is 45 percent, and so the probability that Oddsymandias will win is 55 percent. That makes the odds on The Wiz 13 to 7 and the odds on Oddsymandias 7 to 13. Again, Jeff and Claire are wagering. Jeff is betting on The Wiz to win; Claire is going with Oddsymandias. But this time, there is a bookmaker involved. Jeff bets $7. If The Wiz wins, then the bookmaker will give Jeff $13 plus his "stake," the $7 he put down. Claire bets $13, and if Oddsymandias wins, she will get $7 plus her stake, the $13 she bet. No one else is betting. The horses race, and Oddsymandias wins. Claire gets $7 plus her $13. Jeff gets nothing. Simple enough, but the bookmaker only handled the transaction. He didn't make or lose any money, and he wants to make money.

Let's redo the example. The bookmaker knows the best estimate of each horse's chances of winning, but he sets the odds this way:

the odds on The Wizard of Odds winning are 13 to 8

the odds on Oddsymandias winning are 7 to 14

These make the implied probabilities 38 percent and 67 percent, for a total of 105 percent. But breaking the rule that mutually exclusive probabilities add up to 1.0 will benefit this bookmaker. Jeff now puts down $8, and Claire puts down $14. So, before the race, the bookmaker is holding $22. The horses race, Oddsymandias wins, and Claire gets $7 plus her $14. That's $21, and so the bookmaker keeps $1.

Of course, it is not always that easy for bookmakers to ensure a profit. To do so, they must carefully gauge the true probabilities—using both their own knowledge and information about how people are betting. Then they try to set the odds, or adjust them if need be, so that no matter the outcome, they will make a little bit of money.

The one thing that bookmakers want to avoid is setting the odds so that someone can put a bet on each possible outcome and be certain of making a profit. This happens when the probabilities given by the odds total less than 1.0. For instance, in a horse race with three horses, if the odds on the first horse to win are 3 to 2, the odds on the second horse are 5 to 3, and the odds on the third horse are 7 to 1, then the probabilities are 40 percent, 37.5 percent, and 12.5 percent, which sum to only 90 percent. Now, if one person bets on all three horses, that bettor can be certain of making money. With the amounts given in table B.2—$64 on horse 1, $60 on horse 2, and $20 on horse 3—no matter which horse wins, the bettor will make a $16 profit, which is equivalent to a $16 loss for the bookmaker.

So, almost always, the odds given by a bookmaker are slightly higher than the true odds and total more than 1.0. That allows the bookmaker to make a profit and prevents a situation where he or she is guaranteed a loss.

### B.2 Expected Value

In his book *The Signal and the Noise*, the political forecaster Nate Silver describes a bet made by a Canadian college student who also worked as a skycap at the Winnipeg International Airport. Bob Voulgaris had saved $80,000 and, a few weeks after the

**Table B.2**
A single bettor puts \$64 on horse 1, \$60 on horse 2, and \$20 on horse 3. If horse 1 wins, then for every \$2 she bet on horse 1, she gets \$3 (and she gets her stake back). Having bet \$64, she gets \$96—her winnings—plus the \$64, for a total of \$160. Since all three of her bets only totaled \$144, she makes a \$16 profit. Similarly, if either of the other two horses win instead, she will make a \$16 profit.

| | | | | If that horse wins | |
|---|---|---|---|---|---|
| | Odds | Probability | Amount bet (\$) | Winnings (\$) | Total payoff (\$) |
| Horse 1 | 3 to 2 | .40 | 64 | $^{64}/_{2} \times 3 = 96$ | 160 |
| Horse 2 | 5 to 3 | .375 | 60 | $^{60}/_{3} \times 5 = 100$ | 160 |
| Horse 3 | 7 to 1 | .125 | 20 | $^{20}/_{1} \times 7 = 140$ | 160 |
| Totals | | .90 | 144 | | |

