

# E1 254 Game Theory

**JAN-APRIL 2023**

**SIDDHARTH BARMAN**

**Y. NARAHARI**

**<http://gtl.csa.iisc.ac.in/hari>**

**Computer Science and Automation  
Indian Institute of Science, Bangalore**



# Organisation of the Course

## **Part 1: Noncooperative Game Theory**

**Games where agents play individual actions**

## **Part 2: Cooperative Game Theory**

**Games where agents form coalitions and play joint actions**

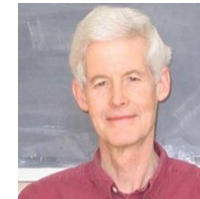
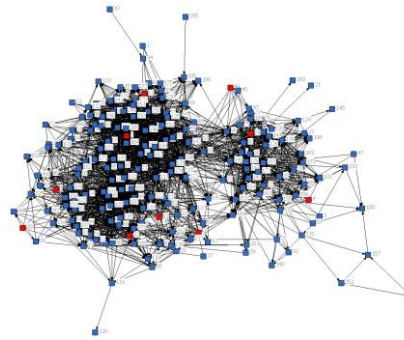
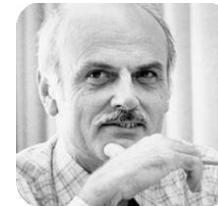
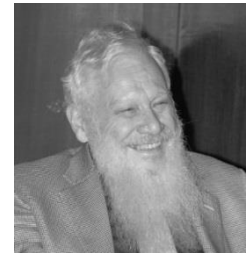
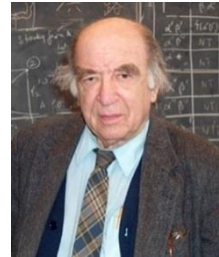
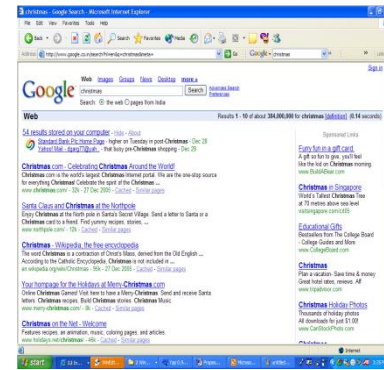
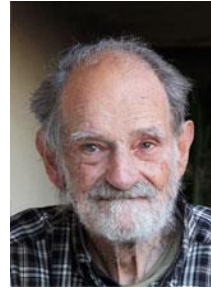
## **Part 3: Mechanism Design**

**Designing games that have a desired equilibrium behavior**

## **Part 4: Algorithmic Game Theory**

**Algorithmic and complexity theoretic issues in game theory**

# Game theory Meets Computer Science





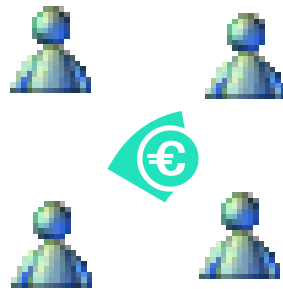
# Game Theory

**Mathematical framework for rigorous study of conflict and cooperation among rational, intelligent agents**

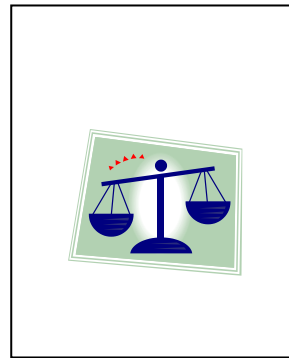
**Game: Mathematical model of a situation where every player strives to obtain the best possible outcome, knowing fully well that all other players also choose their strategies to obtain their best possible outcomes**



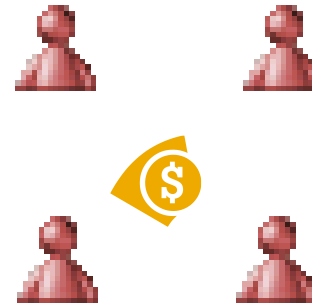
**John von Neumann  
Oskar Morgenstern**



