A MOTIVIC POISSON FORMULA FOR SPLIT ALGEBRAIC TORI AND APPLICATIONS TO MOTIVIC HEIGHT ZETA FUNCTIONS

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Abstract. We prove a motivic version of the Poisson formula and apply it to the study of the motivic height zeta function of split projective toric varieties.

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Introduction

Harmonic analysis stands as one of the foundational tools in the number theorist's arsenal for addressing counting problems. Within the framework of the Batyrev-Manin-Peyre conjectures—which predict the distribution of rational points on Fano varieties over global fields—this method has been instrumental since the field's inception.

Classically, the counting problem is encoded into a height zeta function on which harmonic analysis is performed. In that context, an important role is played by the Poisson formula.

The goal of the present work is to state and prove a geometric-motivic analogue of the Poisson formula for split algebraic tori and apply it to the study of the motivic height zeta function of toric varieties over function fields of curves. In particular, we recover a motivic stabilisation result for the class of the moduli space of high degree morphisms from a smooth and geometrically irreducible projective curve to a smooth split projective toric variety that was previously obtained by the second author using a different approach [Fai25b, Fai25a]. This work also extends residue-type results of Bourqui [Bou09] and Bilu–Das–Howe [BDH22].

The classical setting. Let us recall the classical Poisson formula in its form used by Batyrev and Tschinkel in their proof of Manin's conjecture for toric varieties over number fields [BT95, BT98] and function fields of curves over finite fields [Bou11a]. Let $\mathscr{G}$ be a locally compact topological commutative group, endowed with a certain Haar measure $\mathrm{d}x$, and let $\mathscr{H} \subset \mathscr{G}$ be a cocompact discrete subgroup of $\mathscr{G}$. We give ourselves a function $f : \mathscr{H} \to \mathbf{R}$ and assume that