# MILP-Based Algorithm for the Global Solution of Dynamic Economic Dispatch with Valve-Point Effects

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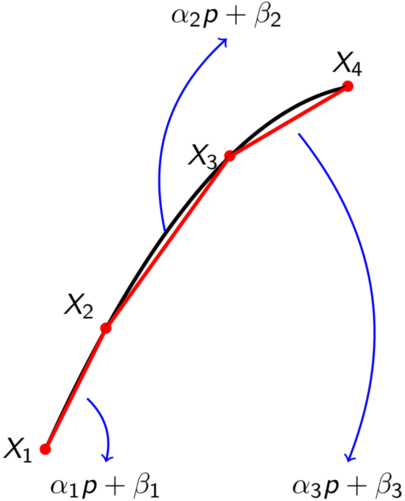


$\alpha_2 p + \beta_2$
$X_4$
$X_3$
$X_2$
$X_1$
$\alpha_1 p + \beta_1$
$\alpha_3 p + \beta_3$

1. **Introduction** - On the use of gas energy in today's and tomorrow's power mix.

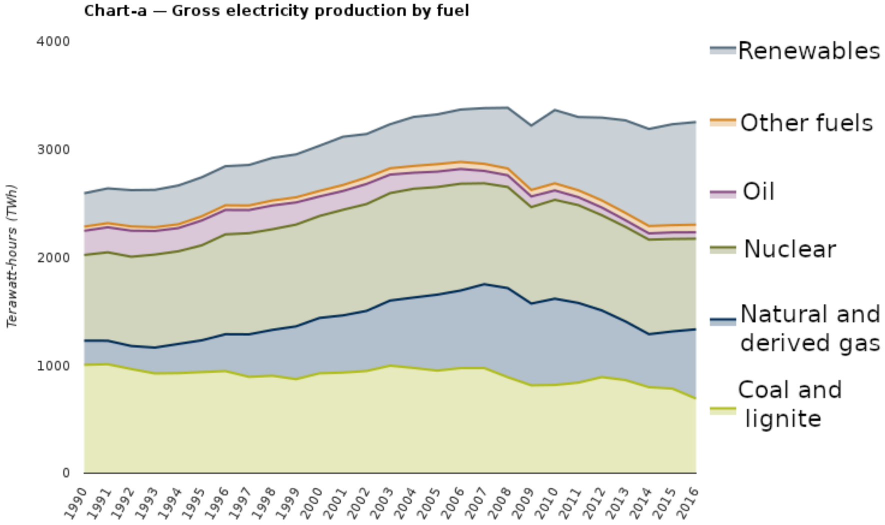
2. **Problem statement** - Economic Dispatch with Valve Point Effect.

3. **Description of the algorithm** - An Adaptive Piecewise-Linear Approximation.

4. **Study case** - A 10-units dispatch over 24 hours.

5. **Further work.**

# On the place of gas energy in today's power mix




Source: European Environment Agency (EEA) - Gross electricity production by fuel in Europe.

# On the place of gas energy in today's power mix

European plan on climate change consists in the 20-20-20 targets:

By 2020...

- Reduce by 20% the emissions of greenhouse gases (GHB) compared to 1990 levels;
- Reach 20% of renewables energy;
- Increase by 20% the energy efficiency.

Source: Climate strategies & targets - 2020 climate & energy package.

# On the place of gas energy in today's power mix

European plan on climate change consists in the 20-20-20 targets:

By 2020...

- ✓ Reduce by 20% the emissions of greenhouse gases (GHB) compared to 1990 levels; **OK since 2016**
- ▶ Reach 20% of renewables energy; **Still on track: 17.52 % in 2017**
- ✗ Increase by 20% the energy efficiency. **Not on track: 4 % above objective in 2016**

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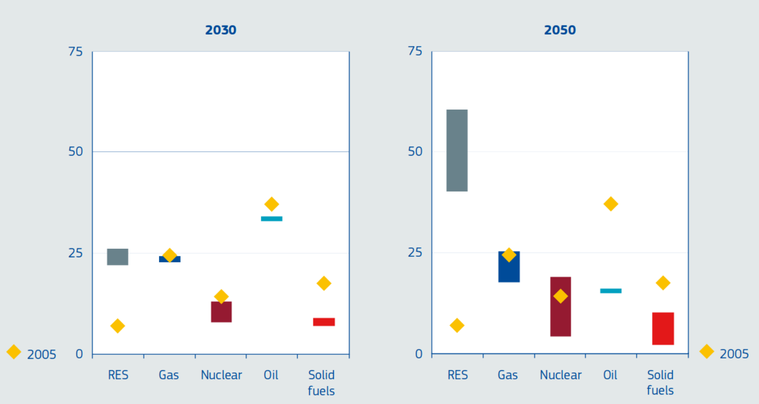
Source: European Environment Agency (EEA), Eurostats.