**Buying  
Agents**



**Market**



**Selling Agents**

**Game Theory  
is the  
Science of  
Microeconomics**

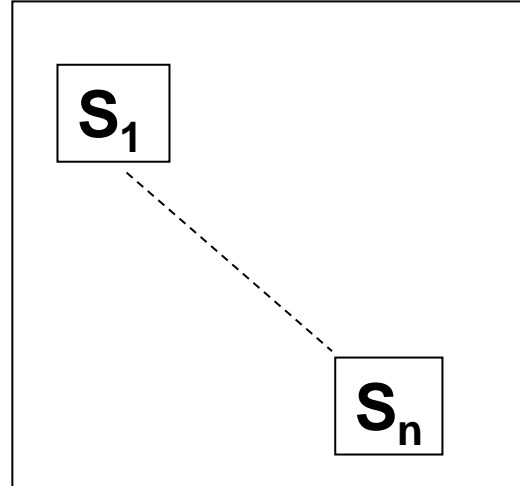
# Noncooperative Game Theory

# Noncooperative Games: Strategic Form Games



$N = \{1, \dots, n\}$

Players



$S_1, \dots, S_n$

Strategy Sets

$S = S_1 \times \dots \times S_n$

$U_1 : S \longrightarrow R$

$U_n : S \longrightarrow R$

Payoff  
functions

(Utility  
functions)

# Example: Student Coordination Game

A \ B	IISc	M.G. Road
	IISc	M.G. Road
IISc	100,100	0,0
M.G. Road	0,0	10,10

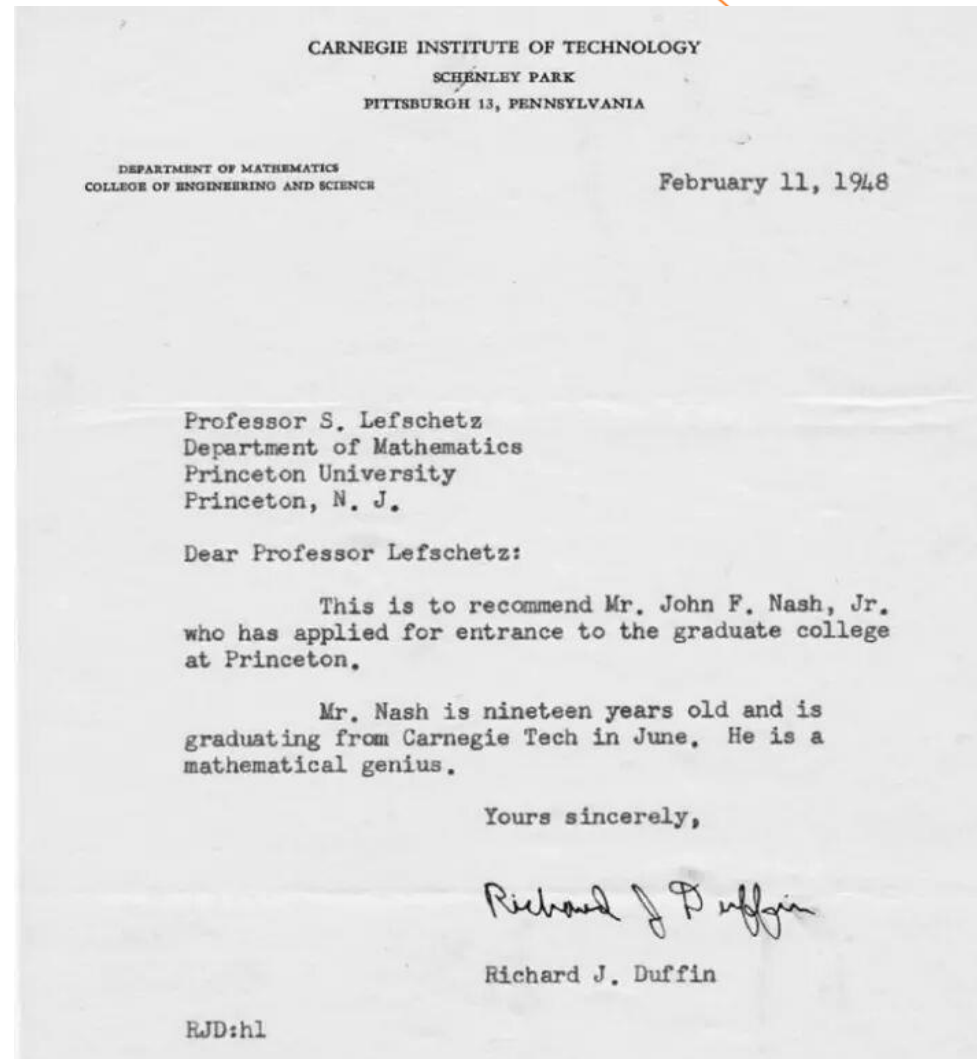
**Models the strategic conflict when two players have to choose their priorities**

