DS 211 Numerical Optimization
Final Exam 2022

The exam is open book/internet. All questions are compulsory. Total available time is 90 mins. Record your answers in this booklet itself. Answers written elsewhere will not be evaluated. Total is 30 points.

Name (As on IISc Records in the same order as on rolls):

IISc E-mail:

Score (For use by the instructor):

Q1 (14)
Q2 (6)
Q3 (5)
Q4 (5)

Question 1 (2*7=14 points). Write True or False against each statement with reasons. Points will be awarded only if the reason is correct.

1. A linear program with more constraints than the state space's dimension cannot be solved.

   False. Constraints can be eliminated and system brought to std form.

2. Dynamic Programming algorithm works worse than Brute Force search.

   False. DP is guaranteed to do better than or equal to brute force search.

3. Genetic Algorithms have a strong mathematical foundation for deciding the update at every iteration.

   False. Absence of a strong math foundation is the disadvantage of GA.

4. First order optimality conditions are not satisfied by the solution found using the dynamic programming algorithm

   False. KKT cond are satisfied

5. Simplex method is an active set method.

   True. Simplex can be considered as an active set method.

6. The KKT matrix for a quadratic programming problem will have exactly no 0 eigen value

   True. KKT matrix has this property.

7. The KKT condition is satisfied at all the vertices of the feasible polytope.

   False. Only at the optimal point.

Question 2 (1+1+1+3 = 6 points)

Consider the following LP:

$$\max 6x_1 + 5x_2 + 2x_3$$
$$s.t.\ 2x_1 + x_2 + x_3 \leq 16$$
$$x_1 + 2x_2 - x_3 \leq 12$$
$$2x_1 - 2x_2 + x_3 \leq 20$$
$$x_1, x_2 \geq 0$$

(i) Write the LP in the standard form

$x_3 = x_3^+ - x_3^-$

$$\min\ -6x_1 - 5x_2 - 2x_3^+ + 2x_3^-$$
$$s.t.\quad 2x_1 + x_2 + x_3^+ - x_3^- + x_4 = 16$$
$$x_1 + 2x_2 - x_3^+ + x_3^- + x_5 = 12$$
$$2x_1 - 2x_2 + x_3^+ - x_3^- + x_6 = 20$$
$$x_1, x_2, x_3^+, x_3^- \geq 0$$
$$x_4, x_5, x_6$$

(ii) Write the dual of the LP

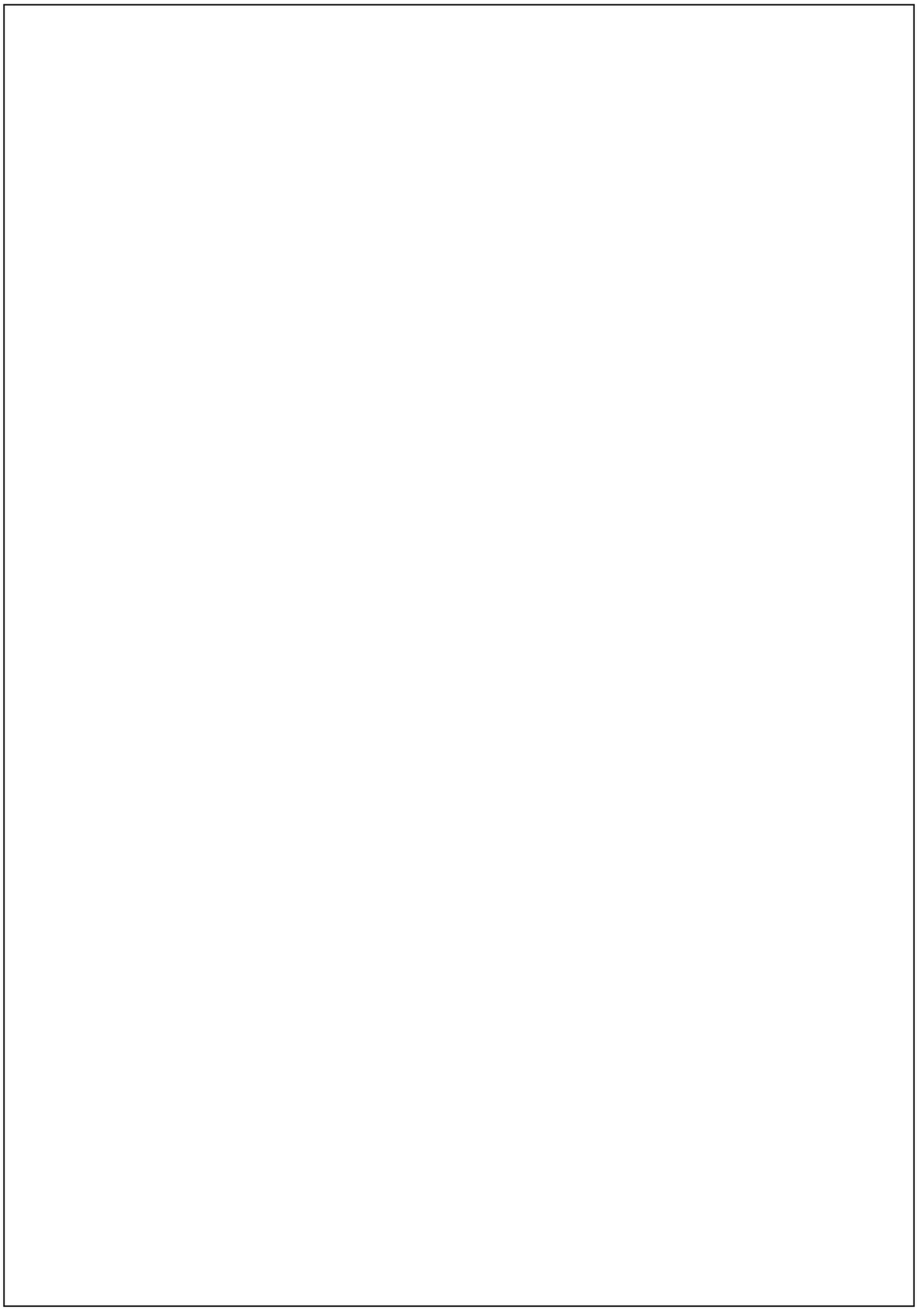
$$\max\ 16\lambda_1 + 12\lambda_2 + 20\lambda_3$$
$$s.t\ \ 2\lambda_1 + \lambda_2 + 2\lambda_3 + s_1 = -6$$
$$\lambda_1 + 2\lambda_2 - 2\lambda_3 + s_2 = -5$$
$$\lambda_1 - \lambda_2 + \lambda_3 + s_3 = -2$$
$$-\lambda_1 + \lambda_2 - \lambda_3 + s_4 = 2$$
$$s_1, s_2, s_3, s_4 \geq 0$$

