Appendix

6.1 Fresnel diffraction and propagation-based phase contrast

The object's influence on the incident wave field $u_{inc}(\mathbf{x})$ can be written as $u_0(\mathbf{x}) = T(\mathbf{x})\ u_{inc}(\mathbf{x})$,where $u_0(\mathbf{x})$ is the wave field at the exit plane of the object. The intensity of the wave at a distance D downstream of the sample is

$$I_D(\mathbf{x}) = |u_D(\mathbf{x})|^2, \tag{6.1}$$

which is the quantity which can be measured directly. Since we assume the incident wave $u_{inc}(\mathbf{x})$ has uniform flux,the wave field at the exit plane of the object is $u_0(\mathbf{x}) = T(\mathbf{x})$.

As the wave interacts with the object, we can consider the phenomenon or effect happening as diffraction. By theory of wave equation which is deduced by Maxwell's equations and boundary conditions, and complex form of monochromatic wave, we obtain Helmholtz equation

$$(\nabla^2 + k^2)u = 0, k = 2\pi/\lambda \tag{6.2}$$

where u represents the field. With the help of Green function G, this equation satisfies

$$(\nabla^2 + k^2)G = -\delta(\vec{r_1} - \vec{r_0}) \tag{6.3}$$

$\vec{r_0}$ and $\vec{r_1}$ respectively represents positions of observation point P_1 and any point P_0 in space. With the help of Green second equation, we obtain

$$u(\vec{r_1}) = \int\int_S (G\frac{\partial u}{\partial n} - u\frac{\partial G}{\partial n}), \tag{6.4}$$

where n represents the normal vector to closed space, and this is the diffraction formula. In free space, since Green function to Helmholtz equation is

$$G = \frac{1}{4\pi}\frac{\exp{(ikr_{10})}}{r_{10}}, \tag{6.5}$$

where r_{10} could be abbreviated as r instead. Substituting it into Sommerfield first diffraction formula, we obtain the corresponding Rayleigh-Sommerfield diffraction integral:

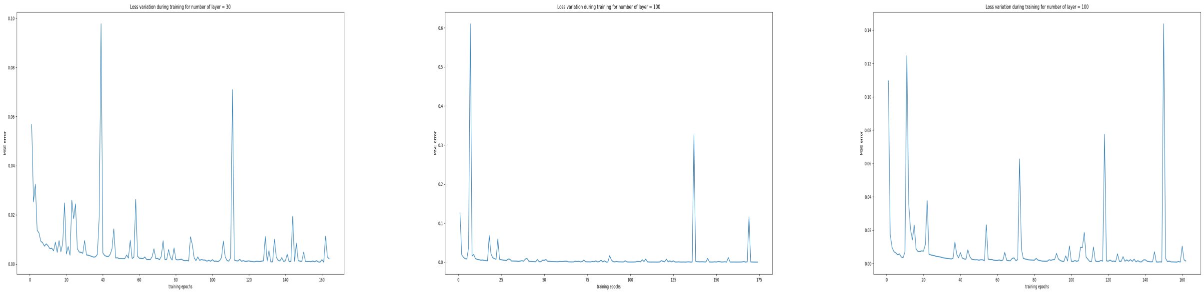
$$u_D(\mathbf{x}) = \frac{1}{i\lambda}\underset{\Sigma}{\int\int} u_0(x_0)\frac{\exp{(ik\,|r|)}\cos\theta}{|r|}d\mathbf{x_0} \tag{6.6}$$

if we take the condition $|r| >> \lambda$ into account, where $|r|$ is the path length between points in the object and diffraction planes and θ is the angle between r and the propagation direction.

For X-rays, θ is small, so we can make the approximations $\cos\theta \approx 1$ and $r \approx D$ in the denominator, moving it outside the integration. In the exponent, $|r| = \sqrt{D^2 + (x-x_0)^2 + (y-y_0)^2}$, can be approximated by the binomial expansion $|r| \approx D[1 + \frac{1}{2}(\frac{x-x_0}{D})^2 + \frac{1}{2}(\frac{y-y_0}{D})^2]$. This is known as the Fresnel approximation, it is valid if the condition $D^3 >> max\frac{\pi}{4\lambda}[(x-x_0)^2 + (y-y_0)^2]^2$ holds. This yields the Fresnel integral:

