

TEN SIMPLE RULES FOR MATHEMATICAL WRITING

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ON WRITING

- **“Easy reading is damn hard writing”
(Hawthorne)**
- **“Word-smithing is a much greater percentage
of what I am supposed to be doing in life than I
would ever have thought” (Knuth)**
- **“I think I can tell someone how to write but I
can’t think who would want to listen” (Halmos)**

WHAT IS MATH WRITING?

- Writing where mathematics is used as a primary means for expression, deduction, or problem solving.
- Examples that are:
 - Math papers and textbooks
 - Analysis of mathematical models in engineering, physics, economics, finance, etc
- Examples that are not:
 - Novels, essays, letters, etc
 - Experimental/nonmathematical papers and reports

WHAT IS DIFFERENT ABOUT MATH WRITING?

- Math writing blends **two** languages (natural and math)
 - Natural language is rich and allows for ambiguity
 - Math language is concise and must be unambiguous
- Math writing requires **slow** reading
 - Often expresses complex ideas
 - Often must be read and pondered several times
 - Often is used as reference
 - Usually must be read selectively and in pieces

WHY THIS TALK?

- Experience is something you get only after you need it ...
- One current model: **The conversational style**
 - “Mathematics should be written so that it reads like a conversation between two mathematicians on a walk in the woods” (Halmos)
 - “Talk to your readers as you write” (Strang)
 - Very hard to teach to others (“Effective exposition is not a teachable art. There is no useful recipe ...” Halmos)
 - Controversial (where do proofs start and end? ... I am not sure what the assumptions are ... I can’t find what I need ... etc)
- **Instead we will advocate a structured style**
 - Offers specific verifiable rules that students can follow and thesis advisors can check
 - Allows room to develop and improve over time

SOURCES

- **General style books**
 - Strunk and White, “The Elements of Style” (www)
 - Fowler and Aaron, “The Little Brown Handbook”
 - Venolia, “Write Right!”
- **Halmos, “How to Write Mathematics”**
- **Knuth, et al, “Mathematical Writing” (www)**
- **Kleiman, “Writing a Math Phase Two Paper,” MIT (www)**
- **Krantz, “A Primer of Mathematical Writing”**
- **Higham, “Handbook of Writing for the Mathematical Sciences”**
- **Alley, “The Craft of Scientific Writing”**
- **Thomson, “A Guide for the Young Economist”**

RULES OF THE GAME

- **Small rules:**
 - Apply to a **single sentence** (e.g., sentence structure rules, mathspeak rules, comma rules, etc)
- **Broad rules:**
 - Apply to the entire document
 - **General** style and writing strategy rules
 - Are **non-verifiable** (e.g., organize, be clear and concise, etc)
- **Composition rules (our focus in this talk):**
 - Relate to how parts of the document connect
 - Apply to **multiple sentences**
 - Are **verifiable**

SOME EXAMPLES OF SMALL RULES I

- Break up long blocks of text into simpler ones
 - **2-3-4 rule:** Consider splitting every sentence of more than 2 lines, every sentence with more than 3 verbs, and every paragraph with more than 4 "long" sentences.
- Mathspeak should be “readable”
 - BAD: **Let $k > 0$ be an integer.**
 - GOOD: **Let k be a positive integer** or **Consider an integer $k > 0$.**
 - BAD: **Let $x \in \mathbb{R}^n$ be a vector.**
 - GOOD: **Let x be a vector in \mathbb{R}^n** or **Consider a vector $x \in \mathbb{R}^n$.**
- Don't start a sentence with mathspeak
 - BAD: **Proposition: f is continuous.**
 - GOOD: **Proposition: The function f is continuous.**

SOME EXAMPLES OF SMALL RULES II

- **Use active voice (“we” is better than “one”)**
- **Minimize “strange” symbols within text**
- **Make proper use of “very,” “trivial,” “easy,” “nice,” “fundamental,” etc**
- **Use abbreviations correctly (e.g., cf., i.e., etc.)**
- **Comma rules**
- **“Which” and “that” rules**
- **... ETC**

SOME EXAMPLES OF BROAD RULES

- **Language rules/goals to strive for: precision, clarity, coherence, directness, simplicity, fluidity, rhythm**
- **Organizational rules (how to structure your work, how to edit, rewrite, proofread, etc)**
- **“Down with the irrelevant and the trivial” (Halmos)**
- **“Honesty is the best policy” (Halmos)**
- **“Defend your style” (against copyeditors - Halmos)**
- **... ETC**

THE TEN COMPOSITION RULES

- **Structure rules (break it into digestible pieces)**
 - Organize in segments
 - Write segments linearly
 - Consider a hierarchical development
- **Consistency rules (be boring creatively)**
 - Use consistent notation and nomenclature
 - State results consistently
 - Don't underexplain - don't overexplain
- **Readability rules (make it easy for the reader)**
 - Tell them what you'll tell them
 - Use suggestive references
 - Consider examples and counterexamples
 - Use visualization when possible

