Prior-Independent Mechs.

A critique of optimal mech for revenue maximization (virtual welf. maximizers): The value distributions $F_1, F_2 \ldots F_n$ are assumed to be known to the mech. designer in advance.

- Still conforming to a Bayesian analysis.
- We continue to assume that the valuations are drawn from distributions. It is just that the dists are unknown to the designer.

[Especially relevant in "thin markets"]

- That is, dists used in the analysis, not in the design.

Prior-Ind. Mechs. — that do not reference the dist in the design.

(Ex. Second Price auction).

[Non Ex. Monopoly price or Virtual Welf. Maximizer

Bulow–Klemperer Thm. '96.

The expected rev. of an opt. single item auction is at most that of a second-price auction with one extra bidder.

(no reserve price)

THM: Let F be a regular dist & n a positive integer. Let p & p^* be the payment vectors of the second price auction with $(n+1)$ bidders and the opt. auction (for F) with n bidders respectively. Then,

$$\mathbb{E}_{v \sim F^{n+1}}\left[\sum_{i=1}^{n} p_i(v)\right] \geq \mathbb{E}_{v \sim F^n}\left[\sum_{i=1}^{n} p_i^*(v)\right]$$

[Informally,

$$\mathop{\mathbb{E}}_{v \sim F^{n+1}} [\text{Rev-Vickrey}(n+1)] \geq \mathop{\mathbb{E}}_{v \sim F^{n}} [\text{Rev-VirtWelf-Max}(n)]$$

Competition better than exact information]

Bulow-Klemperer Thm. gives us a prior-ind. guarantee.

Also, second-price auction simultaneously works for all regular F.

Proof of Thm: Comparison via a fictitious auction $\mathcal{A}$

$\mathcal{A} \leftarrow (n+1)$ DSIC auction: (Single-item setting)

1. Simulate an optimal n-bidder auction for F on the first n bidders. 1, 2, . . . n.

2. If item is not awarded in the first step, then give the item to bidder $(n+1)$ for free.

Two properties of $\mathcal{A}$.

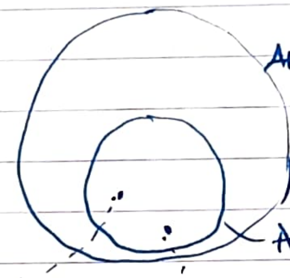
① Exp revenue of $\mathcal{A}$ equals that of an opt. auction with n bidders.

② It always allocates the item.

We note that exp. revenue of a second-price auction (with $n+1$ bidders) is at least that of $\mathcal{A}$.

$\downarrow$

Iid bidders & regular F.



Among All allocations that allocate the item

$\max\limits_{1\le i\le n} \varphi(v_i)$ $\leftarrow$

$\downarrow$ since φ is regular & identical.

(Same as)

$\max\limits_{1\le i\le n} v_i$ $\longrightarrow$ This is exactly the second price auction.

Using equivalence of exp revenue & exp. virtual welf.

Hence, Exp Rev Second Price $(n+1) \geq$ Exp Rev $\mathcal{A}(n+1)$

$\geq$ Exp Rev opt (n).

Multi-Parameter Mechanism Design.

Now, we consider.

multi-param environments

[So far, private val. of each agent i has been a single number. v_i: value per unit of good.]

Here, Cardinal preference of each agent is much more ~~~~ expressive.

for Multi parameter ~~~~ environments, the.

→ Vickrey-Clarke-Groves (VCG) Mechs provide a sweeping positive result: DSIC welfare maximization is possible (in principle) in every multi param env.

General Mech Design Environments

Components

- n bidders
- a finite set Ω of 'outcomes'
- each bidder $i \in (n)$ has a private nonnegative valuation $v_i(\omega)$ for each outcome $\omega \in \Omega$.

[The set Ω is abstract & can be very large.]

