

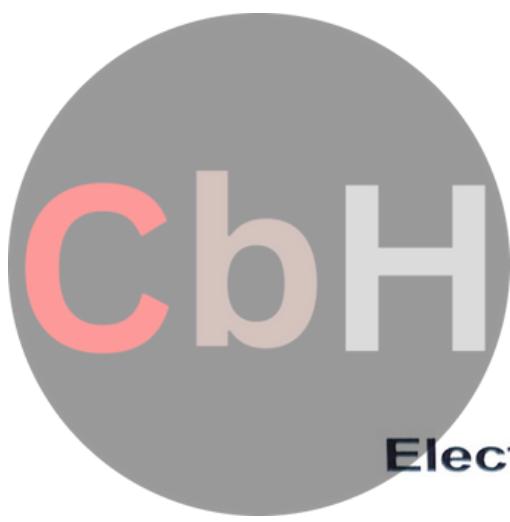
## Unit-I

Electric charge, conductors and insulators, Coulomb's Law, quantization and conservation of electric charge, the electric field, electric lines of force and Gauss' Law of electrostatics, electric potential energy, electric potential, energy and electric power.

Capacitors, capacitance, capacitors in series and parallel, capacitors with dielectric. Electric current resistance, resistivity and conductivity, Ohm's law, electromotive force, series and parallel combination of resistances, current in a single loop, Kirchoff's current law, Kirchoff's Voltage law.

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Chapter

1

## Electrostatics

### 1 Electric Charge

When two insulating objects are rubbed together, electrical charges appear on them and this phenomenon is called frictional electricity. When a glass rod is rubbed with silk cloth, it attracts small paper pieces, leaves, cork etc. The rod is then said to be **charged**. Similarly when ebonite rod is rubbed with cat-skin or fur, it also becomes charged. As is well known uncharged objects have equal number of positively charged and negatively charged electrons. Each electron has the same amount of charge as the proton but of opposite nature. Protons are much heavier than the electrons. Each atom has a central core called the nucleus. Positively charged protons and equally massive neutral particles called neutrons form the nucleus. The negatively charged electrons move around the nucleus. The object from which some electrons are lost due to charging by friction becomes positively charged while the object which gains electrons becomes negatively charged. Thus the glass rod, when rubbed with silk, becomes positively charged and the silk negatively charged. Similarly the ebonite rod rubbed with fur becomes negatively charged and fur positively charged. Thus two types of charges are found in nature, (i) positive charge and (ii) negative charge.

From experiments it is found that like charges repel each other and unlike charges attract each other [Fig. 1.1].

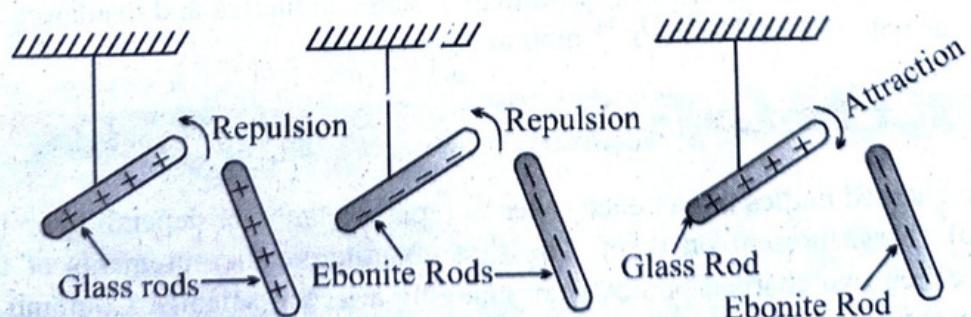


Fig. (1.1) : Electric charges

Repulsion is the best test for determining the presence of electrical uses. Objects gain or lose charge by the electrons only.

## 2. Conductors and Insulators

Different substances behave differently when charge is made to flow through them. Materials through which electric charge flows easily are called conductors. As a general rule metals are good conductors whereas nonmetals are poor conductors. The poorest of conductors in which the charge does not flow under normal conduction are called insulators or non conductors. Amongst the conductors silver is the best conductors followed by copper and aluminium. Examples of conductors and insulators are listed in Table 1.1.

Table 1.1.

Conductors	Insulators
Silver	Mica
Copper	Glass
Aluminum	Paper
Gold	Rubber
Iron	Silk
Mercury	Air

The difference between a conductor and insulator, is that in a conductor there are some free electrons, whereas in an insulator all the electrons are tightly bound to their respective atoms. Flow of electrons in a conductor causes flow of charge and the flow of charge is termed as electric current.

There are some substances which are neither good conductors of electricity nor good insulators. These substances are called semiconductors. Examples of semiconductors are germanium and silicon. In these elements the number of free electrons is very small so conductivity is very low. However by increasing the temperature more electrons can be made free from their parent atoms and so their conductivity increases with increase of temperature.

The electrical conductivities of conductors, semiconductors and insulators are of the order of  $10^8$ ,  $10^{-1}$  and  $10^{-16}$  mho/m respectively.

## 3. Coulomb's Law

Two charged bodies attract each other or repel one another depending on the nature of charge present on them. The first quantitative measurements of the force between two charged bodies were made by a French scientist Coulomb in 1780. On the basis of his experimental observations he gave a law which is called Coulomb's law.

Strictly speaking Coulomb's law describes the interaction between stationary charges in vacuum. According to this law stationary charges repel or attract

each other and the attractive or repulsive force is directly proportional to the magnitude of charges and inversely proportional to the square of the distance between them. If two charges  $q_1$  and  $q_2$  are distance  $r$  apart, then the attractive or repulsive force

$$\begin{aligned} F &\propto q_1 q_2 \\ &\propto \frac{1}{r^2} \\ F &\propto \frac{q_1 q_2}{r^2} \\ \text{or } F &= K \frac{q_1 q_2}{r^2} \end{aligned}$$

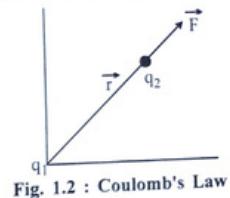


Fig. 1.2 : Coulomb's Law

where  $K$  is a constant whose value depends on the system of units.

In the CGS system (electrostatic system) of units, force is measured in dynes, distance in centimeters and the unit of charge is so chosen that  $K = 1$ . Thus in this system

$$F = \frac{q_1 q_2}{r^2}$$

If  $q_1 = q_2 = 1$  electrostatic unit of charge,  $r = 1$  cm then  $F = 1$  dyne. Thus one electrostatic unit (esu) or Stat Coulomb is that charge which when placed one centimeter away from an identical charge (1esu) repels it by a force of one dyne.

In the MKS system, which is commonly used now, force is measured in newtons, distance in metres, and the unit of charge is called coulomb (C). In this system the constant  $K = 9 \times 10^9$  and thus,

$$F = 9 \times 10^9 \frac{q_1 q_2}{r^2}$$

when  $q_1 = q_2 = 1$  coulomb,  $r = 1$  metre  
then  $F = K = 9 \times 10^9$  newtons.

$1$  coulomb =  $3 \times 10^9$  esu of charge.

To simplify some of the relations derived from Coulomb's law it is convenient to express the constant  $K$  as

$$K = \frac{1}{4\pi\epsilon_0}$$

where  $\epsilon_0$  is a constant called permittivity of free space.

$$\epsilon_0 = \frac{1}{4\pi K} = \frac{1}{4\pi \times 9 \times 10^9}$$

$$= 8.85 \times 10^{-12} \frac{\text{coulomb}^2}{\text{newton} - \text{metre}^2}$$

Using this value of constant K, Coulomb's law becomes

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$

and the system of units is termed as rationalised MKS system.

Coulomb's law is truly valid for stationary point charges in vacuum or free space. If we place a dielectric medium between the charges, then positive and negative charges of the medium get displaced due to the electric field. This event is called **electric polarization**. The electric field produced by the displaced charges change the electric field between the given charges. Thus for describing the interaction between the charges, it becomes necessary to include the electric field due to charges of the medium also. In homogenous medium such as gases or liquids the interaction between the charges can be described by the following law identical to Coulomb's law.

$$F_m = \frac{q_1 q_2}{4\pi\epsilon_0\epsilon_r r^2} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}, \text{ or } F_m = \frac{F_0}{\epsilon_r}$$

Thus in the presence of medium the resultant force  $F_m$  between the charges becomes  $\epsilon_r$  times less relative to the force  $F_0$  in vacuum.  $\epsilon_r$  is called relative permittivity or **dielectric constant** of the medium and  $\epsilon = \epsilon_0\epsilon_r$  is called **absolute permittivity** of the medium.

Coulomb's law alone is not sufficient for the study of interaction between the two charges in presence of other charges. In such a case another important law of nature is needed along with Coulomb's law. This law is called **principle of superposition**. According to this law the **resultant force acting on a particular charge is equal to the vector sum of coulomb forces acting independently on that charge due to other charges**. In the calculation of the force acting on the test charge due to another charge, only the concerned charges are to be taken into consideration.

#### 4. Quantization of Charge

**When exchange of any physical quantity takes place in a discrete manner and not in a continuous manner, the quantity is said to have quantum nature.** For example, if we have some money in the form of paisa and want to exchange them with others, then we can exchange this money in multiples of quantised and not continuous. This exchange of money will be discrete, that is, quantized form, is called quantum of that physical quantity. In the above example one paisa is a quantum of money.

In 1914 Millikan studied the motion of charge oil-drops in gravitational and electric fields and determined the quantity of charge on the oil-drops. Millikan experimented on a very large number of oil-drops and found that the charge on the oil-drops exists in integral multiples of a definite quantity. Millikan called this definite quantity (minimum value of the charge) an atom of electricity. Experimentally its value was found to be  $4.80 \times 10^{-19}$  statcoulomb (esu) or  $1.6 \times 10^{-19}$  coulomb. This value of charges is represented by e and it is now called electronic charge. From Millikan's experiment and many other experiments it has been proved that electric charge exists in nature in integral multiples of  $1.6 \times 10^{-19}$  C charge. Thus **charge is a quantized quantity and the quantum of charge is equal to the electronic charge in magnitude**.

The quantum of charge is so small that the quantization of charge is not experienced ordinarily. Total charge on the charged bodies is very large relative to the quantum of charge and the measurement of variation of charge of the order of electronic charge is beyond the limit of accurate measurement so usually the quantized nature of charge loses significance. However the quantum nature of charge is important in all interactions at atomic level.

#### 5. Conservation of charge

The law of conservation of charge is an empirical law. This law is tested experimentally and no violation of this law has ever been observed in any event. This law is one of the fundamental laws of nature.

**In every isolated system the total charge i.e., the algebraic sum of positive and negative charges does not change in any process taking place in the system, that is, it remains constant.** This law is called the law of conservation of charge.

Following examples prove the validity of this law :

When a glass rod is rubbed with silk, rod and silk jointly form an isolated system. Now if the magnitude and nature of charge produced on the rod and silk are determined, then the positive charge produced on the rod will be equal to the negative charge produced on the silk. Thus the total charge on the system i.e. the rod and the silk before and after rubbing remains zero, or total charge remains constant.

If we take a gas in a tube and ionize it by X-rays, the total charge of the gas and the X-ray photons before ionization is zero. Positive and negative ions are produced in the gas by ionization. Total charge i.e., algebraic sum of charges on positive and negative ions remains zero even after ionization.

In the above mentioned examples total charge is constant (zero). Law of conservation of charge is valid for all values of total charge. Nature has balanced

the negative and the positive charges in the universe in such a way that total charge is zero, i.e., ordinarily all substances are neutral. If it were not so, the forces between the charged objects would have been so strong that their stability would not have been possible.

## 6. Electric Field and Lines of Force

The region in which a stationary charged particle experiences a force (other than the gravitational force) is called electric field. For example, around a charge particle or charged body there is an infinite region in which if a test charge is brought, it experiences a force. The electric field can be produced by one or more static or moving charges or a variable magnetic field. The force acting on a charge or charged particle situated in an electric field is called electrostatic force. This force is a function of the position of the particle. The electric field is a vector field.

The electric field can be depicted by lines of force. An electric line of force is that imaginary smooth curve drawn in an electric field along which a free and isolated unit positive charge will move. A tangent drawn at any point on the line of force shows the direction of electric field intensity at that point.

In Fig. 1.3 (a) the lines of force representing the electric field produced by a positively charged particle are shown. If a positive charge is left free near this charge, then due to repulsion it will move away upto infinity along a straight line path. Hence the lines of force are straight lines from the particle up to infinity. Conversely, in the electric field of an isolated negatively charged particle, a free positive charge will move from infinity to the negative charge due to attraction. Hence lines of force are straight lines from infinity up to the negatively charged particle [Fig. 1.3(b)]

The lines of force of a charged sphere are straight and radial and appear either emerging from the centre or converging at the centre of the sphere.

In Fig. 1.3(c) lines of force representing the electric field of a system of two equal and opposite charges are shown. These lines of force start from the positive charge and end at the negative charge. In Fig. 1.3(d) lines of force representing the electric field of two equal and like charges are shown. In this case at the mid point N of the line joining the charges, the field produced by one charge is equal and opposite to the field produced by the other charge. Therefore the resultant field at this point is zero. This point is called neutral point. No line of force passes through this point.

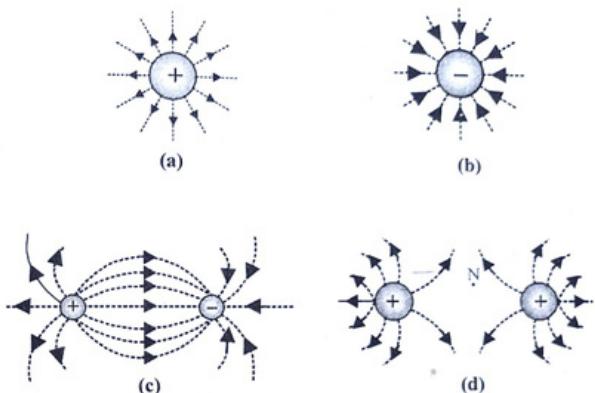


Fig. (1.3) : Electric lines of Force

### Properties of the lines of force :

- Electric lines of force are imaginary and start from a positive charge and end on a negative charge.
- The tangent drawn at any point on the line of force gives the direction of resultant electric field at that point.
- No two lines of force intersect each other, because two tangents can be drawn at the point of intersection which would mean two directions of the field at that point which is not possible.
- These lines have a tendency to contract along the length like a stretched elastic string. This explains the attraction between opposite charges.
- These lines have a tendency to move apart from each other in the direction normal to the their length. This explains the repulsion between like charges.

## 7. Intensity of Electric Field

If a small positive charge  $q_0$  (test charge) is placed at a point in an electric field, then the ratio of the force  $F$  acting on this charge and the amount of charge  $q_0$  is called the intensity of electric field at that point. The test charge should be so small that it does not effect the intensity of electric field at the given point i.e.,  $q_0 \rightarrow 0$ . The intensity of electric field is represented by  $E$ .

By definition of the intensity of electric field.

$$E = \lim_{q_0 \rightarrow 0} \frac{F}{q_0}$$

If  $q_0 = +1$ , then  $E = F$   
i.e., the intensity of electric field at a point is equal to the force on a unit positive charge placed at that point. It is a vector quantity.

Unit and dimensions of electric field :

$$E = \frac{F}{q_0} = \frac{\text{newton}}{\text{coulomb}} \text{ or } \frac{\text{volt}}{\text{metre}}$$

Thus the unit of intensity of electric field is newton/coulomb (N/C) or volt/metre (V/m) and its dimensions are

$$E = \frac{MLT^{-2}}{AT} = MLT^{-3}A^{-1}$$

If a particle of charge  $q$  is placed in an electric field of intensity  $E$ , the force acting on the particle will be

$$F = qE$$

If the electric field intensity in a certain region is same at all points in magnitude and direction, the field is called uniform.