1999–2000 NBA season began, he bet it all on the Los Angeles Lakers to win the NBA championship. Silver writes,

> The Vegas line on the Lakers at the time that Voulgaris placed his bet implied that they had a 13 percent chance of winning the NBA title. Voulgaris did not think the Lakers' were 100 percent or even 50 percent—but he was confident they were quite a bit higher than 13 percent. Perhaps more like 25 percent, he thought. If Voulgaris's calculation was right, the bet had a theoretical profit of \$70,000.[2]

If the Lakers won the championship, Voulgaris would get \$520,000. If they lost, then he would lose his \$80,000. So, +\$520,000 or –\$80,000. Where does \$70,000 come from?

With respect to Voulgaris's bet, there are only two events that matter: (1) the Los Angeles Lakers win the NBA championship, or (2) the Los Angeles Lakers lose the NBA championship. For Voulgaris, the value of *the Lakers win* is \$520,000. But what's really important is the *weighted* or *expected value* of this event. That's the probability that it will happen multiplied by the \$520,000:

$$.25 \times \$520,000 = \$130,000 \tag{B.4}$$

Likewise for *the Lakers lose*:

$$.75 \times -\$80,000 = -\$60,000 \tag{B.5}$$

2. Nate Silver, *The Signal and the Noise* (New York: Penguin Press, 2012), 237.

The expected value of the bet itself, then, is the expected value of *the Lakers win* plus the expected value of *the Lakers lose*.

$$E(bet) = \$130,000 + (-\$60,000) = \$70,000 \tag{B.6}$$

Of course, Voulgaris would never get \$70,000. It's \$520,000 or –\$80,000. But often the expected value is useful when making a decision. Voulgaris's decision was between placing the bet or not placing the bet. The expected value of the latter is simply

$$E(no\ bet) = (.25 \times \$0) + (.75 \times \$0) = \$0 \tag{B.7}$$

Since placing the bet has a higher expected value, it is—or, at least, might be—considered the better choice. Voulgaris went with it, and at the end of the season

> the Lakers disposed of the Indiana Pacers in efficient fashion to win their first NBA title since the Magic Johnson era. And Bob the skycap was halfway to becoming a millionaire.[3]

Another example is in table B.3. There, a choice has to be made between betting that a red marble will be drawn and betting that a blue marble will be drawn. If a red marble is drawn from the jar, the payoffs are \$7 and –\$3, depending on which bet was made. If a blue marble is drawn, the payoffs are –\$4 and \$9. Just looking at these numbers, *bet on blue* looks like the better choice. The positive payoff is higher than it is for bet on red (\$9 versus \$7), and the negative payoff is also higher (–\$3 instead of –\$4).

But since the probability of a red marble being drawn and the probability of a blue marble being drawn are known, it's worth finding the expected value of each option.

$$\begin{aligned} E(bet\ on\ red) &= (.60 \times \$7) + (.40 \times -\$4) = 4.20 + (-1.60) = \$2.60 \\ E(bet\ on\ blue) &= (.60 \times -\$3) + (.40 \times \$9) = -1.80 + 3.60 = \$1.80 \end{aligned} \tag{B.8}$$

**Table B.3**
One marble will be selected from a jar that contains 60 percent red marbles and 40 percent blue marbles. The amount paid to make the bet, whatever it is, and the winnings (or lack there of) are combined into one value for each possible outcome.

| | P(R) = .60 | P(B) = .40 |
|---|---|---|
| | **Red is drawn** | **Blue is drawn** |
| Bet on red | \$7 | –\$4 |
| Bet on blue | –\$3 | \$9 |

3. Silver, *The Signal and the Noise*, 237.

*Bet on red* has a higher expected value, and so most people would agree that it's actually the better choice. The benefit of using the expected values to make a decision is pretty straightforward. The probabilities are extra information, and the expected value for each option combines the possible payoffs and the probabilities for each payoff into one number, which can then be compared to the expected value of the other option or options.

