

## 3 Other concepts of logic

The concept of a valid argument is central to logic. In this section, we will introduce some other important concepts that apply just to sentences, not to full arguments.

### 3.1 Joint possibility

Consider these two sentences:

- B1. Jane's only brother is shorter than her.
- B2. Jane's only brother is taller than her.

Without knowing Jane and her brother, we have no way of knowing which, if either, of these sentences is true. Yet we can say that *if* B1 is true, *then* B2 must be false. Similarly, if B2 is true, then B1 must be false. It is impossible that both sentences are true at the same time. In other words, these sentences are inconsistent. On the other hand, G1 and G2 can both be true at the same time.

- G1. There are at least four giraffes at the wild animal park.
- G2. There are exactly seven gorillas at the wild animal park.

One of these sentences may be false and the other true, but it is *possible* that they are both true at the same time.

#### jointly possible and impossible

A set of sentences are JOINTLY POSSIBLE when, and only when, it is possible for them all to be true at the same time.

A set of sentences are JOINTLY IMPOSSIBLE when, and only when, it is *not* possible for them all to be true at the same time.

So, G1 and G2 are *jointly possible* while B1 and B2 are *jointly impossible*.

We can investigate the joint possibility of any number of sentences. For example, let's add two more sentences to G1 and G2:

- G1. There are at least four giraffes at the wild animal park.
- G2. There are exactly seven gorillas at the wild animal park.
- G3. There are not more than two extra-terrestrials at the wild animal park.
- G4. Every giraffe at the wild animal park is an extra-terrestrial.

Together, G1 and G4 entail that there are at least four extra-terrestrials giraffes at the park. This conflicts with G3, which states that there are no more than two extra-terrestrials there. So, the sentences G1–G4 are jointly impossible. They cannot all be true together. (Notice also that just G1, G3 and G4 are jointly impossible, while G1, G2, and G3 are jointly possible.)

### 3.2 Necessary equivalence

Sentences G1 and G2 – which we said were jointly possible – can both be true at the same time. They can also both be false, or one false and the other true. A stronger relationship holds between these two sentences:

John went to the store after he washed the dishes.

John washed the dishes before he went to the store.

These two sentences must have the same truth value. That is, they must either both be true or both be false. It is impossible for one to be true and one to be false (at the same time). When two sentences *must* (or *necessarily*) have the same truth value, they are **NECESSARILY EQUIVALENT**.

necessarily equivalent

Two sentences are **NECESSARILY EQUIVALENT** if they must have the same truth value. (I.e., they must both be true or they both must be false.)

### 3.3 Necessary truths & falsehoods, and contingency

Consider these sentences:

- a. It is raining.
- b. Either it is raining here, or it is not.
- c. It is both raining here and not raining here.

In order to know if sentence (a) is true, you would need to look outside or check a weather forecasting app. It might be true, or it might be false. A sentence that is capable of being true and capable of being false (in different circumstances, of course) is **CONTINGENT**.

Sentence (b) is different. You do not need to look outside to know that it is true. Regardless of what the weather is, it is either raining or it is not. Thus, this sentence is a **NECESSARY TRUTH**.

Similarly, you do not need to check the weather to determine whether or not sentence (c) is true. It must be false, simply as a matter of logic. It might be raining here and not raining across town. It might be raining now but stop raining before you finish reading this sentence. It is impossible, however, for it to be both raining and not raining in the same place and at the same time. Therefore, this sentence is a **NECESSARY FALSEHOOD**.

sentences: necessary and contingent

A **NECESSARY TRUTH** is a sentence that must be true; it could not possibly be false.

A **NECESSARY FALSEHOOD** is a sentence that must be false; it could not possibly be true.

A **CONTINGENT SENTENCE** is neither a necessary truth nor a necessary falsehood. It may be true or it may not.

Finally, a sentence might always be true and still be contingent. For instance, this sentence is true:

- d. Mary Todd married Abraham Lincoln in 1842.

And there is no way, now, that it will ever be false. But it could have been false. Todd and Lincoln could have gotten married in a different year, or Todd could have married someone else or no one at all. A full analysis of this (and other) contingent truths would be too lengthy to undertake here, but hopefully you can see that things could have worked out in such a way that (d) would be false.

This is in contrast to a sentence like this one: ‘Today, in Starkville, Mississippi, it is Thursday, or it is not Thursday’. Or this one: ‘ $5 + 7 = 12$ ’. These sentences cannot be false, and there is no way to imagine a possible series of events that would make them false. Hence, they are not contingent. They are necessary truths.