Often the electric field is represented by lines of force. These lines are imaginary smooth curves and the tangent at a point on the line gives the direction of electric field at that point. The number of lines is taken in such a way that their number density i.e. number of lines crossing a unit area perpendicular to the lines is proportional to the magnitude of the intensity of electric field. Thus in a uniform field the lines of force are parallel to each other and equally spaced. If the field is non-uniform, the lines of force will have different directions and their spacing will also be unequal at different points.

## B. Electric Field at a Point due to a Point-Charge

The electric field produced by a point charge  $q$  is non-uniform because the electric field intensity at a point depends upon the distance  $r$  of the point from the source charge. Applying Coulomb's law, in free space :

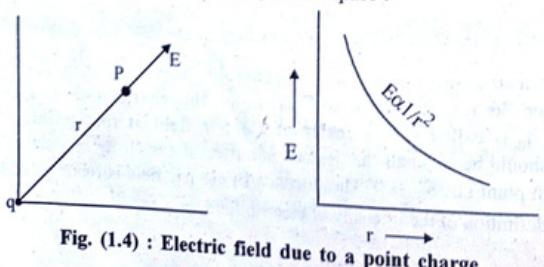


Fig. (1.4) : Electric field due to a point charge

$$\begin{aligned} E &= \frac{F}{q_0} = \frac{1}{q_0} \left[ \frac{1}{4\pi\epsilon_0} \frac{qq_0}{r^2} \right] \\ &= \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} = \frac{Kq}{r^2} \end{aligned}$$

Thus for a point charge  $E \propto \frac{1}{r^2}$  and this dependence is shown in Fig. (1.4).

## 9. Electric Flux

To understand the meaning of electric flux we consider the flow of water on a horizontal surface. Suppose at a particular place the velocity of water is  $\vec{v}$ . If we now hang a small rectangular frame of area 'a' in this flowing water then the amount of water flowing through this frame per second will be called the flux of water. The rate of flow of water through the frame depends on the area 'a' of the frame, velocity of water  $\vec{v}$  and the orientation or inclination of the frame with respect to the direction of flow of water Fig. (1.5).

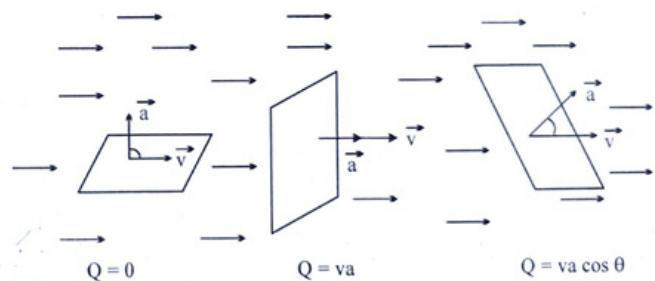


Fig. (1.5) : Inclination of the frame to the direction of flow of water

- (a) If the frame is parallel to the direction of flow, the amount of water flowing through the frame will be zero.
- (b) If the frame is perpendicular to the flow of water then the rate of flow of water through the frame is  $(va)$  which is the maximum value.
- (c) For any other orientation the rate of flow depends on the angle  $\theta$  which the normal to the area makes with the direction of flow and in general the flux through the given frame is

$$Q = v a \cos \theta$$

Here ( $a \cos \theta$ ) is the component of area perpendicular to the direction of flow.

This definition of flux can be applied to all field which can be represented by lines of force, as electric and magnetic fields.

In an electric field  $E$  the flux emerging out of an elementary area  $da$  will be equal to  $E da \cos \theta$  where  $\theta$  is the angle between the outward normal of the area and the electric field. Thus the electric flux through the elementary area is

$$\begin{aligned} d\phi &= E da \cos \theta \\ &= E (\text{component of area perpendicular to the field}) \\ &= da (E \cos \theta) = da (\text{component of the field perpendicular to the area element}) \end{aligned}$$

- (i) If  $\theta = 0^\circ$ ,  $\cos \theta = 1$   $d\phi = Eda$  (maximum value)  
(ii) If  $\theta = 90^\circ$ ,  $\cos \theta = 0$   $d\phi = 0$  (minimum value)

The electric flux  $d\phi$  is a scalar quantity. The unit of electric flux is  $\frac{N \cdot m^2}{C}$  or V-m (volt-meter) and its dimension are  $M^1 L^3 T^{-3} A^{-1}$ .

As the electric field is represented by lines of force, the electric flux will represent the number of lines of force passing through the given area. When  $\theta = 0$  i.e. the area is perpendicular to the lines of force,  $d\phi = Eda$  or  $E = d\phi/da$ . Hence the **number of lines of force passing through an area element per unit area when the area element is normal to the lines of force, defines the intensity of electric field**.

If a surface  $S$  is situated in a vector field  $\vec{E}$  and  $\vec{E}$  is different at different positions then the surface  $S$  can be divided into small area elements and the total flux emerging out of the given area will be equal to the sum of values of flux coming out of the area elements. If the area of the elements is taken infinitesimally small ( $da \rightarrow 0$ ) then the summation can be replaced by integration.

$$\therefore \phi = \lim_{da \rightarrow 0} \sum d\phi = \lim_{da \rightarrow 0} \vec{E} \cdot \vec{da}$$

or

$$\phi = \oint_S \vec{E} \cdot \vec{da}$$

The above integral is called the surface integral of electric field vector  $\vec{E}$ .

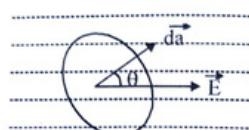


Fig. (1.6) : Electric flux

## 10. Gauss' Law of Electrostatics

Gauss' law relates the total electric flux emerging out of a closed surface situated in an electric field to the net charge enclosed by the surface. According to Gauss' law the total electric flux of an electric field through a closed surface is equal to  $4\pi K$  or  $(1/\epsilon_0)$  times the net charge enclosed by that surface. Thus by Gauss' law

$$\phi = 4\pi K \sum q = \frac{1}{\epsilon_0} \sum q, \quad \dots(1)$$

$$K = \frac{1}{4\pi \epsilon_0} = 9 \times 10^9 N \cdot m^2/C^2 \quad \dots(2)$$

$\epsilon_0$  is primitivity of free space =  $8.85 \times 10^{-12} C^2/N \cdot m^2$ .

If there are number of charges inside the surface, then the net charge is algebraic sum of all the charges

$$\begin{aligned} \Sigma q &= q_1 + q_2 + q_3 + \dots \\ \therefore \phi &= 4\pi K \Sigma q = 4\pi K (q_1 + q_2 + q_3 + \dots) \end{aligned}$$

Using the definition of electric flux

$$\phi = \oint_S \vec{E} \cdot \vec{da} = 4\pi K \Sigma q = \frac{1}{\epsilon_0} \Sigma q \quad \dots(3)$$

$\oint_S$  represents surface integral over a closed surface. The outward flux is taken as positive while the inward flux is taken as negative. From Gauss' law it is clear that (i). The flux  $\phi$  does not depend on the size, shape or area of the closed surface. It only depends on the magnitude and nature of charges inside it. (ii)  $\phi$  does not depend on the distance between the charges or the charge distribution. (iii) If any charge or charges are outside the closed surface they do not contribute to the electric flux through the surface. (iv) If the net charge inside the surface is zero the electric flux through the surface will also be zero. (v) Gauss law is applicable to field which obey inverse square law.

## 11. Electric Potential Energy, Potential Difference and Potential

When a charged particle or a charge is placed in an electric field a force acts on it in accordance with Coulomb's law. If the charge is now displaced from one position to the other work has to be done against the field or by the field. The

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energy spent in doing this work is stored as potential energy of the system, i.e., the potential energy of the charged particle.

For example if an electron is brought towards another electron at rest the potential energy of the system increases as work has to be done on the system against the repulsive force. On the other hand if an electron is brought near a proton the potential energy will be negative as work will be done by the attractive force. Negative potential energy means a bound system as work has to be done by an external agency to break the system. Thus the proton-electron system in an atom is a bound system.

If we take a charge  $q_0$  which may be called test charge and displace it then the ratio of the work done against the field in moving this test charge from one point to another and the magnitude of the test charge defines the potential difference between these points. If the work done in moving a test charge  $q_0$  from a point A to a point B against the field is  $W_{AB}$  then the potential difference between B and A is :

$$V_B - V_A = \frac{W_{AB}}{q_0}$$

If  $q_0 = +1$ , then  $V_B - V_A = W_{AB}$

i.e., the work done against the field in displacing a unit positive charge from one point to another is equal to the potential difference between these points.

To define the potential at a point in an electric field, a reference point is chosen in the field. The potential difference between a given point and the reference point is then called the potential at the given point. Usually a point at infinity is chosen as the reference point. Thus the potential at a point P in the field will be :

$$V = V_p - V_\infty = \frac{W}{q_0}$$

If  $q_0 = +1$ , then  $V = W$  or the potential at a point is the work done against the field in bringing a unit positive charge from infinity to the given point.

The potential at a point in an electric field represents the energy spent in placing a unit positive charge at that point. Therefore, The potential is the potential energy of a unit positive charge placed at the given point.

The rationalized M.K.S. unit of electric potential or potential difference is volt. If the work done in bringing a charge of one coulomb from infinity to the given point is one joule, then the potential at the given point will be

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**one volt.** Similarly if the work done in displacing a charge of one coulomb from one point to another is one joule, then the potential difference between these points will be one volt.

If we consider the electric field produced by a point charge q the potential at a point at a distance r from it, will be

$$V = K \frac{q}{r} = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

## 12. Electric Energy and Electrical Power

We are all familiar with the electrical appliances like electric bulbs, electric heaters, refrigerator etc, commonly used. For the operation of these devices electric energy is supplied by a generator. The amount of energy which is consumed depends on the current passed, applied voltage and the time. Suppose a potential difference, V is applied across a device and a current I flows for a time t. By definition of current

$I = \text{Rate of flow of charge}$

$$= \frac{Q}{t}$$

So charge which has passed through the device will be

$$Q = It$$

Now potential difference V is defined as the work done per unit charge in carrying the given charge from one terminal to the other, so the work done or the energy consumed will be

$$W = VQ = VI$$

If V is in volts, I in amperes and t in second then W will be in joules.

Power is defined as the rate at which energy is consumed or developed.

$$\therefore \quad \text{Power } P = \frac{W}{t} = VI$$

The MKS unit of power is joule per second or watt.

Thus the power in watts is equal to the product of potential difference in volts and current in amperes.

Commercially the consumption of electric energy is expressed in terms of a unit called kilowatt-hour (KWH). One kilowatt-hour (1 unit of electric energy) is the energy consumed in one hour i.e., 3600 s by a device whose power is one kilowatt i.e., 1000 watts.

$$\therefore \quad 1 \text{ kilowatt-hour} = 1000 \text{ watts} \times 3600 \text{ seconds}$$

$$= 1000 \frac{\text{joules}}{\text{second}} \times 3600 \text{ seconds} \\ = 3.6 \times 10^6 \text{ joules.}$$

### 13. Capacity

As a liquid is poured into a vessel the level of liquid goes on rising, in the same way as a conductor is charged its electric level i.e., electric potential rises. The increase in potential is proportional to the charge given to it. Mathematically if the increase in potential is  $V$  when a charge  $Q$  is given to a conductor, then

$$Q \propto V \text{ or } Q = CV \quad \dots(1)$$

$C$  is a constant, which depends on the geometry of the conductor, medium around it and the nearness of other conductors. This physical quantity is called the capacity of the conductor.

From relation (1)

$$C = \frac{Q}{V} \quad \dots(2)$$

i.e., the capacity of a conductor is the ratio of charge given to the conductor and the increase in its potential.

$$\text{If } V = 1, \quad C = Q$$

therefore, numerically the capacity of a conductor is equal to the amount of charge with increases its potential by unity.

**Units of capacity :** In practical or M.K.S. system, the unit of charge is coulomb (C) and unit of potential is volt (V). Hence unit of capacity will be coulomb per volt (C/V). It is called Farad (F).

$$1 \text{ Farad} = \frac{1 \text{ Coulomb}}{1 \text{ Volt}}$$

i.e. 1F is the capacity of a conductor whose potential increases by 1V when a charge of 1C is given to it.

Farad is a very big unit so in practice its sub-multiples are used.

$$1 \mu\text{F} = 10^{-6} \text{ F}$$

and

$$1 \text{ pF} = 10^{-12} \text{ F}$$

### 14. Capacitor or Condenser

The capacity of a conductor is a measure of its ability to hold charge. In the case of a conductor the capacity of the conductor depends on its geometrical size. Further from the relation  $C = Q/V$ , it is clear that the capacity to hold charge can

be increased by increasing the size of the conductor which is not very practical or by decreasing the potential for a given amount of charge  $Q$ .

The potential of a conductor can be reduced by placing another conductor near it. Consider a positively charged plate A. If another plate B is placed near it then by induction an opposite (negative) charge is induced on the inner surface of B and an equal similar (positive) charge is induced on the outer surface, Fig. 1.7. The opposite negative charge on B decreases the potential of A but the similar positive charge on outer surface

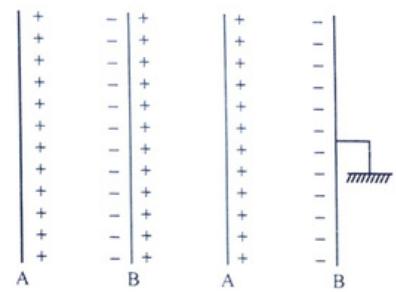


Fig. (1.7) : Capacitor

of B increases the potential of A. The oppositely charged face of B being bound to A and nearer is more effective i.e. the net effect is to decrease the potential of A. This effect can be enhanced if the similar charge induced on B can be removed. This can be done by connecting the outer surface of B to the ground. The charge on the outer surface of B is free while the charge on inner surface is bound to the charge on A the free charge flows to the ground and the bound charge continues to remain on the inner surface of B. Thus by placing another earthed conductor near a given charged conductor the potential can be reduced substantially or the capacity can be increased. This is the principle of a condenser or capacitor.

Thus, the condenser is a arrangement in which the capacity of a conductor is increased by placing another earthed conductor close to it.

The two oppositely charged conductors are called the plates of condenser. An isolated conductor also is a condenser in which the other plate is earth.

The space between the two plates of a condenser is filled with air or any other non-conducting dielectric medium. By using a dielectric of dielectric constant K the capacity of the condenser is increased K times.

### 15. Capacity of a Condenser

If the condenser is given a charge  $+Q$ , one plate acquires the charge  $+Q$  while the other automatically gets an induced charge  $-Q$ . If the potential difference between the plates of the condenser is  $V$  then the capacity of a condenser is defined as the ratio of charge given to the condenser and the potential difference between its plates.

$$\text{i.e., } C = \frac{Q}{V} \text{ farad (F)}$$

Usually the condensers are classified according to their shape e.g., parallel plate condenser, spherical condenser etc. The condensers can also be named according to the dielectric medium used between the plates e.g. paper condensers, mica condensers, etc.

### 16. Parallel Plate Condenser or Capacitor

It consists of two conducting plates  $P_1$  and  $P_2$  of same shape (circular or rectangular) and same area. These are fixed on insulating stands, parallel to each other and at a very small distance apart. In between the plates air or any other dielectric medium is kept. The two plates are connected to two terminals. One plate is connected to earth. A parallel plate condenser is shown in Fig. 1.8 where the plate  $P_1$  is given a charge while the plate  $P_2$  is earthed.