In some cases, however, many people would be wary of making a decision based on the expected values. Consider the choice between lottery A and lottery B in table B.4. This quick calculation shows that the expected value of lottery B is higher, and intuitively that seems like the better choice:

$$\begin{aligned} \mathrm{E}(\textit{lottery A}) &= (.01 \times \$100) + (.10 \times \$100) + (.89 \times \$100) = \$100 \\ \mathrm{E}(\textit{lottery B}) &= (.01 \times \$0) + (.10 \times \$500) + (.89 \times \$100) = \$139 \end{aligned} \tag{B.9}$$

The two lotteries in table B.5 have the same format and the expected values work out the same way, just with more zeros.

$$\begin{aligned} \mathrm{E}(\textit{lottery A}) &= (.01 \times \$1,000,000) + (.10 \times \$1,000,000) + (.89 \times \$1,000,000) \\ &= \$1,000,000 \\ \mathrm{E}(\textit{lottery B}) &= (.01 \times \$0) + (.10 \times \$5,000,000) + (.89 \times \$1,000,000) \\ &= \$1,390,000 \end{aligned} \tag{B.10}$$

Now, however, many people will—very reasonably, it seems—ignore the expected values and make a decision based on just the payoffs. The guarantee of $1 million that comes with choosing lottery A looks a lot more appealing than choosing lottery B and taking a small chance at getting $0. The example just shows that, while often the expected value is important for understanding the value of an option, it's not, for everyone, always going to be the basis for making a decision.

**Table B.4**

A choice between two lotteries. Lottery A and lottery B have the same format. For each, there are 100 tickets, and one ticket will be drawn. The only differences between the two lotteries are the payoffs for tickets 1 – 11.

| | P = .01 | P = .10 | P = .89 |
|---|---|---|---|
| | Ticket # 1 is selected | A ticket # 2 – 11 is selected | A ticket # 12 – 100 is selected |
| Lottery A | $100 | $100 | $100 |
| Lottery B | $0 | $500 | $100 |

**Table B.5**
A choice between two lotteries. These lotteries are identical to the ones described in table B.4 except that the payoffs are increased 10,000-fold.

| | P = .01 | P = .10 | P = .89 |
|---|---|---|---|
| | Ticket # 1 is selected | A ticket # 2 – 11 is selected | A ticket # 12 – 100 is selected |
| Lottery A | $1,000,000 | $1,000,000 | $1,000,000 |
| Lottery B | $0 | $5,000,000 | $1,000,000 |

## B.3 Where Do Probabilities Come From?

The probabilities that were used in chapter 6 were taken directly from the proportions of the populations. Often, however, probabilities are used when there are not any populations available. For example,

- There is a 15 percent chance that the Philadelphia Phillies will win the next World Series.
- There is a 4 percent chance that there is other life, of some kind, in our solar system.
- There is a 60 percent chance that it will rain tomorrow.

In all of these cases, a probability has been established without reference to a proportion of a population. This section explains—in a very nonrigorous manner—three different ways of establishing probabilities, two of which can be used without having a population. What probabilities are and the most basic or fundamental way of establishing them are interesting philosophical questions, but they are beyond the scope of what will be addressed here.

### B.3.1 Proportion of a Population

Although using the proportion of a population to find a probability has been covered, a couple of relevant points were not discussed in chapter 6. First, when a probability is derived from the proportion of a population, the process is actually just an application of the proportional syllogism. For example,

**P1** 90 percent of the marbles in the jar are red.
**P2** One marble is about to be selected from this jar.
**C** The probability is .90 that the selected marble will be red.

Recall from chapter 6 that the conclusion can also be stated this way, which is exactly the probability that is needed:

**C** P(the selected marble will be red) = .90

This way of assigning probabilities is used any time there are a certain number of outcomes and each is equally likely. For instance, when one die is rolled, there are six