

Q.1

Sol: Prior probabilities:

$$P(\text{On time}) = \frac{14}{20}$$

$$P(\text{Late}) = \frac{2}{20}$$

$$P(\text{Very late}) = \frac{3}{20}$$

$$P(\text{cancelled}) = \frac{1}{20}$$

Posterior probabilities:

i) Attribute - Day

Day	On Time	Late	Very late	Cancelled
Weekday	3/14	1/2	3/3	0/1
Saturday	2/14	0/2	0/3	1/1
Sunday	1/14	0/2	0/3	0/1
Holiday	2/14	1/2	0/3	0/0

ii) Attribute - Season

Season	On Time	Late	Very Late	Cancelled
Spring	4/14	0/2	0/3	1/1
Summer	6/14	0/2	0/3	0/1
Autumn	2/14	0/2	1/3	0/1
Winter	2/14	2/2	2/3	0/1

iii) Attribute - Fog

Fog	On Time	Late	Very Late	Cancelled
None	5/14	0/2	0/3	0/1
High	4/14	1/2	1/3	1/1
Normal	5/14	1/2	2/3	0/1

iv) Attribute - Rain

Rain	On Time	Late	Very Late	Cancelled
None	6/14	1/2	1/3	0/1
Slight	5/14	1/2	0/3	0/1
Heavy	2/14	0/2	2/3	1/1

Using Naive Bayes formulae -

$$\begin{aligned}
 P_{NB}(\text{On Time}) &= P(\text{On Time}) \times P(\text{weekday} / \text{On Time}) \\
 &\quad \times P(\text{Winter} / \text{On Time}) \times P(\text{High} / \text{On Time}) \\
 &\quad \times P(\text{None} / \text{On Time}) \\
 &= \frac{14}{20} \times \frac{9}{14} \times \frac{2}{14} \times \frac{4}{14} \times \frac{6}{14} \\
 &= 0.0079
 \end{aligned}$$

$$\begin{aligned}
 P_{NB}(\text{Late}) &= P(\text{Late}) \times P(\text{weekday} / \text{Late}) \times \\
 &\quad P(\text{Winter} / \text{Late}) \times P(\text{High} / \text{Late}) \times \\
 &\quad P(\text{None} / \text{Late}) \\
 &= \frac{2}{20} \times \frac{1}{2} \times \frac{2}{2} \times \frac{1}{2} \times \frac{1}{2} \\
 &= 0.0125
 \end{aligned}$$

$$\begin{aligned}
 P_{NB}(\text{Very Late}) &= P(\text{Very Late}) \times P(\text{weekday} / \text{Very Late}) \\
 &\quad \times P(\text{Winter} / \text{Very Late}) \times P(\text{High} / \text{Very Late}) \\
 &\quad \times P(\text{None} / \text{Very Late}) \\
 &= \frac{3}{20} \times \frac{3}{3} \times \frac{2}{3} \times \frac{1}{3} \times \frac{1}{3} \\
 &= 0.0111
 \end{aligned}$$

$$\begin{aligned}
 P(\text{Cancelled}) &= \frac{1}{20} \times \frac{0}{14} \times \frac{0}{2} \times \frac{1}{3} \times \frac{1}{1} \\
 &= 0
 \end{aligned}$$

Here $P(\text{Late})$ is highest
 \therefore The correct classification is Late

Q. 2

501. We have to test hypothesis that gender and preferred reading are independent. That means there is no correlation between them.

Using Chi-square test
 Contingency table -

	Male	Female
Fiction	250(90)	200(350)
Non-fiction	50(210)	1000(840)

$$\text{Degrees of Freedom} = (2-1) \times (2-1) = 1$$

$$\chi^2 = \sum_{i=1}^2 \sum_{j=1}^2 \frac{(O_{ij} - E_{ij})^2}{E_{ij}}$$

$$= \frac{(250 - 90)^2}{90} + \frac{(50 - 210)^2}{210} + \frac{(200 - 350)^2}{350} + \frac{(1000 - 840)^2}{840}$$

$$= 507.94$$

For degree of freedom 1, significance - 0.01

χ^2 value needed to reject hypothesis is 6.635.

The received value is above this value. Thus, we reject the hypothesis that the gender and preferred reading are independent.