Suppose each plate has an area  $A$  and the distance between the plates is  $d$ . When the plate  $P_1$  is given a charge  $+Q$ , on the inner surface of  $P_2$  (surface towards  $P_1$ ) a charge  $-Q$  is developed by induction and on the outer surface of  $P_2$  a charge  $+Q$  is developed which flows to earth. The charge  $-Q$  on the inner surface remains bound to the charge  $+Q$  in the plate  $P_1$ . The equal and opposite charges on plates  $P_1$  and  $P_2$  produce a uniform electric field in between the plates. At the edges slight non-uniformity of field is present but in the rest of the region the lines of force are parallel to each other and perpendicular to the plates. Outside the plates the electric field is zero. Thus the condenser stores electrostatic energy in the form of electric field in the region bound by the plates.

If the surface charge density on each plate is  $\sigma \text{ C/m}^2$  then for air or vacuum between the plates the electric field intensity in the region will be

$$E = \frac{\sigma}{\epsilon_0} \text{ N/C or V/M} \quad \dots(1)$$

where  $\epsilon_0$  is permittivity of air or vacuum. For the validity of the above relation

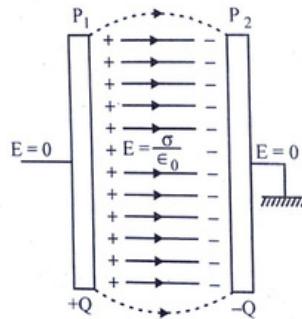


Fig. (1.8) : Parallel plate capacitor

the size of plates compared to the distance between them is assumed very large (theoretically infinite).

As area of each plate is  $A$  and the magnitude of charge on it is  $q$ , therefore,

$$\sigma = \frac{Q}{A} \quad \dots(2)$$

$$\therefore E = \frac{Q}{\epsilon_0 A} \quad \dots(3)$$

If due to this uniform field the potential difference between the plates is  $V$ , then

- $V$  = Work done in moving a unit positive charge from one plate to the other
- = Force on a unit positive charge  $\times$  distance between the plates.
- = Electric field intensity  $\times$  distance between the plates

$$= Ed = \frac{Qd}{\epsilon_0 A}$$

$\therefore$  Capacity of the condenser

$$C = \frac{Q}{V} = \frac{Q}{Qd/\epsilon_0 A} = \frac{\epsilon_0 A}{d} \text{ farad} \quad \dots(4)$$

so that  $C \propto A$  and  $C \propto \frac{1}{d}$ . For increasing the capacity the area of the plates must be increased and distance between them be decreased. Capacity also depends on the dielectric constant of the medium between the plates.

If a medium of dielectric constant  $K$  is placed between the plates then

$$E = \frac{Q}{\epsilon A} = \frac{Q}{K \epsilon_0 A} \text{ and } V = \frac{Qd}{\epsilon A} = \frac{Qd}{K \epsilon_0 A}$$

$$\text{so that } C_m = \frac{Q}{V} = K \frac{\epsilon_0 A}{d} \text{ or } C_m = KC \quad \dots(5)$$

In a standard parallel plate capacitor to remove the non-uniformity of the field at the edges, the plate  $P_1$  is surrounded by a guard ring Fig. (1.9). The guard ring  $G$  and the plate  $P_1$  are connected to each other. Thus the edge effect passes on to the guard ring and the field between  $P_1$  and  $P_2$  remains uniform throughout the region.

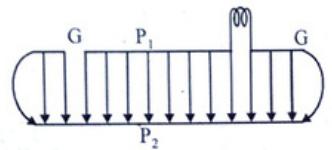


Fig. (1.9) : Standard parallel plate capacitor to remove the non uniformity by a guard ring

### 17. Spherical Capacitor

A spherical capacitor consists of two concentric hollow metal spheres A and B, which do not touch each other at any point Fig. (1.10). The space between the spheres is filled with air or any other dielectric. The outer sphere is earthed. On the inner sphere a metal rod with a knob is fixed. This rod is kept insulated from the outer sphere by a bad conductor.

Suppose the radii of the two spheres A and B are  $r_A$  and  $r_B$  respectively, where  $r_A < r_B$ . If the inner sphere is given a charge  $+Q$ , a charge  $-Q$  is developed on the inner surface

of B and a charge  $+Q$  on the outer surface of B, by induction. The charge  $+Q$  on the outer surface of B being free flows to the earth while the charge  $+Q$  on A and  $-Q$  on B remain bound to each other.

Assuming air as the medium between the spheres the potential of the surface of A due to charge  $+Q$  on it is.

$$V_A = \frac{1}{4\pi\epsilon_0 r_A} (-Q) \quad \dots(1)$$

Now the potential inside a charged hollow sphere is the same as the potential of its surface. Therefore the potential on the surface of A due to the charge  $-Q$  on B is

$$V_B = \frac{1}{4\pi\epsilon_0 r_B} (-Q) \quad \dots(2)$$

$\therefore$  Resultant potential of A is

$$\begin{aligned} V &= V_A + V_B = \frac{1}{4\pi\epsilon_0 r_A} Q - \frac{1}{4\pi\epsilon_0 r_B} Q \\ &= \frac{Q}{4\pi\epsilon_0} \frac{(r_B - r_A)}{r_A r_B} \end{aligned} \quad \dots(3)$$

As the outer surface of B is earthed its potential will be zero and the potential difference between the two conducting spheres will be V.

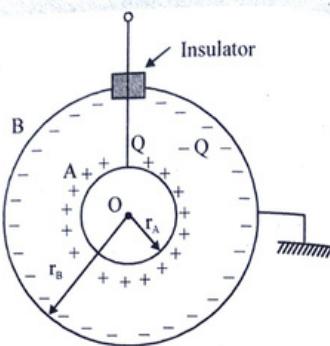


Fig. (1.10) : Spherical Capacitor

$\therefore$  Capacity of the spherical condenser

$$\begin{aligned} C &= \frac{Q}{V} = Q \times \frac{4\pi\epsilon_0 r_A r_B}{Q(r_B - r_A)} \\ &= 4\pi\epsilon_0 \frac{r_A r_B}{r_B - r_A} \end{aligned}$$

### 18. Combinations of Capacitors

In many electrical and electronic circuits combinations of two or more capacitors have to be used to obtain a desired capacitance. Primarily the capacitors can be combined in two ways :

(i) Capacitors in series : In the series combination the first plate of the first condenser is connected to the source of charge and the second plate of last condenser is earthed or connected to the second terminal of source. The second plate of first condenser is connected to first plate of second condenser, the second plate of second condenser is connected to first plate of third condenser and so on. A series combination of three condensers  $C_1$ ,  $C_2$  and  $C_3$  is shown in Fig. (1.11)

Let the charge given to the first plate of first condenser by the battery be  $+Q$ . Due to induction its second plate acquires a charge  $-Q$  and a charge  $+Q$  is set free which goes to the first plate of second condenser. Again the second plate of second condenser develops a charge  $-Q$  at its inner surface and sets free a charge  $+Q$  which goes to third condenser and so on. In this way the first plate of each condenser get a charge

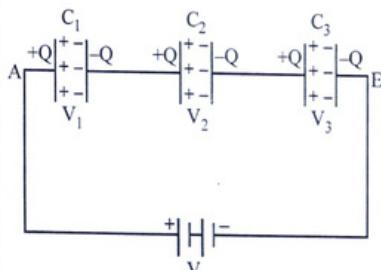


Fig. (1.11) : Series combination of capacitor

$+Q$ , the charge given by the source. The potential difference across these condensers depends on the capacities of these condensers. Suppose the values of potential difference across the condensers  $C_1$ ,  $C_2$  and  $C_3$  are  $V_1$ ,  $V_2$  and  $V_3$  respectively.

$$\therefore V_1 = \frac{Q}{C_1}, V_2 = \frac{Q}{C_2} \text{ and } V_3 = \frac{Q}{C_3} \quad \dots(1)$$

Thus the total potential difference produced by the source is distributed on different condensers. If the potential difference is V then

$$V = V_1 + V_2 + V_3$$

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$$= \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} \quad \dots(2)$$

$$\text{or } \frac{V}{Q} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

If we imagine an equivalent capacitor of capacity  $C$  between the points A and B, then by giving a charge  $Q$  to it the potential difference must be  $V$ , so that

$$C = \frac{Q}{V} \text{ or } \frac{1}{C} = \frac{V}{Q} \quad \dots(3)$$

$$\therefore \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \quad \dots(4)$$

Thus when a number of capacitors are joined in series the reciprocal of the resultant or equivalent capacity is equal to the sum of reciprocals of the capacities of the individual capacitors.

The equivalent series capacitance is always less than the smallest capacitance in the combination.

A series combination is used when a higher potential, which a single capacitor cannot bear, is to be divided on several capacitors or when the capacitance required is less than the capacitance of available capacitors.

If  $n$  capacitors are connected in series then the equivalent capacitance is given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n}$$

If  $C_1 = C_2 = \dots = C_n = C_0$ , then  $C = \frac{C_0}{n}$ .

(ii) **Capacitors in parallel** : In a parallel combination of capacitors the first plate of all capacitors are joined at one point which is connected to one terminal of the source, the second plate of all capacitors are joined at another point which is earthed or connected to the second terminal of the source. A parallel combination of three capacitors is shown in Fig. (1.12). Thus in a parallel combination the potential difference across each condenser is same and equal to the potential difference produced by the source. The charge now gets distributed.

Suppose the charge given by the source is  $+Q$ . This charge is distributed on the condensers in parallel according to their capacities as potential difference  $V$  is same for all. If the amounts of charge received by the condensers are  $Q_1, Q_2$  and  $Q_3$  then

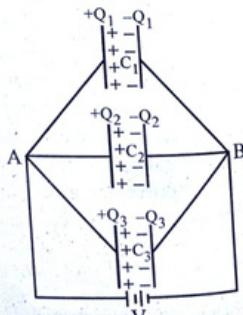


Fig. (1.12): Parallel combination of capacitors

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$$Q = Q_1 + Q_2 + Q_3 \quad \dots(5)$$

But  $Q_1 = C_1 V, Q_2 = C_2 V$  and  $Q_3 = C_3 V$

Substituting the values in (3),

$$Q = C_1 V + C_2 V + C_3 V \\ = (C_1 + C_2 + C_3) V$$

$$\text{or } \frac{Q}{V} = (C_1 + C_2 + C_3) \quad \dots(6)$$

If the equivalent capacity of the combination is  $C$  then

$$C = \frac{Q}{V}$$

so that  $C = C_1 + C_2 + C_3$

For  $n$  condensers in parallel

$$C = C_1 + C_2 + C_3 + \dots + C_n$$

Thus in parallel combination the resultant or equivalent capacitance is equal to the sum of capacitances of all the condensers used in combination.

By a parallel combination the resultant capacitance is more than the capacitances of individual capacitors.

A parallel combination is used when a large capacitance is required or a larger amount of charge at a given voltage is to be stored.

## 19. Effect of Dielectric Medium between the Plates of a Condenser on its Capacity

Some material like glass, mica, oil, ebonite etc. are insulators and are also called dielectrics. The valence electrons of the atoms of these substances are tightly bound with their nuclei. When such a material is placed between the charged plates of a capacitor the centres of the negative and positive charge distributions in the atoms or molecules no longer remain coincident but get separated. The centre of negative charge distribution gets displaced towards the positive plate and the centre of positive charge distribution towards the negative plate. This phenomenon is called **polarisation**. Due to this polarisation negative charge gets accumulated on the surface of the dielectric near the positive plates and an equal positive charge appears on the surface of dielectric near the negative plate. This accumulation of charge on the two surfaces of dielectric reduces the applied

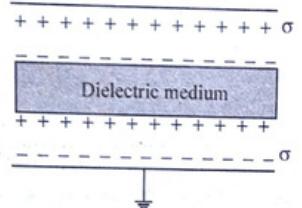


Fig. (1.13) : Effect of dielectric medium between the plates

electric field. If the dielectric constant of the medium is K then the electric field intensity between the charged plates of a parallel plate condenser becomes

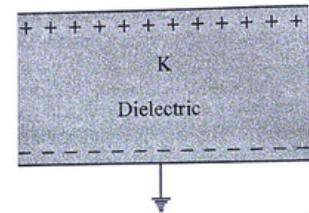
$$E_k = \frac{\sigma}{\epsilon_0} = \frac{\sigma}{\epsilon_0 K} = \frac{Q}{\epsilon_0 KA}$$

where  $\sigma = Q/A$  is the surface charge density on each plate. This intensity is  $1/K$  times the intensity in air or vacuum.

Now we determine the effect of dielectric between the plates of a condenser under different conditions :

**(i) Capacity of a parallel plate condenser when the space between its plates is completely filled by the dielectric :** Let the area of each plate be A and d be the distance between them. When the dielectric occupies all the space between the plates, its thickness will also be d. If the charge on the two plates is  $+Q$  and  $-Q$ , the electric field intensity in the dielectric will be

Fig. (1.14) : Space between its plates is completely filled by the dielectric



$$E_k = \frac{\sigma}{\epsilon_0 K} = \frac{Q}{\epsilon_0 KA} \quad \dots(1)$$

The electric field will be uniform between the plates. The potential difference between the plates will be

$$V = E_k d = \frac{Qd}{\epsilon_0 KA} \quad \dots(2)$$

∴ The capacity of the dielectric filled capacitor

$$C_m = \frac{Q}{V} = \frac{Q}{Qd/\epsilon_0 KA} = \frac{\epsilon_0 KA}{d} \text{ farad} \quad \dots(3)$$

Thus the capacity of the dielectric filled capacitor,  $C_m$

- (i) is directly proportional to the area of each plate,
- (ii) is inversely proportional to the distance between the plates and
- (iii) is directly proportional to the dielectric constant of the medium in between the plates.

Comparing the capacities of a dielectric filled capacitor and an air or vacuum capacitor, the dielectric constant of the medium

$$K = \frac{\text{Capacity of dielectric filled capacitor}}{\text{Capacity of same capacitor with vacuum or air in between the plates}} = \frac{C_m}{C}$$

**(ii) Capacity of a parallel plate capacitor when the space between the plates is partially filled by a dielectric :** Suppose each plate of the capacitor has an area A and their separation is d. A plate of dielectric of thickness t and dielectric constant K is placed parallel to the plates in between the space between them, Fig. (1.15). Thus the thickness of the layer of air between the plates will be  $(d - t)$ .

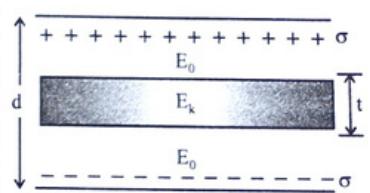


Fig. (1.15) : Space between the plates is partially filled by a dielectric

When a charge  $+Q$  is given to one plate the other plate acquires a charge  $-Q$ . The electric field intensity in air due to these charged plates will be

$$E_0 = \frac{Q}{\epsilon_0 A} \quad \dots(1)$$

while the electric field intensity in the dielectric will be

$$E_k = \frac{Q}{\epsilon_0 KA} \quad \dots(2)$$

If the potential difference between the plates is V, then

$$\begin{aligned} V &= E_0(d - t) + E_k t \\ &= \frac{Q}{\epsilon_0 A}(d - t) + \frac{Q}{\epsilon_0 KA} t \\ &= \frac{Q}{\epsilon_0 A} \left[ d - t + \frac{t}{K} \right] \end{aligned} \quad \dots(3)$$

∴ Capacity of the capacitor

$$\begin{aligned} C &= \frac{Q}{V} = \frac{Q}{\frac{Q}{\epsilon_0 A} \left( d - t + \frac{t}{K} \right)} \\ &= \frac{\epsilon_0 A}{\left( d - t + \frac{t}{K} \right)} \end{aligned} \quad \dots(4)$$

As the dielectric constant  $K$  is always greater than 1 the introduction of the dielectric plate reduces the effective distance between the plates and increase the capacity :

(a) If  $d = t$ , i.e., whole space between the plates is filled by dielectric then

$$C_m = \frac{\epsilon_0 K A}{d}$$

(b) If a plate of metal, for which  $K = \infty$ , of thickness  $t$  is introduced, then

$$C' = \frac{\epsilon_0 A}{(d-t)} = \frac{\epsilon_0 A}{d \left(1 - \frac{t}{d}\right)} = \frac{C}{\left(1 - \frac{t}{d}\right)} \quad \dots(6)$$

(iii) Capacity of a parallel plate capacitor when  $n$  parallel plates of dielectric constants  $K_1, K_2, \dots, K_n$  are placed in between the plates : Suppose the thickness of the plates of dielectric constants  $K_1, K_2, \dots, K_n$  are  $t_1, t_2, \dots, t_n$  respectively. The electric field intensities in these dielectrics will be

$$E_{K_1} = \frac{Q}{\epsilon_0 K_1 A}, \quad E_{K_2} = \frac{Q}{\epsilon_0 K_2 A}, \dots, \quad E_{K_n} = \frac{Q}{\epsilon_0 K_n A}$$

$\therefore$  The potential difference between the plates will be

$$V = (E_{K_1} t_1 + E_{K_2} t_2 + \dots + E_{K_n} t_n)$$

$$= \frac{Q}{\epsilon_0 A} \left[ \frac{t_1}{K_1} + \frac{t_2}{K_2} + \dots + \frac{t_n}{K_n} \right]$$

$$\text{Capacity } C = \frac{Q}{V} = \frac{\epsilon_0 A}{\left[ \frac{t_1}{K_1} + \frac{t_2}{K_2} + \dots + \frac{t_n}{K_n} \right]} \quad \dots(7)$$

## 20. Uses of Condensers

(i) **Storage of charge** : The main use of a condenser is to store charge. When a strong current is to be passed in a circuit momentarily, a condenser

connected in that circuit. By connecting condensers with the coil of an electromagnet a strong magnetic field for a short duration can be produced.