The social welf. of an outcome $\omega \in \Omega$ is defined as

$$\sum_{i=1}^{n} v_i(\omega).$$

Ex 1 (Single Item Auction, revisited)

Here, $|\Omega| = n+1$.

$$\Omega = \{\omega_0, \omega_1, \omega_2 \ldots, \omega_n\}$$

ω_0 – No agent receives the item

ω_i – Agent i receives the item.

Here, $v_i(\omega_i) > 0$

$v_i(\omega_j) = 0 \quad \forall j \neq i$

Ex 2. (Combinatorial Auction) A set of m items (indivisible) and bidders can have complex preferences between different subsets of items.

$\Omega \leftarrow$ set of all n partitions of m

$(S_1, S_2 \ldots S_n) \in \Omega$ denoting that subset $S_i \subseteq [m]$ assigned to bidder i

$v_i(S) \leftarrow$ value of bidder i for subset $S \subseteq (m)$.

Here, Each bidder has 2^m private valuations

Every multi-parameter environment admits a DSIC welfare-maximizing allocation

THM: In every general mechanism design environment, there is a DSIC welfare-maximizing mechanism.

[We follow the two-step approach that worked in the single param env:

1. Assume, without justification, that bidders report their private valuation truthfully. Then, figure out which outcome to pick.

For welf. maximization, using bids as proxies for valuations. (unknown).

Define allocation rule x by

$$x(b) = \underset{\omega \in \Omega}{\operatorname{arg\,max}} \sum_{i=1}^{n} b_i(\omega) \quad \text{— (1)}$$

[b_i, like v_i, is a vector indexed by Ω].

2. The second step is to define payment rule p, which when coupled with x, yields a DSIC mech.

Key Idea: Set payment for i as the "externality" caused by i:

The welfare loss inflicted on the other $(n-1)$ bidders by i's presence.

Specifically,

$$p_i(b) = \left(\max_{\omega \in \Omega} \sum_{j \neq i} b_j(\omega) \right) - \sum_{j \neq i} b_j(\omega^*) \quad \text{— (2)}$$

where $\omega^* = x(b)$ chosen in (1) above.

Alligning the incentive of i with a welf-maximizing decision.

⌈Check:

$p_i(b) \geq 0$ ⌋

Defn: VCG Mech. A mechanism with allocation & payment rules (x, p) as in ① & ②, respectively, is a Vickrey-Clarke-Groves (VCG) mechanism.

⌈Alternate form of payment.

$$p_i(b) = b_i(\omega^*) - \underbrace{\left[\sum_{j=1}^{n} b_j(\omega^*) - \max_{\omega \in \Omega} \sum_{j \neq i} b_j(\omega) \right]}_{\text{rebate}}$$

increase in social welf. attributable to i's presence.

⌈Check:

$p_i(b) \leq b_i(\omega^*)$. ⌋

Hence, individually rational.

Proof of Thm:

By defn, VCG mech (x, p) maximizes swelf. when the bidders are truthful.

We complete the proof by establishing DSIC.

That is, for every bidder i, every set of reports b_{-i} by the other bidders, ~~b_{-i}~~, agent i maximizes her quasilinear utility $v_i(x_i(b)) - p_i(b)$ by setting $b_i = v_i$.

Fix i & b_{-i}, when the chosen outcome $x(b)$ is w^*, agent i's utility is

$$v_i(w^*) - p_i(b) = \underbrace{\left[v_i(w^*) + \sum_{j \neq i} b_j(w^*)\right]}_{A} - \underbrace{\left[\max_{w \in \Omega} \sum_{j \neq i} b_j(w)\right]}_{B}$$

The term (B) is a constant, independent of i's report b_i.

Hence, the problem of maximizing agent i's utility reduces to maximizing (A)

$$b_i = v_i \xrightarrow{x} w^*$$

$$b_i' \xrightarrow{x} w'$$

Thus, truthful reporting aligns i's obj with the mech's objective.