E1 254 - Game Theory & Mechanism Design

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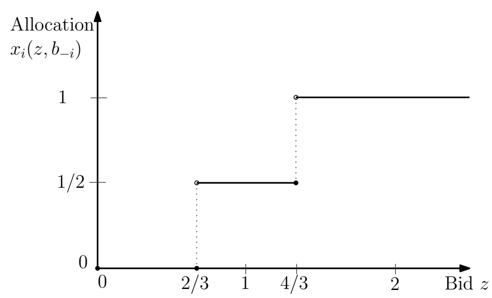
Practice Problems: Mechanism Design

1. In a single-parameter DSIC auction with monotone allocation rule x, consider a bidder i with private valuation $v_i = 2$. With the bids b_{-i} of all other bidders being fixed, if the allocation of i (i.e., $x_i(z, b_{-i})$) varies with her bid z as shown below, then compute bidder i's surplus, payment, and utility when she bids (i) $b_i = 2$, (ii) $b_i = 1$, and (iii) $b_i = 1/2$, respectively.



2. Let $X_1, X_2, \ldots, X_n$ be n identical and independent random variables drawn from the uniform distribution Unif$[a, b]$. Write random variable $Y := \min_{1 \le i \le n} \; X_i$. Show that

$$\mathbb{E}\left[Y\right] = \frac{b + an}{n + 1}.$$

3. Prove that in any single-parameter environment—with feasible set $\mathcal{X}$—the allocation rule, $x^* : \mathbb{R}^n_+ \mapsto \mathcal{X}$, that maximizes social surplus (welfare) $\mathcal{X}$ is always monotone.

4. Given an example of a distribution F—supported on $[0, 1]$—that is *not* regular.[1]

[1] Recall that a distribution (function) F is said to be regular iff the associated virtual valuation function $\varphi(z) = z - \frac{1-F(z)}{f(z)}$ is monotone non-decreasing (in z).