..., h+k$.

4. We move the estimation's window of the variance-covariance matrix to periods $t = k, ..., h+k$. We construct the conditional variance-covariance prediction's matrix for the date $h+k+1$, denoted for the date $\widehat{\sum}_{h+k+1}$.

5. For each strategy, we build the optimal portfolio w^*_{h+k+1} which we keep constant over the periods $t = h+k+1, ..., h+2k$. We store the optimal portfolio return R^*_{pt} for those latter. And so forth (see Fig.1) until the point where, for each strategy, we obtain a continuous series for the optimal portfolio return R^*_{pt} for periods $t = h+1, ..., n$, where n denotes the sample size.

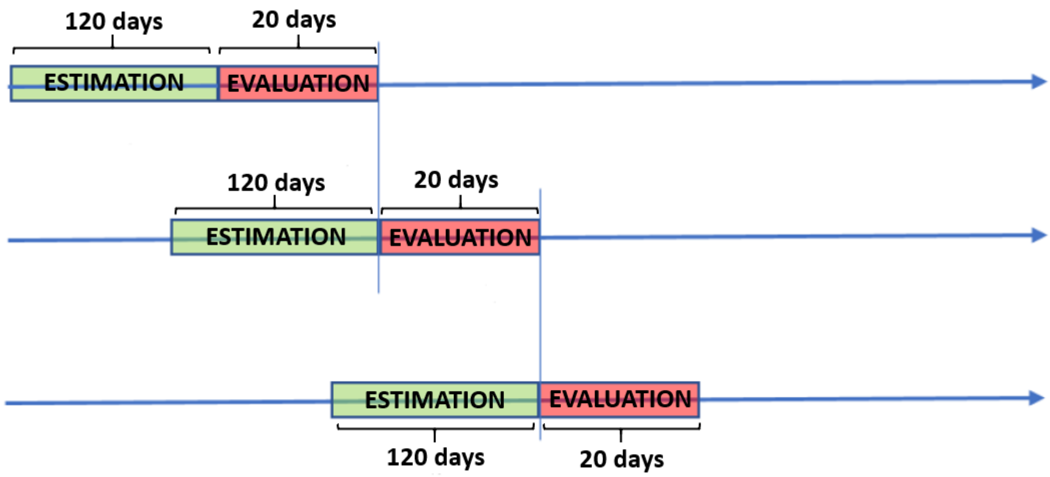

Figure 1.1: Evaluation Strategy

1.3 Estimation of conditional and unconditional moments

Assume $\sum_t = \mathbb{V}(r_t \mid r_{t-1})$ the conditional variance-covariance matrix of the return vector $r_t = (r_{1,t}, ..., r_{m,t})'$. Let us suppose that:

$$r_t = \sum_t^{1/2} \epsilon_t$$

$$\epsilon_t \sim \mathcal{N}(0, I_m)$$

where I_m denotes the identity matrix $(m \times m)$. A **DCC**-Garch (Bauwens, Laurent and Rombouts [2006]) will be estimated, in which the individual conditional variances satisfy a GARCH$(1,1)$. Let $\widehat{\theta}_h$ be the estimated parameters of the DCC-Garch for the period $t = 1, ..., h$ where h denotes the estimation window size. It is then necessary to construct the prediction of the conditional variance-covariance matrix for the period $h+1$ given by:

$$\widehat{\sum}_{h+1} = \sum_t \left(\widehat{\theta}_h\right)$$

1.4 Performance evaluation

In order to compare our models, that is to say the suites of optimal portfolio returns $\left[R^*_{pt}\right]_{t=1}^n$ obtained with conditional MV and RP strategies and the return suite $\left[R_{pt} = \sum_{i=1}^m \frac{1}{m} r_{it}\right]_{t=1}^n$ obtained with the Naive strategy, different indicators and graphs will be computed inspired from De Miguel, Garlappi et Uppal (2007): average returns and the Sharpe Ratio.

In addition to that, the following graphs will be computed:

- Conditional variance graphs of the m asset returns.
- Optimal asset weights graphs for the MV and RP strategies.
- Graphs of the contributions of assets to the volatility of the portfolio (only in the case of the RP portfolio).
- Profitability graph, considering a €1000 investment at date $t = h + 1$.