(ii) **Storage of electrical energy** : A large amount of electrical energy can be stored in condensers. Condensers are used in particle accelerators to give energy to charged particles like electrons, protons etc.

(iii) **In electrical appliances** : Condensers are used in many electrical appliances as motors, electric fans etc. In many circuits where at make and break sparking occurs due to the generation of high e.m.f. on account of self induction, a condenser is connected across it which reduces sparking. This is because the e.m.f. developed by self induction is used for charging the condenser.

(iv) **In scientific studies** : With the help of condensers the electrical behaviour of dielectric materials can be studied which in turn leads to the knowledge of their structure.

(v) **In electronic circuits** : Condensers for a basic component of electrical circuits. However their use in different circuits is different. For example in ac circuits condensers are used to provide reactance, in power supplies as filter components to reduce ripple, in other circuits for the production and detection of electromagnetic waves, for voltage multiplication etc.

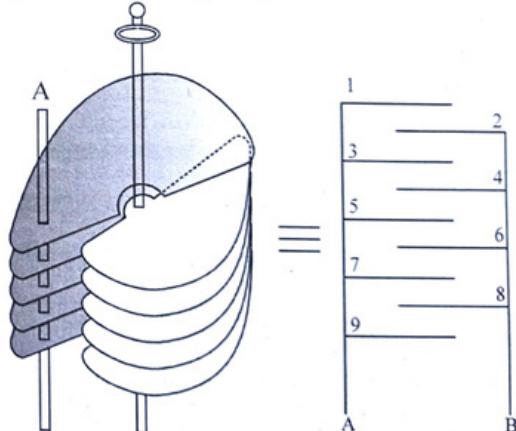


Fig. 1.17 : Gang condenser

In electronic circuits like radio receivers, television sets etc. the condensers are connected in parallel to inductance coils forming resonant circuits called tank circuits. These resonant circuits help in tuning i.e., receiving a signal of desired

frequency. The resonant frequency of such circuits is given by  $f = \frac{1}{2\pi\sqrt{LC}}$ . Variation of inductance L is difficult so the capacitance C is kept variable. For this purpose a special parallel plate condenser is used called **gang condenser**. Fig. (1.17). It consists of two groups of semicircular parallel plates. The alternate plates are connected to one rod A and the other plates to another rod B. One group of plates remains stationary but the other can be rotated with the help of a knob. By rotation of one set of plates the overlapping area of the plates can be varied. The two groups of plates form a combination of condensers in parallel. By changing their effective area the resultant capacitance can be varied. If the effective area of each plate is A and n plates have been used then the resultant capacity

$$C = \frac{(n-1)\epsilon_0 A}{d}$$

where d is the distance between successive plates.

(vi) **For controlling power factor :** In many appliances like fans, motors etc. coils are used. Due to the self inductance of these coils their power factor is less than 1 which results in lesser transfer of electrical energy. To overcome the inductive reactance suitable capacitors are used which can increase the power factor to the desired value.

## 21. Electric Current

When a circuit is connected to a source of emf, the free electrons in the conductors start drifting in a definite direction along with their random motion. In this way charge flows in the circuit and it is drifted as long as the source of emf is connected to the circuit. **The flow of charge in one second or rate of flow of charge through any point of the circuit is called current.** In fact this flow of charge is due to negatively charged electrons and they move from negative electrode of the source towards the positive electrode. But according to the classical convention the direction of current is considered to be in the direction of flow of positive charge, i.e. current flows in the circuit from the positive electrode towards the negative electrode.

If  $\delta Q$  amount of charge flows in the circuit in a small interval of time  $\delta t$ , then the current

$$I = \left( \frac{\delta Q}{\delta t} \right)_{\delta t \rightarrow 0} = \frac{dQ}{dt}$$

Practical unit (rationalized M.K.S. Unit) of current is ampere (A) and it is equal to coulomb per second. When one coulomb charge flows in one second, the current is called one ampere i.e. 1 ampere = 1 coulomb per second. S.I.



unit of current is defined from the magnetic effect of current. According to this definition.

**One ampere is that current which when flows through two infinitely long thin conductors parallel to each other and apart, produces a force of  $2 \times 10^{-7}$  N/m between them.**

The charge of an electron is  $1.6 \times 10^{-19}$  C. Thus for a current of one ampere the number of electrons flowing in one second will be

$$n = \frac{1}{1.6 \times 10^{-19}} = 6.25 \times 10^{18} \text{ electron.}$$

If the current flowing from a cell or battery is studied with time, then it will be observed that the current flows only in definite direction and its direction and magnitude remain constant. Such a current is called direct current.

## 22. Ohm's Law

Ohm applied different potential differences across a conducting wire and determined the amount of current flowing through it. On the basis of these observations he gave a relation between the potential difference across the conductor and the amount of current flowing through it. This relation is called Ohm's law. According to this law if the physical conditions of a conductor (such as temperature etc.) remain same, then the current flowing through it is directly proportional to the potential difference applied across it.

Thus if the potential difference across the ends of the conductor is V and the current flowing through it is I, then according to the ohm's law

$$I \propto V$$

$$\text{or } I = \frac{1}{R}V$$

$$\text{Hence } \frac{V}{I} = R \text{ (constant)}$$

This constant is called **resistance** of the conductor.

If a graph between the applied potential difference V and the current I is plotted, a straight line is obtained which passes through the origin Fig. (1.18).

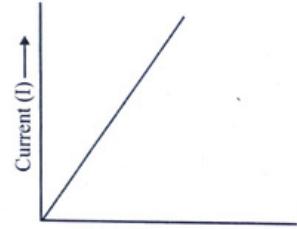


Fig. 1.18 : Variation between the applied potential difference V and the current I

Ohm's law is valid only for metallic conductors in which there are free electrons.

Practical unit of resistance is ohm. If the potential difference across the ends of a conductor is one volt and the current flowing through it is one ampere, then the resistance of the conductor is one ohm. One international ohm is defined as the resistance offered to the flow of current by a column of pure mercury of mass [14.4521] gm, length 106.3 cm and cross-sectional area  $1\text{mm}^2$  at  $0^\circ\text{C}$ .

### 23. Electrical Resistance, Resistivity and Conductivity

Conductors contain a very large number of free electrons. When a potential difference is applied between the ends of the conductor, these free electrons drift from one end to the other end of the conductor along with their random motion, i.e., electric current starts flowing through the conductor. These free electrons collide with the ions during their motion and experience a repulsive force due to bound electrons of the atoms. In this way the motion of these free electrons is obstructed. The obstruction thus produced in the motion of free electrons is called electrical resistance of the conductor. More is the number of free electrons in a conductor more is the current and less is the electrical resistance in it. Number of free electrons in conductors is very large and it is negligible in insulators.

The resistance of a conductor or conducting wire depends on the following factors :

(i) **Length of the wire** : The resistance of a wire is directly proportional to its length, i.e., more is the length of the wire, more will be its resistance. If the resistance of a wire of length  $l$  is  $R$ , then  $R \propto l$ .

(ii) **Cross-sectional area or thickness of the wire** : The resistance of a wire is inversely proportional to the cross-sectional area of the wire. More is the thickness of a wire, more is its area of cross-section and less is its resistance. Thus a thick wire has smaller resistance than a thin wire of same length and material.

$$R \propto \frac{1}{A}$$

where  $A$  is the area of cross-section of the wire.

(iii) **Material of the wire** : If wires of same length and same thickness are made from different materials, then their resistance will be different. Thus the resistance of a wire also depends on the nature of its material. The resistance of silver or copper wire is very small but the resistances of iron, nickel, nichrome, constantan and manganin wires are quite large.

Thus if the length and the cross-sectional area of the wire are  $l$  and  $A$  respectively, then the resistance

$$R \propto \frac{l}{A}$$

$$\text{or } R = \rho \frac{l}{A}$$

where  $\rho$  is a constant called the specific resistance or resistivity of the material of the wire. From the above relation if  $l = 1\text{ m}$  and  $A = 1\text{m}^2$ , then

$$R = \rho$$

Hence the resistivity or specific resistance of the material of a wire is equal to the resistance of a wire of that material having a length of 1 m and cross-sectional area of  $1\text{m}^2$ .

$$\rho = \frac{RA}{l}$$

Unit of specific resistance or resistivity

$$= \frac{\text{ohm} \times \text{m}^2}{\text{m}} = \text{ohm-m}$$

Reciprocal of resistivity is called conductivity, i.e.,

$$\text{Conductivity} = \frac{1}{\text{resistivity}}$$

$$\text{or } \sigma = \frac{1}{\rho}$$

$$\text{Unit of conductivity} = \frac{1}{\text{ohm-m}} = \text{mho/m.}$$

Reciprocal of resistance  $R$  is called conductance  $G$

$$G = \frac{1}{R} \text{ mho.}$$

### 24. Dependence of Resistance on Temperature

The resistance of a conductor depends upon the temperature. As the temperature increases, the random motion of free electrons also increases. If the number density of charge carriers i.e. free electrons remains constant, as in the case of a conductor, then the increase of random motion increases the resistivity and so the resistance. The variation of resistance with temperature is given by the following relation.

$$R_t = R_0(1 + \alpha t + \beta t^2)$$

where  $R_t$  and  $R_0$  are the resistances at  $t^\circ\text{C}$  and  $0^\circ\text{C}$  respectively,  $\alpha$  and  $\beta$  are constants. The constant  $\beta$  is very small so it may be assumed negligible for small variations of temperature.

$$\therefore R_t \approx R_0(1 + \alpha t)$$

$$\text{or } \alpha = \frac{R_t - R_0}{R_0 t}$$

This constant  $\alpha$  is called **temperature coefficient of resistance** of the material.

If  $R_0 = 1 \text{ ohm}$ ,  $t = 1^\circ\text{C}$ , then  
 $\alpha = (R_t - R_0) / (R_0 t)$

Thus the **temperature coefficient of resistance is defined as the increase in resistance of a conductor of resistance 1 ohm on raising its temperature by  $1^\circ\text{C}$ .** From calculations it is found that for most of the metals the value of  $\alpha$  is

nearly  $\frac{1}{273}$  per  $^\circ\text{C}$ . Hence substituting the value of  $\alpha$  in the above relation.

$$\begin{aligned} R_t &= R_0 \left(1 + \frac{1}{273} t\right) \\ &= R_0 \left(\frac{273+t}{273}\right) = R_0 \frac{T}{273} \end{aligned}$$

where  $T$  is the absolute temperature of the conductor.

$$\therefore R_t \propto T$$

Thus the resistance of a pure metal wire is directly proportional to its absolute temperature.

The resistivity of the alloys also increases with the rise of temperature but by a smaller amount in comparison to that of pure metals.

There are certain alloys such as manganin and constantan for which effect of temperature is very small because of their negligible temperature coefficient of resistance. On account of their high resistivity and negligible temperature coefficient of resistance these alloys are used to make resistance wires for resistance boxes, potentiometers, metre bridges etc. The resistivity of copper is very small so copper wires are used as connecting wires in a circuit. The resistivity of copper is very small so copper wires are used as connecting wires in a circuit. The resistivity of nichrome is very high so it is chosen for making heater wires.

The resistivity of insulators such as mica, glass, rubber etc. and semiconductors such as silicon and germanium decreases with the rise of temperature. Thus their temperature coefficients of resistance are negative. In semiconductors the number

of charge carriers increases with the rise of temperature, as a result their resistivity decreases. The effect of increase in number of charge carriers offsets the effect of increase in random motion.

In certain substances an abnormal relation is observed between the resistance and the temperature. As the temperature decreases, the resistance of the substance decreases slowly like pure metals but after attaining a certain low temperature the resistance starts decreasing abruptly and becomes almost zero. For example, the resistance of mercury is almost zero at  $4\text{K}$ . This phenomenon is called **superconductivity**. This phenomenon occurs at low temperature (from  $10\text{K}$  to  $0.1\text{K}$ ). Now a day certain compounds have been discovered which are superconductor at relatively higher temperatures ( $\approx 90\text{K}$ ). Loss of energy occurs in conductors due to resistance when current flows through it. This loss of energy can be minimised by using superconductors.

## 25. Electromotive Force (EMF) and Terminal Voltage

In an electric circuit there has to be a device which makes the charge to move around the circuit. Thus the cause which makes the current to flow is called electromotive force (emf). Actually emf is not a force but energy or work done per unit charge like potential. Cell, electric generator, thermocouple etc. are sources of emf.

When current is drawn from a cell by connecting it to an external circuit, the cell is said to be in closed circuit. If current is not drawn, then the cell is said to be in open circuit. When current is not drawn from the cell i.e., cell is in open circuit, the potential difference  $E$  between the terminals of the cell is called electromotive force (emf). When current is drawn from the cell in the external circuit, i.e. external circuit is closed, the potential difference between the terminals of the cell at that time is called terminal voltage. It is also equal to the potential difference across the external resistance. It is generally represented by  $V$ . The **emf  $E$  of a cell in a closed circuit is equivalent to the work done for the flow of unit positive charge through the external and internal resistance** whereas potential difference or terminal voltage is equivalent to the work done for the flow of unit positive charge through the external resistance only. Thus  $E$  is always greater than  $V$  and their difference  $(E - V)$  is equal to the potential drop across the internal resistance of the cell.

A cell or a generator can be represented by a source of emf  $E$  with an internal resistance  $r$  connected in series. When no current is drawn the internal resistance is ineffective and the potential difference between the terminals is equal to the emf. When the external circuit draws a current  $I$ , there is a potential drop  $Ir$  on the internal resistance so that the potential difference between its terminals becomes  $(E - Ir) = V$ . The emf  $E$  is the amount of work done in driving

a unit positive charge around the whole circuit (external and internal) while the potential difference  $V$  in the closed circuit is the amount of work done in driving a unit positive charge through the external resistance only. If an external resistance  $R$  is connected to the cell or generator and a current  $I$  flows through it, then

$$E = IR + Ir$$

while

$$V = IR \text{ or } I = \frac{V}{R}$$

$\therefore$

$$E = V + Ir$$

or

$$r = \frac{(E - V)}{I}$$

Substituting the value of  $I$

$$r = \frac{(E - V)}{V/R} = \left( \frac{E - V}{V} \right) R$$

## 26. Combination of Resistances

Resistances can be combined in the following two ways in a circuit :

- (i) In series and
- (ii) In parallel.

