Problem 1

November 28, 2022

[1]:
```
import numpy as np
from matplotlib import pyplot as plt
```

[2]:
```
# Markov Transition Matrix
A = np.array([[0, 0,  1,  0,  0,  0,  0,  0,  0,  0 ],
              [0.5, 0,  0,  0,  0,  0,  0,  0,  0.5, 0 ],
              [0,  0.5, 0,  0,  0,  0,  0.5, 0,  0, 0 ],
              [0,  0,  0,  0,  0,  0,  0,  0,  0, 0 ],
              [0,  0,  0,  1,  0,  0,  0,  0,  0, 0 ],
              [0,  0,  0,  0,  0.5, 0,  0,  0.5, 0, 0 ],
              [0,  0,  0,  0,  0,  0.5, 0,  0,  0, 0 ],
              [0,  0.5, 0,  0,  0.5, 0,  0,  0,  0, 1 ],
              [0.5, 0,  0,  0,  0,  0,  0.5, 0.5, 0, 0 ],
              [0,  0,  0,  0,  0,  0.5, 0,  0,  0.5, 0 ]])
A
```

[2]:
```
array([[0. , 0. , 1. , 0. , 0. , 0. , 0. , 0. , 0. , 0. ],
       [0.5, 0. , 0. , 0. , 0. , 0. , 0. , 0. , 0.5, 0. ],
       [0. , 0.5, 0. , 0. , 0. , 0. , 0.5, 0. , 0. , 0. ],
       [0. , 0. , 0. , 0. , 0. , 0. , 0. , 0. , 0. , 0. ],
       [0. , 0. , 0. , 1. , 0. , 0. , 0. , 0. , 0. , 0. ],
       [0. , 0. , 0. , 0. , 0.5, 0. , 0. , 0.5, 0. , 0. ],
       [0. , 0. , 0. , 0. , 0. , 0.5, 0. , 0. , 0. , 0. ],
       [0. , 0.5, 0. , 0. , 0.5, 0. , 0. , 0. , 0. , 1. ],
       [0.5, 0. , 0. , 0. , 0. , 0. , 0.5, 0.5, 0. , 0. ],
       [0. , 0. , 0. , 0. , 0. , 0.5, 0. , 0. , 0.5, 0. ]])
```

[3]:
```
v = np.random.randn(10)
v /= np.linalg.norm(v)

V = np.copy(v)
RQ = []

for k in range(300):
    w = A.dot(v)
    v = w / np.linalg.norm(w)
    RQ.append(np.dot(v,A.dot(v)))
    V = np.vstack((V,v))
```

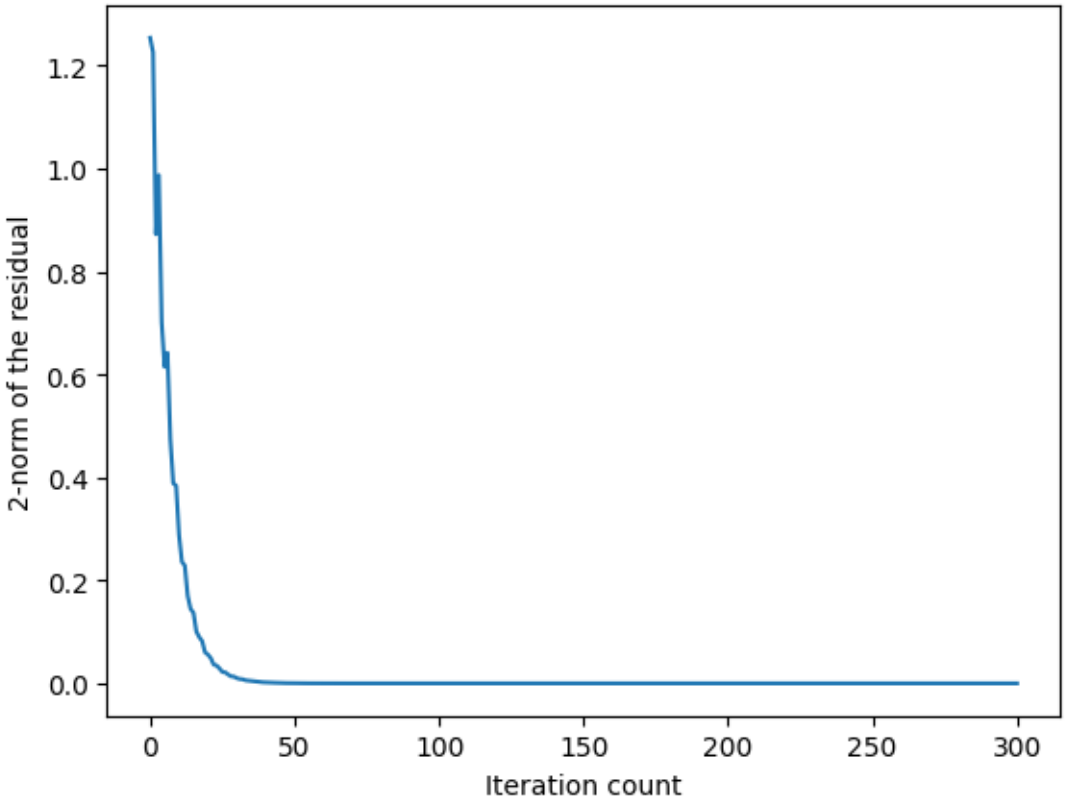
```
print("Rayleigh quotient : ", RQ[-1])
print("Iterations  taken : ", k)
print("Norm of A*v - RQ[-1]*v", np.linalg.norm(A.dot(v) - RQ[-1]*v))
```

```
Rayleigh quotient :  1.0000000000000002
Iterations  taken :  299
Norm of A*v - RQ[-1]*v 1.8824747269678055e-16
```