Chapter 2

Strategies

2.1 Naive Portfolio

The Naive Portfolio also called 1/N Portfolio suggests doing by simply spreading your wealth equally over the different asset classes available. This simple asset allocation rule has been set aside since the rise in popularity of the Modern Portfolio Theory (MPT) developed by Markowitz in 1952. Basically, he discovered that investments (even the volatile ones) if grouped a certain way, could reduce your overall portfolio risk.
Diversification of a portfolio without regard, or with incorrect regard, for the mathematical formulas in the capital asset pricing model. **Naive diversification** rests on the assumption that simply investing in enough unrelated assets will sufficiently reduce risk to make a profit. Alternately, one may diversify naively by applying the capital asset pricing model incorrectly and finding the wrong efficient portfolio frontier. Such diversification does not necessarily decrease risk at a given expected return, and may in fact increase risk[1].

The naive portfolio diversification rule (or $1/n$ portfolio) is such a fraction $1/n$ of wealth is allocated to each of the n assets:

$$w^*_{1/n} = \frac{1}{n}$$

The main argument is that methodological complexity and sophisticated models do not necessarily lead to investment optimality, and for the statistically-minded, complex approaches are seriously constrained by potential estimation errors. There are several advantages of naive diversification. First of all, no parameters have to be estimated. Secondly, the approach is not sensitive to estimation error. As such, a naive portfolio is a more robust approach.

Even if this strategy does not seem optimal at first sight, Victor Demiguel, Raman Uppal, Lorenzo Garlappi [2009] in Optimal Versus Naive Diversification: How Inefficient is the 1/N Portfolio Strategy? studied 14 models and found that none of these models in the literature are consistently better than the 1/N strategy in terms of Sharpe Ratio and turnover. The gain from optimal diversification is more than offset by estimation errors. Therefore, research suggests that a naive portfolio in practice does almost equally well as more sophisticated ones (MV or RP).

[1]Definition from : Farlex Financial Dictionary

2.2 Minimum-Variance Portfolio

The minimum variance approach has recently come to the fore with the introduction of a number of indexes. In contrast to the low volatility approach described earlier, the intention of minimum variance is to create a portfolio of stocks with the lowest overall volatility, subject to defined constraints[2]. A more formal definition would be to say that a Minimum-Variance Portfolio is a portfolio of individually risky assets that, when taken together, results in the lowest possible risk level for the rate of expected return. Such a portfolio hedges each investment with an offsetting investment; the individual investor's choice on how much to offset investments depends on the level of risk and expected return he/she is willing to accept. The investments in a minimum variance portfolio are individually riskier than the portfolio as a whole. The name of the term comes from how it is mathematically expressed in Markowitz Portfolio Theory, in which volatility is used as a replacement for risk, and in which less variance in volatility correlates to less risk in an investment[3]. The idea behind the Minimum-Variance strategy is that it is not the riskiness of an individual stock that matters, it is how the stock affects the riskiness of your entire portfolio that determines whether you should invest or not.

The Efficient Frontier [Figure 2.1] shown below is a graph that rates your portfolio's risk versus expected returns. In other words, it shows you the amount of profit you *expect* from a certain level of risk. Portfolios under the efficient frontier are sub-optimal because your expected returns are too low for according risk-level. An efficient portfolio offers the best expected return for a degree of volatility you are taking on.

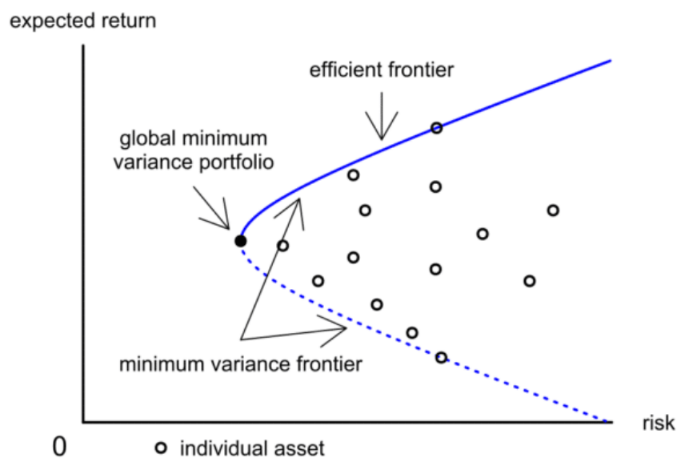

Figure 2.1: Efficient Frontier

One thing worth to mention is that the key ingredient to a Minimum-Variance Portfolio is holding investments with a low-correlation to each other. For example, let's say you are holding 25% of bonds, 25% of any Large Cap Stock, 25% of any Small Cap Stock and 25% of Real Estate. Bonds and Real Estate will not always move in sync with each other. In other words, the impact of a diminution in the Real Estate market will (should) not affect the other three.