# John Forbes Nash Jr.

**Born: 13 June 1928 (West Virginia)**

**BS, MS in CMU in 1948 (Maths)**

**Ph.D. in Princeton Univ (Tucker)**





# John Nash (contd.)

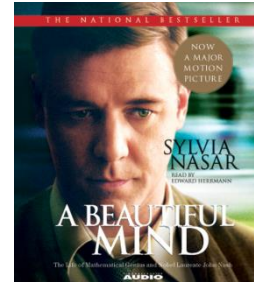
**Ph.D. : Non-Cooperative Games  
(28 pages) (1950)**

**Existence of MSNE in finite  
strategic form games**

**Nash bargaining theorem (1951)**  
**Completely solved the two person  
bargaining problem**

**Asst Professor, MIT (1950s)**  
**Algebraic Geometry,  
Nonlinear Parabolic Differential Eqns**

**1994: Nobel Prize in Economics**  
**Jointly with Reinhard Selten and  
John Harsanyi**



# Cooperative Game Theory

# Cooperative Games: Characteristic Form Game

$$T = (N, v)$$

$N = \{1, 2, \dots, n\}$  set of players

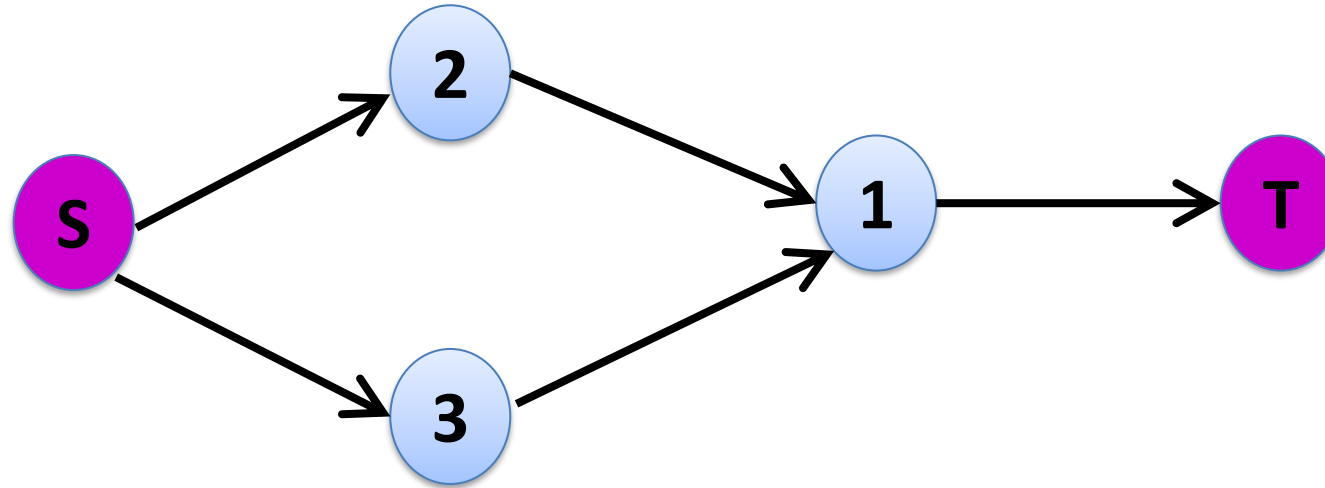
$v: 2^N \rightarrow \mathbb{R}$  characteristic function ;  $v(\emptyset) = 0$

$C \subseteq N$  is called a **coalition**.

