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DS221

Introduction to Scalable Systems [3:1]

Algorithms and Data Structures

Instructor: Chirag Jain
(slides from Prof. Simmhan)

About this Course

- Designed as a introductory course on systems
  - Computer Architecture, Operating Systems [MJT]
  - Data Structures, Algorithms [CJ]
  - Big Data Systems, Concepts and Programming [CJ]
  - Parallel Concepts and Programming [VSS]
- Covers **breadth**, not *depth*. Precursor to:
  - Scalable systems for data science, DS256
  - Parallel programming, DS295
  - Algorithmic foundations of big-data biology, DS202

Prerequisites

- Accessible to non-computer science UG Majors
- Requires some prior *programming knowledge*
  - Will NOT teach programming
- We will use C++, Python
  - Makefile, Standard Template Library
  - IDE, GitHub

Grading Scheme (CJ)

Sessional

Final

- Assignments [30 points]
  - 1 Programming Assignments (15 points)
  - 1 Written Assignment (15 points)
- Final Exams [5 points]

Schedule

- Data Structures and Algorithms, Aug 30 – Sept 22
  - Programming Assignment 1
- Big Data Concepts and Spark, Sept 27 – Oct 4
  - Written Assignment (all topics)
- Final Exam, *TBD*
- *Tutorial sessions will be announced*

Ethics

- **IISc POLICY FOR ACADEMIC INTEGRITY**
- Acknowledge and cite use of others' material
- Acknowledges all contributors to a piece of work
- All work submitted is his or her own in a course
- Produce academic work without the aid of impermissible materials or collaboration.
- Obtain all results by ethical means and report them accurately

https://iisc.ac.in/about/student-corner/academic-integrity/

Ethics

- **Penalties**
  - Warning
  - Community Service
  - Restrictions
  - Monetary Penalty
  - Withholding Grades
  - Suspension
  - Expulsion
  - Ineligibility

Questions?

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Data Structure, Algorithms and Data Systems

Class Resources

- Website
  - http://cds.iisc.ac.in/courses/ds221/
- Textbook
  - Data Structures, Algorithms, and Applications in C++, Sartaj Sahni*
    - http://www.cise.ufl.edu/~sahni/dsaac/
- Other resources
  - The C++ Programming Language, 3rd Edition, Bjarne Stroustrup
  - C++ Standard Template Library, http://www.cplusplus.com/reference/stl/
  - THE ART OF COMPUTER PROGRAMMING (Volume 1 / Fundamental Algorithms), Donald Knuth
  - Introduction to Algorithms, Cormen, Leiserson, Rivest and Stein
  - www.geeksforgeeks.org/data-structures/

*http://www.tatabookhouse.com/data-structures-algorithms-and-applications-in-c-plus-plus--9788173715228?ver=1519259641
**http://www.flipkart.com/data-structures-algorithms-applications-c-english-2nd/p/itmeyf6jvka3kzdu

L1: Introduction

Concepts

- **Algorithm**: Outline, the essence of a computational procedure, with step-by-step instructions
- **Program**: An implementation of an algorithm in some programming language
- **Data structure**: Organisation of data needed to solve the problem (array, list, hashmap)
- **Algorithmic Analysis:** The expected behaviour of the algorithm you have designed, *before you run it*
- **Empirical Analysis:** The behaviour of the program that implements the algorithm, *by running it*

Why not just run it and see how it behaves?

Limitation of Empirical Analysis

- Need to implement the algorithm
  - Time consuming
- Cannot exhaust all possible inputs
  - Experiments can be done only on a limited to set of inputs
  - May not be indicative of running time for other inputs
- Harder to compare two algorithms
  - Same hardware/environments needs to be used

Limitation of Empirical Analysis

- Example?

How do we design an algorithm?

- Intuition
- Mixture of techniques, design patterns
- Experience (body of knowledge)
- Data structures, analysis

How do we implement a program?

- Preferred High Level Language, e.g. C++, Java, Python
- Map algorithm to language, retaining properties
- Use native data structures, libraries

Then why learn about basic data structures?

Basic Data Structures

Lists

Collections of data

- Data Structures to store collections of data items of same type
  - Items also called elements, instances, values…depending on context
- Primitive types can be boolean, byte, integer, etc.
- Complex types can be user or system defined objects, e.g., node, contact, vertex
- Operations on the collection
  - Standard operations to create, modify, access elements
- Properties of the collection
  - **Invariants that must be maintained**, irrespective of operations performed
- **Challenge**: Understand how to pick the right data structure for your application!

