Optimality and Stability in Federated Learning

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2. Hedonic Game (Game theoretic approach)

- different distributions
- join only for benefit

3. Model of federated learning

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  - Uniform federation
  - Coarse-grained federation
  - Fine-grained federation

4. Stability

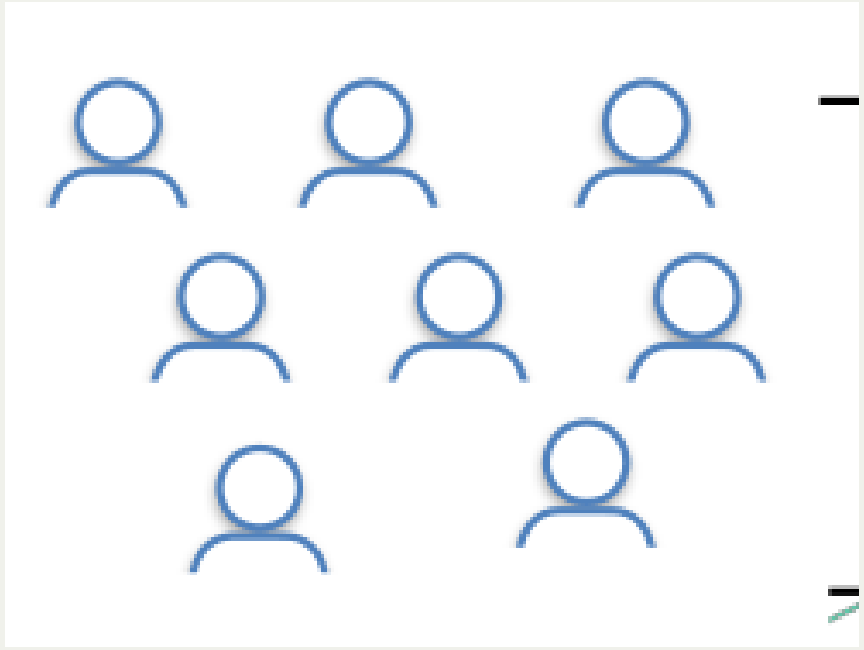
4.1. Uniform Federation

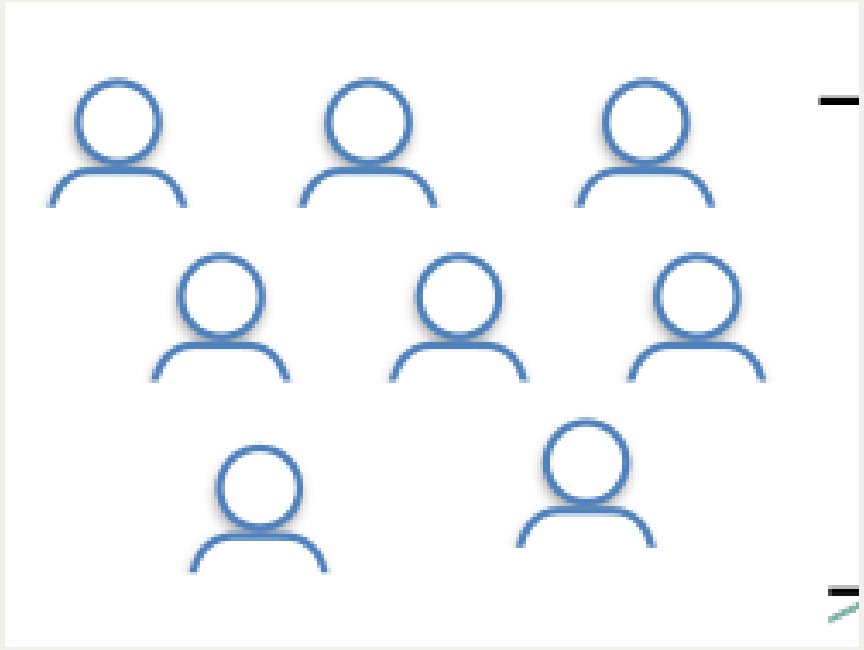
- Same number of samples
- Small/large number of samples

4.2. Coarse-grained federation

- Same number of samples
- Small/large number of samples
- Same number of samples
- Small/large number of samples

4.3. Fine-grained federation





5. Optimality

- Optimal: minimizes weighted sum of errors across all agents
- Algorithm:
  - Start with every agent doing local learning
  - Group the agents togetner in ascending order of size, stopping when the first agent would increase its error by joining the coalition
- Equivalence of player preference and reducing cost

- Swapping
- Monotonicity of joining
- Monotonicity of leaving
- Merging

- Model

$$\hat{\theta}_C = \frac{1}{\sum_{i \in C} n_i} . \sum_{i \in C} n_i . \hat{\theta}_i$$

$$err_j(C) = \frac{\mu_e}{\sum_{i \in C} n_i} + \sigma^2 . \frac{\sum_{i \in C, i \neq j} n_i^2 + \left(\sum_{i \in C, i \neq j} n_i \right)^2}{\left(\sum_{i \in C} n_i \right)^2}$$

6. Price of Anarchy

- for $n_i \geq \frac{\mu_e}{\sigma^2}, \forall i$, the grand coalition π_g is always core stable
- for $n_i \leq \frac{\mu_e}{\sigma^2}, \forall i$, the individually stable or core stable is also optimal
- Π_M is maximum cost IS partition, then $err_i(\Pi_M) \leq \frac{\mu_e}{n_i}$ for all players i

- Error lower bound when a player j joins coalition C

$$err_j(C \cup \{n_j\}) \geq \begin{cases} \frac{1}{2}.\frac{\mu_e}{n_j}, n_j \geq \frac{\mu_e + \sigma^2}{2\sigma^2} \\ \sigma^2, \text{otherwise} \end{cases}$$

- Error upper bound when a player j joins coalition C if total number of samples $N_C \geq \frac{\mu_e}{3\sigma^2}$, then

$$err_j(C \cup \{n_j\} \leq 7.25.\sigma^2$$

- $n_i \leq \frac{\mu_e}{3\sigma^2}, \forall i$, with atleast one player in a coalition with mass of its partners no more than $\frac{\mu_e}{3\sigma^2}$, then the only stable arrangement of these players is to have all of them federating together
- Price of Anarchy

$$PoA = \frac{f_w(\Pi_M)}{f_w(\Pi_{opt})} \leq 9$$

7. Limitations

- Theoretical study, results might be different in practice
- Optimality bound has some assumptions, this may lead to different bounds

8. Related Work

- `Donahue and Kleinberg` studied models of fairness
- `Hu et al. 2023` models clients behaviour in network
- `Cui et al 2021` tries to find collaboration equilibrium
- `Le et al. 2021` analyzes incentives for agents to contribute computational resources while using an auction approach

9. My extension

- Nothing as of now

10. Conclusion

Thank You