E1 254: Mechanism Design

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1 Introduction

- In game theory we inherit a game and are required to reason about it–a lot of economics if concerned with taking existing economic institutions and trying to explain/predict the outcomes that those institutions generate. This is called the *positive economics*.

  By contrast, in mechanism design we start by identifying desirable outcome and work backwards to figure out what institutions would generate those outcomes.

- Mechanism design is inverse game theory, it is the engineering side of game theory where we get to design a game—design rules—such that even with strategic (self interested) participants a desired outcome is obtained.

- Mechanism design is the science of rule making. The board question of mechanism design: What procedures can we design such that even with self-interested agents (strategic participants) the system yields a socially optimal outcome?

- Most situations of interest have private information. When we design a game we do not design it in vacuum. There are parts of the system that the designer does not control, e.g., the participants and their inherent preferences.

- The presence of strategic participants who hold part of the "input" (private information) is what separates mechanism design from algorithm design.

Founding fathers of Mechanism Design: Willian Vickery and Leonid Hurwicz. Seminal work by Myerson, Maskin...

Mechanism Design encompasses: Auctions, Elections, Markets, Government Policy, Financial Regulations,...Range of (potential) applications of mechanism design is quite broad.

The theory of auctions is one of the most successful applications of game theory.

2 Single Item Auctions

Setup:

- 1 good and n strategic bidders. We need a framework to quantitatively understand the behavior of the bidders. That is, we need to model the bidders using game theoretic notions.
- First key concept: **valuation**. Each bidder i has valuation v_i – maximum amount that i is willing to pay of the item. Thus the bidder wishes to acquire the item as cheaply as possible, provided its selling price is at most v_i.
- Here, an important assumption is that the valuations are **private**. v_i is known only to agent i, it is unknown to the auctioneer and the other agents.
- Formally what a bidder wants. Definition of **Quasilinear utility model:**

$$u_i = \left\{ \begin{array}{ll} 0 & \text{if } i \text{ looses the auction} \\ v_i - p & \text{if } i \text{ wins at sale price } p \end{array} \right\}$$

- Throughout the study of auctions, we will *model* the bidders as acting to maximize their quasilinear utility.

2.1 Auction Format: Sealed bid auctions

1. Each bidder submits a bid, b_i, privately to the auctioneer
2. The auctioneer decides who gets the good (if anyone)
3. The auctioneer decides on a selling price

Two decisions made by the auctioneer: allocation and payment.

- Obvious choice of for step 2. Winner = highest bidder. Everyone stated, via their bids, how much they are willing to pay. Allocate the good to the bidder who is willing to pay the most.
- The implementation of step 3 significant affects bidder behavior.

Implementation of Step 3

Example 0 Altruistic. $p = 0$. Bad idea. Since charging nothing turns the auction into who can name the highest number. Leads to an incentive problem.

Example 1 A natural idea. First price auction. Since the auctioneer has asked the bidders the amount they are willing to pay, simply make them pay what they bid. Nontrivial to reason about. Both from the bidder's perspective and the auctioneer's perspective.

Example 2 Right place to start. Second price auction—sealed bid format—highest bidder gets the good and pays the second highest bid.

We will see that the second price auction (Vickery auction) is easy to analyze (i) easy for the bidder to understand what to bid (ii) easy for the auction designer to predict what will happen.

Claim 1 (Vickery 1961)**.** *In a second price auction every bidder has a dominant strategy: set its bid b_i equal to its private valuation v_i. That is, this strategy maximizes (i.e., among all the bids that the bidder can submit, this bid maximizes) the utility of bidder i, no matter what the other bidders do.*

Another important property of the second price auction is that a truth telling bidder will never regret participating in it.

Claim 2. *In second price auction, a truthteling bidder is guaranteed nonnegative utility.*

Theorem 1. *The second price auction simultaneously achieves quite different, but desirable, properties:*

1. *Strong incentive guarantees: It is* **dominant strategy incentive compatible** *(DSIC), i.e., Claims 1 and 2 hold.*
2. *Strong performance guarantee: If bidders bid truthfully, then it maximizes social surplus $\sum_i x_i v_i$, where $x_i = 1$ if i wins and 0 if i loses, subject to feasibility constraints $\sum_i x_i \leq 1$.*
3. *Computationally efficiency: The auction can be implemented in polynomial (linear) time.*

We want to reason about strategic behavior/analyze strategic interaction. This necessitates making behavioral assumptions about how bidders behave. The weaker these assumptions the more plausible the theoretical predictions are for what happens in the system. DSIC is, hence, quite desirable.

3 Design Approach

What's hard about mechanism design problems is that we have to jointly design two things:

- the choice of who wins what (the allocation), and
- the choice of who pays what (the payment)

Interestingly, in many settings, we can tackle this two-prong design problem one step at a time.

Step 1. Assume, without justification, that bidders bid truthfully. Then, how should we assign bidders to slots so that the above properties (2) and (3) hold?

Step 2. Given our answer to Step 1, how should we set selling prices so that the above property (1) holds?

This two-step approach decouples the allocation rule and the payment rule. In particular, we need to design

- Allocation Rule x : Bids $\rightarrow$ Winner. Formally, $x : \mathbb{R}_+^n \mapsto \{0,1\}^n$.
- Payment Rule p : Bides $\rightarrow$ Sale price. Formally, $p : \mathbb{R}_+^n \mapsto \mathbb{R}_+$.

Second Price (Vickery) Auction

- Bids submitted $b \in \mathbb{R}_+^n$. Write $i^* \in \arg\max_{i \in [n]} b_i$ and $B := \max_{i \neq i^*} b_i$.
- Highest bidder wins the item and is charged the second highest bid.
- Allocation rule $x_{i^*}(b) = 1$ and $x_i(b) = 0$ for all $i \neq i^*$. Also, $p_{i^*}(b) = B$ and $p_i(b) = 0$ for all $i \neq i^*$.

4 Auction Theory: Road Map

4.1 Single-parameter environment

- DSIC (Dominant Strategy Incentive Compatible): Characterization via Myerson's Lemma
  Surplus maximization
  - Vickery Auciton
  - Sponsored Search Auction
  - Knapsack Auction

  Revenue maximization
  - Bayesian Model
  - Virtual Surplus = Revenue (Myerson's Auction)
  - Simple Auctions (Prophet Inequality)
  - Bulow-Klemperer Theorem

4.2 Multi-parameter environment

DSIC and VCG Auction for surplus maximization

5 Mechanism Design Basics

Results:

- Second Price (Vickrey Auction) is DSIC
- Vickrey Auction satisfies three desirable, different properties: DSIC, it maximizes social surplus, and runs in polynomial (linear) time.

Case Study: Sponsored Search Auction
Two-Step Design Approach: motivate Myerson's Lemma.

6 Myerson's Lemma

General rule for implementing an allocation rule.

- Define allocation and payment rule
- Define implementable and monotone allocation rule

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- Private Valuation v_i.
- Quasilinear Utility Model. Utility $u_i := v_i - p$, here p is the price/payment incurred by bidder i.
- Bids $b = (b_i, b_{-i})$.
- Feasible set X, allocation rule $x(b) \in X$ and payment rule $p(b) \in \mathbb{R}_+^n$.
- Implementable allocation rule and monotone allocation rule.

8 Not Covered

Deviating from typical treatments in economics, we will not cover the following topics:

- Social Choice Function
- Implementable and Incentive compatible social choice function
- Bayesian Setting..Bayesian-Nash equilibirum
- Arrow's impossibility theorem and Gibbard-Satterthwaite Impossibility Theorem