

## 3 Other concepts of logic

The concept of a valid argument is central to logic. In this section, we will introduce some other important concepts that apply just to sentences, not to full arguments.

### 3.1 Joint possibility

Consider these two sentences:

- B1. Jane's only brother is shorter than her.
- B2. Jane's only brother is taller than her.

Logic alone cannot tell us which, if either, of these sentences is true. Yet we can say that *if* B1 is true, *then* B2 must be false. Similarly, if B2 is true, then B1 must be false. It is impossible that both sentences are true at the same time. In other words, these sentences are inconsistent. On the other hand, G1 and G2 can both be true at the same time.

- G1. There are at least four giraffes at the wild animal park.
- G2. There are exactly seven gorillas at the wild animal park.

One of these sentences may be false and the other true, but it is *possible* that they are both true at the same time. These observations motivate the following definitions.

jointly possible and impossible

Sentences are JOINTLY POSSIBLE when, and only when, it is possible for them all to be true at the same time.

Sentences are JOINTLY IMPOSSIBLE when, and only when, it is *not* possible for them all to be true at the same time.

So,  $G_1$  and  $G_2$  are *jointly possible* while  $B_1$  and  $B_2$  are *jointly impossible*.

We can investigate the joint possibility of any number of sentences. For example, let's add two more sentences to  $G_1$  and  $G_2$ :

- $G_1$ . There are at least four giraffes at the wild animal park.
- $G_2$ . There are exactly seven gorillas at the wild animal park.
- $G_3$ . There are not more than two extra-terrestrials at the wild animal park.
- $G_4$ . Every giraffe at the wild animal park is an extra-terrestrial.

Together,  $G_1$  and  $G_4$  entail that there are at least four extra-terrestrials giraffes at the park. This conflicts with  $G_3$ , which implies that there are no more than two extra-terrestrial giraffes there. So the sentences  $G_1$ – $G_4$  are jointly impossible. They cannot all be true together. (Note that the sentences  $G_1$ ,  $G_3$  and  $G_4$  are jointly impossible.  $G_1$ ,  $G_2$ , and  $G_3$ , meanwhile, are jointly possible.)

### 3.2 Necessary truths, necessary falsehoods, and contingency

Consider these sentences:

- a*. It is raining.
- b*. Either it is raining here, or it is not.
- c*. It is both raining here and not raining here.

In order to know if sentence *a* is true, you would need to look outside or check the weather channel. It might be true; it might be false. A sentence that is capable of being true and capable of being false (in different circumstances, of course) is **CONTINGENT**.

Sentence *b* is different. You do not need to look outside to know that it is true. Regardless of what the weather is like, it is either raining or it is not. Thus, this sentence is a **NECESSARY TRUTH**.

Equally, you do not need to check the weather to determine whether or not sentence *c* is true. It must be false, simply as a matter of logic. It

might be raining here and not raining across town; it might be raining now but stop raining even as you finish this sentence; but it is impossible for it to be both raining and not raining in the same place and at the same time. So, whatever the world is like, it is not both raining here and not raining here. This sentence is a **NECESSARY FALSEHOOD**.

Finally, one thing to note is that a sentence might always be true and still be contingent. For instance, this sentence is true: 'Mary Todd married Abraham Lincoln in 1842'. And there is no way, now, that it will ever be false. But it could have been false. Todd and Lincoln could have gotten married in a different year, or Todd could have married someone else or no one at all. A full analysis here would be too lengthy, but hopefully you can see that things could have worked out in such a way that the sentence would be false. That is in contrast to a sentence like this one: 'Today, in Starkville, Mississippi, it is Thursday, or it is not Thursday'. Or this one: ' $5 + 7 = 12$ '. These sentences cannot be false, and there is no way to imagine a possible series of events that would make them false. Hence, they are not contingent. They are necessary truths.

## Necessary equivalence

We can also ask about the logical relations *between* two sentences. For example:

John went to the store after he washed the dishes.

John washed the dishes before he went to the store.

These two sentences are both contingent, since John might not have gone to the store or even washed dishes. Yet they must have the same truth value. That is, they must either both be true or both be false. When two sentences necessarily have the same truth value, we say that they are **NECESSARILY EQUIVALENT**.

## Summary of some concepts of logic

An argument is (deductively) **VALID** if it is impossible for the premises to be true and the conclusion false. It is **INVALID** oth-

erwise.

A collection of sentences is **JOINTLY POSSIBLE** if it is possible for all these sentences to be true together; it is **JOINTLY IMPOSSIBLE** otherwise.

A **NECESSARY TRUTH** is a sentence that must be true; it could not possibly be false.

A **NECESSARY FALSEHOOD** is a sentence that must be false; it could not possibly be true.

A **CONTINGENT SENTENCE** is neither a necessary truth nor a necessary falsehood. It may be true or it may not.

Two sentences are **NECESSARILY EQUIVALENT** if they must have the same truth value. (I.e., they must both be true or they both must be false.)

### 3.3 Practice exercises

**A.** Determine if each sentence is a necessary truth, a necessary falsehood, or contingent.

1. Caesar crossed the Rubicon.
2. Someone once crossed the Rubicon.
3. No one has ever crossed the Rubicon.
4. If Caesar crossed the Rubicon, then someone has.
5. Even though Caesar crossed the Rubicon, no one has ever crossed the Rubicon.
6. If anyone has ever crossed the Rubicon, it was Caesar.
7. Elephants dissolve in water.
8. Wood is a light, durable substance useful for building things.
9. If wood is a good building material, it is useful for building things.
10. I live in a three story building that is two stories tall.
11. If gerbils are mammals, they nurse their young.