Differentiation of implicit functions.

Ex: ~~y cos x~~ ① $x^3+y^3=6xy$; $y(x)$ ② $y\cos x = x^2+y^2$; $y(x)$

③ $x^3+y^3+z^3+6xyz=1$; $z(x,y)$.

④ $yz + x\log y = z^2$.

Q1. Find $\frac{dy}{dx}$ if y is defined implicitly as a function of x and y by equation

$$x^3+y^3=6xy.$$

Sol^n:

$$x^3+y^3=6xy.$$

$$\therefore \quad 3x^2+3y^2\frac{dy}{dx} = 6y+6x\frac{dy}{dx}.$$

$$\Rightarrow \quad \frac{dy}{dx}(3y^2-6x) = 6y-3x^2.$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{6y-3x^2}{3y^2-6x} = \frac{2y-x^2}{y^2-2x}.$$

Q.2. y is a function of x and $y\cos x = x^2+y^2$

Find $\frac{dy}{dx}$.

$$y\cos x = x^2+y^2.$$

$$\therefore \quad -y\sin x + \cos x\frac{dy}{dx} = 2x+2y\frac{dy}{dx}.$$

$$\Rightarrow \quad \frac{dy}{dx}(\cos x - 2y) = 2x + y\sin x.$$

$$\therefore \quad \frac{dy}{dx} = \frac{2x+y\sin x}{\cos x - 2y}.$$

Q3. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if z is defined implicitly as a function of x and y by equation

$$x^3 + y^3 + z^3 + 6xyz = 1.$$

To find $\frac{\partial z}{\partial x}$, we differentiate implicitly with respect to x, treating y as constant.

$$3x^2 + 3z^2\frac{\partial z}{\partial x} + 6yz + 6xy\frac{\partial z}{\partial x} = 0.$$

$$\therefore \quad \frac{\partial z}{\partial x}(3z^2 + 6xy) = -(3x^2 + 6yz)$$

$$\therefore \quad \frac{\partial z}{\partial x} = -\frac{3x^2 + 6yz}{3z^2 + 6xy} = -\frac{x^2 + 2yz}{z^2 + 2xy}.$$

Q4. $yz + x\log y = z^2$; z is a function of x and y.

Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

Ans:

$$y\frac{\partial z}{\partial x} + \log y = 2z\frac{\partial z}{\partial x}.$$

$$\cancel{\therefore \frac{\partial z}{\partial x} = -\frac{\log y}{y - 2\frac{\partial z}{\partial x}}}$$

$$\Rightarrow \frac{\partial z}{\partial x}(y - 2z) = -\log y.$$

$$\therefore \quad \frac{\partial z}{\partial x} = -\frac{\log y}{y - 2z}$$

$$z + y\frac{\partial z}{\partial y} + \frac{x}{y} = 2z\frac{\partial z}{\partial y}.$$

$$\therefore \quad \frac{\partial z}{\partial y}(y - 2z) = -z - \frac{x}{y}$$

$$\therefore \quad \frac{\partial z}{\partial y} = -\frac{(yz - x)}{y(y - 2z)}$$