Game Theory and Mechanism Design

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Practice Problems: Two Player Zerosum Games

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Problem Set 4

Warm-up

1. Show that a matrix A will have a saddle point if and only if the maxmin value is equal to the minmax value.
2. Given a matrix $A = [a_{ij}]$, show that if a_{ij} and a_{hk} are saddle points, then a_{ik} and a_{hj} are also saddle points.

Workhorse

1. For the following matrix game with $(2x2)$ matrix A with $a_{11} = 1$, $a_{12} = 3$, $a_{21} = 4$ and $a_{22} = 1$, write down the primal and dual LPs and compute all mixed strategy Nash equilibria.
2. Compute maxmin and minmax values in mixed strategies for the game with:
   $N = \{1, 2\}$; $S_1 = S_2 = \{A, B\}$;
   $u_1 = 2, 3, 3, 4$ for $(A, A), (A, B), (B, A)$, and (B, B) respectively
   $u_2 = 1, 2, 4, 3$ for $(A, A), (A, B), (B, A)$, and (B, B) respectively
3. Complete the proof of the theorem (Theorem 9.3 on page 145) that provides NASC for a mixed strategy profile to be an MSNE of a matrix game.

Thought Provoking

1. An (m x m) square matrix is called a "Latin Square" if each row and each column is a permutation of (1,2, ..., m). Compute all saddle points of a Latin Square Matrix game.
2. Consider a square matrix game where the matrix is symmetric. What can you say about the value in mixed strategies of such a game. Repeat your analysis for a skew-symmetric matrix.
3. Consider a matrix game A that is a 2 x 2 matrix with first row elements a,b and second row elements c,d, where a,b,c,d are real numbers. Derive the conditions on a,b,c,d for which the game is guaranteed to have an MSNE. Compute all MSNEs.

4. Suppose you are given a matrix game with 3 pure strategies for each player. Which numbers among 0,1, ..., 9 cannot be the total number of saddle points for the game. Justify your answer.