Lecture 3: Revenue Maximization

Siddharth Barman

March 2023

1 Last Time

- Single Parameter Enviroments
- Myerson's Lemma

  Theorem 1 (Myerson 1981)**.** *For a single-parameter environment*

  1. *An allocation rule is implementable iff it is monotone.*
  2. *If an allocation rule x is monotone, then there exists a unique payment rule p such that the auction (x, p) is DSIC (assuming the normalization, $b_i = 0$ implies $p_i(b) = 0$).*
  3. *The payment rule in (2) is given by an explicit formula.*

2 Revelation Principle

We have rolled in two conditions in the definition of DSIC

1. Every bidder in the auction has a dominant strategy, irrespective of its private valuation.
2. The dominant strategy is *direct revelation*, wherein the bidder truthfully reports her valuation to the auctioneer.

Beyond Dominant-Strategy Equilibria: There are settings where it is necessary, and also desirable, to consider mechanisms which do not admit dominant-strategy equilibria (e.g., implementation in Bayes-Nash Equilibria). DSIC and non-DSIC mechanisms are incomparable in general – the former enjoy strong incentive guarantees, the latter better performance guarantees.

The Revelation Principle states that, if we have (designed) a mechanism with Property 1, then we do not need to forgo Property 2.

Theorem 2. *For every mechanism M in which every bidder has a dominant strategy (no matter what its private valuation), there is an equivalent direct-revelation DSIC mechanism M'.*

Equivalent in the sense that M and M' have identical an outcome for every valuation profile.

Proof. Delegate implementing the dominant strategy, say s_i, to the mechanism itself. $\square$

3 Revenue Maximization

For social surplus we obtained a very strong result: a single DSIC auction which—independent of the inputs, i.e., the private valuations—maximized the social surplus. We obtained an "ex post" (pointwise) guarantee. These auctions did generate revenue (the sum of the prices charged to the bidders), but maximizing revenue was not the prime objective.

Such a strong result is not possible when the main objective is to maximize revenue. Consider the case of a single bidder in a single-item auction...

The optimal auction depends upon the private valuation.

Overall, *we need a model to reason about different inputs*. Revenue maximization requires a model to reason about trade-offs across different inputs.

3.1 Bayesian Analysis

We impose a distribution over the private valuations and seek to do as good as possible in expectation with respect to those distributions. That is, we cannot obtain a single auction which maximizes revenue over the set of all inputs. But, we do have auctions that maximize expected revenue with respect to specified distributions.

The Bayesian model consists of

- Single-parameter environment
- The valuation v_i is *independently* drawn from a distribution F_i (with density function f_i) with support $[0, v_{\max}]$
- The distributions are known to the mechanism designer. Though, as usual, the realization v_i is known only to bidder i.

In this Bayesian environment the "optimal" auction is clearly defined: it is the auction, among all DSIC auctions, which maximizes expected revenue. The expectation is with respect to the given distributions $F_1 \times F_2 \times \ldots \times F_n$ over the valuation profiles (assuming truthful bidding).

3.2 One Bidder and One Item

The expected revenue of a posted price r is simple $r(1 - F(r))$. Given a distribution F, finding the optimal value of r leads to the optimal auction.

The optimal posted price is called the *monopoly price* of the distribution F.