There are  $2^{|N|} - 1$  possible coalitions

Players form coalitions and play correlated (joint) strategies

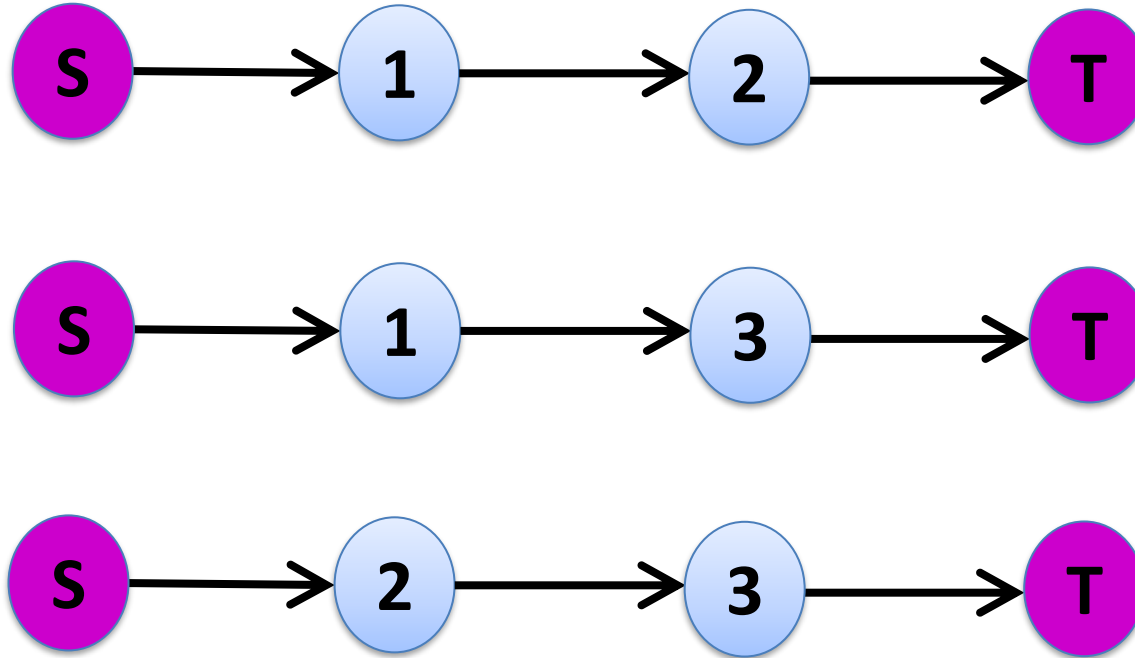
# Divide the Dollar Game: Version 1



$$v(\{1\}) = v(\{2\}) = v(\{3\}) = v(\{2,3\}) = 0$$

$$v(\{1,2\}) = v(\{1,3\}) = v(\{1,2,3\}) = 300$$

# Divide the Dollar Game: Version 2



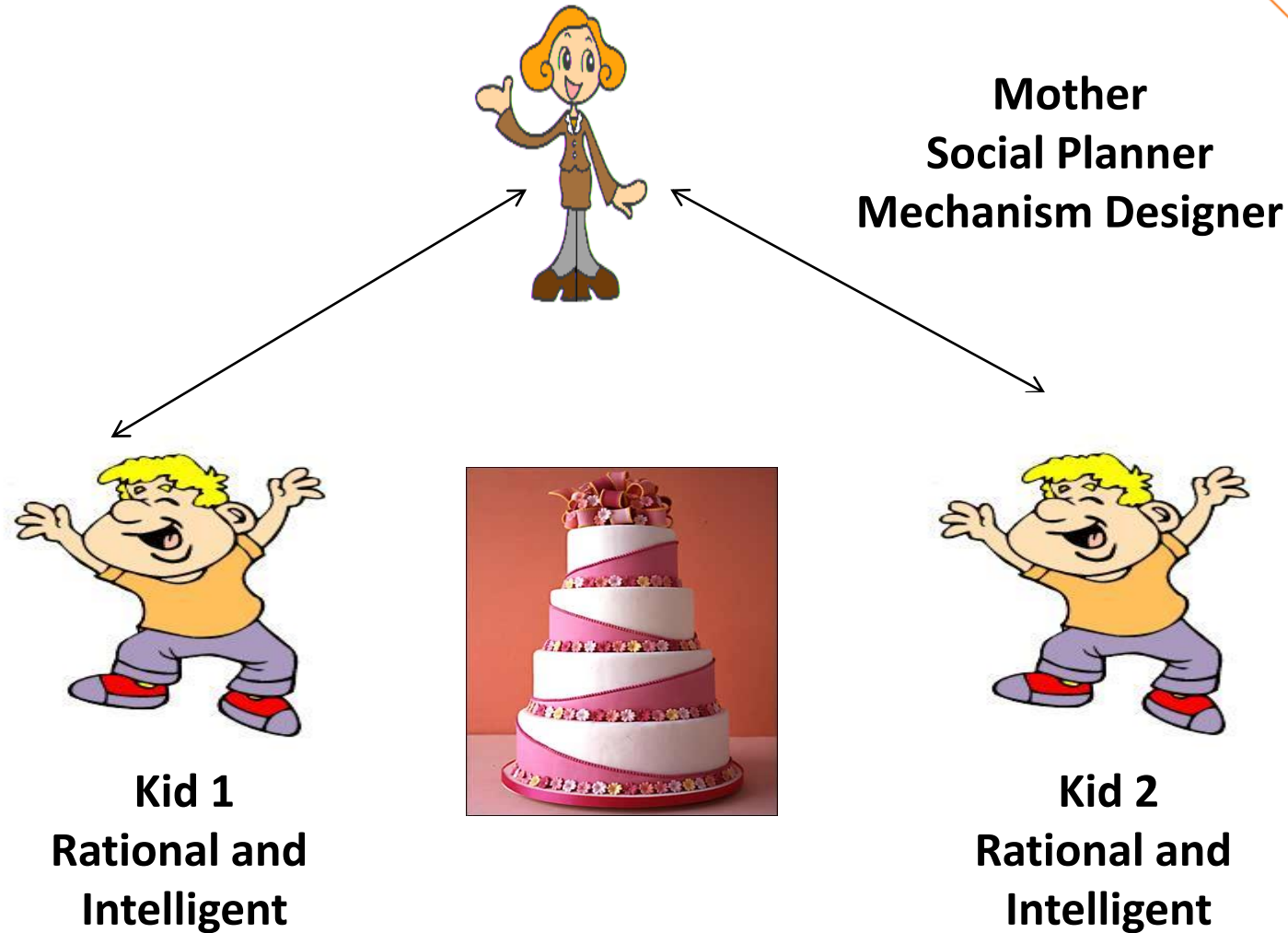
$$v(\{1\}) = v(\{2\}) = v(\{3\}) = 0$$

$$v(\{1,2\}) = v(\{1,3\}) = v(\{2,3\}) = v(\{1,2,3\}) = 300$$



# Mechanism Design

# Mechanism Design: Cake Cutting



# Some Impact Applications

# Example 1: Auction for Selling an Item



**William Vickrey**  
(1914 – 1996 )  
Nobel Prize: 1996

1		40
2		45
3		60
4		80

**Buyers/  
Bidders**

**Winner = 4  
Payment = 60**

# Example 2: Reverse Auction to Buy an Item Indivisible



**William Vickrey**  
**(1914 – 1996 )**  
**Nobel Prize: 1996**

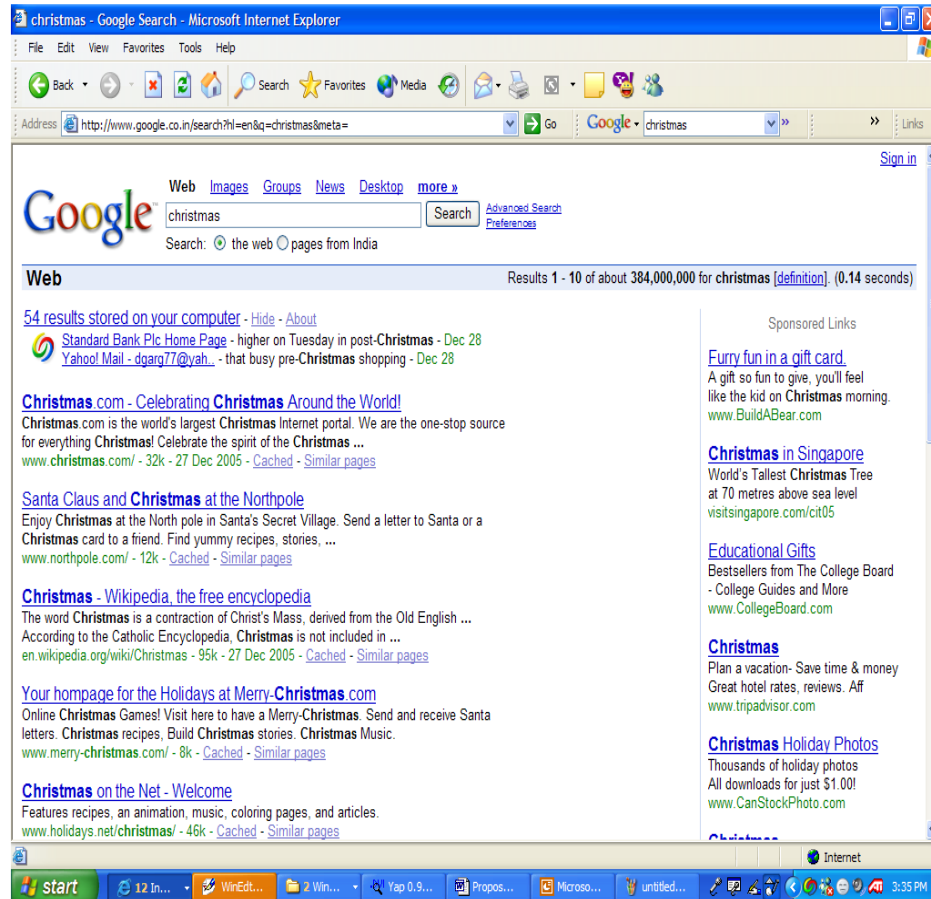
1		40
2		50
3		60
4		80

**Sellers  
are the  
Bidders**

**Winner = 1  
Payment = 50**