Sometimes some resistances can be combined in series and rest in parallel. Such type of combination is called a mixed combination.

**(i) Series combination :** In this combination the second end of the first resistance is connected to the first end of the second resistance and so on. For the flow of current the first end of the first resistance and second end of the last resistance are connected at the battery. In such a combination same amount of current flows in all the resistances but the potential difference across the resistances changes according to their resistance. In Fig. (1.19) AB, BC and CD are three resistances connected in series. Suppose their resistances are  $R_1$ ,  $R_2$  and  $R_3$  and their equivalent resistance is  $R$ .

Equivalent resistance is that resistance which when connected in place of the combination of resistances, no change of current takes place in the electrical circuit.

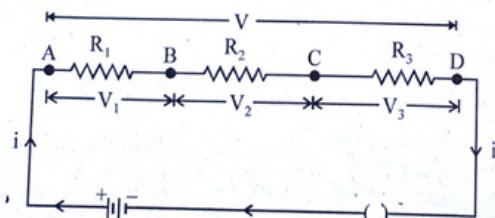


Fig. (1.19): Series combinations of resistances

Suppose current  $i$  is flowing through these resistances  $R_1$ ,  $R_2$  and  $R_3$  and the potential differences across these resistances are  $V_1$ ,  $V_2$  and  $V_3$  respectively. From Ohm's law,

$$V_1 = iR_1, V_2 = iR_2 \text{ and } V_3 = iR_3$$

If the total potential difference applied by the battery is  $V$ , then

$$\begin{aligned} V &= V_1 + V_2 + V_3 \\ &= iR_1 + iR_2 + iR_3 \\ &= i(R_1 + R_2 + R_3) \end{aligned} \quad \dots(1)$$

If the equivalent resistance of the combination is  $R$ , then from Ohm's law

$$V = iR \quad \dots(2)$$

Comparing equations (1) and (2)

$$iR = i(R_1 + R_2 + R_3)$$

$$\text{or } R = R_1 + R_2 + R_3$$

Thus the equivalent resistance of the resistances connected in series is equal to the sum of individual resistances.

In general, the equivalent resistance of  $n$  resistances connected in series is

$$R = R_1 + R_2 + R_3 + \dots + R_n \quad \dots(3)$$

**(ii) Parallel combination :** When two or more than two resistances are connected in such a way that one end of all resistances are connected together at one point and other end of all resistances are connected together at another point, this combination is called parallel combination. In this combination same potential difference exists across all resistances but the currents flowing through these resistances are different.

Three resistances  $R_1$ ,  $R_2$  and  $R_3$  are connected between the points A and B in parallel, as shown in Fig. (1.20). Battery terminals are connected to these points. Suppose the current supplied by the battery is  $i$ . This current  $i$  is divided into three parts  $i_1$  flowing through  $R_1$ ,  $i_2$  flowing through  $R_2$  and  $i_3$  flowing through  $R_3$ .

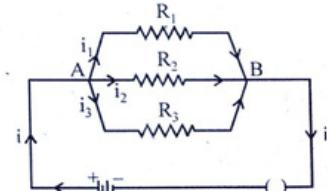


Fig. (1.20) : Parallel combinations of resistances

These currents meet at point B and gives the total current  $i$ , so that

$$i = i_1 + i_2 + i_3 \quad \dots(1)$$

If the potential difference between A and B is  $V$ , then same potential difference will be across all the resistances. Thus from Ohm's law,

$$i_1 = \frac{V}{R_1}, i_2 = \frac{V}{R_2} \text{ and } i_3 = \frac{V}{R_3}$$

Using these values  $i = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$  .....(2)

If the equivalent resistance between A and B is R, then from Ohm's law

$$i = \frac{V}{R} \quad \dots\text{.....}(3)$$

Comparing equation (2) and (3).

$$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

or  $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$

Thus the reciprocal of the equivalent resistance of the resistances connected in parallel is equal to the sum of the reciprocals of individual resistances.

In general, if R is the equivalent resistance of n resistances connected in parallel, then

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_n}$$

The equivalent resistance of the resistances connected in parallel is always less than the least resistance connected in the parallel combination.

If two resistances are connected in parallel, then the currents flowing through them are inversely proportional to their resistances. According to Fig. (1.21)

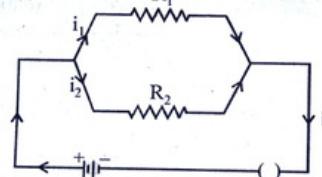


Fig. (1.21) : Two resistors are connected in parallel.

$$i_1 = \frac{V}{R_1} \text{ and } i_2 = \frac{V}{R_2}$$

and  $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

$\therefore \frac{i_1}{i_2} = \frac{R_2}{R_1} \text{ and } R = \frac{R_1 R_2}{R_1 + R_2}$

As  $V = iR$

∴

$$i_1 = \frac{V}{R_1} = \frac{iR}{R_1} = \frac{i}{R_1} \left( \frac{R_1 R_2}{R_1 + R_2} \right)$$

or

$$i_1 = \frac{R_2}{(R_1 + R_2)} i$$

and

$$i_2 = \frac{R_1}{(R_1 + R_2)} i$$

## 27. Current in a Single Loop

The closed path of the current in a circuit having certain elements like resistance, inductance etc. is called a loop. In a single loop the current is same at every point. This happens because charge is conserved (it can not be destroyed or created). Thus in a given time whatever charge enters an element

of the circuit the same amount of charge leaves it i.e. it can not get accumulated. The current which starts from the positive terminal is not consumed or used up in the path before reaching the negative terminal.

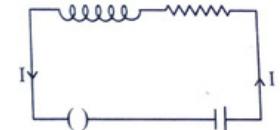


Fig. (1.22) : Current in a single loop

## 28. Electrical Power Consumption

Electric current is produced by the flow of free electrons in conductors. A source of electromotive force is used for maintaining the flow of free electrons in the conductors. During their motion the free electrons collide with the atoms of the conductors. The obstruction so produced in their flow is called resistance. Some energy of the free electrons is lost due to these obstructions. This energy increases their random motion i.e., it is converted in the form of heat.

In daily life there are several appliances such as heater, electric press, electric kettle, electric furnace etc. which consume electrical energy.

Suppose a current I is passed through a conductor of resistance R for t seconds. The potential difference across the conductor is V. If the amount of charge flowing in time t second is q, then

$$q = I \times t$$

because rate of flow of charge is current.

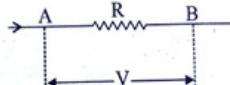


Fig. (1.23) : Electrical power consumption

From the definition of potential difference, if one joule work is done in carrying a charge of one coulomb through the conductor, then the potential difference across the conductor will be one volt. Thus the amount of work done or energy consumed in carrying a charge of  $q$  coulomb when the potential difference is  $V$  volt, will be  $qV$ . Hence,

$$W = qV = It V \text{ joule}$$

From Ohm's law,

$$V = IR$$

$$\therefore W = I^2Rt \text{ joule}$$

Now rate of consumption or production of energy is called power.

$\therefore$  Electrical power consumption

$$\begin{aligned} P &= \frac{W}{t} = \frac{VIt}{t} = VI \\ &= I^2R = \frac{V^2}{R} \text{ joule/s or watt.} \end{aligned}$$

Thus electrical energy is dissipated in a resistor or a resistive device at the rate of  $I^2R$ . Every resistor as well as appliance has a power rating which is the maximum power the device can dissipate without getting overheated and damaged.

## 29. Multiloop Circuits

In general an electric circuit may contain one or more loops each having several branches and junctions. Such a circuit is called multiloop circuit. Fig. (1.24) shows a three loop circuit. In this circuit B, C, E and F are junctions

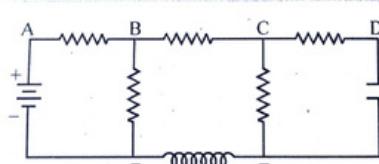


Fig. (1.24) : Multiloop loop

or nodes where two or more branches of the circuit meet. The part of the circuit through which same amount of current flows every where is called a branch. Thus BAF is one branch BF, BC, CE and CDE are other branches.

Analysis of multiloop circuits involves the determination of values of current in different branches and potential at different junctions.

## 30. Kirchhoff's Laws

Ohm's law gives the relation between the potential difference across a conductor and the current flowing through it. In complex circuits, i.e., electrical networks direct use of Ohm's law is not possible. Kirchhoff in 1842 proposed two laws to

determine the current flowing through any branch of the network and the voltage at the node. These laws can be used in any complex network. These laws are basically alternate forms of laws of conservation of charge and energy. These laws are :

(i) **Kirchhoff's first law or current law** : According to this law the algebraic sum of the currents meeting at any junction or node is zero, i.e.,

$$\sum i = 0 \quad \dots(1)$$

The currents towards the junction are taken as positive and going away from the junction are taken as negative.

Suppose five conductors meet at the junction O, as shown in Fig. (1.25). Let the currents flowing through these be  $i_1$ ,  $i_2$ ,  $i_3$ ,  $i_4$  and  $i_5$ . Currents  $i_1$  and  $i_5$  are flowing towards the junction and  $i_2$ ,  $i_3$ ,  $i_4$  are flowing away from the junction. Thus according to the Kirchhoff's law,

$$i_1 - i_2 - i_3 - i_4 + i_5 = 0$$

$$\text{or} \quad i_1 + i_5 = i_2 + i_3 + i_4 \quad \dots(2)$$

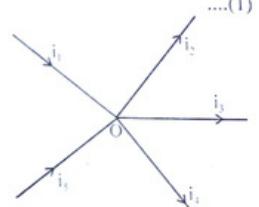


Fig. (1.25) Kirchhoff's current law

In other words, the sum of the currents flowing towards a junction is equal to the sum of the currents flowing away from the junction. Thus according to this law if a steady current is flowing in a circuit, then the charge does not accumulate at any junction or point of the circuit i.e., the rate of flow of charge towards the junction is equal to the rate of flow charge away from the junction. In this way Kirchhoff's first law or current law is equivalent to the law of conservation of charge.

(ii) **Kirchhoff's second law or Voltage law** : According to Kirchhoff's voltage law the algebraic sum of the voltages in a specified direction along a closed loop of an electrical circuit is zero. For this law the voltage drop in a definite direction is taken as negative. For example, a resistance network is shown in Fig. (1.26). The voltage drops across resistances  $R_1$  and  $R_2$  are  $V_1$  and  $V_2$  which are positive as these are in the direction of current flow. Battery emf,  $E_1$  will be negative as the direction of the current in the battery is from the negative electrode at lower potential to positive electrode at a higher potential i.e., there is potential rise. Similarly battery emf  $E_2$  will be positive as the  $(-E_1)$  direction of current is in the direction of voltage drop. Thus from voltage law, starting from point A,

$$V_1 + V_2 + E_2 - E_1 = 0 \quad \dots(3)$$

Equation (3) can be rewritten as

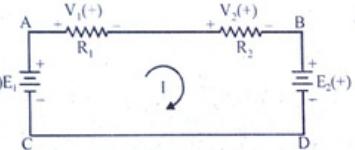


Fig. (1.26) : Kirchhoff's voltage law

$$V_1 + V_2 = E_1 - E_2 \quad \dots(4)$$

Thus the algebraic sum of the voltage drops in a closed loop is equal to the sum of emfs in that loop. For the Kirchhoff's second law in this form, if the current given by a source is in the direction of specified current, then its emf is taken as positive and if the current given by the source is in a direction opposite to that of specified current, then the emf is taken as negative. Thus in relation (4) emf  $E_1$  will now be taken as positive and emf  $E_2$  as negative. According to relation (4)

$$\Sigma V = \Sigma IR = \Sigma E \quad \dots(5)$$

This law is based on the law of conservation of energy because the emf  $E$  of a source represents the energy supplied by the source for the flow of unit positive charge in the closed circuit and this energy is equivalent to the energy used in the voltage drops on different components of the circuit. Use of Ohm's law in this law is essential for the determination of a voltage drop. Thus this law is also based on the Ohm's law.

As an example, consider the closed loops ABCD and BEFC of the circuit given in Fig. (1.27).

Using second law for the loop ABCD.

$$i_1 R_1 + i_2 R_2 + i_1 R_3 = E \quad \dots(6)$$

$$\text{or } i_1 (R_1 + R_3) + i_2 R_2 = E \quad \dots(7)$$

and for the loop BEFC,

$$i_3 R_4 + i_3 R_5 - i_2 R_2 = 0 \quad \dots(8)$$

Using first law at the node B,

$$i_1 = i_2 + i_3$$

Substituting  $i_3 = i_1 - i_2$  in equation (8), we get

$$(i_1 - i_2)(R_4 + R_5) - i_2 R_2 = 0$$

$$\text{or } i_1(R_4 + R_5) - i_2(R_2 + R_4 + R_5) = 0 \quad \dots(9)$$

In equation (7) and (9),  $i_1$  and  $i_2$  are only two unknown quantities. On solving these equations one can get the values of  $i_1$  and  $i_2$  and current in all branches of the circuit can be obtained.

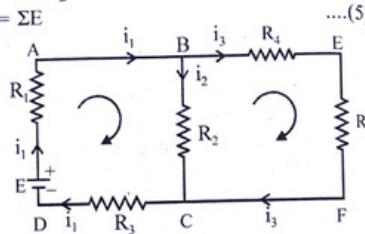


Fig. (1.27) : Closed loop of the circuit

### Numericals

■ Example 1. Two charges of  $1\mu\text{C}$  each are located 10 cm apart. What is the force of repulsion between them?

Solution : Given  $q_1 = q_2 = 1\mu\text{C} = 10^{-6}\text{C}$

$$d = 10 \text{ cm} = 0.1 \text{ m}$$

$$F = K \frac{q_1 q_2}{r^2} = 9 \times 10^9 \frac{10^{-6} \times 10^{-6}}{(0.1)^2} 9 \times 10^{-1} = 0.9 \text{ Newton}$$

■ Example 2. In a hydrogen atom the nucleus has a proton and outside it an electron moves around it in a circular path of radius  $0.5 \times 10^{-10} \text{ m}$  ( $0.5\text{\AA}$ ). Magnitude of charge on both proton and electron is  $1.6 \times 10^{-19} \text{ C}$ . Calculate the attracting force on the electron due to the proton.

Solution : Charge on proton  $q_1 = + 1.6 \times 10^{-19} \text{ C}$

Charge on electron  $q_2 = - 1.6 \times 10^{-19} \text{ C}$

Distance between them  $r = 0.5 \times 10^{-10} \text{ m}$

$$\begin{aligned} \text{Force of attraction} \quad F &= K \frac{q_1 q_2}{r^2} = \frac{9 \times 10^9 \times 1.6 \times 10^{-38}}{(0.5 \times 10^{-10})^2} \\ &= 0.92 \times 10^{-7} \text{ newton} \\ &= 9.2 \times 10^{-8} \text{ N} \end{aligned}$$

■ Example 3. Repulsive force between two charges in vacuum is  $2.5 \times 10^{-7} \text{ N}$ . If a mica sheet is placed between them the force becomes  $5.0 \times 10^{-8} \text{ N}$ . What is the relative permittivity (dielectric constant) of mica?

Solution :

$$F_0 = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$

$$F_m = \frac{1}{4\pi\epsilon_0\epsilon_r} \frac{q_1 q_2}{r^2}$$

$$\frac{F_0}{F_m} = \epsilon_r \text{ (dielectric constant)}$$

Given  $F_0 = 2.5 \times 10^{-7} \text{ N}$  and  $F_m = 5.0 \times 10^{-8} \text{ N}$

$$\therefore \epsilon_r = \frac{2.5 \times 10^{-7}}{5.0 \times 10^{-8}} = 5$$

■ Example 4. A point charge of  $3\mu\text{C}$  is placed at the origin. Find the intensity of electric field at a distance of 0.5m from this charge.