In mathematical terms, the Global Minimum Variance portfolio has the smallest variance of returns, and is defined by:

$$\begin{aligned} \omega^*_{GMV} &= \arg\min_{\omega \in \Theta} \omega^T \sum \omega \\ u.c.\ \omega^T e &= 1 \text{ and } \omega_{it} \geq 0 \end{aligned} \tag{2.1}$$

Without the short-selling constraint, the optimal allocation ω^*_{GMV} has a closed-form expression given by :

$$\omega^*_{GMV} = \frac{\sum^{-1} e}{e^T \sum^{-1} e} \tag{2.2}$$

[2]Definition from : Low Volatility or Minimum Variance - FTE Russell
[3]Definition from : Farlex Financial Dictionnary

2.3 Risk Parity Portfolio

Risk parity (or **risk premia parity**) is an approach to investment portfolio management which focuses on allocation of risk, usually defined as volatility, rather than allocation of capital. The risk parity approach asserts that when asset allocations are adjusted (leveraged or deleveraged) to the same risk level, the risk parity portfolio can achieve a higher Sharpe ratio and can be more resistant to market downturns than the traditional portfolio.
Roughly speaking, the approach of building a risk parity portfolio is similar to creating a minimum-variance portfolio subject to the constraint that each asset (or asset class, such as bonds, stocks, real estate, etc.) contributes equally to the portfolio overall volatility[4].
Risk parity is a conceptual approach to investing which attempts to provide a lower risk and lower fee alternative to the traditional portfolio allocation of 60% stocks and 40% bonds which carries 90% of its risk in the stock portion of the portfolio as below [Figure 2.2].

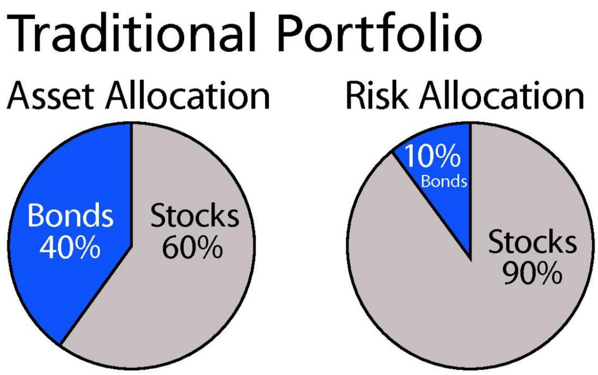

Figure 2.2: Comparison of asset and risk allocation

2.3.1 Portfolio risk contributions

We consider $w_t = (w_{1t}, ..., w_{mt})'$ an arbitrary weight vector of the m assets in the portfolio, with $w_{it} \geq 0$ (no short-selling) and $\sum_{i=1}^{N} w_{it} = 1$. Let's denote $R_{mt} = w_t^T r_t$ the portfolio return at date t. At date t the conditional volatility of the portfolio returns R_{mt} is equal to :

$$\sigma_t(x) = \sqrt{w_t^T \, \Sigma_t} \tag{2.3}$$

where $\sum_t$ denotes the conditional variance-covariance matrix of the asset returns from the DCC-Garch. The contribution $S_{it}(\omega_t, \sum_t)$ (in %) of the asset i at date t is defined by the weight product of this given asset in the portfolio, w_{it}, times the marginal contribution $\partial\sigma_t/\partial\omega_{it}$

$$S_{it}(\omega_t, \sum\nolimits_t) = \frac{\omega_{it}}{\sigma_t(x)} \frac{\partial\sigma_t(x)}{\partial\omega_{it}} \tag{2.4}$$

We can show that :

$$S_{it}(\omega_t, \sum_t) = \omega_{it} \frac{(\sum_t \omega_t)}{\omega_t^T \sum_t \omega_t} \tag{2.5}$$

where $(\sum_t \omega_t)$ denotes the i^{th} line of the vector $(m \times 1)$ defined by the product $\sum_t \omega_t = 1$

[4]Sébastien Maillard, Thierry Roncalli, Jérôme Teiletche, "On the properties of equally-weighted risk contributions portfolios"

2.3.2 Risk Parity strategy

The Risk Parity strategy consists in determining an index such that the risk contributions $S_{it}(\omega_t, \sum_t) = (S_{1t}(\omega_t, \sum_t), ..., S_{nt}(\omega_t, \sum_t))^T$ are equal to a target risk contributions, denoted $b = (b_1, ..., b_n)^T$ with $\sum_{i=1}^{n} = b_i$ and $b_i > 0$, fixed by the investor.