# On the place of gas energy in **tomorrow's** power mix

## European targets for 2030 and 2050




"Natural gas will continue to play a key role in the EU's energy mix in the coming years and gas can gain importance as the *back-up fuel* for variable electricity generation." (European Commission's Communication Energy 2020)

Source: Energy roadmap 2050

# 2. Problem statement - Economic Dispatch with Valve Point Effect.

## Data

Load demand: $D_t$ $\quad t \in T$

Spinning reserve: $S_t$ $\quad t \in T$

Ramp constraint: $R_i^U$, $R_i^D$ $\quad i \in I$

Set of producers with cost function $f_i$ $\quad i \in I$

## Decision variables

Production: $p_{it}$ $\quad i \in I\,, t \in T$

Reserve: $s_{it}$ $\quad i \in I\,, t \in T$

## Problem

How to optimally *dispatch* the power between producers ?

# Optimization model

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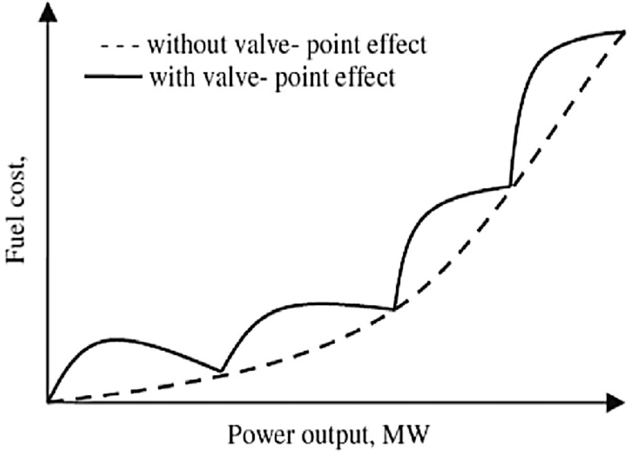
# Valve-Point Effect

The VPE is a natural characteristic of a gas turbine. Operating off a valve point increases the throttling losses, therefore rising the heat rate.

$$f(p) = a\,p^2 + b\,p + c + d\,\left|\sin e(p - P^{\min})\right|$$



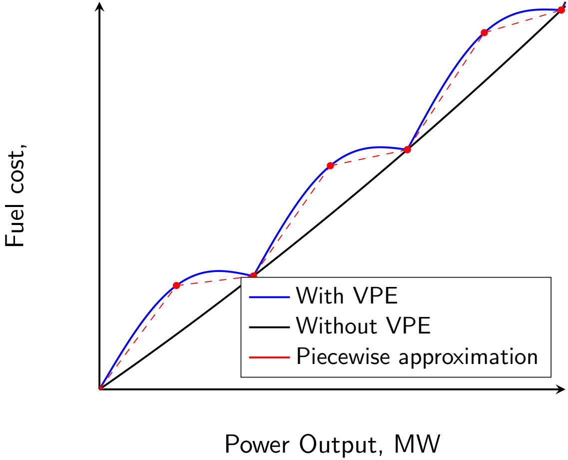
# Proposed approach

## **A**daptive **P**iecewise-**L**inear **U**nder-**A**pproximation

Idea: a sequence of piecewise approximations.

We could use an uniform grid ...

... but there are too many integer variables !



# 3. Description of the algorithm - An Adaptive Piecewise-Linear Approximation.

## **A**daptive **P**iecewise-**L**inear **U**nder-**A**pproximation

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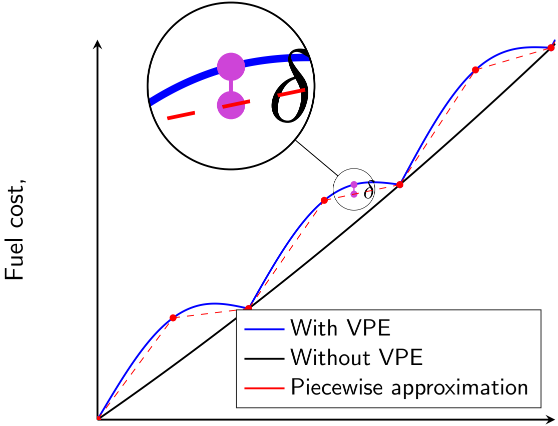

