An N-body Investigation of Halo Density Profiles under Various Initial Conditions

Yihan Li

June 2025

Contents

1 Clarification of The Process

1.1 Construction of Initial Conditions

The initial state of the system is set to represent a quiescent, isolated overdensity, which serves as the progenitor of the collapsed halo. The generation process begins by populating a unit sphere with a particle distribution that is uniform by volume, achieved by sampling a radial coordinate $r \approx u^{1/3}$ where u is a random number from a uniform distribution U(0,1). This spherical distribution then undergoes a linear transformation to produce the desired initial geometry, either a sphere of a specified radius or a triaxial ellipsoid defined by three semi-axes (a,b,c). To investigate the initial distribution, optional small-scale perturbations are introduced by applying a Gaussian random displacement vector, with a specified standard deviation σ_{pert}, to the position of each particle. Finally, an isotropic velocity field is created by assigning each particle a velocity vector drawn from a 3D Gaussian distribution with a specified dispersion, v_{disp}, which sets the initial virial ratio of the system.

1.2 Dynamical Evolution of the N-body System

The time evolution of the particles is governed by the N-body equations of motion, which are integrated using the REBOUND gravitational dynamics code. I initially utilized the ias15 integrator, a high-accuracy adaptive time-step method, but it would take too long to run a small size simulation. So I turned to whfast, which only has fixed time steps, yet much faster for this project. The simulation lasts from t=0 to a specified final time, t_{end}. Throughout the integration, the full phase-space information (three-dimensional positions and velocities) of all particles is recorded at set time points. This sequence of "snapshots" provides a complete history of the system's evolution, capturing the essential physics of gravitational collapse, violent relaxation, and the eventual settling into a quasi-virialized equilibrium state.

1.3 Post-Simulation Analysis

To characterize the structure of the final relaxed halo, a series of analysis steps are performed on the final snapshot. First, the halo's center is determined using an iterative algorithm $find_robust_com$. This will calculate the center of mass within a progressively shrinking sphere, thus finding the potential minimum of the primary bound structure. With the center chosen, the radial distance of each particle is computed. The system is then binned into a series of concentric spherical shells of either linear or logarithmic width. By dividing the number of particles in each shell by the shell's volume, the spherically-averaged density profile, $\rho(r)$, is robustly measured. The analysis radius is dynamically scaled relative to the halo's estimated virial radius r_{200} to ensure the entire structure is captured. For comparison and clarity, I also plot the data points in a parameter space $\rho - r$, so that different final distributions would be easily identified.

1.4 Fitting With NFW Profile

The final step involves a comparison of the simulated halo's structure with established theoretical models. Using the $scipy.optimize.curve_fit$ function, a non-linear least squares fit is performed to match an analytical density profile to the measured data points from the simulation. I have implemented both the standard two-parameter $Navarro - Frenk - White$ (NFW) profile and a more flexible three-parameter generalized NFW (gNFW) profile, where the inner logarithmic slope, γ, is treated as a free parameter. Although I finally gave up this three-parameter fitting method, as the data points are not enough for a complex fitting like that. By comparing these data points and fitting results across simulations with varying initial conditions, we can test hypotheses regarding the impact of formation history on the final equilibrium structure of dark matter halos (the parameter space we explored is very limited tho).