### 3.4 Practice exercises

**A.** Determine if each sentence is a necessary truth, a necessary falsehood, or contingent.

1. Caesar crossed the Rubicon.
2. No one has ever crossed the Rubicon.
3. If Caesar crossed the Rubicon, then someone has.
4. Even though Caesar crossed the Rubicon, no one has ever crossed the Rubicon.
5. If anyone has ever crossed the Rubicon, it was Caesar.
6. Elephants dissolve in water.
7. Wood is a light, durable substance useful for building things.
8. If wood is a good building material, it is useful for building things.
9. I live in a three story building that is two stories tall.
10. If gerbils are mammals, they nurse their young.

**B.** Which of the following pairs of sentences are necessarily equivalent?

1. Elephants dissolve in water.  
If you put an elephant in water, it will dissolve.
2. All mammals dissolve in water.  
If you put an elephant in water, it will dissolve.
3. George Bush was the 43rd president.  
Barack Obama was the 44th president.
4. Barack Obama was the 44th president.  
Barack Obama was president immediately after the 43rd president.
5. Thelonious Monk played with John Coltrane.  
John Coltrane played with Thelonious Monk.

**C.**

- G1. There are at least four giraffes at the wild animal park.
- G2. There are exactly seven gorillas at the wild animal park.
- G3. There are not more than two Martians at the wild animal park.
- G4. Every giraffe at the wild animal park is a Martian.

Determine if each set of sentences is jointly possible or jointly impossible.

- 1. Sentences G2, G3, and G4
- 2. Sentences G1, G3, and G4
- 3. Sentences G1, G2, and G4
- 4. Sentences G1, G2, and G3

**D.**

- M1. All people are mortal.
- M2. Socrates is a person.
- M3. Socrates will never die.
- M4. Socrates is mortal.

Determine if each set of sentences is jointly possible or jointly impossible.

- 1. Sentences M1, M2, and M3
- 2. Sentences M2, M3, and M4
- 3. Sentences M2 and M3
- 4. Sentences M1 and M4
- 5. Sentences M1, M2, M3, and M4

**E.** For each statement, determine whether or not it is possible. If it is possible, give an example that illustrates the statement. If it is not possible, explain why not.

- 1. A valid argument, the conclusion of which is a necessary falsehood
- 2. An invalid argument, the conclusion of which is a necessary truth
- 3. A necessary truth that is contingent
- 4. Two necessarily equivalent sentences, both of which are necessary truths
- 5. Two necessarily equivalent sentences, one of which is a necessary truth and one of which is contingent

6. Two necessarily equivalent sentences that together are jointly impossible
7. A jointly possible collection of sentences that contains a necessary falsehood
8. A jointly impossible set of sentences that contains a necessary truth
9. A valid argument with premises that are all necessary truths and with a conclusion that is contingent
10. A valid argument with true premises and a false conclusion
11. A jointly possible collection of sentences that contains two sentences that are not necessarily equivalent
12. A jointly possible collection of sentences, all of which are contingent
13. A false necessary truth
14. A valid argument with false premises
15. A necessarily equivalent pair of sentences that are not jointly possible

### 3.5 Answers

A. For each of the following: Is it necessarily true, necessarily false, or contingent?

1. Caesar crossed the Rubicon.  
Contingent
2. No one has ever crossed the Rubicon.  
Contingent
3. If Caesar crossed the Rubicon, then someone has.  
Necessarily true
4. Even though Caesar crossed the Rubicon, no one has ever crossed the Rubicon.  
Necessarily false
5. If anyone has ever crossed the Rubicon, it was Caesar.  
Contingent
6. Elephants dissolve in water.  
Contingent
7. Wood is a light, durable substance useful for building things.  
Contingent

8. If wood is a good building material, it is useful for building things.  
Necessarily true
9. I live in a three story building that is two stories tall.  
Necessarily false
10. If gerbils are mammals, they nurse their young.  
This sentence is necessarily true. (*Mammalia* is defined as the class of animals wherein the females have mammaries and nurse their young. Hence, 'If gerbils are mammals, they nurse their young' is necessarily true.)

**B.**

1. Elephants dissolve in water.  
If you put an elephant in water, it will dissolve.  
These sentences are necessarily equivalent.
2. All mammals dissolve in water.  
If you put an elephant in water, it will dissolve.  
These sentences are *not* necessarily equivalent.
3. George Bush was the 43rd president.  
Barack Obama was the 44th president.  
These sentences are *not* necessarily equivalent.
4. Barack Obama was the 44th president.  
Barack Obama was president immediately after the 43rd president.  
These sentences are necessarily equivalent.
5. Thelonious Monk played with John Coltrane.  
John Coltrane played with Thelonious Monk.  
These sentences are necessarily equivalent.

**C.**

- G1. There are at least four giraffes at the wild animal park.
- G2. There are exactly seven gorillas at the wild animal park.
- G3. There are not more than two Martians at the wild animal park.
- G4. Every giraffe at the wild animal park is a Martian.