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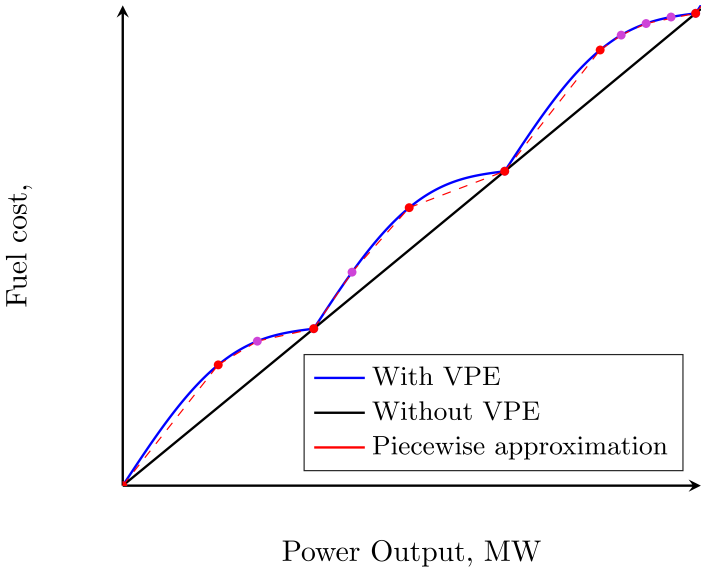
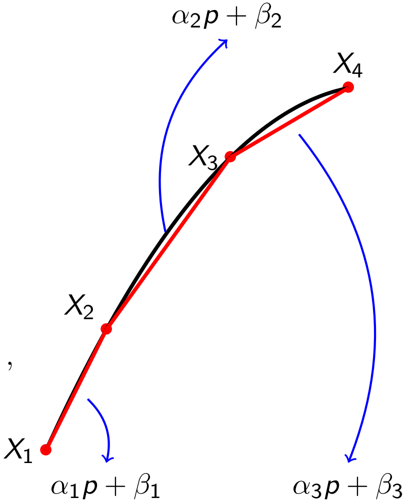


# Piecewise linearization of objective

## First model: binary variables

$$g(p,\boldsymbol{\xi},\boldsymbol{\eta}) := \begin{cases} \sum_{j=1}^{n^{\text{knot}}-1} \alpha_j \xi_j + \eta_j \beta_j \,, \\ \text{with } \sum_{j=1}^{n^{\text{knot}}-1} \xi_j = p \,, \\ \qquad \sum_{j=1}^{n^{\text{knot}}-1} \eta_j = 1 \,, \\ \qquad X_j\, \eta_j \le \xi_j \le X_{j+1}\, \eta_j \,, \\ \qquad \eta_j \text{ binary} \,. \end{cases}$$



# Optimality gap

- Gap between surrogate and true solution

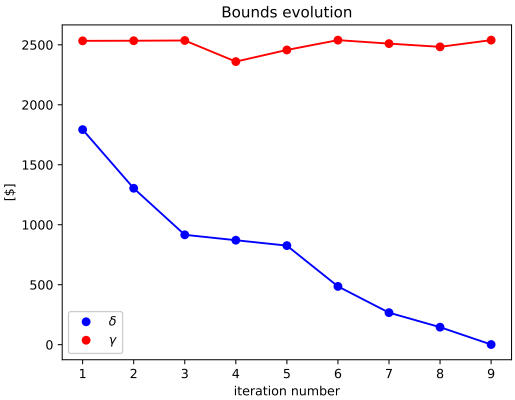
$$\overbrace{\left| f(\mathbf{p}^*) - \mathbf{f}(\mathbf{p^k}) \right|}^{\text{Optimality gap}} \leq \delta^k + \gamma^k + \epsilon^k$$

- Solver tolerance error
- Over-approximation error

# What about the convergence ?

- $\gamma^k$ is bounded below by $\gamma f(\mathbf{p}^*)$ ;
- $\epsilon^k$ is virtually negligible since the "convex zones" are smaller than 0.1% of the domain ;
- $\delta^k$ converges to zero.

## A practical example



# What about the convergence ?

Theorem

$\lim_{k\to\infty} \delta^k = 0$

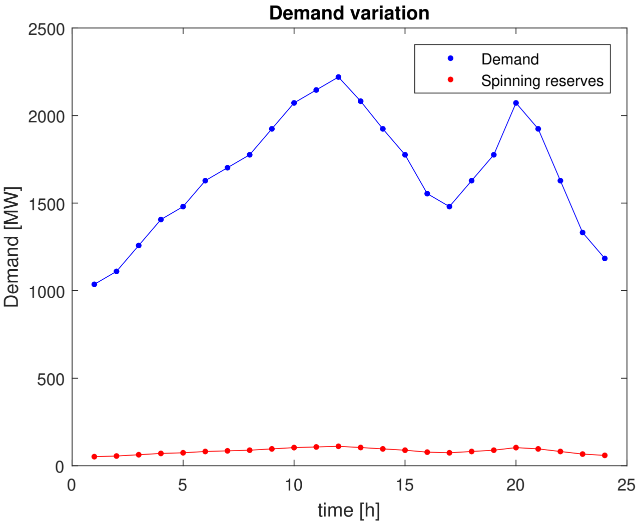
Proof (sketch).