2 Result Analysis

2.1 Hot/Cold, Sphere/Ellipsoid, Perturbation

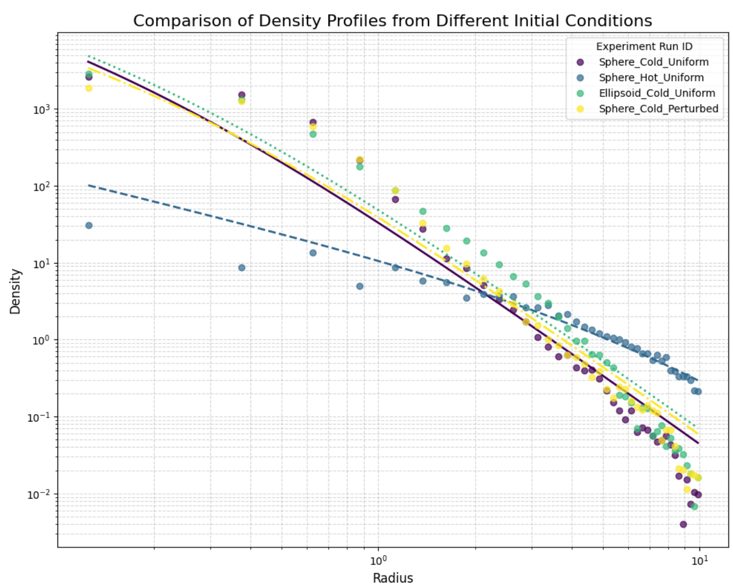

Figure 1: DM halo density profiles under different initial conditions

Dots are data points collected as described above, and lines are fitted NFW profiles. First we can notice that when radius is small, the fitted lines are not very consistent with the data points. This is because central part of the DM halo is so dense and gravitational interactions between particles is so strong that our simple simulation cannot properly handle.

This baseline run, "Sphere Cold Uniform", representing a quiescent spherical collapse, produced the most compact and centrally dense halo, as evidenced by its high density at small radii and steep profile. As for the "Sphere Hot Perturbed", although it looks very close to the previous one, it shows some inclination to be slightly more diffused. This could be explained by the role of dynamical heating during the hierarchical merging of substructures, which puffs up the central region and reduces its density. Same for the "Ellipsoid Cold Uniform", where the non-spherical initial condition leads to a more chaotic relaxation and eventually a more puffed distribution. The blue dots, representing "Sphere Hot Uniform", have the most distinct difference, with a significantly diffused halo. The high initial kinetic energy counteracted gravitational collapse, leading to a structure with a density that is orders of magnitude lower.

I did try to tweak the parameters to get a more easy-to-tell result, but those three cases still look very close. Perhaps our simulation, as many simplification as it already had, is not very sensitive to these parameters.

2.2 GIF(fake)

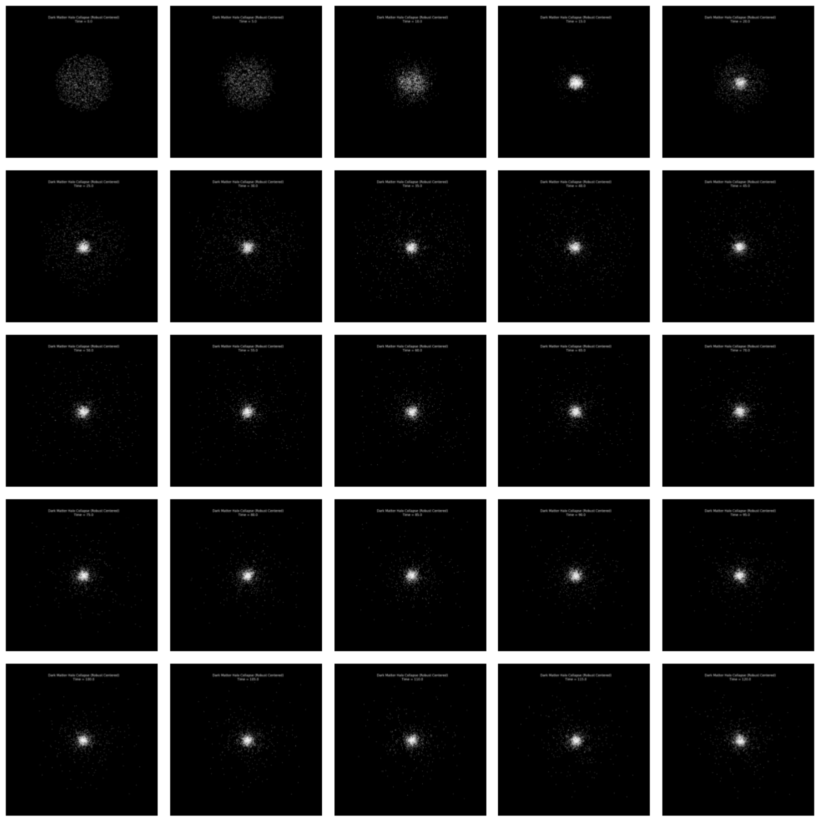

Figure 2: fake GIF plot of dark matter halo relaxation

Honestly, the first reason for me to pick up n-body simulation as my final project is the GIF image of two dark matter halo colliding shown in class. The vivid dynamic is just so captivating. So here I show one GIF I made through this simulation. From this, we can see that the evolution time is enough for these particles to relax and form a bound structure.