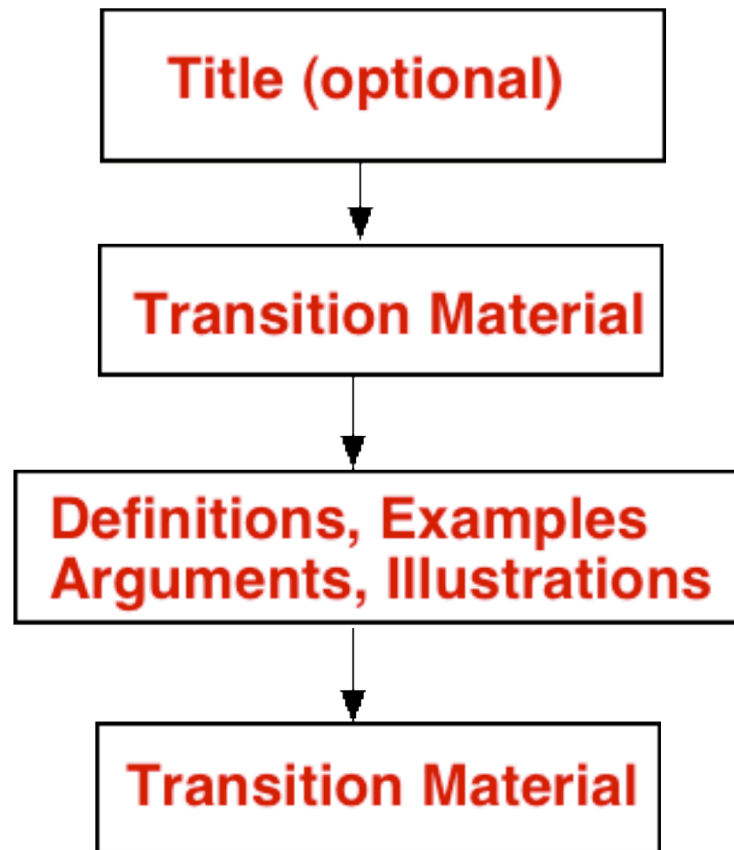
1. ORGANIZE IN SEGMENTS

- “Composition is the strongest way of seeing” (Weston)
- Extended forms of composition have a fundamental unit:
 - Novel Paragraph
 - Film Scene
 - Slide presentation Slide
 - Evening news program News report
- Key Question: What is the fundamental unit of composition in math documents?
- Answer: **A segment**, i.e., an entity intended to be read comfortably from beginning to end
- Must be not too long to be tiring, not too short to lack content and unity

SEGMENTATION PROCESS

- **Examples of segments:**
 - **A mathematical result and its proof**
 - **An example**
 - **Several related results/examples with discussion**
 - **An appendix**
 - **A long abstract**
 - **A conclusions section**
- **A segment should “stand alone” (identifiable start and end, transition material)**
- **Length: 1/2 page to 2-3 pages**

SEGMENT STRUCTURE



EXAMPLE OF SEGMENTATION: A SECTION ON PROB. MODELS

- **Sample space - Events** (1 page)
- **Choosing a sample space** (0.5 page)
- **Sequential models** (0.75 page)
- **Probability laws - Axioms** (1.25 page)
- **Discrete models** (2 pages)
- **Continuous models** (1 page)
- **Properties of probability laws** (2 pages)
- **Models and reality** (1.25 page)
- **History of probability** (1 page)

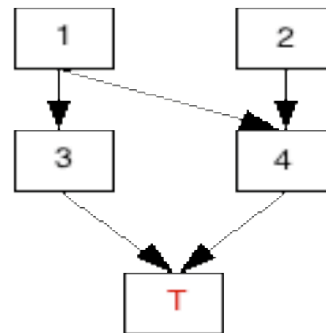
See Sec. 1.2 of Bertsekas and Tsitsiklis probability book

2. WRITE SEGMENTS LINEARLY

- Question: What is a good way to order the flow of deduction and dependency?
- General rule: **Arguments should be placed close to where they are used (minimize thinking strain)**
- Similarly, definitions, lemmas, etc, should be placed close to where they are used
- View ordering as an optimization problem
- A **linear/optimal** order is one that positions arguments (definitions, lemmas) so as to minimize the total number of “crossings” over other arguments (definitions, lemmas), subject to the dependency constraints. **Depth-first** order is usually better.

