Q3. This is an illustration of convergence rate of methods that are similar to Monte Carlo method, consider the estimation of $z = \pi = 3.1415\ldots$ via the estimator $Z \stackrel{\text{def}}{=} 4\mathbb{1}\{U_1^2 + U_2^2 < 1\}$, where U_1, U_2 are independent uniform$(0,1)$ r.v.'s. Here $\mathbb{1}\{U_1^2 + U_2^2 < 1\}$ is a Bernoulli r.v. with mean $\pi/4$, so the variance of Z is $4^2(\pi/4)(1 - \pi/4) \approx 2.70$. Thus, to get the leading 3 in π correct w.p. 95% we need $R \approx 1.96^2 2.70/0.5^2$ 42, whereas for the next 1 we need R 100 times more.

Q5.

(a) Test for significance ($H_0 : \mu_d = 0$)
Letting $d_i = y_i - z_i$, $\bar{d} = 1.80$, $S_d = 3.60$

$$t_0 = 1.80/(3.60/\sqrt{4}) = 1.0$$

For $\alpha = 0.05$, $t_{3,0.025} = 3.18$
Since $|t_0| < 3.18$, do not reject the null hypothesis.

(b) Sample size needed for $\beta \le 0.20$
$\delta = 2/3.60 = 0.556$
For $\alpha = 0.05$, $\beta \le 0.20$ and $\delta = 0.556$
$n = 30$ observations.

Q2.

For this problem the true optimal solution can be computed analytically: $x^* = 2.611$ years, giving an expected cost of \$11,586. This solution is obtained by minimizing the expected cost, which can be written as

$$2000x + \int_0^\infty 20000\, I(y \le 1) \frac{e^{-y/x}}{x}\, dx$$

where I is the indicator function.

Q1. Let X represent a process S -T such that S has a distribution that complements the exponential arrival times such that X is normally distributed with zero mean and standard deviation σ . Then:

$$W_{n+1} - w = \phi(W_n - w) + X_n$$

$$W_{n+1} - w = \phi^{n+1}(W_0 - w) + \sum_{j=0}^{n} \phi^{n-j-1} X$$

$$E(W_{n+1}) = w + \phi^{n+1}(E(W_0) - w) \to w \quad as \quad n \to \infty$$

$$Var(W_{n+1}) = \phi^{2(n+1)} + \sigma^2 \sum_{j=0}^{n} \phi^{(2j)} \to \sigma^2/(1-\phi^2) \quad as \quad n \to \infty$$

The first two lines in the above can be obtained by induction.

Q4. In the calculation below, substitute epsilon = 3 and t = 2.821 for 9 degrees of freedom and at 1% confidence level obtained as 0.04/(5-1). Also use S12, S23, S24, S25 in the calculations below.

$\bar{Y}_{\cdot i}$	1	2	3	4	5
	8.9464	6.5448	7.1387	9.5497	11.1213

S_{ij}^2	1	2	3	4	5
1		0.0251	0.0375	0.0330	0.0620
2			0.0028	0.0104	0.0226
3				0.0051	0.0104
4					0.0046

$$t_{0.04/(5-1),(10-1)} = \cancel{2.685} \rightarrow \boxed{2.821}$$

$$\bar{Y}_{\cdot 1} = 8.9464 > \bar{Y}_{\cdot 2} + t\sqrt{S_{12}^2/10} = \cancel{6.6793}\ \boxed{6.6861}$$

$$\bar{Y}_{\cdot 3} = 7.1387 > \bar{Y}_{\cdot 2} + t\sqrt{S_{13}^2/10} = \cancel{6.5895}\ \boxed{6.71755}$$

$$\bar{Y}_{\cdot 4} = 9.5497 > \bar{Y}_{\cdot 2} + t\sqrt{S_{14}^2/10} = \cancel{6.6314}\ \boxed{6.70685}$$

$$\bar{Y}_{\cdot 5} = 11.1213 > \bar{Y}_{\cdot 2} + t\sqrt{S_{15}^2/10} = \cancel{6.6724}\ \boxed{6.76693}$$

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Thus, there was adequate data to select the best, policy 2, with 96% confidence.

[illegible] $\widehat{S^2} = \max_{i \neq j} S_{ij}^2 = 0.0620$. The seconde-stage sample size,

$$R = \max\left(R_0, \left\lceil \frac{t^2 \widehat{S^2}}{\epsilon^2} \right\rceil\right) = \max\left(10, \left\lceil \frac{(2.82^2)(0.0620)}{3^2} \right\rceil\right) = 10$$

Thus, 10 replication is sufficient to make statistical comparisons.

Since $\min_{i=1}^{5} \{\bar{Y}_{\cdot i}\} = \bar{Y}_{\cdot 2}$, there was adequate data to conclude that policy 2 has the least expected cost per day with 96% confidence.

However, the conclusion remains as above. The answer does not change.