# Example 3: Sponsored Search Auction



Advertisers

$\theta_1$

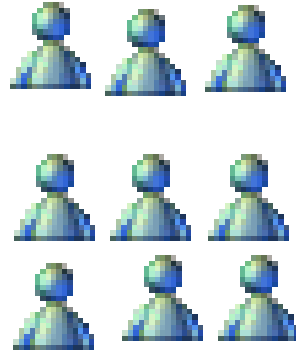
$\theta_2$

$\theta_n$

CPC

Design an auction that maximizes social utility

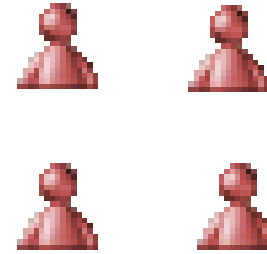
# Example 4: Matching Market



**Students**  
**Employees**  
**Doctors**  
**Farmers**



**Matching  
Market**



**Colleges**  
**Companies**  
**Hospitals**  
**Consumers**

**Design a matching market that maximizes the  
social welfare and induces honest behavior**

# Example 5: Mechanism for Geospatial Exploration



**Mechanism Design Meets Computer Science, Communications of the ACM, Aug-2010**

Image Courtesy: CACM, Aug-2010



# Example 6: Procurement Auction by Agri FPO



किसान उत्पादक संगठन

**FPO**

2000 packets of Variety A; 1000 of B; 1000 of C; 1000 of D; and 2500 of E



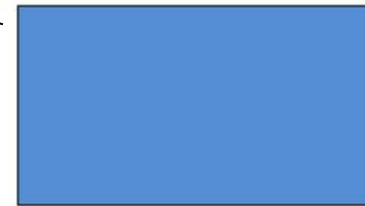
**SUPPL-1**



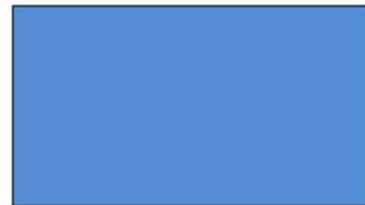
**SUPPL-2**



**SUPPL-3**



**SUPPL-4**



**SUPPL-n**

# The Challenge

The problems involve strategic agents which can manipulate the algorithms and solutions in different ways

