

E1 254 Game Theory-2022  
(Y. Narahari)

There are four questions with weightage of 3, 4, 4, 4 points respectively. Attempt all questions. Max Marks 15.

Duration: 75 minutes.

Write legibly and as dark as possible. Upload a neat, scanned pdf document which is named after your full name. Use page #  $i$  for question #  $i$  ( $i=1, 2, 3, 4$ ).

1. Given an  $m \times n$  payoff matrix of a strategic form game, what would be:

- (a) Min # of PSNE
- (b) Max # of PSNE
- (c) Max # of MSNE
- (d) Max # of VWDSNE
- (e) Max # of WDSNE
- (f) Max # of SDSNE.

2. A potential game is a strategic form game  $\langle N, (S_i), (u_i) \rangle$  such that  $\exists$  a function  $F: S \rightarrow \mathbb{R}$  such that

$$u_i(s_i, \underline{s}_i) - u_i(t_i, \underline{s}_i) = F(s_i, \underline{s}_i) - F(t_i, \underline{s}_i) \\ \forall s_i, t_i \in S_i \quad \forall \underline{s}_i \in \underline{S}_i \quad \forall i \in N$$

Given a finite potential game, I claim that it will always have a PSNE. Prove or disprove.

3. (Third Price Auction). Consider an auction for an indivisible object where the winner is whoever bids highest and has the smallest index among highest bidders. The winner will pay an amount equal to the third highest bid. Consider  $N = \{1, 2, 3\}$  with the players 1, 2, 3 having valuations  $v_1, v_2, v_3$ , respectively, such that  $v_1 > v_2 > v_3$ . What can you say about the bid profile  $(b_1, b_2, b_3)$ . Does this reveal any interesting fact about third price auction?

4. Consider a strategic form game with

$$N = \{1, 2, \dots, n\}$$

$$S_1 = \dots = S_n = \{0, 1\}$$

$$u_i(s_1, s_2, \dots, s_n) = s_i - \frac{k}{n} \sum_{j=1}^n s_j$$

Where  $k$  ~~is~~ is any positive integer.

Compute all dominant strategy equilibria for this game under the following three conditions:

(a)  $n > k$

(b)  $n < k$

(c)  $n = k$

All the best!

Charalain  
28/01/2022