Game Theory and Mechanism Design

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Practice Problems: MSNE

Problem Set 2

Warm-up

1. Let S be any finite set with n elements. Show that the set $\Delta(S)$, the set of all probability distributions over S, is a convex set.

2. Using first principles, find all MSNE of the matching pennies game.

3. Using first principles, find all MSNE of the prisoner's dilemma game.

Workhorse

1. Find all mixed strategy Nash equilibria, applying the NASC, of the following game:
   $N = \{1, 2\}$; $S_1 = S_2 = \{A, B\}$;
   U_1 is $4, 0, 1, 1$ for $(A, A), (A, B), (B, A)$, and (B, B) resp.
   U_2 is $1, 4, 5, 1$ for $(A, A), (A, B), (B, A)$, and (B, B) resp.

2. Find all mixed strategy Nash equilibria, applying the NASC, of the following game:
   $N = \{1, 2\}$; $S_1 = S_2 = \{A, B\}$;
   U_1 is $2, 1, 2, 1$ for $(A, A), (A, B), (B, A)$, and (B, B) resp.
   U_2 is $2, 2, 1, 1$ for $(A, A), (A, B), (B, A)$, and (B, B) resp.

3. Show that any strictly dominant (mixed) strategy in a strategic form game must be a pure strategy.

4. I claim that in the theorem that provides NASC for a strategy profile to be MSNE, I can replace the ">=" sign in the second condition by a strict ">". Is my claim right? Prove or disprove.

Thought Provoking

1. Consider a zerosum, strategic form game with $N = \{1, 2\}$; $S_1 = S_2 = [a, b]x[a, b]$ where a and b are positive real numbers such that a is strictly less than b. Essentially, each player picks simultaneously a point in the square $[a, b]x[a, b]$. Define the utility function $u_1(s_1, s_2) = -u_2(s_1, s_2) = d(s_1, s_2)$ where $d(s_1, s_2)$ is the Euclidean distance between the two points. Compute all MSNE.

2. Consider the following strategic form game where the numbers a, b, c, d are real numbers:
   $N = \{1, 2\}$; $S_1 = S_2 = \{A, B\}$;
   U_1 is a, b, c, d for $(A, A), (A, B), (B, A)$, and (B, B) resp.
   U_2 is a, c, b, d for $(A, A), (A, B), (B, A)$, and (B, B) resp.
   It is known that the game has a strongly dominant strategy equilibrium. Prove or disprove: The above SDSE is the only possible MSNE for this game.