1. Sentences G2, G3, and G4 Jointly possible
2. Sentences G1, G3, and G4 Jointly impossible

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|---|------------------|
| 3. Sentences G <sub>1</sub> , G <sub>2</sub> , and G <sub>4</sub> | Jointly possible |
| 4. Sentences G <sub>1</sub> , G <sub>2</sub> , and G <sub>3</sub> | Jointly possible |

**D.**

- M<sub>1</sub>. All people are mortal.  
 M<sub>2</sub>. Socrates is a person.  
 M<sub>3</sub>. Socrates will never die.  
 M<sub>4</sub>. Socrates is mortal.

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| 1. Sentences M <sub>1</sub> , M <sub>2</sub> , and M <sub>3</sub> | Jointly impossible |
| 2. Sentences M <sub>2</sub> , M <sub>3</sub> , and M <sub>4</sub> | Jointly impossible |
| 3. Sentences M <sub>2</sub> and M <sub>3</sub>                    | Jointly possible   |
- Person*, at least in the philosophical sense, is different than *human being* (although the two concepts generally overlap). *Person* means, basically, *moral agent*, and so, for instance, God, if he exists, is a person. Consequently, just the sentence 'Socrates is a person' doesn't tell us whether or not Socrates will die.
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| 4. Sentences M <sub>1</sub> and M <sub>4</sub>                                     | Jointly possible   |
| 5. Sentences M <sub>1</sub> , M <sub>2</sub> , M <sub>3</sub> , and M <sub>4</sub> | Jointly impossible |

**E.**

1. A valid argument, the conclusion of which is a necessary falsehood  
 Yes, this is possible. This is a valid argument, and the conclusion is a necessary falsehood:  
 P<sub>1</sub>. If today is Tuesday, then  $1 + 1 = 3$ .  
 P<sub>2</sub>. Today is Tuesday.  
 C. Therefore,  $1 + 1 = 3$ .
2. An invalid argument, the conclusion of which is a necessary truth  
 No, this is not possible. If the conclusion is necessarily true, then there is no way to make it false, and hence no way to make it false whilst making all the premises true.
3. A necessary truth that is contingent  
 No, this is not possible. If a sentence is a necessary truth, it cannot possibly be false, but a contingent sentence can be false.



4. Two necessarily equivalent sentences, both of which are necessary truths  
Yes, this is possible, e.g., '4 is even' and '4 is divisible by 2'.
5. Two necessarily equivalent sentences, one of which is a necessary truth and one of which is contingent  
No, this is not possible. A necessary truth cannot possibly be false, while a contingent sentence can be false. So in any situation in which the contingent sentence is false, it will have a different truth value from the necessary truth. Thus, they will not necessarily have the same truth value, and so they will not be equivalent.
6. Two necessarily equivalent sentences that together are jointly impossible  
Yes, this is possible, e.g., ' $1 + 1 = 4$ ' and ' $1 + 1 = 3$ '.
7. A jointly possible collection of sentences that contains a necessary falsehood  
No, this is not possible. If a sentence is necessarily false, there is no way to make it true, let alone make it true along with all the other sentences.
8. A jointly impossible set of sentences that contains a necessary truth  
Yes, this is possible. ' $1 + 1 = 4$ ' and ' $1 + 1 = 2$ '.
9. A valid argument with premises that are all necessary truths and with a conclusion that is contingent \*  
This is not possible. In a deductively valid argument, the information contained in the conclusion is information that is in the premises. If all of the premises are necessary truths, then the conclusion will be as well.
10. A valid argument with true premises and a false conclusion  
This is not possible. A valid argument is one where if the premises are true, then the conclusion has to be true. Thus, if the premises are true, the conclusion has to be as well.
11. A jointly possible collection of sentences that contains two sentences that are not necessarily equivalent  
Yes, this is possible. G1 and G2 on p. 19 are jointly possible, but they are not necessarily equivalent.
12. A jointly possible collection of sentences, all of which are contingent

Yes, this is possible. G1 and G2 on p. 19 are both contingent.

13. A false necessary truth

This is not possible. A necessary truth is a sentence that has to be true, and so it could not be false.

14. A valid argument with false premises

Yes, this is possible. This argument has false premises, and it is valid:

P1. Mississippi is in Canada.

P2. New York City is in Mississippi.

C. Therefore, New York City is in Canada.

15. A necessarily equivalent pair of sentences that are not jointly possible

Yes, this is possible (although it isn't the standard case). These two sentences are both necessary falsehoods:

(a) It is November and it is not November.

(b) Jeff is in Texas and he is not in North America.

Since they are both always false, they are necessarily equivalent. But, at the same time, since they are both always false, they cannot be jointly possible.