The objective is to determine at each period t of validation, the portfolio weights associated with the conditional Risk Parity strategy (based on $\sum_t$) and unconditional (based on $\sum$). Concerning the conditional Risk Parity, at each date t, we determine the optimal portfolio $\omega_t^* = (w_{1,t}^*, ..., w_{5,t}^*)$ such as:

$$\begin{aligned} S_{1t}(\omega_t, \sum_t) = ... = S_{mt}(\omega_t^*, \sum_t) = \frac{1}{m} \\ w_{i,t}^* \geq 0 \\ \sum_{i=1}^{m} w_{i,i}^* = 1 \end{aligned} \tag{2.6}$$

Without weight constraints, this program results in an analytical solution[5]. However, when in presence of constraints, the optimal portfolio need to be determined by numerical optimization. How to build a 1/n portfolio which takes into account risk? The simplest Risk Parity strategy is the Equal Risk Contribution (ERC) which consists in finding a portfolio such that the risk contribution is the same for all the assets. It can be viewed as a special case with $b_i = 1/n$ for $i = 1, ..., n$

$$S_i(\omega_{ERC}^*, \sum) = \frac{1}{n} \ \forall i = 1, ..., n \tag{2.7}$$

The ERC portfolio may be viewed as a portfolio *between* the 1/n portfolio and the minimum variance portfolio. In other terms, the ERC portfolio may be viewed as a form of variance-minimizing portfolio subject to a constraint of sufficient diversification in terms of components weights[6].

However, numerical optimization is necessary in most cases due to the endogeneity of the solutions. All in all, determining the ERC solution for a large portfolio might be a computationally intensive task, something to keep in mind when compared with the minimum variance and, even more, with the 1/n competitors.

2.4 The Sharpe Ratio

The Sharpe ratio was developed by Nobel laureate William F. Sharpe and is used to help investors understand the return of an investment compared to its risk. The ratio is the average return earned in excess of the risk-free rate per unit of volatility or total risk.
The Sharpe ratio has become the most widely used method for calculating the risk-adjusted return. Modern Portfolio Theory states that adding assets to a diversified portfolio that has low correlations can decrease portfolio risk without sacrificing return. Adding diversification should increase the Sharpe ratio compared to similar portfolios with a lower level of diversification. For this to be true, investors must also accept the assumption that risk is equal to volatility which is not unreasonable but may be too narrow to be applied to all investments[7].

$$SR = \frac{R_p - R_f}{\sigma_p}$$

where R_p is the return of the portfolio, R_f is the risk-free rate and σ_p is the standard deviation of the portfolio's excess return. In a nutshell, the Sharpe ratio adjusts a portfolio's past performance—or expected future performance—for the excess risk that was taken by the investor. A high Sharpe ratio is good when compared to similar portfolios or funds with lower returns. However, there is some limitations in the use of that ratio. Indeed, the Sharpe Ratio assume that investment returns are normally distributed. Moreover, choosing a period for the analysis with the best potential Sharpe Ratio rather than a neutral look-back period, can be used to boost the apparent risk-adjusted returns history.

[5] Roncalli [2013], Martellini, Milhau and Tarelli [2014]

[6] Sébastien Maillard, Thierry Roncalli and Jéerôme Teiletche, "Equally-weighted Risk contributions: a new method to build risk balanced diversified portfolios"

[7] Sharpe, W. F. [1966], "Mutual Fund Performance". Journal of Business.