**Solution :**

$$E = K \frac{q}{r^2}$$

According to the question  $q = 3 \mu\text{C} = 3 \times 10^{-6} \text{ C}$ ,  $r = 0.5 \text{ m}$  and  $K = 9 \times 10^9 \text{ Nm}^2/\text{C}^2$ . In MKS units.

$$\therefore E = 9 \times 10^9 \frac{3 \times 10^{-6}}{(0.5)^2} = 1.08 \times 10^5 \text{ N/C}$$

**Example 5.** Two point charges of  $1\mu\text{C}$  and  $2\mu\text{C}$  are placed  $10 \text{ cm}$  apart. Determine the position on the line joining the charges where the electric field is zero.

**Solution :** For all points in between the charges on the lines joining them the electric field due to one charge will be in a direction opposite to the electric field due to the other charge. At a certain position the two field will be equal and opposite and so the resultant electric field will be zero. Suppose the distance of such a point from the charge  $1\mu\text{C}$  is  $x$  metre then the distance from the other charge will be  $(0.1 - x)$  metre.

$$\therefore K \frac{(10^{-6})}{x^2} = K \frac{(2 \times 10^{-6})}{(0.1-x)^2}$$

$$\text{or } (0.1 - x) = x\sqrt{2} = 1.414 x$$

$$\text{or } 2.414 x = 0.1$$

$$\text{or } x = \frac{0.1}{2.414} = 0.041 \text{ m} = 4.1 \text{ cm}$$

Hence the electric field will be zero at the point which is at a distance of  $4.1 \text{ cm}$  from the charge of  $1 \mu\text{C}$ .

**Example 6.**  $2.4 \times 10^{-5} \text{ J}$  work is done in carrying a charge of  $3.0 \times 10^{-6} \text{ C}$  up to a charged body. What is its potential?

**Solution :** Potential  $V = \text{work done per unit positive charge in bringing a charge to a given point.}$

$$\text{Given work done} = 2.4 \times 10^{-5} \text{ J}$$

$$\text{charge} = 3.0 \times 10^{-6} \text{ C}$$

$$\therefore V = \frac{2.4 \times 10^{-5}}{3.0 \times 10^{-6}} = \frac{24}{3} = 8 \text{ volts.}$$

**Example 7.** A spherical conductor of radius  $1 \text{ cm}$  has a charge of  $25 \times 10^{-9} \text{ C}$ . Calculate the potential at a point  $10 \text{ cm}$  away from the centre.

**Solution :** Considering the lines of force of a charged spherical conductor, it behaves as if the total charge is concentrated at the centre. At points outside it the potential due to a charge spherical conductor is therefore

$$V = K \frac{q}{r}$$

where  $r$  is the distance from the centre

As  $q = 25 \times 10^{-9} \text{ C}$ ,  $r = 10 \text{ cm} = 0.1 \text{ m}$ ,  $K = 9 \times 10^9 \text{ Nm}^2/\text{C}^2$

$$\therefore V = \frac{9 \times 10^9 \times 25 \times 10^{-9}}{0.1} = 2250 \text{ volts.}$$

**Example 8.** In a hydrogen atom the electron moves round the proton in an orbit of radius  $0.53 \times 10^{-10} \text{ m}$ . What is the potential energy of the electron?

**Solution :** Charge on proton  $= +1.6 \times 10^{-19} \text{ C}$

Charge on electron  $= -1.6 \times 10^{-19} \text{ C}$

The potential due to the proton at a distance  $0.53 \times 10^{-10} \text{ m}$  will be

$$V = K \frac{q}{r} = \frac{9 \times 10^9 \times 1.6 \times 10^{-19}}{0.53 \times 10^{-10}} \text{ volt}$$

As potential is equal to potential energy of a unit positive charge placed at the given point so the potential energy of electron will be

$$\begin{aligned} \text{P.E.} &= -V \times 1.6 \times 10^{-19} \\ &= -\frac{9 \times 10^9 \times 1.6 \times 10^{-19}}{0.53 \times 10^{-10}} \times 1.6 \times 10^{-19} \\ &= -43.5 \times 10^{-19} \text{ J} \\ &= -27.2 \text{ eV} \quad (1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}) \end{aligned}$$

**Example 9.** A hostel has 10 rooms. Each room is provided with a fluorescent tube of  $40 \text{ W}$  and a fan of  $60 \text{ W}$ . If on the average the tube is operated for 8 hours and fan for 10 hours per day how much electric energy is consumed in a month of 30 days?

**Solution :** Consumption of electric energy in the tube per day

$$= 40 \times 8 = 320 \text{ watt-hour}$$

Consumption of electric energy per day in the fan

$$= 60 \times 10 = 600 \text{ watt-hour.}$$

Total consumption per day  $= 320 + 600 = 920 \text{ watt-hour}$

Total consumption in a month  $= 920 \times 30$

$$\begin{aligned} &= 27600 \text{ watt-hour} \\ &= 27.6 \text{ kilo-watt-hour} \end{aligned}$$

**■ Example 10.** An electric generator supplies a current of 20A at 220V. What is the power of the generator ?

Solution : Power  $P = VI$

Given  $V = 220$  volts,  $I = 2$ - amp.

$$\begin{aligned} P &= 220 \times 20 \\ &= 440 \text{ watt} \\ &= 4.4 \text{ kilo-watt.} \end{aligned}$$

**■ Example 11.** A 250 volt battery is connected to a  $6\mu\text{F}$  capacitor. Find the charge on each plate.

Solution : Given  $C = 6\mu\text{F} = 6 \times 10^{-6} \text{ F}$ ,  $V = 250$  volt

Charge on each plate  $Q = CV$

$$\begin{aligned} &= 6 \times 10^{-6} \times 250 \\ &= 1.5 \times 10^{-3} \text{ coulomb} \end{aligned}$$

**■ Example 12.** Two flat metal sheets  $20 \text{ cm} \times 50 \text{ cm}$  are mounted parallel to each other  $0.5\text{cm}$  apart. These are then immersed in oil of dielectric constant 2. Find the capacitance.

Solution : According the question :

$$\begin{aligned} A &= 20 \times 50 = 1000 \text{ cm}^2 \\ &= 1000 \times 10^{-4} = 0.1 \text{ m}^2 \\ d &= 0.5 \text{ cm} = 5 \times 10^{-3} \text{ m} \end{aligned}$$

$$K = 2 \quad \text{and} \quad \epsilon_0 = 8.85 \times 10^{-12} \text{ MKS units.}$$

Capacitance of a parallel plate condenser

$$\begin{aligned} C &= \frac{K\epsilon_0 A}{d} \\ &= \frac{2 \times 8.85 \times 10^{-12} \times 10^{-1}}{5 \times 10^{-3}} \\ &= 3.54 \times 10^{-10} \text{ F} \\ &= 354 \text{ pF} \end{aligned}$$

**■ Example 13.** The area of each plate of a parallel plate condenser is  $0.01 \text{ m}^2$  and the distance between the plates is  $0.5 \text{ mm}$ . On filling the space between the plates by a dielectric its capacity becomes  $3.54 \times 10^{-10} \text{ F}$ . Determine the dielectric constant of the dielectric material.

Solution : The capacity of a parallel plate capacitor with a dielectric of

## Electrostatics

dielectric constant K is

$$C_K = \frac{\epsilon_0 K A}{d}, \text{ so that } K = \frac{C_K d}{\epsilon_0 A}$$

Given

$$\begin{aligned} \epsilon_0 &= 8.85 \times 10^{-12} \text{ F/m}, d = 0.5 \text{ mm} = 5 \times 10^{-4} \text{ m}, \\ A &= 10^{-2} \text{ m}^2 \text{ and } C_K = 3.54 \times 10^{-10} \text{ F} \end{aligned}$$

$$\therefore K = \frac{3.54 \times 10^{-10} \times 5 \times 10^{-4}}{8.85 \times 10^{-12} \times 10^{-2}} = 2$$

**■ Example 14.** The area of each plate of parallel plate condenser is  $0.20 \text{ m}^2$  and the distance between the plates is  $0.01 \text{ m}$ . The potential difference between the plates is  $3000 \text{ V}$ . When a plate of a dielectric material of thickness  $0.01 \text{ m}$  is placed between the plates of the condenser, the potential difference decreases to  $1000 \text{ V}$ . Calculate (i) the capacity before placing the dielectric plate, (ii) the charge on each plate, (iii) dielectric constant of the medium and (iv) the capacity after placing the dielectric plate.

Solution : (i) The capacity of the condenser before placing the dielectric plate

$$\begin{aligned} C &= \frac{\epsilon_0 A}{d} \\ \text{Given} \quad \epsilon_0 &= 8.85 \times 10^{-12} \text{ F/m}, A = 0.20 \text{ m}^2 \text{ and } d = 0.01 \text{ m} \\ &\quad \therefore C = \frac{8.85 \times 10^{-12} \times 0.20}{0.01} \\ &= 177 \times 10^{-12} \text{ F} \\ &= 177 \text{ pF} \end{aligned}$$

$$\begin{aligned} \text{(ii) Charge on each plate} \quad Q &= CV, V = 3000 \text{ V} \\ Q &= 177 \times 10^{-12} \times 3000 \\ &= 5.31 \times 10^{-7} \text{ C} \end{aligned}$$

(iii) Dielectric constant

$$\begin{aligned} K &= \frac{C_k}{C} = \frac{Q/V_k}{Q/V} \\ &= \frac{V}{V_k} = \frac{3000}{1000} = 3 \end{aligned}$$

(iv) The capacity of the condenser with dielectric

$$\begin{aligned} C_k &= KC \\ &= 3 \times 177 \text{ pF} \\ &= 531 \text{ pF.} \end{aligned}$$

**Example 15.** Three capacitors have capacities 0.50, 0.30 and 0.20  $\mu\text{F}$  respectively. How will you connect these to get (i) minimum capacitance and (ii) maximum capacitance? Calculate their values also.

**Solution :** By connecting the condenser in series the capacity is reduced while by connecting them in parallel the capacity increases.

(i) For minimum capacitance the capacitors are connected in series. If the resultant capacitance is  $C_s$  then

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$C_1 = 0.50 \mu\text{F}, C_2 = 0.30 \mu\text{F} \text{ and } C_3 = 0.20 \mu\text{F}$$

$$\therefore \frac{1}{C_s} = \frac{1}{0.50} + \frac{1}{0.30} + \frac{1}{0.20} = \frac{31}{3}$$

$$\text{or } C_s = \frac{3}{31} \mu\text{F} = 0.097 \mu\text{F} \text{ (Minimum value)}$$

(ii) For maximum capacitance the capacitors are connected in parallel. If equivalent capacitance is  $C_p$ , then

$$C_p = C_1 + C_2 + C_3 \\ = 0.50 + 0.30 + 0.20 = 1.0 \mu\text{F} \text{ (maximum value)}$$

**Example 16.** Three identical capacitors each of capacity 1  $\mu\text{F}$  are to be connected in such a way that the resultant capacitance is 1.5  $\mu\text{F}$ . Explain the method with reasons.

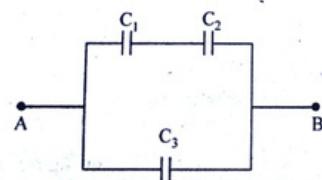
**Solution :** By connecting the three given capacitors in parallel the resultant capacitance becomes 3  $\mu\text{F}$  while by connecting them in series the resultant capacitance is 0.33  $\mu\text{F}$ . Hence to obtain a resultant capacitance of 1.5  $\mu\text{F}$  a mixed combination has to be used as shown in the figure.

Two condensers  $C_1$  and  $C_2$  are connected in series to give an equivalent capacity  $C'$ .

$$\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{1} + \frac{1}{1} = 2$$

$$\text{or } C' = \frac{1}{2} = 0.5 \mu\text{F}.$$

The third capacitor  $C_3$  of 1  $\mu\text{F}$  when is connected in parallel to the series



combination of  $C_1$  and  $C_2$  the resultant capacitance will be

$$C'' = C_3 + C' = 1 + 0.5 = 1.5 \mu\text{F}$$

**Example 17.** In the given diagram the capacity of each condenser is 1  $\mu\text{F}$ . Find the equivalent capacitance between A and B.

**Solution :** In the arrangement shown the capacitors in the various parallel branches are in series. Suppose the capacitance of these branches are  $C_1, C_2, C_3, \dots$  etc. respectively. Hence

$$C_1 = 1 \mu\text{F}.$$

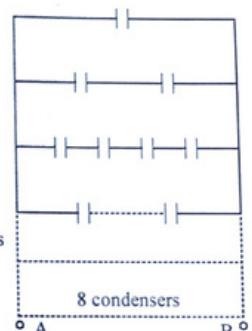
$C_2$  = capacity of two condensers in series

$$= \frac{1}{2} \mu\text{F}$$

$C_3$  = Capacity of four condensers in series

$$= \frac{1}{4} \mu\text{F}$$

$$C_4 = \frac{1}{8} \mu\text{F}, \dots$$



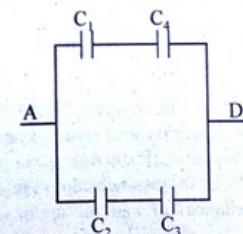
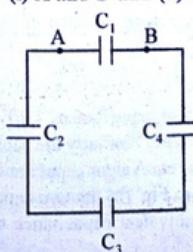
As  $C_1, C_2, C_3, \dots$  are in parallel, the resultant equivalent capacitance is

$$C = C_1 + C_2 + C_3 + C_4 + \dots$$

$$= \left( 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots \right)$$

$$= \frac{1}{\left( 1 - \frac{1}{2} \right)} = 2 \mu\text{F}$$

**Example 18.** In the figure given four identical capacitors each of capacity 3  $\mu\text{F}$  are connected. Find the equivalent capacitance between the points (i) A and B and (ii) A and D.



**Solution :** (i) Across the points A and B, the capacitors  $C_2$ ,  $C_3$  and  $C_4$  are in series and this series combination is in parallel to  $C_1$ . If the resultant capacitance of  $C_2$ ,  $C_3$  and  $C_4$  in series is  $C'$  then

$$\frac{1}{C'} = \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$$

or

$$C' = 1\mu F$$

∴ Equivalent capacitance between A and B

$$C = C' + C_1 = 1 + 3 = 4\mu F$$

(ii) Across the points A and D,  $C_1$  and  $C_4$  are in series and  $C_2$  and  $C_3$  are in series. These two series combinations are in parallel. Let the two series combinations have equivalent capacitances  $C'$  and  $C''$  then

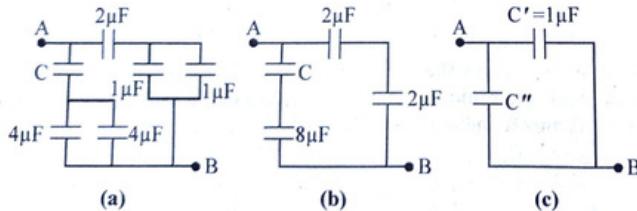
$$\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_4} = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}, \text{ so that } C' = \frac{3}{2}\mu F$$

$$\frac{1}{C''} = \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}, \text{ so that } C'' = \frac{3}{2}\mu F$$

∴ Resultant capacitance between A and D

$$\begin{aligned} C &= C' + C'' \\ &= \frac{3}{2} + \frac{3}{2} = 3\mu F. \end{aligned}$$

**Example 19.** In the figure given below the equivalent capacitance between A and B is 5  $\mu F$ . Find the value of capacitance C.



