Some Examples

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An example to illustrate the computation of MSNE

Theorem: A strategy profile $(\sigma_1^*, \dots, \sigma_n^*)$ is a MSNE iff $\forall i \in N$,

(1) $u_i(s_i, \sigma_{-i}^*)$ is the same $\forall s_i \in \delta(\sigma_i^*)$

(2) $u_i(s_i, \sigma_{-i}^*) \geq u_i(s_i', \sigma_{-i}^*)$ $\quad \forall s_i \in \delta(\sigma_i^*)$ $\forall s_i' \in S_i \setminus \delta(\sigma_i^*)$

1 \ 2	A	B
A	2, 2	1, 2
B	2, 1	1, 1

Define

$$b_i(s_{-i}) = \{ s_i \in S_i : u_i(s_i, s_{-i}) \geq u_i(s_i', s_{-i}) \ \forall s_i' \in S_i \}$$

$$b_1(A) = \{A, B\}$$
$$b_1(B) = \{A, B\}$$
$$b_2(A) = \{A, B\}$$
$$b_2(B) = \{A, B\}$$

Let us compute MSNEs by considering supports one at a time

Support $\{A\} \times \{A\}$

This leads to the MSNE $((1,0), (1,0))$

Condition (1) vacuously true since

Condition (1) vacuuosly true since there is only one pure strategy in the support of the mixed strategies here:

$$\delta(\sigma_1^*) = \delta(\sigma_2^*) = \{A\}$$

Condition (2)

Player 1 :

$u_1(A, A) = 2$ $\quad A \in \delta(\sigma_1^*)$

$u_1(B, A) = 2$ $\quad B \notin \delta(\sigma_1^*)$

	A	B
A	2, 2	1, 2
B	2, 1	1, 1

Player 2 :

$u_2(A, A) = 2$ $\quad A \in \delta(\sigma_2^*)$

$u_2(A, B) = 2$ $\quad B \notin \delta(\sigma_2^*)$

Note that equality is satisfied in both the cases

Thus condition (2) is satisfied.
Hence $(A, A) = ((1,0), (1,0))$ is a MSNE

Support $\{A\} \times \{A, B\}$

$\sigma_1^* = (1,0)$ $\quad \delta(\sigma_1^*) = \{A\}$

$\sigma_2^* := (y, 1-y)$ with $0 < y < 1$ $\quad \delta(\sigma_2^*) = \{A, B\}$

Let us see if (σ_1^*, σ_2^*) leads to a MSNE

Condition (1)
Vacuuosly true for Player 1 since $\delta(\sigma_1^*) = \{A\}$
Player 2 : $\delta(\sigma_2^*) = \{A, B\}$

$u_2(A, A) = 2$

$u_2(A, B) = 2$

Hence condition (1) is satisfied.

	A	B
A	2, 2	1, 2
B	2, 1	1, 1

Condition (2)
vacuously true for player 2 since $\delta(\sigma_2^*) = \{A, B\}$
Player 1 : $A \in \delta(\sigma_1^*)$; $B \notin \delta(\sigma_1^*)$

$u_1(A, (y, 1-y)) = 2y + (1-y) = y+1$
$u_1(B, (y, 1-y)) = 2y + (1-y) = y+1$

Thus condition (2) is true for $\forall y : 0 < y < 1$

Hence $(A, (y, 1-y))$ is a MSNE $\forall\, 0 < y < 1$

Support $\{A, B\} \times \{A, B\}$

Let $\sigma_1^* = (x, 1-x) \qquad 0 < x < 1$
$\sigma_2^* = (y, 1-y) \qquad 0 < y < 1$

We explore if (σ_1^*, σ_2^*) leads to any MSNE

Condition (1)
Player 1 : $\delta(\sigma_1^*) = \{A, B\}$

$u_1(A, (y, 1-y)) = 2y + 1 - y = y+1$
$u_1(B, (y, 1-y)) = 2y + 1 - y = y+1$
$\therefore$ Satisfied

	A	B
A	2, 2	1, 2
B	2, 1	1, 1

Player 2 : $\delta(\sigma_2^*) = \{A, B\}$
$u_2((x, 1-x), A) = 2x + 1 - x = x+1$
$u_2((x, 1-x), B) = 2x + 1 - x = x+1$
$\therefore$ Satisfied

Condition (2) is vacuously true for both the players.

$\therefore ((x, 1-x), (y, 1-y))$ is a

for both the players.

Thus $((x, 1-x), (y, 1-y))$ is a MSNE $\forall\, 0<x<1 \;\; \forall\, 0<y<1$

In fact, all supports here lead to MSNE.

Prisoner's Dilemma Problem

	NC	C
NC	−2, −2	−10, −1
C	−1, −10	−5, −5

We know that (C, C) is a SDSE.
Let us see if there is a MSNE:

$$((x, 1-x), (y, 1-y)) \qquad \begin{array}{l} 0<x<1 \\ 0<y<1 \end{array}$$

Condition (2) vacuously satisfied

Condition (1)

Player 1:

$$u_1(NC, (y, 1-y)) = -2y - 10(1-y) = 8y - 10$$

$$u_1(C, (y, 1-y)) = -y - 5(1-y) = 4y - 5$$

If these have to be equal,

$$8y - 10 = 4y - 5 \Rightarrow 4y = 5 \Rightarrow y = \frac{5}{4}$$

$$8y - 10 = 4y - 5 \Rightarrow 4y = 5 \Rightarrow y = \frac{5}{4}$$

Impossible !

So, condition (1) can never be satisfied for any of the players.

Consider the support $\{C\} \times \{C\}$

Condition (1) is vacuously satisfied.

Condition (2)

Player 1:

$u_1(C, C) = -5$

$u_1(NC, C) = -10$

	NC	C
NC	-2,-2	-10,-1
C	-1,-10	-5,-5

Player 2:

$u_2(C, C) = -5$

$u_2(C, NC) = -10$

Strict inequality holds in both the cases