Tutorial 1 solutions

1) $Q_1(1) = 0$

$$Q_2(1) = \frac{1}{1} = 1$$

$$Q_3(2) = \frac{1}{1} = 1$$

$$Q_4(2) = \frac{3}{2}$$

$$Q_5(2) = \frac{5}{3}$$

$$Q_6(3) = \frac{0}{1} = 0$$

$$\boxed{3, 6}$$

$$\boxed{2, 3, 4, 5, 6}$$

2)

$$\frac{n\,\mu^*(V) - E\left[\sum_{t=1}^{n} X_t\right]}{n}$$

$$\mu^*(V) - \frac{E\left[\sum_{t=1}^{n} X_t\right]}{n}$$

$$\lim_{n\to\infty} \frac{E\left[\sum_{t=1}^{n} X_t\right]}{n} = \mu^*(V)$$

$$\sum_{t=1}^{n} \sum_{a=1}^{k} \mu_a\, E\left[\mathbb{1}_{\mu_{A_t}=\mu_a}\right] / n$$

$$\frac{\sum_{a=1}^{k} \mu_a\, E\left[\sum_{t=1}^{n} \mathbb{1}_{\mu_{A_t}=\mu_a}\right]}{} = \mu^*(V)$$

n

$$\lim_{n\to\infty} \frac{\sum_{a=1}^{k} E\left[\sum_{t=1}^{n} \mathbb{1}_{\hat{\mu}_{A_t}=\mu_a}\right]}{n} = 1$$

RTP

$$\lim_{n\to\infty} \frac{E\left[\sum_{t=1}^{n} \mathbb{1}_{\mu_{A_t}=\mu^*}\right]}{n} = 1$$

If
$$\lim_{n\to\infty} \frac{E\left[\sum_{t=1}^{n} \mathbb{1}_{\mu_{A_t}=\mu^*}\right]}{n} < 1$$

$= K_1$ (suppose)

$$\therefore \lim E\left[\sum^{n} \mathbb{1} \quad \right]$$

$$n \to \infty \quad \frac{\quad {}_{t=1} \quad \mathbb{1}_{\mu_{A_t} \neq \mu^*}]}{n} > 0$$

$$= K_2 \text{ (suppose)}$$

$$\frac{\sum_{d=1}^{K} \mu_d \, E\left[\sum_{t=1}^{n} \mathbb{1}_{\mu_{A_t} = \mu_d}\right]}{n}$$

$$\leq \mu^* K_1 + \mu^\# K_2 < \mu^*$$

where $\mu^\#$ is the 2nd largest mean.

which is a contradiction

[Note that the above is the weighted avg of all the μ_d]

$$\lim_{n\to\infty} \sum_{t=1}^{n} P(\mu^* - \mu_{A_t} > 0) = $$

$$\cancel{P(\mu^* > \mu_{A_t}) = 0 \ \forall t}$$

$$\lim_{n\to\infty} \frac{E\left[\sum_{t=1}^{n} \mathbb{1}\{\mu^* \neq \mu_{A_t}\}\right]}{n} = 0$$

It can happen that

$$P(\mu^* \neq \mu_{A_t}) > 0$$

for some t

still the above relation satisfies,

$\therefore$ (b) is not true

3)

$$J(x,y) = \max_{i=1,\dots,m} k_i \left(r_i + J_{y-1}(x-1) \right) + (1-k_i)\left(J_{y-1}(x) \right)$$

$$= k_i r_i + k_i J_{y-1}(x-1) + J_{y-1}(x) - k_i J_{y-1}(x)$$

$$J_1(x) = \max_i \left(k_i r_i + J_0(x-1) \right) + (1-k_i)\left(J_0(x) \right)$$

$$= k_m r_m$$

$$J_2(x) = \max_i \left(R_i r_i + J_1(x-1) + (1-R_i) J_1(x) \right)$$

$$= \max_i \left(R_i r_i + R_m r_m + (1-R_i) R_m r_m \right)$$

$$= 2 R_m r_m$$

$\therefore$ It is ideal to select the highest rate r_m

b)

$$J(x,y) = \sum_{i=1}^{m} R_i \max\left(r_i + J_{y-1}(x-1), \right.$$

$$+ \quad (\quad , \quad)$$

$J_{y-1}(x)\big)\Big)$

$$J_1(x) = \sum_{i=1}^{m} k_i \max\left(r_i + J_0(x-1),\ J_0(x)\right)$$

$$= \sum_{i=1}^{m} k_i r_i, \quad \bar{r} = 0$$

$$J_2(x) = \sum_{i=1}^{m} k_i \max\left(r_i + J_1(x-1),\ J_1(x)\right)$$

$$, \quad \bar{r} = 0$$

$$J_3(x) = \sum_{i=1}^{m} k_i \max\left(r_i + J_2(x-1),\ J_2(x)\right)$$

$\therefore$ The threshold $\bar{r} = 0$

Since $\bar{r} > 0$, always accept

[illegible]

... customers offer.

4)

$$E\left(X_N^{+} \sum_{k=0}^{N-1} (1-U_k) X_k\right.$$

$$\left(X_{N-1} + w_{N-1} u_{N-1} X_{N-1}\right.$$

$$J_1(X) = E\Big[(1-U_0)X + J_0(X + w_0 U_0 X)\Big]$$

$$= E\left[(1-U_0)X + X + w_0 U_0 X\right]$$

$$= (2 - u_0 + \bar{w} u_0) x$$

$$= (2 + (\bar{w} - 1) u_0) x$$

If $\bar{w} > 1$, $u^*_0 = 1$

If $\bar{w} < 1$, $u^*_0 = 0$

If $\bar{w} = 1$, $u^*_0 = 0$

$$J_2(x) = E\left[(1 - u_1) x + J_1(x + w_1 u_1 x)\right]$$

$$= (1 - u_1) x + (2 + (\bar{w} - 1) u_0)(1 + \bar{w} u_1) x$$

$$= (1 - u_1 + 2 + 2\bar{w} u_1 + (\bar{w} - 1) u_0$$

$(-2 \quad \bar{w}) u_0 u_1)$

$$+(\bar{w} \; - \; \ldots \;)x$$

$$= \big(3 + (2\bar{w} - 1)v_1 + (\bar{w} - 1)v_0 + (\bar{w}^2 - \bar{w})v_0 v_1\big)x$$

If $\bar{w} > 1$, $u^*_1 = 1$

If $\bar{w} < \frac{1}{2}$, $u^*_1 = 0$

If $\frac{1}{2} < \bar{w} \le 1$, $u^*_1 = 1$

$$J_3(x) = (1 - v_2)x + \big(3 + (2\bar{w} - 1)v_1 + (\bar{w} - 1)v_0 + (\bar{w}^2 - \bar{w})v_0 v_1\big)(1 + \bar{w} v_2)x$$

$$= 4 + (2\bar{w} - 1)v_1 + (\bar{w} - 1)v_0 + (\bar{w}^2 - \bar{w})v_0 v_1$$

$$(3\bar{\omega}-1)v_2 + \bar{\omega}(2\omega-1)v_2 v_1$$

$$+\bar{\omega}(\bar{\omega}-1)v_0 v_2$$

$$+\bar{\omega}^2(\bar{\omega}-1)v_0 v_1 v_2$$

If $\bar{\omega} > 1$, $u^*_2 = 1$

If $\bar{\omega} < \frac{1}{3}$, $u^*_2 = 0$

If $\frac{1}{3} \leq \bar{\omega} \leq 1$, $u^*_2 = 1$

So as N increases, we conclude (a), (b), (c) satisfies