Solutions for Test 1 (28-01-2022)

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Problem 1

We are given a two player game with m strategies for player 1 and n strategies for player 2. There are therefore mn strategy profiles.

(a) Min # of PSNE $= 0$

(b) Max # of PSNE $= mn$

(c) Max # of MSNE $= \infty$ (infact, uncountably so)

(d) Max # of VWDSE $= mn$

(e) Max # of WDSE $= 1$

(f) Max # of SDSE $= 1$

Problem 3 (Third Price Auction)

Note that in a third price auction, the payment is equal to the third highest bid. Also note that this is the second highest bid among losers. If the bid profile happens to be (100, 80, 60), the payment is 60. If the bid profile is (100, 80, 80), the payment is 80. If the bid profile is (100, 100, 100), the payment is 100.

We are given $N = \{1, 2, 3\}$ with $v_1 > v_2 > v_3$. These valuations are private and known only to the respective players.

We are given the bid profile (v_1, v_1, v_1).

We know that $S_1 = S_2 = S_3 = [0, \infty)$.

Player 1 : Note that

$u_1(v_1, v_1, v_1) = v_1 - v_1 = 0$ (1 wins)

$u_1(b_1, v_1, v_1) = v_1 - v_1 = 0 \quad \forall b_1 > v_1$ (1 wins)

$u_1(b_1, v_1, v_1) = 0 \quad \forall b_1 < v_1$ since 1 loses

Thus $u_1(v_1, v_1, v_1) \geq u_1(b_1, v_1, v_1) \quad \forall b_1 \in (0, \infty)$

Player 2 : Note that

$u_2(v_1, v_1, v_1) = 0$ (2 loses)

$u_2(v_1, b_2, v_1) = v_2 - v_1 < 0 \quad \forall b_2 > v_1$ (2 wins)

$u_2(v_1, b_2, v_1) = 0 \quad \forall b_2 < v_1$ (2 loses)

Thus $u_2(v_1, v_1, v_1) \geq u_2(v_1, b_2, v_1) \quad \forall b_2 \in (0, \infty$

Player 3 : Note that

$u_3(v_1, v_1, v_1) = 0$ (3 loses)

$u_3(v_1, v_1, b_3) = v_3 - v_1 < 0 \quad \forall b_3 > v_1$ (3 w

$u_3(v_1, v_1, b_3) = 0 \quad \forall b_3 < v_1$ (3 l

Thus $u_3(v_1, v_1, v_1) \geq u_3(v_1, v_1, b_3) \quad \forall b_3 \in ($

The above analysis shows that $(v_1, v_1, v$
is a pure strategy Nash equilibrium.
In this equilibrium, the maximum util
for any player is zero. Further, players
and 3 have to bid higher than their
respective valuations in order to win
and unfortunately end up with negativ

but unfortunately end up with [illegible] utility.

Problem 2 (Potential Game)

It is given that $\langle N, (S_i), (u_i) \rangle$ is a finite game such that $\exists$ a function $F : S_1 \times \dots \times S_n \rightarrow \mathbb{R}$ such that $\forall i \in N$

$$u_i(s_i, \underline{s}_{-i}) - u_i(t_i, \underline{s}_{-i}) = F(s_i, \underline{s}_{-i}) - F(t_i, \underline{s}_{-i})$$
$$\forall s_i, t_i \in S_i \quad \forall \underline{s}_{-i} \in \underline{S}_{-i}$$

Since the game is finite, we can safely assume that F is bounded and therefore F has a maximum. Let this maximum be attained at $(s_1^*, s_2^*, \dots, s_n^*) \in S_1 \times S_2 \times \dots$

We clearly have $\forall i \in N$,

$$F(s_i^*, \underline{s}_{-i}^*) \geq F(s_i, \underline{s}_{-i}) \quad \forall s_i \in S_i \ \forall$$

$$\Rightarrow F(s_i^*, \underline{s}_{-i}^*) \geq F(s_i, \underline{s}_{-i}^*) \quad \forall s_i \in S_i$$

$$\Rightarrow u_i(s_i^*, \underline{s}_{-i}^*) \geq u_i(s_i, \underline{s}_{-i}^*) \quad \forall s_i \in S_i$$

$\Rightarrow (s_1^*, \dots, s_n^*)$ is a PSNE since the above inequality is satisfied for all $i \in N$

Problem 4 (Tragedy of the Common)

We have $N = \{1, 2, \ldots, n\}$

$S_1 = S_2 = \cdots = S_n = \{0, 1\}$

$$u_i(s_1, s_2, \ldots, s_n) = s_i - \frac{k}{n} \sum_{j=1}^{n} s_j$$

$$= \left(\frac{n-k}{n}\right) s_i - \frac{k}{n} \sum_{j \neq i} s_j$$

Case 1: $n < k$. for any $\underline{s}_{-i} \in \underline{S}_{-i}$ and $\forall i \in N$

$$u_i(0, \underline{s}_{-i}) = -\frac{k}{n} \sum_{j \neq i} s_j$$

$$u_i(1, \underline{s}_{-i}) = \left(\frac{n-k}{n}\right) - \frac{k}{n} \sum_{j \neq i} s_j$$

clearly $u_i(0, \underline{s}_{-i}) > u_i(1, \underline{s}_{-i}) \quad \forall \underline{s}_{-i} \in \underline{S}_{-i}$
$\forall i \in N$

This means $(0, 0, \ldots, 0)$ is a SDSE.

Case 3: $n > k$. for any $\underline{s}_{-i} \in \underline{S}_{-i}$ and $\forall i \in$

$$u_i(0, \underline{s}_{-i}) = -\frac{k}{n} \sum_{j \neq i} s_j$$

$$u_i(1, \underline{s}_{-i}) = \left(\frac{n-k}{n}\right) - \frac{k}{n} \sum_{j \neq i} s_j$$

clearly $u_i(1, \underline{s}_{-i}) > u_i(0, \underline{s}_{-i}) \quad \forall \underline{s}_{-i} \in \underline{S}_{-i}$
$\forall i \in N$

this implies $(1, 1, \ldots, 1)$ is a SDSE

Case 3: $n = k$. ~~$\forall \underline{s}_i \in \underline{S}_i$~~ $\forall i \in N$, we ha

$$u_i(0, \underline{s}_{-i}) = u_i(1, \underline{s}_{-i}) = -\frac{k}{n} \sum_{j \neq i} s_j$$

Here, it can be seen ~~that~~ every strategy profi is a VWDSE