Game Theory and Mechanism Design

Y. Narahari

Practice Problems: Minmax and Maxmin Values

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Problem Set 3

Warm-up

1. Find the maxmin values, minmax values, maxmin strategies, minmax strategies of the following game:
   $N = \{1, 2\}$; $S_1 = S_2 = \{A, B\}$
   u_1 is $0, 1, 1, 1$ for $(A, A), (A, B), (B, A)$, and (B, B) resp.
   u_2 is $1, 1, 1, 0$ for $(A, A), (A, B), (B, A)$, and (B, B) resp.

2. Find the maxmin values, minmax values, maxmin strategies, minmax strategies of the following game:
   $N = \{1, 2\}$; $S_1 = S_2 = \{A, B\}$
   u_1 is $4, 0, 1, 1$ for $(A, A), (A, B), (B, A)$, and (B, B) resp.
   u_2 is $1, 4, 5, 1$ for $(A, A), (A, B), (B, A)$, and (B, B) resp.

3. Find the maxmin values, minmax values, maxmin strategies of the Rock-Paper-Scissors game.

Workhorse

1. Consider the following two player zero-sum game where a, b, c, d are real numbers with $a > b, d > c, d > b$, and $a > c$. Compute all mixed strategy Nash equilibria for this game. Also compute the maxmin value and minmax value in mixed strategies. Determine the maxmin mixed strategies of each player and the minmax mixed strategies against each player.

	A	B
A	a,-a	b,-b
B	c,-c	d,-d

2. Prove the following propositions.

(a) Suppose a strategic form game $\Gamma =< N, (S_i), (u_i) >$ has a mixed strategy Nash equilibrium $(\sigma_1^{*,...,\sigma_n^*)}$. Then

$$u_i(\sigma_1^*, ..., \sigma_n^*) \geq \underline{v_i} \; \forall \; i \in N$$

where $\underline{v_i}$ is the maxmin value in mixed strategies of player i.

(b) Consider a strategic form game $\Gamma =< N, (S_i), (u_i) >$. Then

$$\overline{v_i} \geq \underline{v_i} \; \forall \; i \in N$$

where $\underline{v_i}$ is the maxmin value in mixed strategies of player i and $\overline{v_i}$ is the minmax value in mixed strategies of player i.

(c) Suppose a strategic form game $\Gamma =< N, (S_i), (u_i) >$ has a mixed strategy Nash equilibrium $(\sigma_1^*, ..., \sigma_n^*)$. Then

$$u_i(\sigma_1^*, ..., \sigma_n^*) \geq \overline{v_i} \; \forall i \in N$$

where $\overline{v_i}$ is the minmax value in mixed strategies of player i.

(d) Given a two player strategic form game, the maxmin value in mixed strategies is equal to the minmax value in mixed strategies.

Thought Provoking

1. Consider a single player game with $N = \{1\}$ and $S_1 = [0, 1]$. Define the utility function as a discontinuous map:

$$u_i(s_i) = \begin{cases} s_i, \text{ if } 0 \leq s_i < 1 \\ 0, \text{ if } s_i = 1 \end{cases}$$

Show that the above game does not have a mixed strategy equilibrium.

2. There are two sellers 1 and 2 and there are three buyers A, B and C.

   (a) A can only buy from seller 1.

   (b) C can only buy from seller 2.

   (c) B can buy from either seller 1 or seller 2.

   (d) Each buyer has a budget (maximum willingness to pay) of 1 and wishes to buy one item.

   (e) The sellers have enough items to sell.

   (f) Each seller announces a price as a real number in the range $[0, 1]$. Let s_1 and s_2 be the prices announced by sellers 1 and 2, respectively.

   (g) Naturally, buyer A will buy an item from seller 1 at price s_1 and buyer C will buy an item from seller 2 at price s_2.

   (h) In the case of buyer B, if $s_1 \leq s_2$, then he will buy an item from seller 1, otherwise he will buy from seller 2.

We have shown in Chapter 6 that the above game does not have pure strategy Nash equilibrium. Does this game have a mixed strategy Nash equilibrium?