How do we realize social goals in the presence of rational agents? How do we make agents behave honestly?

Game theory and mechanism design have principled answers to these challenges and fill an important gap



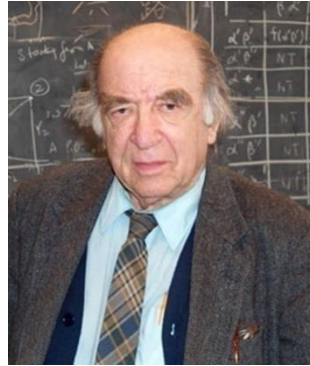
# Hall of Fame: Game Theory Nobel Laureates (2005-)

**Robert Aumann**  
Nobel 2005



**Thomas Schelling**  
Nobel 2005

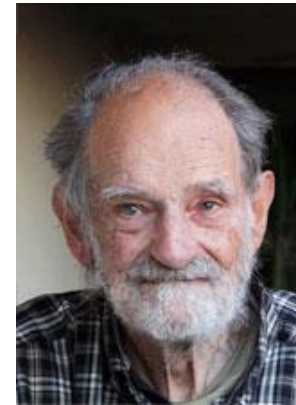
**Roger Myerson**  
Nobel 2007



**Leonid Hurwicz**  
Nobel 2007



**Eric Maskin**  
Nobel 2007



**Lloyd Shapley**  
Nobel 2012



**Alvin Roth**  
Nobel 2012



**Paul R. Milgrom and Robert B. Wilson**  
Nobel 2020

# Why is GTMD Exciting to Study?

**Deep Science:** Algorithmic Game Theory, Market Design, Computational Social Choice (fair division, voting, matching), Interface of ML and GT; Esoteric and intellectually deep theory

**Deep Tech:** Internet advertising, crowdsourcing, Smart Grids, Autonomous Systems, Traffic, Social Networks, Matching Markets, Cyber Security, Agriculture, Education, ... all involve strategic agents and strategic decision making

**AI++:** AI toolkit without GTMD is incomplete due to strategic nature of interactions in modern AI powered systems

# To Probe Further

**Martin Osborne. Introduction to Game Theory.**  
Oxford University Press, 2003

**Roger Myerson. Game Theory and Analysis of Conflict.** Harvard University Press, 1997

**A, Mas-Colell, M.D. Whinston, and J.R. Green.**  
**Microeconomic Theory**, Oxford University Press, 1995

**N. Nisan, T. Roughgarden, E. Tardos, V. Vazirani**  
**Algorithmic Game Theory**, Cambridge Univ. Press, 2007

**T. Roughgarden. Twenty Lectures on Algorithmic Game Theory**, Cambridge Univ. Press, 2016

# To Probe Further (contd.)

**Y. Narahari**

**Game Theory and Mechanism Design**  
**IISc Press and World Scientific, 2014**

**Y. Narahari, Dinesh Garg, Ramasuri, and Hastagiri**  
**Game Theoretic Problems in Network Economics**  
**and Mechanism Design Solutions, Springer, 2009**