Collections of data

- Can have different implementations for same abstract data type
  - All offer **same** operations and invariant guarantees
  - Differ in performance (space/time complexity)
- **Challenge**: Understand how to pick the right implementation!

- Also, do we need to have different copies of code for different data types (e.g., integer, floats)?

Try yourself!

- Learn **templates/generics**. In many collections, the item type does not matter for invariants and operations, and can be replaced by a place-holder type "T".
- Learn C++ **Standard template library (STL)**. Read up examples of abstract collections and their implementations.
  - http://www.cplusplus.com/reference/stl/

Linear List (abstract data type)

- Properties
  - **Ordered** list of items…precedes, succeeds; first, last
  - **Index** for each item…lookup or address item by index value
  - **Well-defined size** for the list at a point in time…can be empty, size may vary with operations performed
  - Items of **same type** present in the list
- Operations
  - Create, destroy
  - Add, remove item
  - Lookup by index, item value
  - Find size, check if empty
  - *Precise name of operation may vary with language, but semantics remain same/similar….READ THE DOCS!*

Type = int, Size = 7

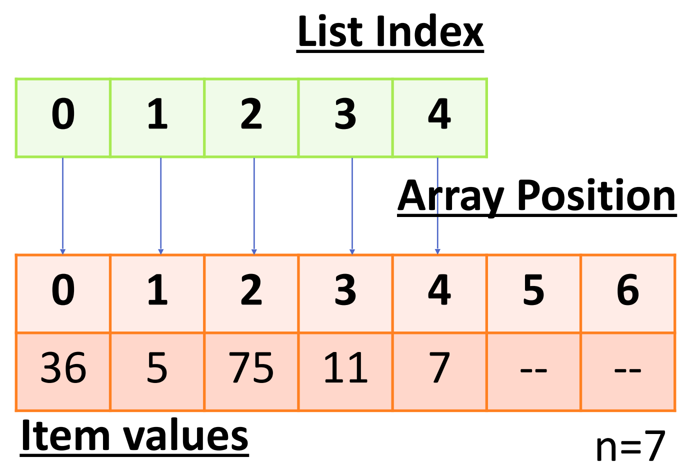
Index	**0**	**1**	**2**	**3**	**4**	**5**	**6**
Item	36	5	75	11	7	19	-1

1-D Array (implementation of list)

- List implementation using arrays, in a prog. language
- Arrays are contiguous memory locations with fixed capacity
  - Contiguous locations mean **locality** matters!
  - **Capacity** is different from **size**. Size is current number of items in list. Capacity denotes max possible size.
- Allow elements of same type to be present at specific positions in the array
  - Position is the **offset** from the start of array memory location, while accounting for **data type size**

Mapping Function

- Index in a **List** can be mapped to a Position in the **Array**
  - **Mapping function** from index to position



- Say **n** is the capacity of the array
- Simple mapping
  - **position**(index) = index

List Operations

- item **get**(index)
- void **set**(index, item)

- int **size**()
- int **capacity**()
- boolean **isEmpty**()
- int **indexOf**(item)

List Operations using Arrays

- void **create**(initCapacity)
  - Create array with initial capacity *(optional hint)*
- void **set**(index, item)
  - Use mapping function to set value at position
  - Sanity checks?
- item **get**(index)
  - Use mapping function to set value at position
  - Sanity checks?

```
class List {        // list with index starting at 1
    int arr[]           // backing array for list
    int capacity        // current capacity of array
    int size            // current occupied size of list

    /**
     * Create an empty list with optional
     * initial capacity provided. Default capacity of 15
     * is used otherwise.
     */
    void create(int _capacity){
        capacity = _capacity > 0 ? _capacity : 15
        arr = new int[capacity]       // create backing array
        size = 0        // initialize size
    }
```

```
// assuming pos = index-1 mapping fn.
void set(int index, int item){
    if(index > capacity) { // grow array, double it at least
        arrNew = int[MAX(index, 2*capacity)]
        // copy all items from old array to new
        // source, target, src start, trgt start, length
        copyAll(arr, arrNew, 0, 0, capacity)
        capacity = MAX(index, 2*capacity) // update var.
        delete(arr)      // free up memory
        arr = arrNew
    }
    if(index < 1) {
        cout << "Invalid index:" << index << "Expect >=1"
    } else {
        int pos = index – 1
        arr[pos] = item
        size++
    } // end if
  } // end set()
} // end List
```

Try yourself!