Chapter 3

Results

3.1 Share Prices Evolution

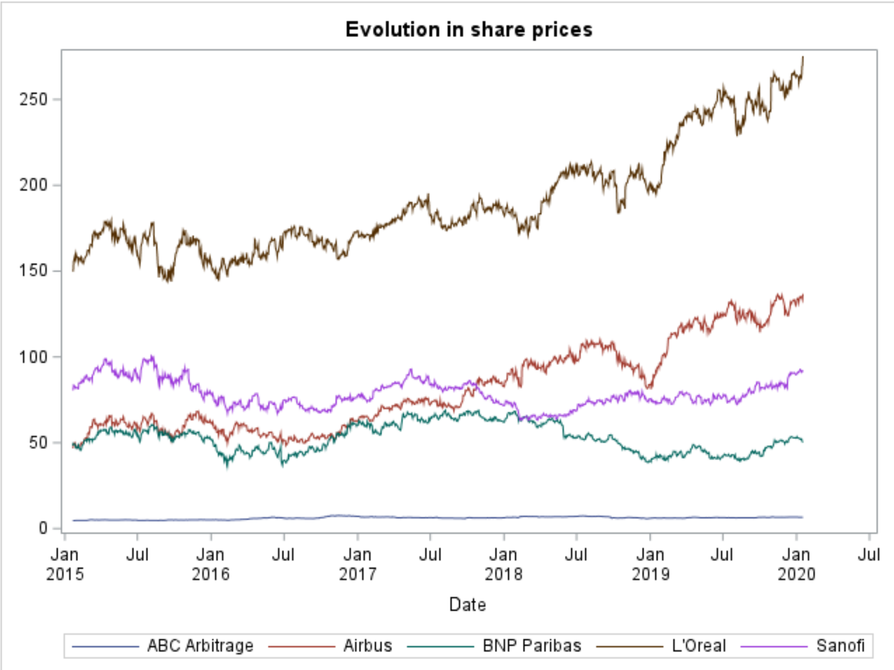

Figure 3.1: Share Prices Evolution

We can see on [Figure 3.1] that most of our shares are stable over time, except for Airbus and L'Oréal. Indeed, Airbus nearly tripled in our time-period going from €46.82 on the 01.21.2015 to €136.78 on the 01.17.2019. This latter is probably marked by the 737 MAX crisis for Boeing, resulting in catastrophic turnover for Boeing on 2019. The indicators for the 4^{th} quarter of 2019 in particular are very bad: negative operating cash flow of 2.22 billion dollars over the last three months of the year and a net profit of -1.010 billion dollars (i.e. 1.79 dollar loss per share). Over the full year, sales stood at 76.56 billion dollars (-24% year-on-year), net profit at -636 million dollars (or 1.12 dollar loss per share) and operating cash flow at -2.45 billion dollars.
L'Oréal also underwent a notable success, increasing from €149.80 on the 01.21.2015 to €275.29 on the 01.17.2019. L'Oréal jumped and registered a new historic high. Investors welcome the particularly robust quarterly turnover revealed by them. In the 3^{th} quarter, the group led by Jean-Paul Agon achieved a turnover of 7.18 billion euros over the period, up 11% on a year-on-year basis and 7.8% on a comparable basis.

3.2 Conditional Variance of returns

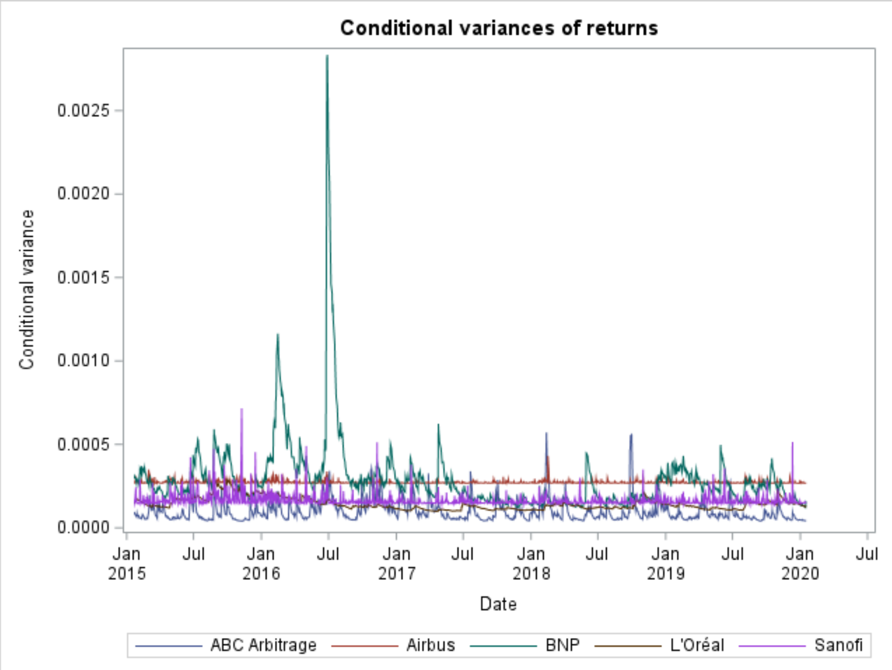

Figure 3.2: Conditional Variance of returns

For four of our five assets, the conditional variance of returns seems stable. However for BNP Paribas, there are different spikes to which we do not have an answer for, notably the one in early 2016 and in July 2016.