3 Encountered Problems

3.1 Runaway Halo

As prof. Jiang expected, although I've already set the initial coordinates to its barycentric coordinates and its total momentum to 0, the system would still try to run away from the center in the simulation.

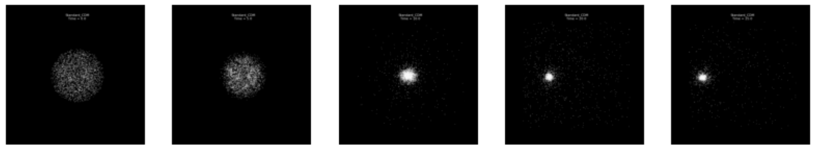

Figure 3: just a super introverted halo

The reason might be that little residual created during the numerical process add up to a visible effect. To solve that, I refined the code so that it would recalculate the center of mass of all particles and move to its barycentric coordinates every time I try to capture the system. That worked for a while, but then failed again. After a careful examine, the reason might be caused by my decreasing the softening length, and the two-body relaxation process got so violent that some of the particles got accelerated to a very high orbit, so when calculating the center of mass, the result would be confusing.

In order to solve that, I adapted a new method to iteratively find the center of mass, which would exclude those faraway particles and recalculate a center then iterate this for a few times. This seems to work fine.

3.2 Wait, why is there a CORE?

The first successful plot I made looks like this:

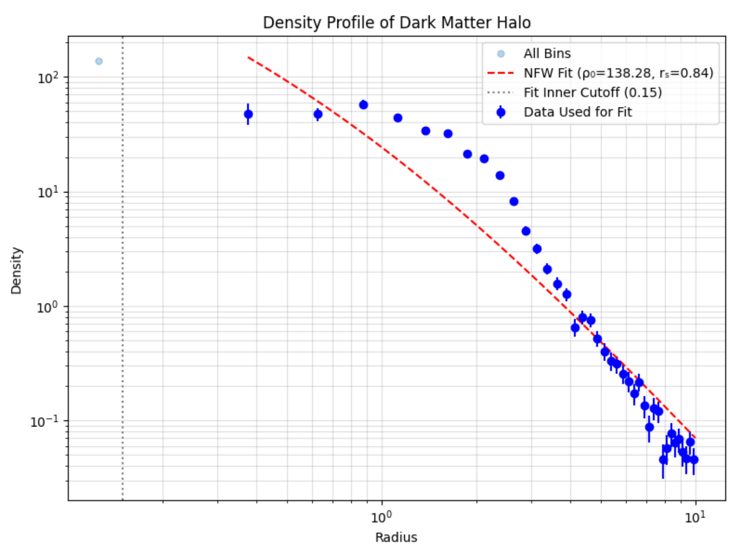

Figure 4: data points look like a core?

I was shocked as the inner part looks a lot like a core. There must be something wrong. After a few more test, I found out the key is softening length.

For this one, the softening length is so big that within a not-so-small region around each particle gravity gets milder, which prevents the particles near the center from being too close, thus giving out a flat core.

To determine a more optimized softening length, I read (Power et al. 2003), which gives a equation for this (it also mentioned that a big softening length would cause this core):

$$\epsilon \approx \frac{4r_{200}}{\sqrt{N_{200}}}$$

After consulting Fangxiong, I chose to estimate the virial radius via the following:

$$r_{virial} \approx \frac{GM^2}{2E}$$

, so I could use a more reasonable softening length and avoid this problem.

4 Expectation & Reality

4.1 Let's Look At Some REAL Stuff

Once I noticed there is barely a way for me to simulate something closer to reality using REBOUND, I searched for some published papers to have a peep at what real cosmological simulations should look like.

4.1.1 Cosmological N-Body Simulations of Cold Dark Matter Halos

This is actually Jürg Diemand's doctor's thesis (Diemand 2004), where he simulates dark matter halo relaxation and evolution of sub-halo and analyzed related numerical and theoretical problems using PKDGRAV3, a package uses a multipole method, and exploits the MPI library, together with multi-threading and GPGPU acceleration.

In Chapter 2, they find through isolated dark matter halo evolution simulation that the so-called "two-body relaxation" could lead to energy transferred into the central region, thus creating a core instead of a cusp, which we have actually seen above in our simulation. The paper also point out that at a fixed softening length, more particles and galaxy-mass instead of cluster-mass halo could reduce this kind of numerical effect. However, as the relation between the accumulated amount of relaxation and resolution (number of particles, N) is $\propto N^{-0.3}$, that is, the help from increasing N is actually limited, so we didn't explore it in our simulation.