Implement **get**(index)

List Operations using Arrays

- int **indexOf**(item)
  - Get "first" index of item with given value
  - Sanity checks?
- void **remove**(index)
  - May replace the item with a NULL or shift all items at (index+1) left by 1

List Operations using Arrays

- Increasing capacity
- Start with initial capacity given by user, or default
- When capacity is reached
  - Create array with more capacity, e.g. double it
  - Copy values from old to new array
  - Delete old array space
- Can also be useful to shrink space
  - Why?
- **Pros & Cons of List using Arrays**

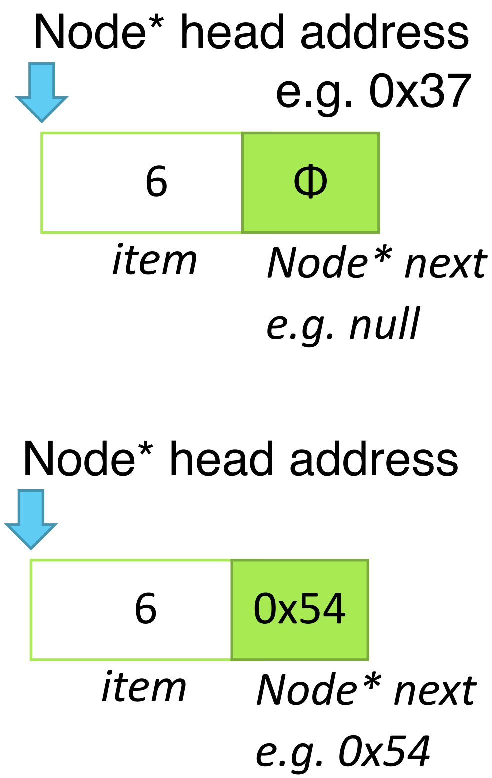
Linked List Representation

- **Problems with array**
  - Pre-defined capacity, under-usage, cost to move items when full. Fixed-size items (primitives)
- **Solution:** Grow data structure dynamically when we add or remove ➲ Only use as much memory as required
- *Linked lists* use ***pointers*** to contiguous chain items
  - **Node** structure contains **item** and pointer to **next** node in List
  - Add or remove nodes when setting or getting items

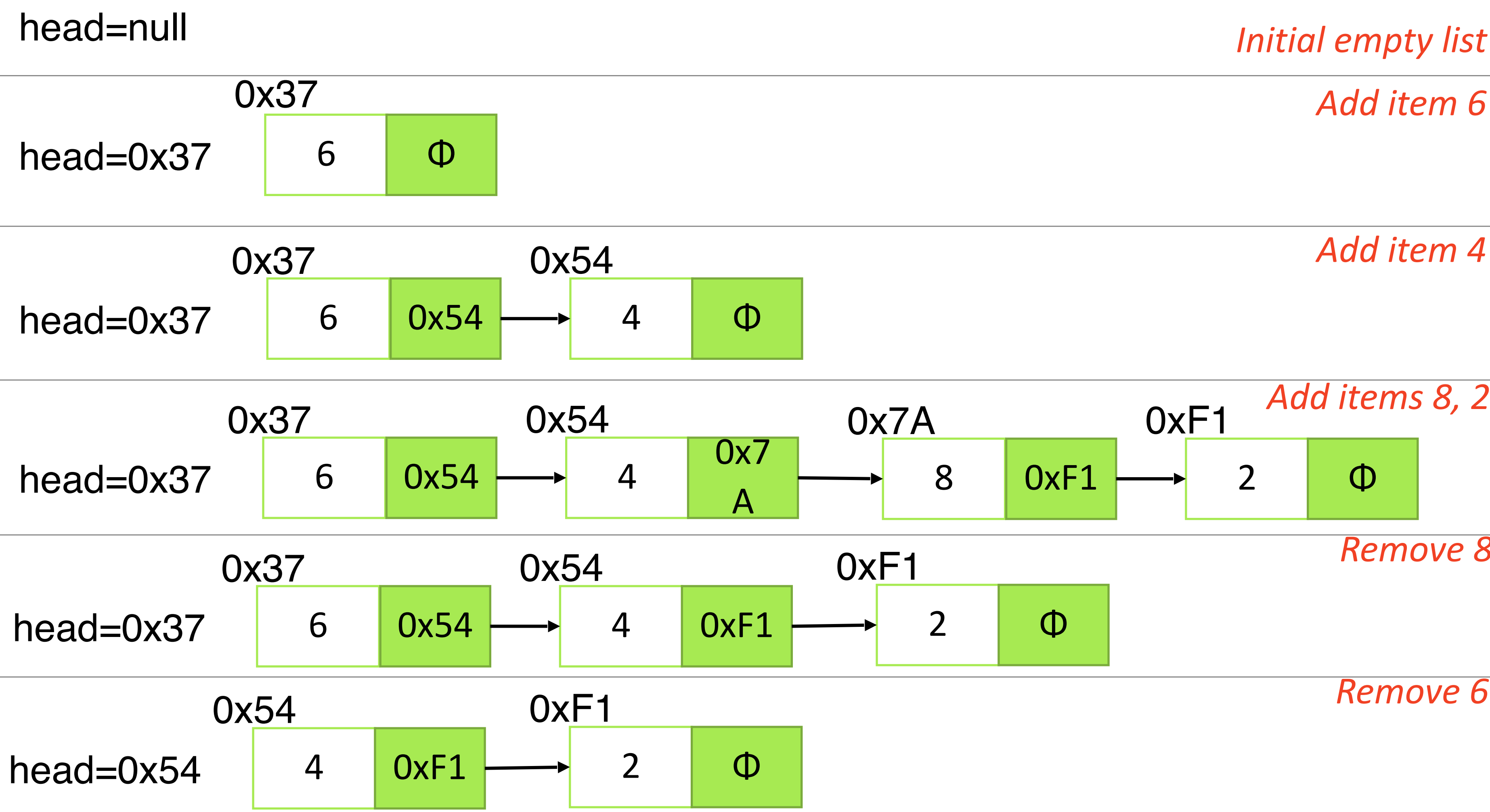
Try yourself!
Print the values of **pointer locations** for array and linked list items. Is there any pattern?