**Michael Maschler, Eilon Solan, Shmuel Zamir**  
**Game Theory. Cambridge University Press, 2013**

**Yoam Shoham and Kevin Leyton-Brown**  
**Multiagent Systems: Algorithmic, Game Theoretic,**  
**And Logical Foundations, CUP, 2009**

# To Probe Further (contd.)

**Harold W. Kuhn. Classics in Game Theory.**  
**Princeton University Press, 1997**

**Philip D. Straffin Jr.**  
**Game Theory and Strategy**  
**The Mathematical Association of America, 1993**

**David C. Parkes and Sven Seuken. Economics and Computation.** Cambridge University Press, 2016

**Jorg Rothe (Editor).**  
**Economics and Computation: An Introduction to Algorithmic Game Theory, Computational Social Choice, and Fair Division,** Springer, 2016



# To Probe Further (contd.)

**Tilman Borgers, Daniel Krahmer, Roland Strausz.**  
**An Introduction to the Theory of Mechanism Design**  
**Springer, 2014**

**Rakesh V. Vohra**  
**Mechanism Design: A Linear Programming Approach**  
**The Mathematical Association of America, 1993**

**Vijay Krishna.**  
**Auction Theory**  
**Second Edition, Academic Press, 2010**

# To Probe Further (contd.)

**<http://www.gametheory.net>**

**A rich source of material on game theory and game theory courses**

**<https://agtb.wordpress.com>**

**Turing's Invisible Hand:  
Economics, Computation, and Game Theory**

**<http://gtl.csa.iisc.ac.in/hari>**

**Course material and  
several survey articles can be downloaded**

# To Probe Further (contd.)

## CONFERENCES

ACM EC, WINE, AAMAS, AAAI, IJCAI, UAI, WWW,  
FOCS, STOC, SODA, PODC,  
ICML, NeurIPS, COLT, ICLR, ICTS

## JOURNALS

ACM TEAC, JAIR, AI, JMLR, Economics Journals  
such as Econometrica, Journal of Economic Theory,  
Games and Economic Behaviour, Journal of Mathematical  
Economics, American Economic Review

# Schedule and Logistics

**Classes on Monday, Wednesday, Friday;  
(Including Tutorials, Special Lectures)**

**Test 1: Feb 20, 2023 (20)**  
**Test 2: March 29, 2023 (20)**

**Miniproject Finalisation: Jan 31, 2023**  
**2 page paper submission : Feb 15, 2023 (5)**  
**Term Paper Submission: Apr 1, 2023 (10)**  
**Submission of Video Presentation and slides: Apr 10, 2023 (15)**

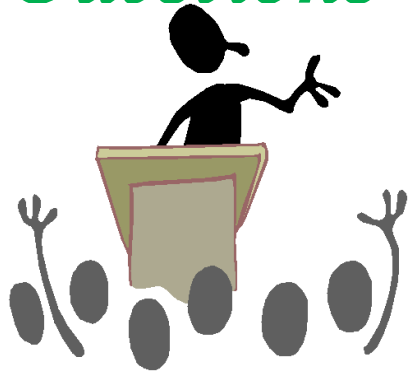
**Final Exam: April final Week (30)**

**Problem Sets will be shared regularly and tutorials will be held**

# Teaching Assistants

**Chaitanya Chennam**  
**Kaushik Kukadia Hareshbhai**  
**Prathik Diwakar**  
**Abhishek Chadhary Kumar**  
**Kishan Mittal**  
**Sneha Negi**  
**Rutwik Pasani**  
**Burhanuddin Kamlapur Wala**  
**Shreepranav Varma**  
**Jaydeep Pawar Vasudev**

*Questions ...*



*Answers ...*