3.3 Optimal Weights

3.3.1 Minimum-Variance Portfolio

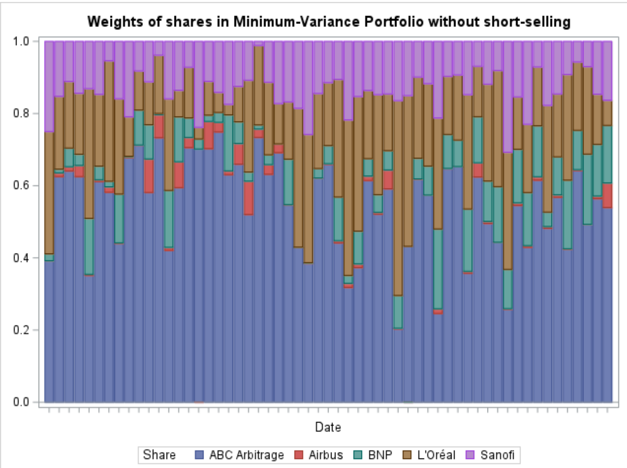

Figure 3.3: Optimal Weights - No Short-Selling

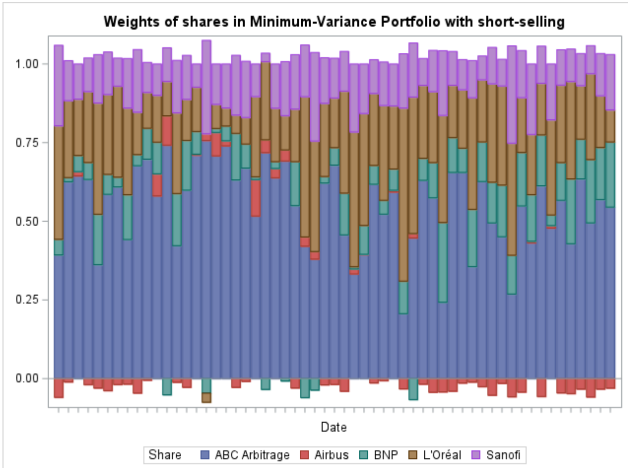

Figure 3.4: Optimal Weights - With Short-Selling

[Figure 3.4] shows an interesting result. Indeed, when we authorize short-selling, almost at all times Airbus is the asset we short-sell. With a little hindsight this result seems logical as Airbus is the most volatile asset we have in our portfolio. Hence, as we are applying the Minimum-Variance strategy, we will obviously not take much of this asset (cf. [Figure 3.3]). ABC Arbitrage is the asset with the most weight because this is the least volatile asset of our portfolio. Numerical optimization has been used to find those optimal weights through **PROC OPTMODEL**.

3.3.2 Risk Parity Portfolio

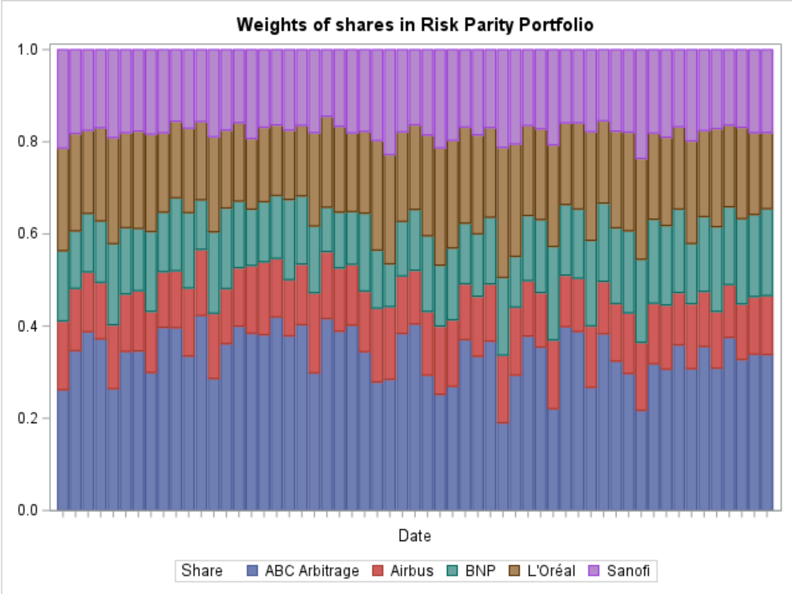

Figure 3.5: Optimal Weights - Risk Parity

[Figure 3.5] shows that at almost all time, ABC Arbitrage has more than 20% and Airbus less than 20%. This result is logical as Airbus contributes way more to the risk taken than ABC Arbitrage (whose variance is really low). L'Oréal and Sanofi are both stable around 20% at each time. BNP Paribas however, finds itself at 15% most of the time. Numerical optimization has been used to find those optimal weights through **PROC OPTMODEL**.

