Notes on chapter 1

We are starting with **decision theory**, and then we will move on to **game theory**. The distinction between the two is merely technical. In decision theory, there is just one decision maker who confronts the decision problem. In game theory, there are two or more decision makers (or players).

Notes for pp. 6 – 9

As explained on pp. 6 – 7, a decision problem has three parts: *options* (or *acts* or *possible actions*), *outcomes*, and *states* (or *states of the environment*).

Here is an example of a decision problem:

> After a hard day at MSU, you decide to go to the racetrack. By the time you arrive, there is only one race left. Three horses are racing and you are going to bet on which one will win. The horses are Sloppy Stan, Wobbly Pete, and Dave. If you pick either Sloppy Stan or Wobbly Pete and that horse finishes first, you get $10, otherwise you lose $4. If you pick Dave, and Sloppy Stan finishes first, you lose $12; if you pick Dave and Wobbly Pete finishes first, you get $5; and if you pick Dave and he finishes first, you get $10.

When we create a *decision table*, the options go on the left side of the table and the states go on the top.

	state 1	**state 2**	**state 3**
option 1	A	B	C
option 2	D	E	F
option 3	G	H	I

The outcomes, which are option-state pairs, are the A through I in this table. B, for instance, is the outcome that the decision maker gets if he or she chooses option 1 and state 2 occurs.

See if you can create a decision table, which will be just like this one, for the decision problem about betting on the horse race. The completed table is at the end of these notes.

One thing to be careful with when creating a decision table is identifying the states (i.e., what's at the top of the table). Sometimes people are tempted to make the states in this problem "I win" and "I lose," but that's wrong. (Almost always) the states are independent of what the decision maker chooses. They are just the possibilities that can happen no matter what the decision maker does. But "I win" or "I lose" are dependent on what the decision maker chooses (and so they are not independent).

In fact, "win" and "lose" usually won't be included in a decision table. *Winning* just means getting the outcome that the decision maker wants, and *losing* means getting the outcome that he or she doesn't want.

In the middle of p. 8, Resnik says, "For a problem specification to be definite and complete, its states must be mutually exclusive and exhaustive; that is, one and only one of the states must obtain." There are a few further things to say about this.

(1) *Mutually exclusive* and *exhaustive* are separate concepts. Taken together they mean that only one of the states will happen. But let's look at the definition of each concept.

> ***Mutually exclusive*****:** States are mutually exclusive when one of them occurring prevents any of the other states from occurring. In other words, one state occurring *excludes* the others from occurring.
>
> ***Exhaustive:*** A set of states are exhaustive when one of them must occur. In other words, the states that have been specified (and are given at the top of the table) exhaust all of the possibilities.

(2) If we are analyzing a real-world decision problem, it's always possible that we will make a mistake and the states will not be mutually exclusive and exhaustive. But for the purposes of learning decision theory, we will assume that the states in our decision problems are mutually exclusive and exhaustive. (Resnik says this at the bottom of p. 8.)

(3) On p. 8, Resnik is discussing states being mutually exclusive and exhaustive, but the same two concepts also apply to the options. At least for the purposes of learning the theory, the options given will be mutually exclusive (picking one excludes the possibility of picking any other one) and exhaustive (they are the decision maker's only options).

The dominance principle is defined at the bottom of p. 9. This is the first of several principles and rules for making decisions that we will examine. Consider this decision problem. Which option would you choose?

	A red marble is drawn	**a blue marble is drawn**	**a green marble is drawn**
bet on red	\$4	\$3	\$1
bet on blue	\$5	\$8	\$3
bet on green	\$2	\$1	\$3

To apply the dominance principle, we proceed as follows.

(1) Just consider 'bet on red' and 'bet on blue'. If a red marble is drawn, you will do better if choose 'bet on blue' (\$5 vs. \$4). If a blue marble is drawn, you will do better if you choose 'bet on blue' (\$8 vs. \$3). And if a green marble is drawn, you will do better if you choose 'bet on blue' (\$3 vs. \$1). **Therefore, 'bet on blue' dominates 'bet on red'.** Consequently, as you can see, it would never make sense to choose 'bet on red' when you can choose 'bet on blue'.

(2) When we compare 'bet on blue' and 'bet on green', we find that 'bet on blue' does better if either a red or a blue marble is drawn. If a green marble is drawn 'bet on blue' and 'bet on green' will have the same outcome (\$3). **So, 'bet on blue' dominates 'bet on green.** And, similarly, it would never make sense to choose 'bet on green' when you can choose 'bet on blue'.

(3) Since 'bet on blue' dominates all of the other options, the dominance principle tells us to choose it.

In many decision problems, one option won't dominate any of the others (much less all of the others), and so we can't always use the dominance principle. But when we can, we typically will want to do so.

Notes on pp. 13 – 14

Decision problems are classified based on what is known about the states occurring. There are three categories: *decisions under certainty*, *decisions under risk*, and *decisions under ignorance*. Find the definition of each on pp. 13 – 14.

I will illustrate these three concepts with an example about betting on the color of a marble that is about to be selected from a jar.

(1) Let's say a marble will be drawn from a jar containing only red marbles. (And you, the decision-maker, know that all of the marbles in the jar are red.) This is a decision under certainty.

	A red marble is drawn
bet on red	\$10
bet on blue	-\$1,000

(2) A marble will be randomly selected from a jar containing 60 percent red marbles and 40 percent blue marbles. So, you know the chance that a red marble will be drawn (60 percent chance) and the chance that a blue marble will be drawn (40 percent chance). This is a decision under risk. (*Risk* here doesn't mean that the decision is necessarily risky. It just means that we know how likely each state is to occur.)

	probability = .60 **A red marble is drawn**	probability = .40 **a blue marble is drawn**
bet on red	\$10	-\$4
bet on blue	-\$5	\$8

(3) A marble will be selected from a jar containing red and blue marbles, but you (the decision-maker) don't know how likely it is that either color will be drawn. This is a decision under ignorance.

	A red marble is drawn	**a blue marble is drawn**
bet on red	$10	-$4
bet on blue	-$5	$8

	Sloppy Stan wins	**Wobbly Pete wins**	**Dave wins**
bet on Sloppy Stan	$10	-$4	-$4
bet on Wobbly Pete	-$4	$10	-$4
bet on Dave	-$12	$5	$10