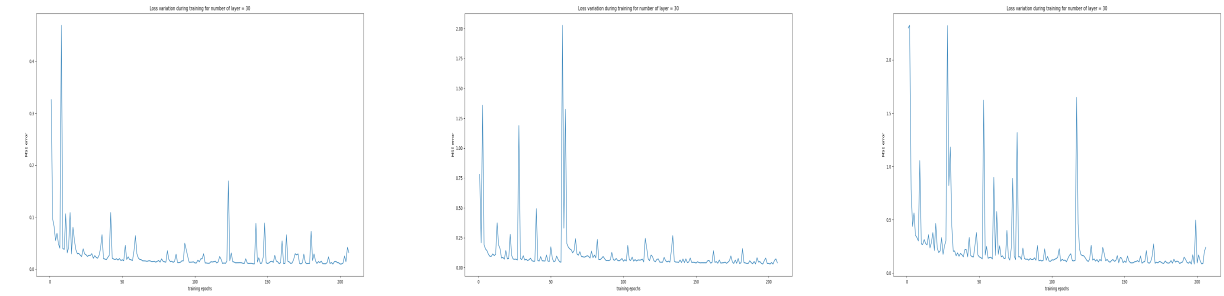
$$u_D(\mathbf{x}) = \frac{exp(ikD)}{i\lambda D}\underset{\Sigma}{\int\int} u_0(\mathbf{x_0})\exp{(i\frac{\pi}{\lambda D}[(x-x_0)^2 + (y-y_0)^2]^2)}d\mathbf{x_0} \tag{6.7}$$

6.2 ADAM Non-convergence Phenomenon in Training



(a) Single Density; Layer=30; No Noise (b) Single Density;Layer=100; No Noise (c) Single Density; Layer=30; No Noise

Figure 6.1: Training Loss Evolution of ADAM Optimizer for Non-noisy Case



(a) Single Density; Layer=30; Noise = 24dB (b) Single Density; Layer=30; Noise = 12dB (c) Single Density; Layer=30; Noise = 6dB

Figure 6.2: Training Loss Evolution of ADAM Optimizer for Noisy Case

6.3 Normalization Model Graphical Comparison

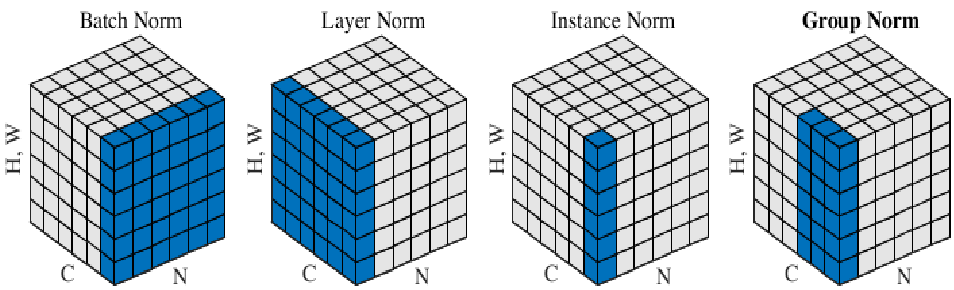


Figure 6.3: Normalization methods. Each subplot shows a feature map tensor, with N as the batch axis, C as the channel axis, and (H, W) as the spatial axes. The pixels in blue are normalized by the same mean and variance, computed by aggregating the values of these pixels [3].

6.4 Network Degradation Problem

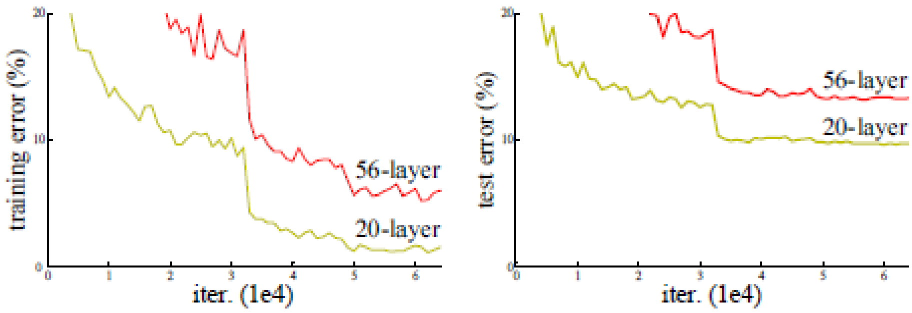


Figure 6.4: Network Degradation