Computational & Data Sciences
Indian Institute of Science, Bangalore

AUGUST–DECEMBER SEMESTER, 2022

DS290: Modeling and Simulation

(Time allowed: 90 minutes)

NOTE: Full Marks: 25. All questions carry equal points. Please write your name and SR No. on the answer scripts. Please upload your work to the *Assignments* on the MS Teams DS290 Modelling and Simulation.

Midterm Test 1

1. Show that the geometric distribution for a random variable X (taking values x) given by $p(x) = q^{x-1}(1-q)$ (where $0 \leq q \leq 1$ is a real number) for $x = 1, 2, \cdots$ and $p(x) = 0$ otherwise, is memoryless.

2. For a linear congruential generator with $c = 0$ in $X_{i+1} = (aX_i + c) \bmod m$, show that $X_{i+n} = (a^n X_i) \bmod m$.

3. Let there be two queues, Q1 and Q2. At Q1, the arrival and service rates are 6 and 12 per hour, respectively. At Q2 arrival and service rates are 12 and 24 respectively. Assume that the arrivals occur in Poisson processes and that service times have exponential distributions for both the queues and that the queues have infinite calling population and capacity.
(a) If the two queues operate independently of each other, compare the two queues in terms of the the mean waiting time for a customer in the line for service.
(b) If Q1 and Q2 are merged into a single queue Q0 with arrival rate 18 per hour and service rate 18 per hour with Poisson arrivals and exponential service times as before, then find out if Q0 will have lesser or more waiting time than each of Q1 and Q2.

4. Suppose there is an $M/M/1/3$ queue with a Poisson arrival rate of 1 per minute and service times are exponentially distributed with mean 2.5 minutes.
Hint: Note that when the system is in equilibrium, the probability that the system is filled to capacity is $P_K,\ K = 3$; and when there are $n < K$ in the system, the joint probability that there are n in the system and an arrival can happen is given by $\frac{P_n}{1-P_K}$. Also, note that waiting happens when n exceeds the number of servers.
(a) From the hint write the formula for computing the probability that a customer has to wait more than t minutes in an $M/M/c/K$ queue.
(b) What is the probability that the customer has to wait for more than 1.5 minute before getting served ?
(c) If the queue is converted into $M/M/2/3$, what happens to the probability that the wait is more than 1.5 minutes?

5. (a) Give a compact expression that is computationally efficient to evaluate for generating the random variate distributed with the probability density function $|x|,\ x \in [-1, 1] \subset \mathbb{R}$ using a random number generator for the uniform distribution $U(0, 1)$ on $[0, 1]$.
(b) Using the random variate generated with the probability distribution function, $|x|,\ x \in [-1, 1] \subset \mathbb{R}$, give an acceptance-rejection scheme, with the maximum efficiency possible, for generating the random variate distributed as $\frac{3}{2}x^2,\ x \in [-1, 1] \subset \mathbb{R}$.