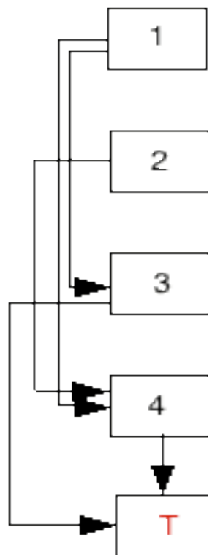
EXAMPLES OF ORDERING

Dependency
Graph of
Arguments



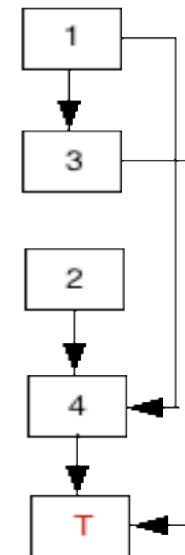
Level 1
Arguments

Level 2
Arguments



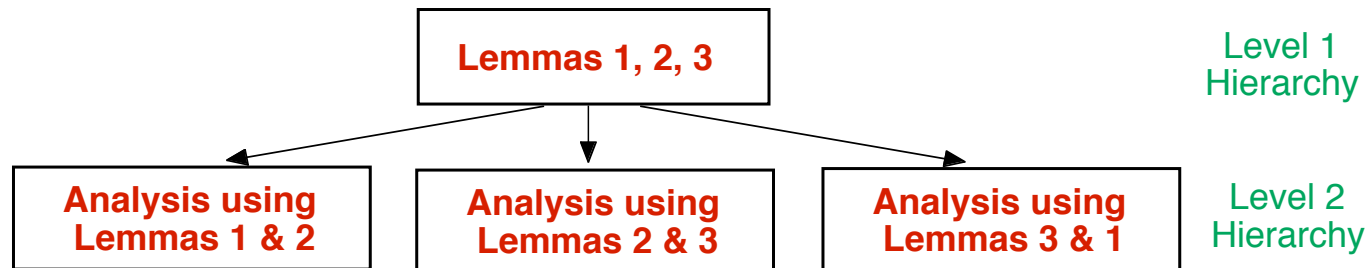
Nonlinear

Linear



3. CONSIDER A HIERARCHICAL DEVELOPMENT

- Arguments/results used repeatedly may be placed in special segments for efficiency



- Possibly create special segments for special material (e.g., math background, notation, etc)
- Analogy to subroutines in computer programs

4. USE CONSISTENT NOTATION

- Choose a notational style and stick with it
- Examples:
 - Use capitals for random variables, lower case for values
 - Use subscripts for sequences, superscripts for components
- Use suggestive/mnemonic notation. Examples: S for set, f for function, B for ball, etc
- Use simple notation. Example: Try to avoid parenthesized indexes: $x(m,n)$ vs x_{mn}
- Avoid unnecessary notation:
 - BAD: Let X be a compact subset of a space Y. If f is a continuous real-valued function over X, it attains a minimum over X.
 - GOOD: A continuous real-valued function attains a minimum over a compact set.

5. STATE RESULTS CONSISTENTLY

- Keep your language/format simple and consistent (even boring)
- Keep distractions to a minimum; make the interesting content stand out
- Use similar format in similar situations
- Bad example:
 - Proposition 1: If A and B hold, then C and D hold.
 - Proposition 2: C' and D' hold, assuming that A' and B' are true.
- Good example:
 - Proposition 1: If A and B hold, then C and D hold.
 - Proposition 2: If A' and B' hold, then C' and D' hold.

6. DON'T OVEREXPLAIN - DON'T UNDEREXPLAIN

- Choose a **target audience level** of expertise/background (e.g., undergraduate, 1st year graduate, research specialist, etc)
- Aim your math to that level; don't go much over or under
- Explain potentially unfamiliar material in separate segment(s)
- Consider the use of appendixes for background or difficult/specialized material

7. TELL THEM WHAT YOU'LL TELL THEM

- **Keep the reader informed** about where you are and where you are going
- Start each segment with a short introduction and perhaps a road map
- Don't string together seemingly aimless statements and surprise the reader with "we have thus proved so and so"
- Announce your intentions/results, e.g., "It turns out that so-and-so is true. To see this, note ..."
- Tell them what you told them

8. USE SUGGESTIVE REFERENCES

- Frequent numbered equation/proposition referencing is a **cardinal sin**
- It causes page flipping, wastes the reader's time, and breaks concentration
- Refer to equations/results/assumptions by content/name (in addition to number), e.g., Bellman's equation, weak duality theorem, etc
- Repeat simple math expressions
- Remind the reader of unusual notation, and earlier analysis
- Dare to be repetitive (but don't overdo it)

9. CONSIDER EXAMPLES AND COUNTEREXAMPLES

- **“Even a simple example will get three-quarters of an idea across” (Ullman)**
- **Examples should have some spark, i.e., aim at something the reader may have missed**
- **Illustrate definitions/results with examples that clarify the boundaries of applicability**
- **Use counterexamples to clarify the limitations of the analysis, and the need for the assumptions**

10. USE VISUALIZATION WHEN POSSIBLE

- **“A picture is worth a thousand words”**
- **Keep figures simple and uncluttered**
- **Use substantial captions**
- **Captions should reinforce and augment the text, not repeat it**
- **Use a figure to illustrate the main idea of a proof/argument with no constraint of math formality**
- **Prefer graphs over tables**

THE END

**“Bad thinking never produces good writing”
(Lamport)**

Good writing promotes good thinking ...