

1. SingularValueDecomposition( $ADA^T$ ) =  $U S^2 U^T$  where  $U$  is a unitary matrix and  $S$  is a diagonal matrix and  $\Sigma = S$ .

2. Expectation is zero, obviously. Let the unitary matrix be  $Q$ . Then the covariance matrix is  $I t$ ,  $t$  being the time as with the original Brownian motion. The probability measure does not change.

3. (a) null, (0), (1), (1,1) . Milstein Method.

(b)  $\frac{1}{2} X$  (c) It does not converge to the SDE either strongly or weakly since the approximate numerical scheme has the extra drift term that the original SDE lacks.

4. The maximum time step size is zero. No time step satisfying the contraction condition thus exists.

5. Simply identify the drift in the modified Wiener process.

$$dQ = \exp(+3 W_t^{(1)} - W_t^{(2)} - 5t) dP;$$

$$\theta_t = 1 + \int_0^t -(-31) \theta_s dW_s$$

6. The change of state from  $i$  to  $j$  driven by the SDE along  $(i, j)$  is:

$$((V_j - V_i) \exp(d_{ij} x) - (V_j - V_i)) \approx (V_j - V_i) (d_{ij} x + (1/2) d_{ij} x d_{ij} x + \dots)$$

Note that  $(V_j - V_i)/1$  is constant along the transition  $i$  to  $j$  and second order derivative along the transition is zero.

Then:

$$\begin{aligned} E(\delta V_i) &= \delta t (\dot{V}_i + \sum_{j \in \text{Adj}(i)} (V_j - V_i) (d_{ij} x + (1/2) d_{ij} x d_{ij} x)) \\ &= \delta t (\dot{V}_i + \sum_{j \in \text{Adj}(i)} (V_j - V_i) (a_{ij} + (1/2) \sigma_{ij}^2)) \\ \text{Var}(\delta V_i) &= \delta t \sum_{j \in \text{Adj}(i)} (V_j - V_i)^2 \sigma_{ij}^2 \end{aligned}$$

$\text{Adj}(i)$  is the set of possible states reachable given the state  $i$  at  $t$