Node & Chain

```
class Node {
    int item
    Node* next
}
class LinkedList {
    Node* head
    int size
    append() {...}
    get() {...}
    set() {...}
    remove {...}
}
```



Linked List Operations

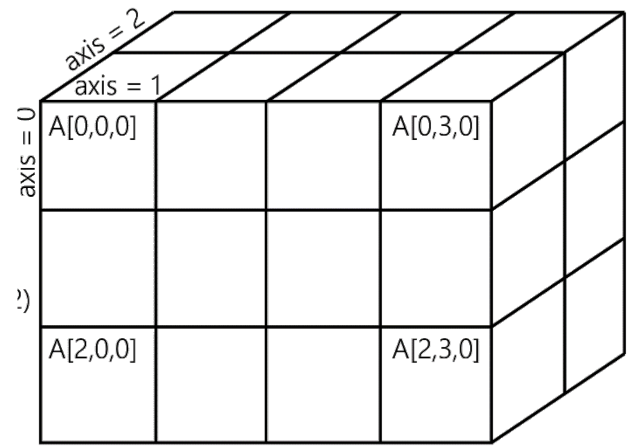


Matrices & n-D Arrays

Matrices & n-D Arrays

- Arrays can have more than 1-dimension
  - 2-D Arrays are called **matrices,** higher dimensions called **tensors**
- Arrays have as many **indexes** for access as dimensions
  - A[i], B[i][j], C[i][j][k]
- Dimensions may have **different lengths**

	Column 0	Column 1	Column 2	Column 3
Row 0	a[0][0]	a[0][1]	a[0][2]	a[0][3]
Row 1	a[1][0]	a[1][1]	a[1][2]	a[1][3]
Row 2	a[2][0]	a[2][1]	a[2][2]	a[2][3]



- Mapping from **n-D to 1-D array**
  - Items **n** dimensions “flattened” into **1** dimension
  - Contiguous memory locations in 1-D
  - Native support in programming languages

https://www.tutorialspoint.com/cprogramming/c_multi_dimensional_arrays.htm
https://ipython-books.github.io/13-introducing-the-multidimensional-array-in-numpy-for-fast-array-computations/

n-D Arrays as 1-D Arrays

- Convert A[i][j] to B[k] … i=row index, j=column index, C=number of cols, R=number of rows
  - **Row Major Order** of indexing: k=map(i,j)=i*C+j
  - **Column Major Order** of indexing: k=map(i,j)=j*R+i
- *How does this look in memory location layout?*
- *How can you extend this to higher dimensions (tensors)?*