**Solution :** In the given figure (a) the capacitors of capacitances 1  $\mu F$  and 1  $\mu F$  are in parallel so their resultant capacitance will be 2  $\mu F$ . Similarly the capacitors of capacitance 4  $\mu F$  and 4  $\mu F$  are in parallel and their equivalent capacitance will be 8  $\mu F$ . Fig. (a) thus reduces to Fig. (b). According to Fig. (b) the two capacitors of capacitance 2  $\mu F$  and 2  $\mu F$  are in series so their equivalent capacitance will be

$C' = \frac{2 \times 2}{2+2} = 1\mu F$ . The capacitors of capacitance C and 8  $\mu F$  are in series and their equivalent capacity will be  $C'' = \frac{8C}{8+C}\mu F$ .

$C'$  and  $C''$  are in parallel across A and B. Fig. (c). Hence the resultant capacitance between A and B will be

$$C''' = C' + C'' = \left( 1 + \frac{8C}{8+C} \right)$$

According to the question  $C''' = 5\mu F$

$$\therefore I + \frac{8C}{8+C} = 5$$

so that  $C = 8\mu F$ .

**Example 20.** A capacitor is made of 16 sheets of tin foil 6 cm  $\times$  10 cm separated by mica sheets 0.25 mm thick. If alternate sheets of tin foil are connected together find the capacitance. Dielectric constant of mica is 4.8.

**Solution :** The given system is a multiple condenser with  $n = 16$ ,  $A = 6 \times 10 = 60 \text{ cm}^2 = 60 \times 10^{-4} \text{ m}^2$ ,  $d = 0.25 \text{ mm} = 0.25 \times 10^{-3} \text{ m}$  and  $K = 4.8$ .

$$\text{Capacitance } C = (n-1) \frac{\epsilon_0 K A}{d}$$

$$\begin{aligned} &= (16-1) \frac{8.85 \times 10^{-12} \times 4.8 \times 6 \times 10^{-3}}{0.25 \times 10^{-3}} \\ &= 1.53 \times 10^{-8} \text{ F} \\ &= 1.53 \times 10^{-2} \mu \text{F} \end{aligned}$$

**Example 21.** A charge of 20  $\mu C$  flows through a circuit in 2 millisecond. What is the value of current in the circuit?

**Solution :** Given  $Q = 20\mu C = 20 \times 10^{-6} \text{ C}$   
 $t = 2 \text{ millisecond} = 2 \times 10^{-3} \text{ s}$

current is rate of flow of charge

$$\therefore I = \frac{Q}{t} = \frac{20 \times 10^{-6}}{2 \times 10^{-3}} = 10^{-2} \text{ A} = 10 \text{ mA}$$

**■ Example 22.** A current of 0.5A flows through a conductor for 4 minutes. How much charge and how many electrons flow through any cross-section of the conductor in this time ?

**Solution :** Given  $I = 0.5\text{A}$  and  $t = 4 \text{ minutes} = 240\text{s}$

$$\text{Charge } Q = It = 0.5 \times 240 = 120 \text{ C}$$

Number of electrons flowing in this time

$$= \frac{120}{1.6 \times 10^{-19}} = 7.5 \times 10^{20}$$

**■ Example 23.** The current in an electron beam of a CRO is 200  $\mu\text{A}$ . How many electrons strike the screen per second ?

**Solution :** If the number of electrons passing across any cross-action of the beam per second is  $n$ , then charge flowing per second is  $ne$ . So current  $I = ne$  or

$$n = \frac{I}{e}$$

$$\therefore n = \frac{200 \times 10^{-6}}{1.6 \times 10^{-19}} = 1.25 \times 10^{15} \text{ per second}$$

**■ Example 24.** A current of 0.5A produces a potential difference of 2.0 V across a resistance wire. Determine its resistance.

**Solution :** Given  $I = 0.5\text{A}$  and  $V = 2.0 \text{ volts}$

By Ohm's law  $V = IR$

$$\text{or } R = \frac{V}{I} = \frac{2.0}{0.5} = 4 \text{ ohms}$$

**■ Example 25.** How much potential difference will be produced across a resistance wire of resistance 60 ohm when a current of 5 ampere flows through it ?

**Solution :** Given  $R = 60 \text{ ohm}$ ,  $I = 5\text{A}$

$$V = IR \\ = 5 \times 60 = 300 \text{ V}$$

**■ Example 26.** A conducting wire has a radius of 0.5mm and length 2.0m. If its resistance is 0.05 ohm, determine its resistivity.

**Solution :** Resistivity  $\rho = \frac{RA}{l}$

Given :  $R = 0.05 \text{ ohm}$ ,  $A = \pi r^2 = \pi(0.5 \times 10^{-3})^2$ ,  $l = 2.0 \text{ m}$

$$\begin{aligned}\rho &= \frac{0.05 \times \pi \times (0.5 \times 10^{-3})^2}{2.0} \\ &= 196.25 \times 10^{-10} \\ &= 1.96 \times 10^{-8} \Omega\text{-m}\end{aligned}$$

**■ Example 27.** A wire of resistance 5.0 ohm is drawn out through a die so that its new length is three times its original length. Find its new resistance.

**Solution :** Let the length of the wire be  $l$ , area of cross-section be  $A$  and the resistivity be  $\rho$ . Then

$$R = \rho \frac{l}{A} = 5.0 \text{ ohms}$$

After drawing out of the die if its length becomes  $l'$ , area  $A'$  and resistance  $R'$ , then

$$R' = \rho \frac{l'}{A'}, l' = 3l$$

As by drawing the volume does not change

$$\therefore IA = I'A'$$

$$\text{or } A' = \frac{IA}{I'} = \frac{IA}{3l} = \frac{A}{3}$$

$$\therefore R' = \rho \frac{(3l)}{(A/3)} = \rho \frac{9l}{A} = 9R \\ = 9 \times 5 = 45 \text{ ohms}$$

**■ Example 28.** Two conductors of the same material have the same length. One conductor is a solid wire of diameter 1.0 mm while the other is a hollow tube having external and internal diameters as 2.0 mm and 1.0 mm respectively. Determine the ratio of their resistance.

**Solution :** Resistance  $R = \rho \frac{l}{A}$

For Solid Conductor  $r = \frac{1.0}{2} = 0.5 \text{ mm}$

$$\therefore R_1 = \rho \frac{1}{\pi(0.5 \times 10^{-3})^2}$$

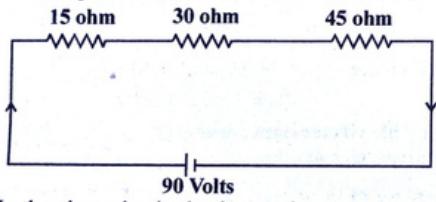
For hollow conductor  $r_1 = \frac{2.0}{2} = 1.0$  mm,  $r_2 = \frac{1.0}{2} = 0.5$  mm

$$\therefore R_2 = \rho \frac{l}{\pi(r_1^2 - r_2^2)}$$

$$= \rho \frac{l}{\pi[(1.0)^2 - (0.5)^2] \times 10^{-6}}$$

$$\therefore \frac{R_1}{R_2} = \frac{(1.0 - 0.25) \times 10^{-6}}{0.25 \times 10^{-6}} = \frac{0.75}{0.25} = \frac{3}{1}$$

**Example 29.** Three resistance wires are connected as shown in the figure. Determine its equivalent resistance and the current flowing through it.



**Solution :** In the given circuit, the three resistances are connected in series. Hence its equivalent resistance will be

$$R = R_1 + R_2 + R_3 \\ = 15 + 30 + 45 = 90 \text{ ohm}$$

Current flowing through the circuit

$$I = \frac{V}{R} = \frac{90}{90} = 1 \text{ A}$$

**Example 30.** Two resistances wires of 10 ohm and 30 ohm are connected in parallel. Determine its equivalent resistance. Calculate the current flowing through the resistance of 30 ohms when this combination is connected to a voltage source of 15V.

**Solution :** Given  $R_1 = 10$  ohm,  $R_2 = 30$  ohm

These resistance are connected in parallel, so the equivalent resistance

$$R = \frac{R_1 R_2}{R_1 + R_2} = \frac{10 \times 30}{10 + 30} = \frac{300}{40} \\ = 7.5 \text{ ohm}$$

Current flowing through the resistance of 30 ohm

$$I = \frac{V}{R_2} = \frac{15}{30} = \frac{1}{2} = 0.5 \text{ A}$$

**Example 31.** Two resistances of 1 ohm and 2 ohm are connected in series with a 30 volt battery. Determine the potential difference across the resistor of 20 ohm.

**Solution :** Given  $R_1 = 10$  ohm,  $R_2 = 20$  ohm

The resistors of 10 ohm and 20 ohm are connected in series, so their equivalent resistance is

$$R = R_1 + R_2 = 10 + 20 = 30 \text{ ohm}$$

$$\text{Total current in the circuit} = \frac{\text{Potential difference}}{\text{Total resistance}}$$

$$I = \frac{30}{30} = 1 \text{ A}$$

$\therefore$  Potential difference across the resistor of 20 ohm  
 $V = IR_2 = 1 \times 20 = 20 \text{ V}$

**Example 32.** Determine the following in the given circuit :

- (i) Total resistance of the circuit, and
- (ii) Current flowing through the resistor of 10 ohm.

**Solution :** The three resistors are connected in parallel. If  $R$  is their equivalent resistance, then

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

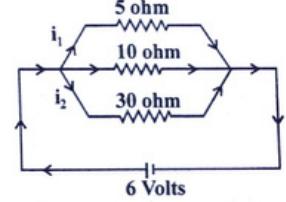
$$\frac{1}{R} = \frac{1}{5} + \frac{1}{10} + \frac{1}{30}$$

$$= \frac{6+3+1}{30} = \frac{10}{30}$$

$$\therefore R = \frac{30}{10} = 3 \text{ ohm}$$

Current flowing through the resistance of 10 ohm will be

$$I_2 = \frac{V}{R_2} = \frac{6}{10} = 0.6 \text{ A}$$



**Example 33.** An X-ray tube operates at a current of 5.0 mA while the potential difference applied is 60 kV. Determine the power dissipated.

**Solution :** Power dissipated  $P = IV$

$$\text{Given : } I = 5.0 \text{ mA} = 5.0 \times 10^{-3} \text{ A}, V = 60 \text{ kV} = 60 \times 10^3 \text{ volts}$$

$$\therefore P = 5.0 \times 10^{-3} \times 60 \times 10^3 = 300 \text{ watts.}$$

**Example 34.** When a small bulb is connected to a 3.0 V battery the power consumption is 0.54 watt. The same bulb is now connected to a 1.5 V battery. Calculate the resistance of bulb and power consumed in the second case.

**Solution :** Let the resistance of the bulb be  $R$  ohms.

$$\text{Power consumed } P = IV = \frac{V^2}{R}$$

In the first case

$$\begin{aligned} 0.54 &= \frac{(3.0)^2}{R} \\ \therefore R &= \frac{3 \times 3}{0.54} \\ &= \frac{50}{3} = 16.67 \text{ ohms} \end{aligned}$$

In the second case

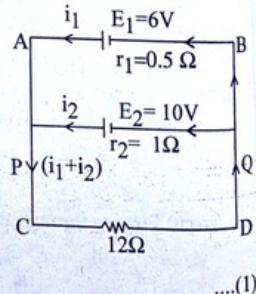
$$\begin{aligned} P &= \frac{(1.5)^2}{50/3} \\ &= \frac{2.25 \times 3}{50} = 0.135 \text{ watt.} \end{aligned}$$

**Example 35.** Two cells whose emfs and internal resistances are 6V, 0.5 ohm and 10V, 1 ohm are connected in parallel with a resistance of 12 ohm. Calculate the currents supplied by each cell.

**Solution :** Let the currents supplied by the cells of emfs  $E_1 = 6V$  and  $E_2 = 10V$  be  $i_1$  and  $i_2$  respectively. From Kirchhoff's first law the current flowing through the resistance of 12 ohm will be  $(i_1 + i_2)$ . Each cell behaves like a source of emf connected in series with its internal resistance.

Applying Kirchhoff's second law for the closed mesh ACDBA

$$\begin{aligned} i_1 \times 0.5 + (i_1 + i_2) \times 12 &= 6 \\ \text{or } 12.5i_1 + 12i_2 &= 6 \end{aligned}$$



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Applying Kirchhoff's second law for the closed loop PCDQP

$$i_2 \times 1 + (i_1 + i_2) \times 12 = 10$$

$$\text{or } 12i_1 + 13i_2 = 10 \quad \dots(2)$$

Multiplying equation (1) by 13 and equation (2) by 12

$$162.5i_1 + 156i_2 = 78 \quad \dots(3)$$

$$144i_1 + 156i_2 = 120 \quad \dots(4)$$

Subtracting equation (4) from equation (3)

$$\therefore 18.5i_1 = -42$$

$$\therefore i_1 = -\frac{42}{18.5} = -2.27 \text{ A}$$

Substituting the value of  $i_1$  in equation (2),

$$-12 \times 2.27 + 13i_2 = 10$$

$$i_2 = \frac{10 + 27.24}{13} = 2.86 \text{ A}$$

Thus 2.86 A current will be supplied by the cell  $E_2$  and the cell  $E_1$  will get 2.27 A current.

**Example 36.** A battery of emf 10V and internal resistance 1 ohm is connected in parallel with a battery of emf 20V and internal resistance 2 ohm. Current is allowed to flow through a 8 ohm resistance from this combination. Calculate the currents flowing through each battery and the external resistance.

**Solution :** Let the currents supplied by the batteries  $E_1 = 10V$  and  $E_2 = 20V$  be  $i_1$  and  $i_2$  respectively. Thus applying voltage law for the closed loop ACDBA

$$i_1 \times 1 + (i_1 + i_2) \times 8 = 10$$

$$\text{or } 9i_1 + 8i_2 = 10 \quad \dots(1)$$

Applying voltage law for the closed loop PCDQP

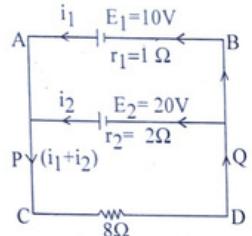
$$i_2 \times 2 + (1 + i_2) \times 8 = 20$$

$$\text{or } 8i_2 + 10i_2 = 20 \quad \dots(2)$$

$$\text{or } 4i_1 + 5i_2 = 10 \quad \dots(2)$$

Comparing equations (1) and (2),

$$9i_1 + 8i_2 = 4i_1 + 5i_2$$



$$\therefore 5i_1 = -3i_2$$

$$\text{or } i_2 = -\frac{5}{3}i_1$$

Using the value of  $i_2$  in equation (1),

$$9i_1 - \frac{8 \times 5}{3}i_1 = 10$$

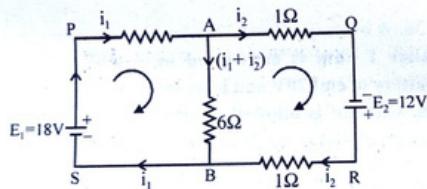
$$\text{or } i_1 = -\frac{30}{13} \text{ A}$$

$$\therefore i_2 = -\frac{5}{3}i_1 = \frac{5}{3} \times \frac{30}{13} = \frac{50}{13} \text{ A}$$

$\therefore$  The current flowing through the resistance of 8 ohm will be

$$i_1 + i_2 = -\frac{30}{13} + \frac{5}{13} = \frac{2}{13} \text{ A}$$

**Example 37.** Calculate the current in each branch of the following circuit :



**Solution :** Suppose the current supplied by the 18V battery is  $i_1$  and by the 12V battery is  $i_2$ .

According to Kirchhoff's first law at the node A the current flowing through the resistance of 6 ohm will be  $(i_1 - i_2)$ .