Next, they implement a high resolution cluster forming simulation with a mass resolution of 2.4 million $M_{\odot}$ and conclude that down to the resolved scale, the density profile could be well fitted by a power-law $\rho \propto r^{-\gamma}$.

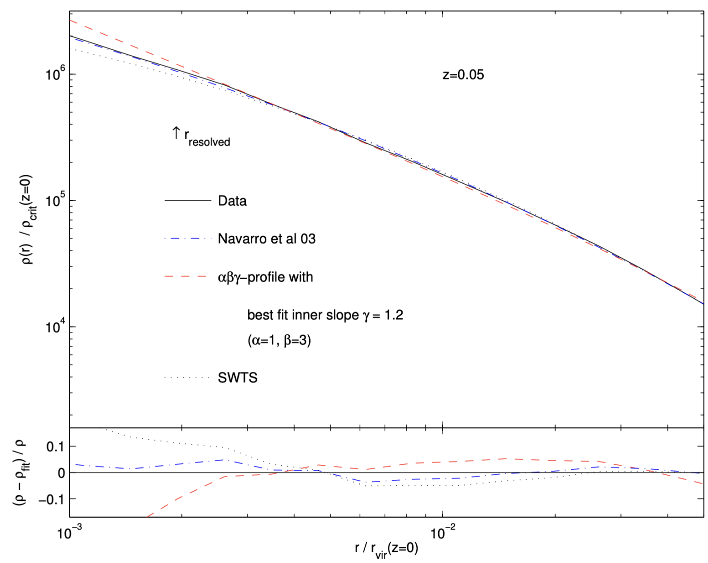

Figure 5: density profile of data and simulations from different models (Diemand 2004)

Look at that simulation! That's what we call a good fitting!

4.1.2 The Central Cusps In Dark Matter Halos: Fact or Fiction

This is another paper I found (Baushev and Pilipenko 2020).

This paper is intereating as it proposed a new perspective towards core-cusp problem – that the formation of cusp and its later transformation into core caused by two-body relaxation during a simulation could entirely be a numerical effect instead of a true physical process.

To test this, they use two different method to calculate gravity, tree algorithm and direct summation. They find out that the tree algorithm is what causes the core to form instead of relaxation and the numerical violent relaxation is falsely taken as a real physical process and thus leading to a cusp to form. So there is no physics, just purely technical problem.

4.1.3 Inner Structure of Dark Matter Halos At High z In Cosmological Models With Non-power-law Primordial Spectra

I find this paper interesting because I wanted to explore the effect of initial perturbation on halo density profile, but find it quite degenerate.

In this paper, they run simulations under different initial primordial spectra and find out that despite significant differences in the initial conditions, the mean density profiles of halos in all tested models remain close to the NFW profile at the final redshift of z=8 (no wonder I didn't see differences!). Also they test a toy model to which is added random velocities, and this leads to a flattened central cusp and even a core. However, the velocity amplitude required for this effect is higher than what is theoretically expected to compensate for the missing power, and it also leads to an artificial disruption of small-scale halos.

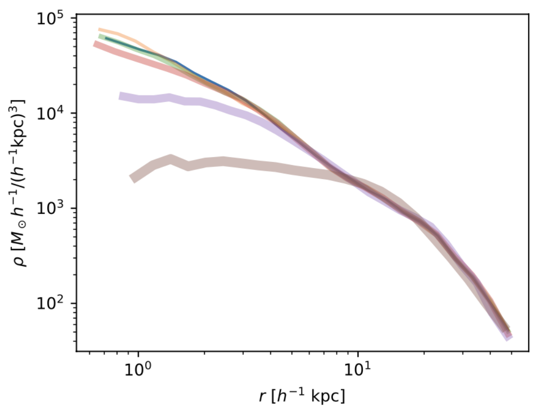

Figure 6: Density profiles at z = 8 for the most massive halo in simulations with artificially added velocities of σ_v = 0.5, 1.0, 2.0, 4.0, 8.0 and 16.0 km/s at z = 25. The thicker the line, the higher the velocity dispersion (Tkachev et al. 2024)