0	1	2	3	4	5
6	7	8	9	10	11
12	13	14	15	16	17

(a) Row-major mapping

0	3	6	9	12	15
1	4	7	10	13	16
2	5	8	11	14	17

(b) Column-major mapping

Figure 7.2 Mapping a two-dimensional array

Sahni Textbook, Chapter 7

Matrix-Vector Multiplication

<u>Try yourself!</u>

Write optimised codes for matrix-vector multiplication with: (a) matrix stored as 1D array in row-major order, and (b) with column-major order

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} ax + by + cz \\ dx + ey + fz \\ gx + hy + iz \end{bmatrix}$$

Hint: Reading memory in contiguous locations is significantly faster than jumping around among locations.

n-D Arrays

- **Array of Arrays** representation
- First find pointer for row array
- Then lookup value at column offset in row array
- *How does this look in memory location layout?*
- *Pros & cons relative to using 1-D array representation?*

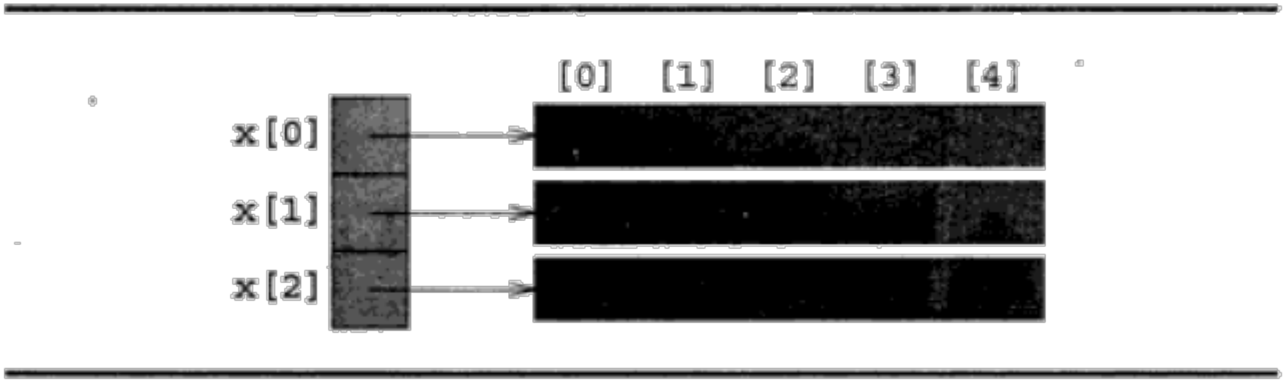
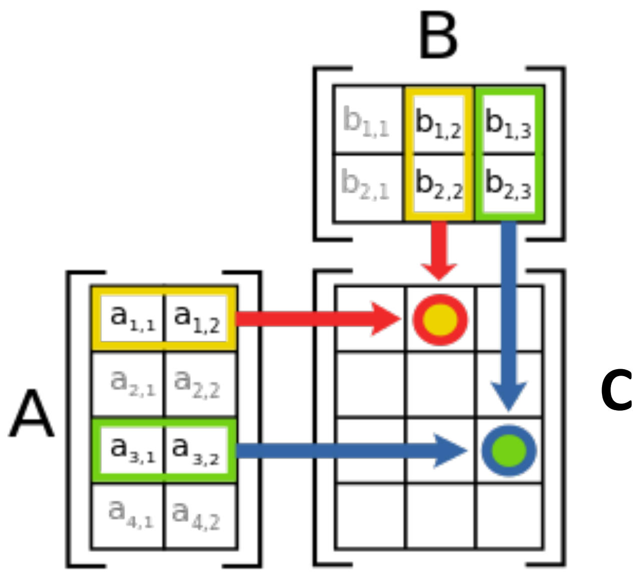


Figure 7.3 Memory structure for a two-dimensional array

Sahni Textbook, Chapter 7

Matrix Multiplication

```
// Given a[n][n], b[n][n]
// c[n][n] initialized to 0
for (i = 0; i < N; i++)
  for (j = 0; j < N; j++)
    for (k = 0; k < N; k++)
      c[i][j] += a[i][k] * b[k][j];
```



https://en.wikipedia.org/wiki/Matrix_multiplication

Sparse Matrices

- Only a small subset of items are populated in matrix
  - Students and courses taken; faculty and courses taught
  - Adjacency matrix of social network graph
    - vertices are people, edges are “friends”
    - Rows and columns are people, cell has 0/1 value
- Why not use regular 2-D matrix?
  - 1-D representation OR
  - Array of arrays representation

