3 Other concepts of logic

Valid is a central concept in logic. In this section, we will introduce some other important concepts that apply just to sentences, not to full arguments.

3.1 Concepts for single sentences

Consider these three sentences about you:

a. I am in Mississippi.
b. Either I am in Mississippi, or I am not in Mississippi.
c. I am in Mississippi, and I am not in Mississippi.

In order to know if sentence (*a*) is true, you would need to check where you are. Depending on where that is, the sentence might be true, or it might be false. A sentence that is capable of being true and capable of being false (in different circumstances, of course) is CONTINGENT.

Sentence (*b*) is different. You do not need to check your location to know that it is true. Wherever you are, this sentence will be true. Thus, this sentence is a NECESSARY TRUTH.

Similarly, you do not need to check anything to determine whether or not sentence (*c*) is true. It must be false, simply as a matter of logic. You might be jumping back and forth over the Mississippi state line, or you might even be standing with one foot in Mississippi and one foot in Tennessee (in which case you are still in Mississippi). It is impossible, however, for you, at any one moment, to be both in and not in Mississippi. Therefore, this sentence is a NECESSARY FALSEHOOD.

sentences: necessary and contingent

A NECESSARY TRUTH is a sentence that must be true; it could not possibly be false.

A NECESSARY FALSEHOOD is a sentence that must be false; it could not possibly be true.

A CONTINGENT SENTENCE is neither a necessary truth nor a necessary falsehood. It may be true or it may not.

An easily misunderstood aspect of this analysis is that a sentence might always be true and still be contingent. For instance, this sentence is true:

d. Mary Todd married Abraham Lincoln in 1842.

And there is no way, now, that it will ever be false. But it could have been false. Todd and Lincoln could have gotten married in a different year, or Todd could have married someone else or no one at all. Thus, it is contingent. A full analysis of this (and other) contingent truths would be too lengthy to undertake here, but hopefully you can see that things could have worked out in such a way that (*d*) would be false.

This is in contrast to sentences like these:

e. Today, in Starkville, Mississippi, it is Thursday, or it is not Thursday.
f. Five plus seven equals twelve.
g. Every oncologist is a doctor.

These sentences cannot be false, and there is no way to imagine a possible series of events that would make them false. Hence, they are not contingent. They are necessary truths.

3.2 Joint possibility

Consider these two sentences:

B1. Jane's only brother is shorter than her.
B2. Jane's only brother is taller than her.

Without knowing Jane and her brother, we have no way of knowing which, if either, of these sentences is true. Yet we can say that *if* B1 is true, *then* B2 must be false. Similarly, if B2 is true, then B1 must be false. It is impossible that both sentences are true at the same time. On the other hand, G1 and G2 can both be true at the same time.

G1. There are at least four giraffes at the wild animal park.
G2. There are exactly seven gorillas at the wild animal park.

Both of these sentences might be false. One of them might be false while the other is true. But it is *possible* that they are both true.

jointly possible and impossible

A set of sentences are JOINTLY POSSIBLE when, and only when, it is possible for them all to be true at the same time.

A set of sentences are JOINTLY IMPOSSIBLE when, and only when, it is *not* possible for them all to be true at the same time.

So, G1 and G2 are *jointly possible* while B1 and B2 are *jointly impossible*.

We can investigate the joint possibility of any number of sentences. For example, let's add two more sentences to G1 and G2:

G1. There are at least four giraffes at the wild animal park.
G2. There are exactly seven gorillas at the wild animal park.
G3. There are not more than two extra-terrestrials at the wild animal park.
G4. Every giraffe at the wild animal park is an extra-terrestrial.

Together, G1 and G4 entail that there are at least four extra-terrestrials giraffes at the park. This conflicts with G3, which states that there are no more than two extra-terrestrials there. So, G1–G4 are jointly impossible. They cannot all be true together. (Notice also that just G1, G3 and G4 are jointly impossible, while G1, G2, and G3 are jointly possible.)

3.3 Necessary equivalence

Sentences G1 and G2 (which we said were jointly possible) can both be true at the same time. They can also both be false, or one false and the other true. A stronger relationship holds between these two sentences:

> John went to the store after he washed the dishes.
> John washed the dishes before he went to the store.

These two sentences must have the same truth value. That is, they must either both be true or both be false. It is impossible for one to be true and one to be false (at the same time). When two sentences *must* have the same truth value, they are NECESSARILY EQUIVALENT.

necessarily equivalent

Two sentences are NECESSARILY EQUIVALENT if they must have the same truth value. That is, they must both be true or they both must be false.

3.4 Practice exercises

A. Determine if each sentence is a necessary truth, a necessary falsehood, or contingent.

1. Caesar crossed the Rubicon.
2. No one has ever crossed the Rubicon.
3. If Caesar crossed the Rubicon, then someone has.
4. Even though Caesar crossed the Rubicon, no one has ever crossed the Rubicon.
5. If anyone has ever crossed the Rubicon, it was Caesar.
6. Elephants dissolve in water.
7. Wood is a light, durable substance useful for building things.
8. If wood is a good building material, it is useful for building things.
9. I live in a three story building that is two stories tall.
10. If gerbils are mammals, they nurse their young.

B. Which of the following pairs of sentences are necessarily equivalent?