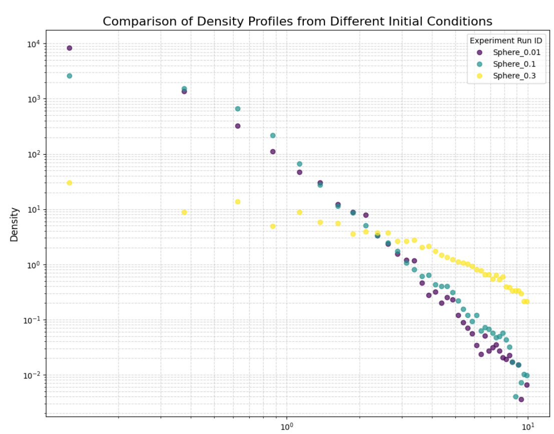

Figure 7: different density profiles from different velocity dispersions from my simulation, has the same overall trend as the image above, but doesn't extend to bigger radius

4.1.4 Axions in Andromeda: Searching for minicluster-neutron star encounters with the Green Bank Telescope

This paper is here because it is written by a grad student here at UVA (where I'm currently doing my summer research). He would spend half of his time at work in the lab, which is very new to me since we don't have a lab for astronomy back at PKU.

The paper describes the first dedicated search for radio transients produced by collisions between axion miniclusters (AMCs) and neutron stars (NS). Axions are compelling dark matter candidates that can theoretically convert into photons in the strong magnetic fields of neutron star magnetospheres. Encounters with dense AMCs are expected to enhance this conversion, producing a unique, time-variable radio signal that can be used to detect them.

The search targeted the core of the Andromeda Galaxy, which was chosen because it allows for the simultaneous observation of a vast number of neutron stars. The observations were conducted with the 100-meter Green Bank Telescope (GBT) using the X-band receiver (8-10 GHz) and the VEGAS spectrometer. This setup is sensitive to axions with masses between 33 and 42 μeV. Also a custom data analysis pipeline was developed to search for narrow, stable spectral lines consistent with an axion signal. Although it is a null result, it is consistent with the most current theoretical models, which predict that detectable AMC-NS events in the X-band should be rare. The team is continuing the search in other frequency bands (C-band and L-band) that target a predicted event-rate peak near 3 GHz.

4.1.5 Lessons Learned

Through searching for papers, I learned more about the history of core-cusp problem.

Early N-body simulations, based on the standard Lambda-Cold Dark Matter (ΛCDM) model, consistently predicted that dark matter halos should have "cuspy" centers. However, observations of actual galaxies, particularly dark-matter-dominated dwarf galaxies, revealed that their rotation curves were more consistent with a flat, constant-density "core". Initially, this discrepancy led to questions about the reliability of both methods. Some people suggested the simulated cusps might be numerical artifacts, while others argued that observational effects, like viewing a non-spherical halo from a specific angle, could make a real cusp appear like a core. Two main categories of solutions then emerged – Baryonic Feedback and Alternative Dark Matter Models. The modern frontier of the problem focuses on distinguishing between these viable solutions. Both strong baryonic feedback and SIDM can produce cores, so researchers are now looking for unique observational signatures to tell them apart. People now are also focusing on a more complicated question rather than a simple core-cusp problem – why do galaxies of similar mass exhibit a wide variety of density profiles, from cored to cuspy?

4.2 Final Thoughts

In the very beginning I was intended to simulate a dark matter halo relaxation, and try fitting it with a NFW profile. However as I proceed, I found that the best I can do without a supercomputer is to simulate a simplest even technically meaningless case. Also I find it very messy and tricky and time-consuming to manipulate so many parameters in order to get a set of optimal ones, almost like finding a needle in the hay.

I spent the past week anxious over this assignment, as the fitting result did not look good and I felt like there are so many things about this topic that I don't understand. But the slow life in this small town and the chill yet enthusiastic atmosphere at this department cleared my mind – this simulation does not have the same background as the NFW profile, so it might be fine to have bad fitting results. More importantly, the deadline of this assignment is not the end of world, and I have a lifetime to learn and explore more about dark matter.

Although my simulation is quite simplified from real situation, I still managed to learn more about N-body simulation and core-cusp problem, as well as what researchers are doing to try to understand dark matter. Hopefully future me could have a chance to run real cosmological simulations.

Thanks to prof. Jiang and my warm-hearted classmates, this semester passed fun and rewarding. I feel very lucky to have a chance to learn and understand dark matter, which has always been a mystery since I fell in love with astrophysics in middle school.

References

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