Introduction to the Theory of Complex Systems

Chapter 1: **Introduction to Complex Systems**

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Complex Systems span, and borrow tools from, three broad subjects: Physics, Life Sciences and the Social Sciences.

1 Ideas across these three broad disciplines

1.1 Ideas borrowed from Physics:

- Research in the Physical Sciences - experimentation, followed by quantification and prediction
- Essentially the study of the 4 fundamental forces
- Interactions are non-specific in nature with respect to the individual entities
- Most systems follow a standard Newton-Laplace for Dynamical Systems - write some Differential Equations and then solve them, and finally impose some boundary/initial conditions
- Notion of predictability has evolved from Classical Physics, to Statistical Mechanics, followed by Quantum Mechanics, then Non-Linear Dynamics, and finally Complex Systems
- Follows a largely Analytical paradigm, which is to start from the ground fundamentals and build up from there

1.1.1 A note from Chemistry:

- Follow reaction kinetics that tend toward equilibrium
- Usually a large number of molecules, and therefore follows mass-action and rate laws

1.2 Ideas borrowed from Biology

- No mass action rule or equilibrium observed
- Systems are usually driven away from equilibrium in order to sustain

- Living systems essentially follow three rules:
  1. Self-replicating
  2. Involves a Carnot Cycle
  3. Localised
- Across scales in Biology (Gene Regulation, Metabolism, Neurons, Ecological Communities, and so on), the interactions are almost completely network-based

1.2.1 Evolution:

- Essentially consists of three processes - Replication, Variation and Selection
- Population undergoes random mutations, and those with a fitness advantage get 'selected'
- However change in the population affects changes in the environment, which in turn affects the population - BOTH the system and the environment are changing with time
- Impossible to analyse or solve analytically
- No fixed boundary conditions + the phase space evolves with time
- **'Adjacent Possible'** - subset of all possible 'worlds' that are reachable within the next time step (useful idea to keep in mind when studying evolutionary systems)

1.2.2 Edge of Chaos - Adaptiveness and Robustness:

- Systems have a characteristic Lyapunov exponent λ which is given by

$$|\delta X(t)| \sim e^{\lambda t}|\delta X(0)|$$

- The behaviour is different for three cases:
  1. $\lambda > 0$ is the chaotic regime/strongly-mixing case (adaptiveness) - trajectories will try to find a new state
  2. $\lambda < 0$ will display attractor-dynamics/convergence (robustness) - adjacent trajectories will converge
  3. $\lambda = 0$ is a quasi-periodic regime
- Famous example: the logistic map
- Most natural systems tend to show both robustness and adaptiveness, and one explanation for this behaviour is that the quasi-periodic regime exists for extended regions in parameter space, which is why natural systems are able to easily find these states.
- There are also systems that show this behaviour spontaneously, such as self-organised critical systems (have a critical point/line as an attractor)

- There are also systems in the life sciences whose trajectories over time show path-dependence, implying that they have memory, and are therefore neither Ergodic nor Markovian (the idea of the adjacent possible becomes useful here)

1.3 Ideas from the Social Sciences

- Social systems have a vast range of features and properties that all play out once. Hence the invoking of superimposed multilayer interaction networks
- Game Theory was developed in the context of Economics to study the outcome of rational agents interacting with each other (i.e. the outcome of rational interactions between agents or entities trying to optimise a target payoff/utility)
- Social systems show a richness of co-evolution (which is the joint dynamics between reconstructing networks and changing individual states)

2 Complex Systems - Ten Commandments

In one line, **Complex Systems are Co-evolving Multilayer Networks**

Note: These are simply ideas of what properties can give rise to complexity - there is no hard-and-fast rule to what discriminates a complex system from other systems.

1. many agents/elements, $i = 1, 2, 3, \cdots$
2. multiple interaction types, α, β, γ
3. specificity of interactions (as well as their time evolution - may be deterministic or even stochastic), $M_{ij}^{\alpha}(t)$
4. characteristic states for each node, $\vec{\sigma}_i(t)$
5. Co-evolutionary dynamics of the links and the nodes
6. typically on-linear dynamics followed on the network links
7. Context-dependence (which is when the context of one layer is given by the other layers/rest of the network)
8. follows an algorithmic approach since analytical approaches are impossible
9. exhibits path-dependence - therefore non-Markovian, non-Ergodic
10. Memory may be encoded within the nodes or the structure of the network across layers

2.1 What exactly is Co-Evolution?

1. The states of the network (i.e. the topology and weights) determine the future states of the nodes.

2. The states of the nodes determine the future state of the links of the network.

$$\frac{d}{dt}\sigma_i(t) \sim \mathrm{F}\left(M_{ij}^{\alpha}(t), \sigma_j(t)\right) \tag{1}$$

$$\frac{d}{dt}M_{ij}^{\alpha}(t) \sim \mathrm{G}\left(M_{ij}^{\beta}(t), \sigma_j(t)\right) \tag{2}$$

3. Note that a system that is sufficiently described using only equation (1) encompasses the historically analytical nature of traditional physical systems.

2.2 The role of Computers and Data Science

- The availability of data enables corroboration of models with actual results
- Computers enable the creation of ensembles of histories/statistically-equivalent copies (especially useful when data is sparse, or when an event say in a social system takes place only once in real-life)
- Many of these complex systems do not have closed-form solutions, requiring the use of simulations

3 Structure of the Book

1. **An introduction to Complex Systems** - where the ideas come from, and what systems they may be (or quite aren't)

2. **Probability and Random Processes** - Limit Theorems, useful distribution functions and scaling laws, and Stochastic Processes

3. **Scaling** - mechanisms leading to scaling and power laws

4. **Networks** - basics of networks, random networks, communities, functional networks, network dynamics, generalised networks

5. **Evolutionary Processes** - Complex Dynamics in evolutionary processes, general evolution algorithm and its connection to co-evolving network structures, fitness, evolutionary models

6. **Statistical Mechanics and Information Theory** - notions of Entropy, Entropy and phase space in different processes, Maximum entropy principle