[4]:
```
residual_norm = np.linalg.norm(A.dot(V.T) - RQ[-1]*V.T, axis=0)

plt.xlabel("Iteration count")
plt.ylabel("2-norm of the residual")
plt.plot(residual_norm)
```

[4]: [<matplotlib.lines.Line2D at 0x7f173d0b29b0>]

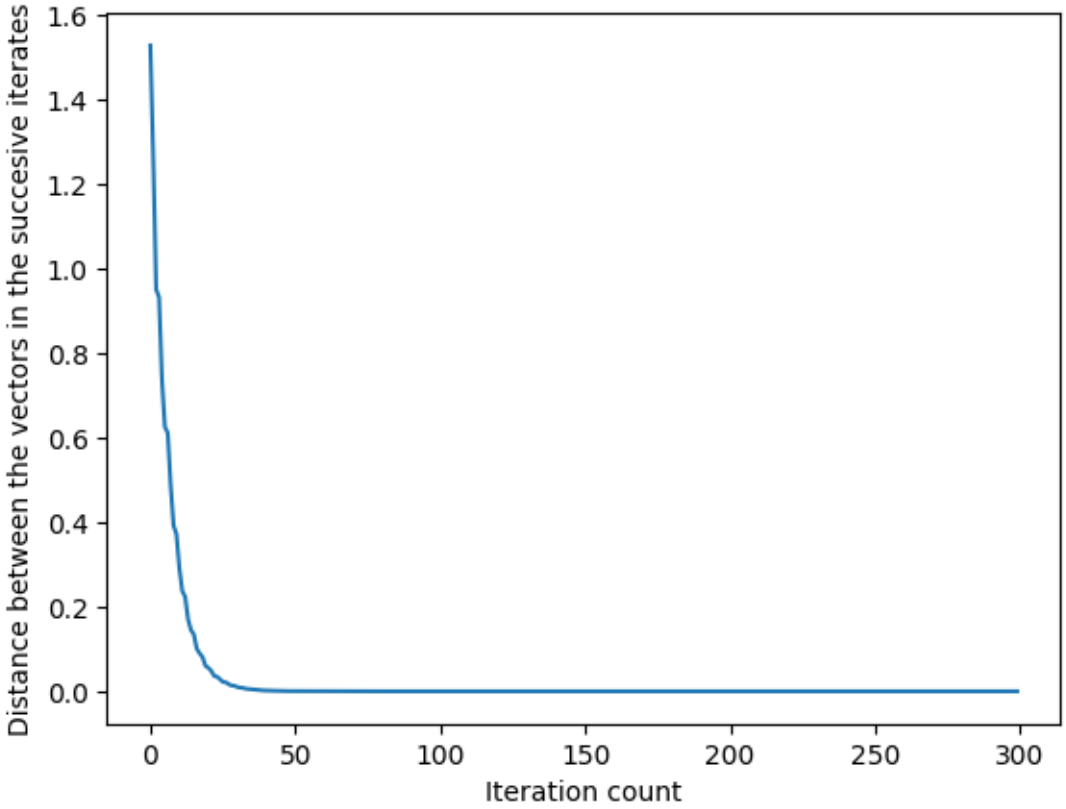


[5]:
```
diff_norm = np.linalg.norm(V[:-1,:] - V[1:,:], axis=1)

plt.xlabel("Iteration count")
plt.ylabel("Distance between the vectors in the succesive iterates")
```

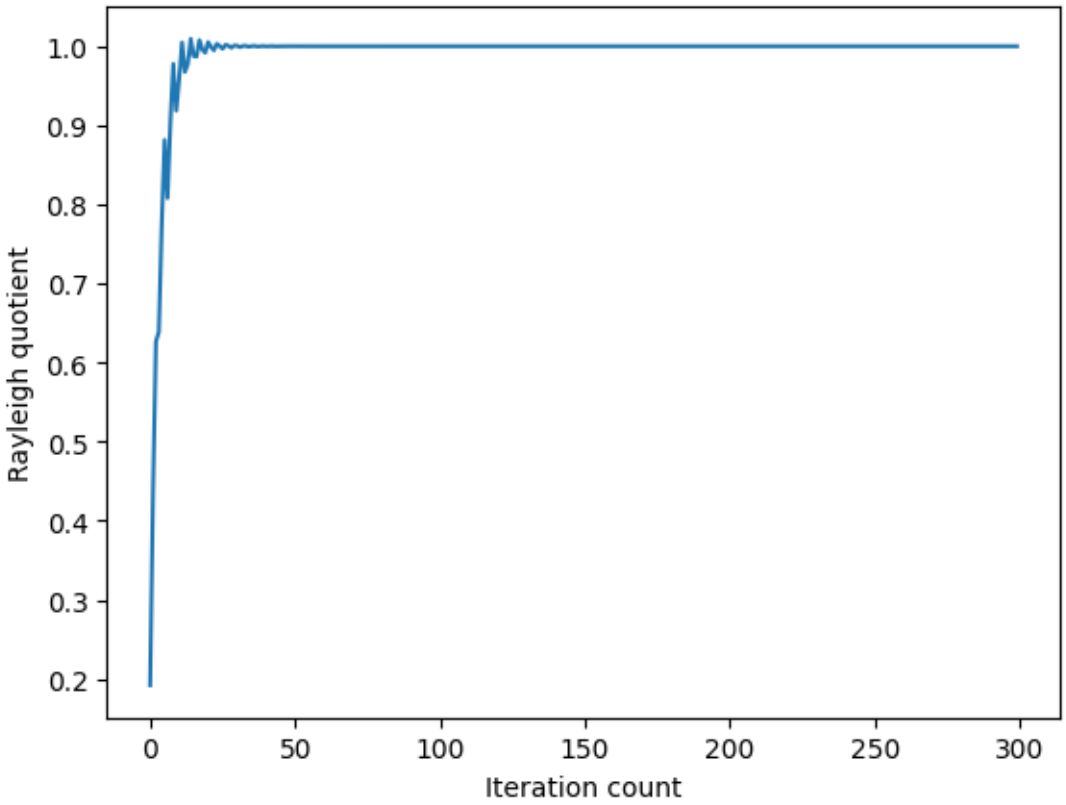
```
plt.plot(diff_norm)
```

[5]: [<matplotlib.lines.Line2D at 0x7f173cfcc6d0>]



[6]:
```
plt.xlabel("Iteration count")
plt.ylabel("Rayleigh quotient")
plt.plot(RQ)
```

[6]: [<matplotlib.lines.Line2D at 0x7f173ce68040>]



[7]:
```
page_rank = np.absolute(v)
page_rank /= np.sum(page_rank)

print("Highest page rank: ", np.argmax(np.absolute(v)))
print("Lowest page rank: ", np.argmax(-np.absolute(v)))
```

```
Highest page rank:  7
Lowest page rank:  3
```