STAT400HW

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1 Problem 1

Suppose that X has the uniform distribution on the interval [a, b]. Determine the m.g.f. of X.

$M_X(t) = \int_a^b e^{tx} \cdot \frac{1}{b-a}\, dx$

2 Problem 2

Suppose that X is a random variable for which the m.g.f. is as follows: $\psi(t) = e^{t^2+3t} \quad \text{for } -\infty < t < \infty$

Find the mean and the variance of X

The mean for this is 3 and the variance is 1/2 because the coefficient of t^2 is 1, we square root it, and we divide it by 2 and the mean is 3 because the coefficient of t is 3.

3 Problem 3.A

$Pr(|X|1) = 1$

Since, in this case, X is bounded by [-1, 1], X exists

4 Problem 3.B

$Pr(|X|1) = 1$

Since, in this case, the probablility that x is an element of 0, 1 = 1. In this case n = 10, p = 0.2, so E(X) = 5 and E(X^4) = 625 > E(X^2) = 25

5 Problem 3.C

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6 Problem 4

The probability mass function (PMF) of the binomial distribution is:

$$f(x|n,p) = \binom{n}{x} p^x (1-p)^{n-x}$$

The ratio $\frac{f(x+1|n,p)}{f(x|n,p)}$ is:

$$\frac{f(x+1|n,p)}{f(x|n,p)} = \frac{\binom{n}{x+1} p^{x+1}(1-p)^{n-x-1}}{\binom{n}{x} p^x (1-p)^{n-x}}$$

$$= \frac{n!}{(x+1)!(n-x-1)!} \cdot \frac{p^{x+1}}{p^x} \cdot \frac{(1-p)^{n-x-1}}{(1-p)^{n-x}}$$

$$= \frac{n!}{(x+1)!(n-x-1)!} \cdot p$$

If we set it equal to 1,

$$1 = \frac{n!}{(x+1)!(n-x-1)!} \cdot p$$

We get, x = n - 1 = mode

7 Problem 5.A

$P(X = k) = \frac{e^{-\mu}\mu^k}{k!}$

$P(X = 6) = \frac{e^{-4.5}4.5^6}{6!}$

8 Problem 5.B

$P(X <= 6) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5) + P(X = 6)$

$P(X <= 6) = \frac{e^{-4.5}4.5^0}{0!} + \frac{e^{-4.5}4.5^1}{1!} + \frac{e^{-4.5}4.5^2}{2!} + \frac{e^{-4.5}4.5^3}{3!} + \frac{e^{-4.5}4.5^4}{4!} + \frac{e^{-4.5}4.5^5}{5!} + \frac{e^{-4.5}4.5^6}{6!}$