- $f_i$ is L-continuous: $|f_i(p+\Delta) - f_i(p)| \leq \overbrace{(2a_i P_i^{\max} + b_i + d_i e_i)}^{:=K_i} \Delta$
- $f = \sum_{i=1}^n f_i$ is L-continuous with constant $K := \sum_{i=1}^n K_i$
- $g$ is also Lipschitz continuous with same constant

$$\begin{aligned}
0 \leq \delta^k &= \min_{l\in 1\ldots k} f(\mathbf{p}^l) - g^k(\mathbf{p}^k) \\
&\leq f(\mathbf{p}^k) - g^k(\mathbf{p}^k) \\
&= f(\mathbf{p}^k) - g^k(\mathbf{p}^{k-1}) + g^k(\mathbf{p}^{k-1}) - g^k(\mathbf{p}^k) \quad \text{(trick: +1 -1)} \\
&= f(\mathbf{p}^k) - f(\mathbf{p}^{k-1}) + g^k(\mathbf{p}^{k-1}) - g^k(\mathbf{p}^k) \quad \text{(knot updating)} \\
&\leq 2K \left|\left|\mathbf{p}^k - \mathbf{p}^{k-1}\right|\right| \quad \text{(Lipschitz continuity)}
\end{aligned}$$

$\square$

# 4. Study case - A 10-units dispatch over 24 hours




Spinning reserves set at 5% of the demand.
10 units with valve-point loading effect.

# Results table

## Previous results

## And our approach ?

TABLE III
SUMMARY RESULTS FOR THE 10-UNIT SYSTEM WITHOUT LOSS

| Method | Total generation cost ($) Minimum | Average | Maximum | S-time(min) |
|---|---|---|---|---|
| SQP [3] | 1051163 | NA | NA | 0.42 |
| EP [3] | 1048638 | NA | NA | 15.05 |
| CDE [16] | 1036756 | 1040586 | 1452558 | 0.20 |
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| CDE[16] | 1019123 | 1020870 | 1023115 | 0.32 |
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| EBSO [21] | 1017147 | 1017526 | 1017891 | 0.15 |
| MILP | 1016316 | | | 0.94 |
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NA denotes that the value was not available in the literature.

Pan, Jian, and Yang, "A hybrid MILP and IPM approach for dynamic economic dispatch with valve-point effects", 2018.

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## And our approach ?

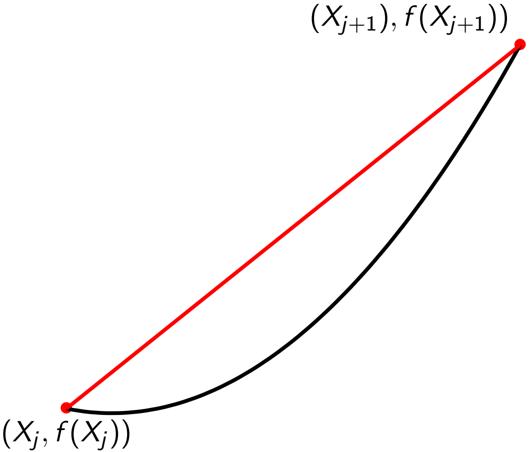
| | | |
|---|---|---|
| APLUA | 1016276$ | 15(min) |
| APLUA + Local Heuristic | 1016207$ | 1.5(min) |

Pan, Jian, and Yang, "A hybrid MILP and IPM approach for dynamic economic dispatch with valve-point effects", 2018.

# 5. Further work

## Important condition of the proof: **Under**-approximation

$\Rightarrow$ The proof is not valid for convex functions (e.g. without valve point effect)



# Further work: non-convex case

Important condition of the proof: **Under**-approximation

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- Possible to prove that $g^{k+1} \geq g^{k}$ and that we cannot do better with that number of points
- Number of integer variables rises linearly ($\sim$ factor 2)

# Further Work: power losses and network constraints

## (Revisited) demand constraints

$$\sum_{i=1}^{n} p_{it} = D_t + p^L(\mathbf{p}_t)$$

$p^L(\mathbf{p}_t)$ models the transmission losses computed as

$$p^L(\mathbf{p}_t) = \mathbf{p}_t^T \mathbf{B} \mathbf{p}_t + \mathbf{B}_0 \mathbf{p}_t + \mathbf{B}_{00}$$

with **B** symmetric matrix.

## Network constraints

# Conclusion

- APLUA manages to find a good candidate ...
- ... it takes more time but same range number ...
- ... and we are limited by the solver tolerance gap.
- ? How to take the quadratic transmission lost into account?

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## Contact

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