

DS290 Modelling and Simulation. Aug-Dec 2022 Semester.

Full Marks: 25.

Expected Time Required to Work on the Paper: 90 Minutes.

Important:

Release on Assignments on Microsoft Teams: 1530 hours, 19 Oct 2022.

Submission Due through Microsoft Teams at or before: 1700 hours, 19 Oct 2022.

Please use the Turn-in feature of the Assignments on Microsoft Teams.

Please answer all questions. All questions carry equal points.

Please indicate approximately what percentage of lectures you could attend from the beginning of the course until 19 Oct 2022. This information is needed for filling in the attendance data on the SAP SLCM at the end of the semester.

Please make sure to write your name and SR no. at the beginning of your answer script.

The tables for t-distribution and the standard normal distribution are attached at the end of the question paper.

Q1. Let $W_{n+1} = \max(W_n + S_n - T_n, 0)$ be the recursive state update of an M/G/1 queue with infinite calling population and infinite system capacity, such that W_n is the waiting time at the n th time tick (that is, n th arrival), S_n , the service time and T_n , the inter-arrival time at the n th time tick. Let the service times and inter-arrival times at each time tick (arrival) be identically and independently distributed. Given the correlation between any W_n and W_{n+1} is ϕ , and W , the waiting time, has a long run average of w . Show that the above recursion generates an AR(1) time series with suitable assumptions (state clearly) consistent with M/G/1 queue. Derive the convergence of the AR(1) process to the long run average and the long run variance of the waiting times W .

Q2. An electronic component with mean to failure of x years can be purchased for $2x$ thousand Rupees (thus, more reliable the component, the more expensive it is). The value of x is restricted to be between 1 to 10 years and the actual time to failure is modeled as exponentially distributed. The mission for which the component is used lasts for 1 year; if the component fails in less than one year, then there is a cost of 20,000 Rupees for early failure. What value of x should be chosen to minimize the expected total cost (purchase plus early failure)?

Q3.

Consider the estimation of $z = \pi = 3.1415 \dots$ via the estimator $Z := 4 \times \mathbf{1}\{U_1^2 + U_2^2 < 1\}$, where U_1, U_2 are independent uniformly distributed random variables over the interval $(0, 1)$ and $\mathbf{1}$ is the indicator function. Here $\mathbf{1}\{U_1^2 + U_2^2 < 1\}$ is a Bernoulli random variable with mean $\pi/4$, so that the variance of Z is $4^2(\pi/4)(1 - \pi/4) \approx 2.70$. To get the leading significant digit 3 in π correct with probability 95% how many simulations would we need using a normally distributed ("z-statistic") confidence interval? How many simulations shall we need for obtaining the digit 1 after the decimal point correct with probability 95% assuming the same statistic?

Q4. The following table shows the analysis of the *average cost* output from 10 replications of 5 models that are being considered for the simulation of a system. The objective is to select the model that outputs the *least average cost*. Calculate whether the following table has enough number of replications for selecting the model that outputs the least average cost with a 96% confidence level. Also, compute if the number of replications is adequate for the above selection if the practically significant difference is 3.

Replication	Average Cost 1	Average Cost 2	Average Cost 3	Average Cost 4	Average Cost 5
1	7.6373	5.2587	5.8389	8.2064	9.7678
2	6.5244	4.2275	4.8392	7.2047	8.8036
3	15.3749	13.1426	13.7859	16.1918	17.8318
4	12.9925	10.6375	11.2746	13.7282	15.3572
5	6.0970	3.9108	4.5280	6.8655	8.5000
6	6.2691	3.8416	4.3754	6.6956	8.1976
7	4.6737	2.0640	2.5447	4.9386	6.3736
8	6.8304	4.1287	4.7100	7.2374	8.7528
9	16.7394	14.2877	14.9347	17.4566	19.0692
10	6.3254	3.9487	4.5558	6.9718	8.5595
average	8.9464	6.5448	7.1387	9.5497	11.1213

Q5. Let Z_i be the actual average number of jobs in a shop for the i th period. Let Y_i be the average number of jobs *predicted* by a simulation model for the same i th period. For 4 periods, the data are as follows:

i	1	2	3	4
Z_i	21.7	19.2	22.8	19.4
Y_i	24.6	21.1	19.7	24.9

- Conduct a statistical test to check the consistency of the actual system output and the simulation output. Use a 5% level of significance.
- If the difference of 2 jobs is regarded as important to detect, what sample size (that is, how many periods of observation) is required to guarantee a probability of at least 0.8 for detecting the difference if it indeed exists. The level of significance may be taken as 5%.

cum. prob one-tail two-tails	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$
	0.50	0.25	0.20	0.15	0.10	0.05	0.025	0.01	0.005	0.001	0.0005
	1.00	0.50	0.40	0.30	0.20	0.10	0.05	0.02	0.01	0.002	0.001
df											
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.92
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.000	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.000	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.000	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.000	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.000	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.000	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.000	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.000	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.000	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.000	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.000	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.000	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.000	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.000	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.000	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.000	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	0.000	0.681									

Standard Normal Cumulative Probability Table



Cumulative probabilities for NEGATIVE z-values are shown in the following table:

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641