Q3 Assignment 3

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November 2022

Normal Equation

$$
\begin{aligned}
\hat{\mathbf{Y}} &= \mathbf{X}\hat{\mathbf{w}} \\
\text{least squares: } &\min_{\hat{\mathbf{w}}} \frac{1}{2}(\mathbf{Y} - \hat{\mathbf{Y}})^T(\mathbf{Y} - \hat{\mathbf{Y}}) \\
\implies &\min_{\hat{\mathbf{w}}} \frac{1}{2}(\mathbf{Y} - \mathbf{X}\hat{\mathbf{w}})^T(\mathbf{Y} - \mathbf{X}\hat{\mathbf{w}}) \\
\implies &\frac{d}{d\hat{\mathbf{w}}}\frac{1}{2}(\mathbf{Y} - \mathbf{X}\hat{\mathbf{w}})^T(\mathbf{Y} - \mathbf{X}\hat{\mathbf{w}}) = 0 \\
\implies &\frac{d}{d\hat{\mathbf{w}}}(\mathbf{Y}^T\mathbf{Y} - \mathbf{Y}^T\mathbf{X}\hat{\mathbf{w}} - \hat{\mathbf{w}}^T\mathbf{X}^T\mathbf{Y} + \hat{\mathbf{w}}^T\mathbf{X}^T\mathbf{X}\hat{\mathbf{w}} = 0 \\
\implies &\frac{d}{d\hat{\mathbf{w}}}(\mathbf{X}\hat{\mathbf{w}})^T(\mathbf{X}\hat{\mathbf{w}}) - (\mathbf{X}\hat{\mathbf{w}})^T\mathbf{Y} = 0 \\
\implies &\mathbf{X}^T\mathbf{X}\hat{\mathbf{w}} - \mathbf{X}^T\mathbf{Y} = 0 \\
\implies &\hat{\mathbf{w}} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{Y}
\end{aligned}
$$

Linear Regression 1D

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\begin{aligned}
\hat{y} &= \hat{w}_0 + x\hat{w}_1 \\
\text{As per least squares method, } \hat{\mathbf{w}} &= (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{Y} \\
\implies &(\mathbf{X}^T\mathbf{X})\hat{\mathbf{w}} = \mathbf{X}^T\mathbf{Y} \\
\implies &\begin{pmatrix} N & \sum x^i \\ \sum x^i & \sum (x^i)^2 \end{pmatrix}\begin{pmatrix} \hat{w}_0 \\ \hat{w}_1 \end{pmatrix} = \begin{pmatrix} \sum y^i \\ \sum x^i y^i \end{pmatrix} \\
\implies &N\hat{w}_0 + \hat{w}_1 \sum x^i = \sum y^i, \hat{w}_0 \sum x^i + \hat{w}_1 \sum (x^i)^2 = \sum x^i y^i \\
&\text{Solving the above 2 equations, we get} \\
&\hat{w}_0 = \frac{\sum y^i \sum (x^i)^2 - \sum x^i \sum x^i y^i}{N \sum (x^i)^2 - (\sum x^i)^2} \\
&\hat{w}_1 = \frac{N \sum x^i y^i - \sum x^i \sum y^i}{N \sum (x^i)^2 - (\sum x^i)^2}
\end{aligned}
$$