Applying voltage law for the closed loop PABSP,

$$i_1 \times 12 + (i_1 - i_2) \times 6 = 18$$

$$\text{or } 2i_2 + (i_1 - i_2) = 3$$

$$3i_1 - i_2 = 3 \quad \dots(1)$$

Now using voltage law for the closed loop AQRB,

$$i_2 \times 1 + i_2 \times 1 - (i_1 - i_2) \times 6 = 12$$

$$\text{or } -3i_1 + 4i_2 = 6 \quad \dots(2)$$

Adding equation (1) and (2),  $i_2 = 3 \text{ A}$

Using the value of  $i_2$  in equation (1),  $i_1 = 2 \text{ A}$

$\therefore$  Current flowing through the resistance of 6 ohm

$$= (i_1 - i_2) = 2 - 3 = -1 \text{ A}$$

Thus 1A current will flow through the resistance 6 ohm in the direction BA.

**Example 38.** In the given circuit  $E_1 = 3V$ ,  $E_2 = 2V$  and  $E_3 = 1V$  and  $R_1 = R_2 = R_3 = 1 \text{ ohm}$ . Calculate the current flowing through each branch.

**Solution :** Suppose the currents flowing through the batteries are  $i_1$ ,  $i_2$  and  $i_3$  as shown in the circuit

Applying current law at the junction a,

$$i_1 - i_2 - i_3 = 0$$

$$\text{or } i_3 = (i_1 - i_2) \quad \dots(1)$$

Applying voltage law in the closed loop badcb

$$i_1 \times 1 \times i_2 \times 1 = 3 - 2$$

$$\text{or } i_1 + i_2 = 1 \quad \dots(2)$$

Again applying voltage law in the closed loop bfecb

$$i_1 \times 1 + i_3 \times 1 = 3 - 1$$

$$i_1 + i_3 = 2$$

Using the value of  $i_3$  from equation (1),

$$2i_1 - i_2 = 2 \quad \dots(3)$$

Adding equation (2) and (3),

$$3i_1 = 3$$

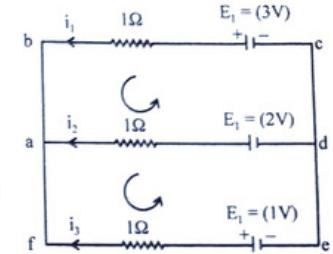
$$i_1 = 1 \text{ A}$$

$\therefore$  From equation (3),

$$i_2 = 0$$

and from equation (1),

$$i_3 = (i_1 - i_2) = 1 - 0 = 1 \text{ A}$$



## Questions

### Very Short Answer Type Questions

1. What is charge conservation ?
2. How is a body positively charged ?
3. What are the carriers of charge in conductors ?
4. Which is the best conductor ?
5. Why there is no flow of charge in insulators ?
6. What is the quantum of charge ?
7. What is an electric line of force ?
8. What is the unit of electric field intensity ?
9. What is the definition of electric field intensity ?
10. What is meant by electric potential ?
11. How does the electric potential due to a point charge varies with distance from the point charge ?
12. If an electron approaches another electron at rest, what happens to the potential energy of the system ?
13. The potential energy of the electron-proton system in a hydrogen atom is negative. What does it indicate ?
14. Masses of two spheres of same material are same. One is given a positive charge of magnitude  $Q$  and the other is given a negative charge of the same magnitude. Their masses will be different on charging, why, explain.
15. The power of an electric bulb is 100 W. What will be the time taken to consume 1 unit of electric energy ?
16. Define capacity of a capacitor.
17. If the second plate of a condenser is not earthed, how will its capacity be affected ?
18. While charging a capacitor by a constant voltage battery energy is consumed. Where does this energy go ?
19. What is the effect of using a dielectric medium in place of air ?
20. If three capacitors of capacitances  $C$ ,  $2C$  and  $3C$  are connected in series what will be the resultant capacitance of the combination ?
21. In a parallel combination of capacitors which physical quantity is same for all capacitors and which one is distributed ?
22. If the voltage rating of the capacitors available is less than the available voltage how the capacitors are to be connected ?

### Electrostatics

23. If by immersing a parallel plate capacitor in oil its capacitance becomes 2.5 times, what is the dielectric constant of oil ?
24. A charged condenser is connected to a voltmeter. If its plates are moved apart what will be the effect on the reading of voltmeter ?
25. Three identical capacitors are first connected in series and then in parallel. What will be the ratio of equivalent capacitances in the two cases ?
26. What do you mean by the resistance of a conductor ?
27. Which device is used for increasing or decreasing the current in a circuit ?
28. On combining resistances in parallel, the total resistance of the circuit decreases or increases.
29. What is the effect of heating a conductor on its resistance ?
30. What are the necessary conditions for the validity of Ohm's law ?
31. The radius of a wire is reduced to half of its original value by stretching it. What will be its resistance now ?
32. Explain the difference between ohmic and non-ohmic resistances by giving one example for each.
33. Why are resistance wires made from manganin or constantan alloys generally used in resistance box, standard resistor etc. ?
34. Why are copper wires used as connecting wire in a circuit ?
35. How does the resistance of filament of a bulb depend on its power rating ?
36. On which conservation law is Kirchhoff's first law based ?
37. What is coulomb's law ?
38. Define electric power and write different units to measure it. [Raj. 2008]

### Long Answer type Question

1. What is meant by quantization ? How was it ascertained that charge is quantized ? What is the quantum of charge ? [Raj., 2003, 2006, 2010, 2009]
2. What is an electric line of force ? How is the intensity and direction of electric field represented by the lines of force ? With the help of lines of force show that a charged spherical conductor will behave as if the charge is concentrated at the centre. [Raj., 2006, 2007]
3. State Gauss's law of electrostatics. What will be the total electric flux from a closed surface when (i) a charge  $q$  is inside it and (ii) a charge  $q$  is outside it. [Raj. 2004, 2007 2009]
4. What is meant by electric potential ? Explain the difference between potential and potential difference. [Raj., 2003, 2008, 2011]

5. What is electric potential energy ? If the potential energy of a system is negative what does it signify ? In a hydrogen atom the electron moves round the proton in an orbit of radius  $r$ . What will be the potential energy of the electron in the field of proton ? [Raj., 2004]
6. What do you understand by quantization and conservation of electric charge ? Define electric field intensity, electric potential and electric potential energy. [Raj., 2005]
7. How is electric energy consumed in a device related to the potential difference and current flowing through it ? Define electrical power and the unit kilo watt-hour. [Raj. 2009]
8. What is a condenser ? Explain its principle. How is the capacity to store charge is increased ? [Raj., 2003, 2009, 2011]
9. Describe a parallel plate capacitor. On what factor does its capacitance depend ? [Raj., 2003, 2010]
10. What is a capacitor ? Show that capacity of a parallel plate capacitor is given as :  

$$C = \epsilon_0 A/d$$

How does the capacity of a capacitor increases when dielectric medium is inserted between plates ?
11. Calculate the capacitance of series and parallel combination of three capacitors. [Raj. 2007, 2008, 2010, 0211]
12. If a number of capacitors are connected in series show that the equivalent capacitance of the combination is less than the smallest capacitance in the combination.
13. What is the effect of placing a dielectric medium between the plates of a capacitor ? How are the charge given potential difference developed, electric field between the plates and capacitance affected ? [Raj., 2004, 2006, 2007]
14. Explain the use of condensers in electronic circuits ? What is a gang condenser ? Explain its construction and working. [Raj. 2011, 2009]
15. State Ohm's law and define resistance of a conductor. On what factors and how does the resistance of a conductor depend ? [Raj. 2008, 2009, 2010 2011, ]
16. Define specific resistance or resistivity. What are its units ? Explain with reasons why :
  - (i) Alloys as manganin and constantan are used to make resistance wires of resistance boxes.

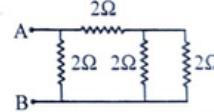
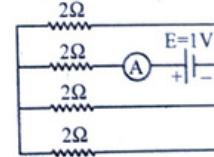
**Electrostatics**

- (ii) Copper wires are used as connecting wire in a circuit.
  17. Explain the series and parallel combination of resistances. Obtain expression for the equivalent resistances in the two cases. [Raj., 2004, 2006, 2009, 2011]
  18. State Kirchhoff's laws and explain their application with the help of examples. [Raj., 2003, 2007, 2008, 2009, 2011]
  19. Define electric field intensity. Derive an expression for electric field due to a point charge. [Raj. 2010]z
- ### Numerical Questions
1. Two equal charges each of  $0.9 \mu\text{C}$  are 6 cm apart. Calculate the repulsive force between them. [Answer : 2.025 N]
  2. Two equal point charges are 24 cm apart. A repulsive force of  $4 \times 10^{-3} \text{ N}$  acts between them. Determine the magnitude of each charge. [Ans. :  $0.16 \mu\text{C}$ ]
  3. A proton has mass of  $1.672 \times 10^{-27} \text{ kg}$  and charge of  $1.602 \times 10^{-19} \text{ C}$ . If the proton is placed in an electric field of 5000 V/m, then calculate the force acting on it and compare this force with its weight [ $g = 9.8 \text{ m/s}^2$ ]. [Ans.,  $8.01 \times 10^{-16} \text{ N}$ ,  $4.89 \times 10^{10} : 1$ ]
  4. A charge of  $10^{-9} \text{ C}$  is placed in an electric field of intensity 6000 V/m. What will be the force acting on the charge due to the electric field ? [Ans.,  $6 \times 10^{-6} \text{ N}$ ]
  5. The electric field at a distance of 20 cm from a point charge is 10 V/m. What will be the electric field at a distance of 8 cm from the charge ? [Ans., 62.5 V/m]
  6. Positive charge on the proton is  $1.6 \times 10^{-19} \text{ C}$ . Determine the electric field and the potential at a distance of 1 Å from the proton. (Proton is assumed to be a point charge.) [Ans.,  $1.44 \times 10^{11} \text{ V/m}$ ,  $14.4 \text{ V}$ ] [Raj. 2005]
  7. In a nucleus two protons are  $6 \times 10^{-15} \text{ m}$  apart. Determine the electric potential energy of the system in joule and eV. [Ans.,  $3.84 \times 10^{-14} \text{ J}$ ,  $2.4 \times 10^5 \text{ eV}$ ] [Raj. 2004]
  8. A bulb is rated as 220 V, 100 W. What is the current drawn by the bulb when it is connected to a 220 V line ? [Ans., 0.45 A]
  9. An electric kettle of 2000 W is used to heat water for half an hour every day. What will be the consumption of electric energy in a month (30 days). [Ans., 30 kWh] [Raj., 2003]

**Elementary Physics**

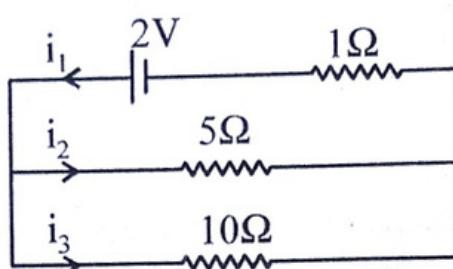
10. An electric motor running at 220 volts draws 5A current. What is the power of the motor ?  
[Ans., 1.1 kilo-watt]
11. A 100 volt battery is connected to a  $10 \mu\text{F}$  capacitor. What is the charge on each plate ?  
[Ans.:  $10^{-3}$  coulomb] [Raj., 2005]
12. A parallel plate capacitor with glass as dielectric has a capacitance of 0.05  $\mu\text{F}$ . Dielectric constant of glass is 8. If glass is replaced by mica of dielectric constant 3, what will be its capacitance.  
[Ans. :  $0.0189 \mu\text{F}$ ]
13. Three capacitors of capacitance 8, 12 and  $24 \mu\text{F}$  are connected in series. Determine the equivalent capacitance of the combination. [Ans. :  $4 \mu\text{F}$ ]
14. The capacity of a charged parallel plate capacitor is  $5 \mu\text{F}$ . If a glass plate is placed between the plates of the capacitor the potential difference across the plates reduces to  $1/8$  times the initial value, find the dielectric constant of glass.  
[Ans. : 8]
15. Two capacitors of  $1 \mu\text{F}$  and  $2 \mu\text{F}$  are connected in series and the combination is connected to a battery of 120V. What will be potential difference across the  $1 \mu\text{F}$  capacitor.  
[Ans. : 80V]
16. Two capacitors of capacitances  $2 \mu\text{F}$  and  $4 \mu\text{F}$  are connected in series. Calculate the equivalent capacitance.  
[Ans. :  $\frac{4}{3} \mu\text{F}$ ]
17. A capacitor of  $10 \mu\text{F}$  is charged to 100 V. It is then connected to another uncharged capacitor in parallel. Now the potential difference becomes 40V. Calculate the capacitance of the second capacitor.  
[Ans. :  $15 \mu\text{F}$ ]  
[Raj., 2004]
18. A number of capacitors of  $2 \mu\text{F}$  are available. To obtain a capacitance of  $5 \mu\text{F}$  what will be the minimum number of capacitors used ?  
[Ans. : Four]  
[Raj., 2003]
19. Three capacitors each of  $1 \mu\text{F}$  are connected in parallel and a fourth capacitor of  $1 \mu\text{F}$  is connected in series with the combination. Calculate the equivalent capacitance.  
[Ans. :  $\frac{3}{4} \mu\text{F}$ ]
20. A gang condenser is made of 15 semicircular plates 8 cm in diameter. If the air gap between adjacent plates is 0.5mm. Calculate its maximum capacitance.  
[Ans. :  $622.5 \text{ pF}$ ]
21. The resistance of a wire of length 13.78 cm and diameter 0.2 cm is 2.15 ohm. Find the specific resistance of the material of wire. [Ans. :  $49 \times 10^{-6} \text{ ohm-m}$ ]

**Electrostatics**

22. The specific resistance of the material of a wire is  $2.62 \times 10^{-8} \text{ ohmmetre}$ . If the length of wire is 1.2 m and diameter is 0.04 mm, find its resistance.  
[Ans.: 25 ohms]
23. Three wires of resistances 10 ohm, 20 ohm and 30 ohm are connected in series and then in parallel. Find the equivalent resistance in each case.  
[Ans. : 60 ohm, 5 ohm]
24. Two wires of resistances 30 ohm and 60 ohm are connected in parallel. This combination is connected to a voltage source of 30 volt. Determine (i) the total resistance of the circuit, (ii) the current flowing through each resistance.  
[Ans. : (i) 20 ohm, (ii) 1.0 A and 0.5A]
25. Three resistances of 10 ohm, 15 ohm and 30 ohm are connected in parallel and then connected to a battery of 15 volt. Find the total current in the circuit and the current in each resistance.  
[Ans.: 3A, 1.5A, 1A, 0.5A]
26. Four resistance wires of 10 ohm each are connected in the form of a square. What will be the equivalent resistance between the ends of a diagonal of the square ?  
[Ans. : 10 ohm]
27. A network of four resistances is shown in the figure. What is the equivalent resistance between A and B ?  
[Ans. : 1 ohm]
- 
28. What will be the reading in the ammeter connected in the circuit in the figure ?  
[Ans. :  $\frac{3}{8} \text{ A}$ ]
- 
29. A student has two resistors. By joining these resistors in different ways he can obtain the resistances of 3, 4, 12 and 16 ohm. What are the resistance of the two resistors ?  
[Ans. : 4 and 12 ohm] [Raj. 2004]
30. How can three resistances of 2, 3, 6 ohm be connected to give an equivalent resistance of 4 ohm ?  
[Ans. : Connect 3 ohm and 6 ohm resistors in parallel and then the parallel combination is to be connected series with the resistance of 2 ohm.]

31. Find the current flowing through the resistances of 5 ohm and 10 ohm in the given circuit.

[Ans. :  $i_2 = \frac{4}{13} \text{ A}$ ,  $i_3 = \frac{2}{13} \text{ A}$ ]



32. How much charge will be stored in the capacitor C connected in the given circuit ? Internal resistance of the cell is negligible.

[Ans. :  $Q = \frac{2}{3} CE$ ]