Sparse Matrices as 2-D arrays

- Each non-zero item has one entry in list
  - index: <row, column, value>
  - index is the (i-1)*th* non-zero item in row-major order
  - Space taken in **3*NNZ** (number of non zero), compared to **n*m** for non-sparse representation

0	0	0	2	0	0	1	0
0	6	0	0	7	0	0	3
0	0	0	9	0	8	0	0
0	4	5	0	0	0	0	0

(a) A 4 × 8 matrix

terms	0	1	2	3	4	5	6	7	8
row	1	1	2	2	2	3	3	4	4
col	4	7	2	5	8	4	6	2	3
value	2	1	6	7	3	9	8	4	5

(b) Its linear list representation

Figure 7.14 A sparse matrix and its linear list representation

Sahni Textbook, Chapter 7

Sparse Matrix Addition

```
while(p < pMax && q < qMax) {              // C is no. of cols in orig. matrix
    p1 = A[p].r*C + A[p].c        // get index for A in orig. matrix
    q1 = B[q].r*C + B[q].c
    if(p1 < q1)                   // Only A has that index
        C[k] = <A[p].r, A[p].c, A[p].val>              // Copy val
        p++
    else if(p1==q1)               // Both A & B have that index
        C[k] = <A[p].r, A[p].c, A[p].val+B[q].val>     // Add vals
        p++
        q++
    else                                       // Only B has that index
        C[k] = <B[q].r, B[q].c, B[q].val>              // Copy vals
        q++
    k++
}
```


See Sahni, Program 7.17

Compressed Sparse Row (CSR)

- Similar to 2-D array, but more **space efficient**
- 3 arrays *(first 2 same as 2-D array representation)*
  - **A[nnz]** stores non-zero values in row-major order
  - **JA[nnz]** stores column index of nnz in A
  - **IA[m+1]** stores cumulative counts of non-zero values
    - IA[i] = IA[i-1] + number of NNZ in (i-1)th row
    - Always, **IA[0] = 0** and **IA[m+1] = NNZ**
  - ith row elements from **A[IA[i]].** Existence of jth col elements in JA
- Space taken = 2*NNZ + (m + 1)

https://en.wikipedia.org/wiki/Sparse_matrix#Compressed_sparse_row_(CSR,_CRS_or_Yale_format)
https://www.geeksforgeeks.org/sparse-matrix-representations-set-3-csr/

Compressed Sparse Row (CSR)

- **A[nnz]** non-zero values in row-major order
- **JA[nnz]** column index of nnz in A … *Column offset* within the row group for a non zero value
- **IA[m+1]** stores cumulative count of non-zero values till (i-1)th row … Offset to the start of nnz values for the *i*th row in array A

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 5 & 8 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 6 & 0 & 0 \end{pmatrix}$$

$$A = [\,\boxed{5\ 8}\ \boxed{3}\ \boxed{6}\,]_{nnz}$$

$$JA = [\,0\ 1\ 2\ 1\,]_{nnz}$$

$$IA = [\,\boxed{0}\ \boxed{0}\ \boxed{2}\ \boxed{3}\ \boxed{4}\,]_{m+1}$$

Row groups

Column offset

Offset to row group

To access X[i,j]:

- *Row i* values must be present between A[s] and before A[e], where s= IA[i] and e= IA[i+1]
- Check corresponding column offset, JA[s] until JA[e-1]. If any of these JA values match j, the value is present in the corresponding index of A.
- If (e-1 < s) OR no JA matches, then the value is 0

<u>Try yourself!</u>
Matrix-matrix addition using CSR

https://en.wikipedia.org/wiki/Sparse_matrix#Compressed_sparse_row_(CSR,_CRS_or_Yale_format)
https://www.geeksforgeeks.org/sparse-matrix-representations-set-3-csr/

Tasks

- Self study (Sahni Textbook)
  - Chapters 5 & 6 “Linear Lists—Array & Linked Representations”
  - Chapter 7, Arrays and Matrices
- Programming Self Study
  - Try out **list** data structure in **C++ STL**
  - Define your own **abstract list interface** using **templates/ generics** in C++. Implement create, set, get, front and back using a **1-d array** representation.
  - Try out **matrix-matrix multiplication, matrix-vector multiplication** in C++

Upcoming tutorials

- **Date TBD**: Tutorial on Makefile, C++ STL
- **Date TBD**: Tutorial on GitHub
- **Date TBD**: Release programming assignment