Total Time: **30 minutes**   Total Marks: **15 Marks**

1. [Question 1] Let $u(x,y) = 2x^3 - 6xy^2 + 18xy - 12x$ for all $z = x + iy \in \mathbb{C}$. Let f be an analytic function whose real part is u. Then the value of $f'(3i)$ is   (**4** points)

   (A) 12 (B) -12 (C) 0 (D) $3i$

2. [Question 2] Let $C : |z| = 2$ be the circle oriented positively. Then the value of   (**3** points)

$$\int_C 2 \cos z \; e^{\left(|z|^2 + z\right)} \; Re \; (z) \; dz \quad \text{is}$$

   (A) $(2\pi i) \; e^4$ (B) $(2\pi i) \; e^2$ (C) $(8\pi i) \; e^4$ (D) $(4\pi i) \; e^4$

3. [Question 3] Which of the following statements is/are NOT true ?   (**4** points)

   (A) There exists a non-constant entire function f such that e^f is bounded.

   (B) There exists a non-constant entire function f such that f is bounded for all real x.

   (C) There exists a non-constant entire function f such that $Re \; (f)$ is bounded.

   (D) There exists a non-constant entire function f such that $|f(z)| > 1$ for all $|z| > 1$ and $f(z) \neq 0$ for all $|z| \leq 1$.

4. [Question 4] Let $f(z) = |Re \; (z) \; Im \; (z)|^{1/2}$. If $f = u + iv$, then which of the following statements is/are TRUE ?

   (**2** points)

   (A) u, v satisfies the Cauchy-Riemann equations (C-R equations) at the origin.

   (B) u_x exists at all points in some neighbourhood of the origin.

   (C) Every neighbourhood of the origin contains a point where u_y does not exist.

   (D) f is differentiable at the origin.

5. [Question 5] Which of the following statements is/are TRUE? $\qquad$ (**2** points)

(A) $i^{1/\pi} = e^{i(4k+1)/2}, \quad k \in \mathbb{Z}$

(B) $1^{\sqrt{2}} = e^{2\sqrt{2}k\pi i}, \quad k \in \mathbb{Z}$

(C) $\text{Log } i^{1/8} = i\pi/8$

(D) One of the values of $\log\left[(1+i)^{2i}\right]$ is $\frac{7\pi}{2} + i(\ln 2 - 2\pi)$.

****** Paper Ends ******