(iii) Write a basic fesible solution

Set 4 out of 7 to 0, then you will get a basic feasible soln.

(iv) Do one iteration of the revised simplex method starting from the above basic feasible solution (only the method done in class using the terminology used in class will be awarded points, i.e., if you open a revised simplex youtube video or other resource and use that method, we will not evaluate your answer)

1. Pricing - 1 pt
2. Identify entering & leaving variable - 1 pt
3. Calculate x_B, x_a, x^+ - 1 pt



Question 3 (5 points)

Consider Covishield and Covaxin distribution to a town in Palakkad district of Kerala (K). Covishield is manufactured in Pune (P) and Covaxin in Hyderabad (H). Government has to make a decision of which vaccine to distribute to K.

Bangalore (B) and Chennai (C) are distribution hubs with connections from both P and H and connection to COK and CJB, the two airports near K. Between all cities with airports, the government can use air, or land transportation. Since the route of land transportation from B and C to COK crosses through CJB and K, the option of land transportation to COK need not be considered. The cost of tranporting one vial of vaccine per km of air transportation is Rs 5 and the cost per km of road transportation is Rs 2. The speed of road transportation is 60 kmph and that of the air transportation is 600 kmph. When transporting by air, an additional time of 6 hours is required for unloading and reloading to another plane or truck, which also costs Rs 50 per vial for unload+reload operations.

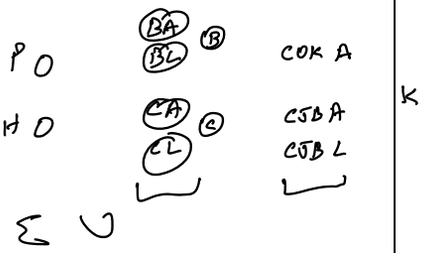
Now, let us define a cost function as 0.5 * time in hours + 0.5 * cost in Rs. Find the minimum cost path of both vaccines to K using Dynamic Programming and suggest which one to distribute at K.

Table : Land Distance Between the Different Cities in km

	Pune (P)	Hyderabad (H)	COK	CJB
Bangalore (B)	839	575		364
Chennai (C)	1192	626		506
Palakkad (K)			110	66

Table : Air Distance Between the Different Cities in km

	Pune (P)	Hyderabad (H)	COK	CJB
Bangalore (B)	723	456	368	252
Chennai (C)	913	508	518	404



① Draw the DP diagram with correct weights - 1 pt

② Set up the state, stage, action, reward - 2 pt

③ Solve - 2 pts

Question 4 (5 points)

IRCTC wants to revamp their dynamic pricing strategy for a premium train between Delhi and Bengaluru. Booking opens 90 days before the departure date from Delhi. There are N stations (including Delhi and Bengaluru) along the route and the total number of seats (tickets) are divided according to a fraction x_1, where $i = 1, \cdots, n$. For simplicity, assume that a seat can be booked only once (i.e., two bookings 1-4 and 5-N are not allowed on the same seat). Under the dynamic pricing, the price increases by 10% of base fare (bij , where $i = 1, \cdots, n; j = i, \cdots, n$) 10 times during the 90 day period. The expected demand for tickets between each station pair is available as $dijkp$, where $i = 1, \cdots, n; j = i, \cdots, n; k = 1, \cdots, 90; p = 1, \cdots, 10$. k is the index for the number of days before departure and p for the number of increments from base price. Formulate an optimization problem to find the times $tijp$ (in days before departure) for imposing increments that maximize revenue. What is the shape you expect for d on a fixed i, j, k and varying p.

1. Identify objective 1.5

2. Identify constraints 1.5

3. Set up the optimization problem 1

4. Sketch the graph of $d_{i,j,k,p}$ vs p 1