3.4 Assets contribution to the portfolio volatility

3.4.1 Risk Parity Portfolio

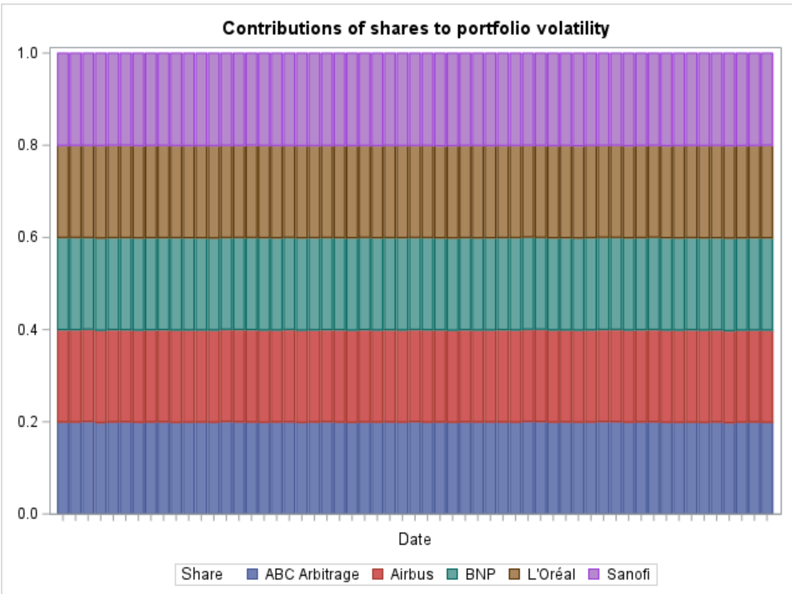

Figure 3.6: Assets Contribution to the portfolio volatility

[Figure 3.6] shows the obvious result we expected. Indeed no asset contributes more than its peers to the total risk of the portfolio. Numerical optimization has been used to find those optimal weights through **PROC OPTMODEL**.

3.5 Portfolio Evolution

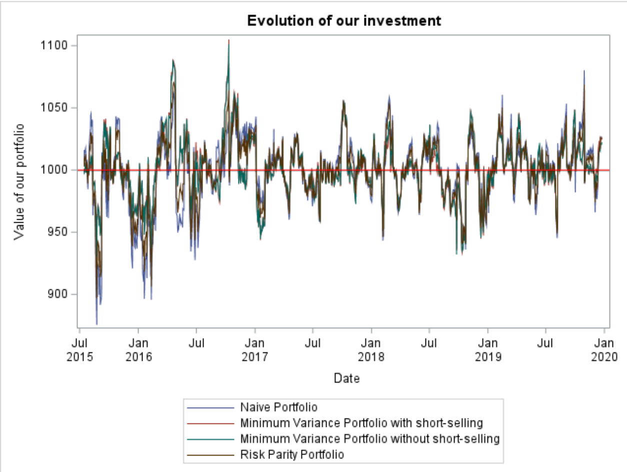

Figure 3.7: Evolution of our investment of 1000 €

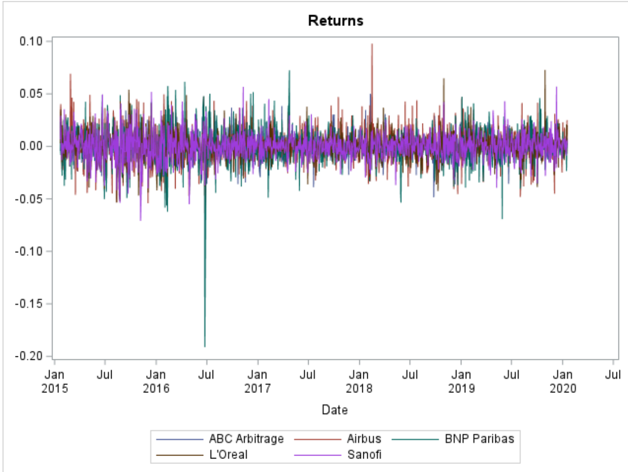

Figure 3.8: Evolution of our returns

Date	Naive	Minimum-Variance (NO SS)	Minimum-Variance (SS)	Risk Parity
01.17.2015	€1000	€1000	€1000	€1000
07.15.2015	**€1004.33**	€1001.04	€1000.28	€1002.79
04.25.2016	€1056.16	**€1082.48**	€1080.26	€1066.18
05.29.2017	€1011.22	€1012.64	**€1013.62**	€1010.51
11.17.2017	€1000.55	€998.82	€998.77	**€1001.61**
02.28.2018	**€1053.36**	€1025.34	€1031.79	€1047.45
11.26.2018	**€1012.02**	€1009.08	€1010.58	€1011.07
02.05.2019	€1045.23	€1042.53	€1041.57	**€1045.38**
12.25.2019	**€1026.32**	€1025.59	€1022.04	€1025.59

Figure 3.9: Investment at different times for the different strategies

	Naive	Minimum-Variance (NO SS)	Minimum-Variance (SS)	Risk Parity
Number of days	**562**	351	202	25

Figure 3.10: Number of days where a strategy outperformed the other

[Figure 3.10] shows that the Naive Strategy outperformed every other strategies 562 days out of the 1140 estimated days whereas Risk Parity only outperformed the other 25 days and Minimum-Variance with no short-selling 351 days. [Figure 3.9] shows that at the end of our evaluation's time-period, that is, the 12.25.2019 the strategy that got us the best return is the Naive as we end up with €1026.32 instead of €1025.59 for the Minimum-Variance with no short-selling, €1022.04 for the Minimum-Variance with short-selling and €1025.59 for Risk Parity.

3.6 Evaluation

	Naive	Minimum-Variance (NO SS)	Minimum-Variance (SS)	Risk Parity
Mean	0.063314	0.060344	0.056065	**0.065919**
Variance	0.02307	**0.01420**	0.01421	0.01773
Volatility	15.19%	**11.92%**	11.92%	13.31%
Sharpe Ratio	0.41685	**0.50639**	0.47038	0.49510

Figure 3.11: Performance comparison

[Figure 3.11] shows that Risk Parity has the best annualized mean return with 0.065919. The Minimum-Variance has obviously the minimum annualized volatility with 11.92%, followed by the Risk Parity with an annualized volatility of 13.31%. Last but not least, the strategy that has the highest volatility is the Naive one with a volatility of 15.19%. This confirm the theoretical order $\sigma_{GMV} < \sigma_{ERC} < \sigma_{1/N}$ that Thierry Roncalli suggests.
Concerning the Sharpe Ratio:

- If the ratio is negative, we conclude that the portfolio underperforms a risk-free investment and therefore it does not make sense to invest in such a portfolio.
- If the ratio is between 0 and 1, it means that the excess return over the risk-free rate is lower than the risk taken.
- If the ratio is greater than 1, then the portfolio outperforms a risk-free investment and therefore generates higher profitability.

In our case the risk-free rate is obviously 0. None of our strategy were able to give a Sharpe Ratio above one, with the MV's Sharpe Ratio being the highest one with 0.50639. Therefore we conclude that none of our strategy is optimal as our returns are lower than the risk taken. However we sould keep in mind that it remains nevertheless limited when it comes to comparing a portfolio of stocks, whose profitability is expressed as a percentage, with a stock market index.

Chapter 4

Conclusion

Our project aimed to check if conditional moments-based Minimum-Variance and Risk Parity portfolios perform better than the Naive one. Throughout this project, we showed that our results strongly depend on the evaluation criteria used. Indeed, even if the 1/N strategy outperforms the other strategies 562 days out of the 1140 estimated days, the annualized mean of returns is better for the Risk Parity, even tough this latter outperforms only 25 days the other strategies. When looking at the Sharpe Ratio, the best portfolio is the Minimum-Variance with no short-selling. However, we should also keep in mind that the Sharpe Ratio is limited.

Our finding suggests that the Risk Parity strategy cannot be considered consistently superior relative to the Minimum-Variance and the Naive strategy. Those results are in line with those from Demiguel, Uppal and Garlappi [2009].

Nonetheless, we only had securities for our analysis. That is, we only focused ourselves on one market. Choosing different assets from different markets (bonds, Large Cap Stocks, Small Cap Stocks and so on) would have probably changed our results because the correlation would have changed between those assets. Moreover, in order to affirm our conclusion it would have been interesting to conduct our study several times with different assets.

Annexes

Annexes

WARNINGS

If you want to run the code, please change the path and list macro-variables as well as the libname.
If you wish to change the data, please use the right names in the list macro-variable.
The import macro adapt itself to .csv coming from Yahoo Finance.

1 Application

All the codes can be found on the following GitHub under the SAS section.

2 Data

All the codes can be found on the following GitHub under the DATA section.

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