

# BOARD QUESTION PAPER : MARCH 2019

## MATHEMATICS AND STATISTICS

### Note:

- (1) All questions are compulsory.
- (2) Figures to the right indicate full marks.
- (3) The Question paper consists of **30** questions divided into **FOUR** sections **A, B, C, D**.
  - **Section A** contains **6** questions of **1 mark** each.
  - **Section B** contains **8** questions of **2 marks** each. (One of them has internal option)
  - **Section C** contains **6** questions of **3 marks** each. (Two of them have internal options)
  - **Section D** contains **10** questions of **4 marks** each. (Three of them has internal options)
- (4) For each **MCQ**, correct answer must be written along with its **alphabet**,  
e.g., (A) ..... / (B) ..... / (C) ..... / (D) ..... etc.  
In case of **MCQs**, (Q. No. 1 to 6) evaluation would be done for the first attempt only.
- (5) Use of logarithmic table is allowed. Use of calculator is **not** allowed.
- (6) In L.P.P. only rough sketch of graph is expected. Graph paper is **not** necessary.
- (7) Start each section on new page only.

### SECTION A

Select and write the most appropriate answer from the given alternative for each question:

1. The principal solutions of  $\cot x = -\sqrt{3}$  are \_\_\_\_\_. [1]  
(A)  $\frac{\pi}{6}, \frac{5\pi}{6}$  (B)  $\frac{5\pi}{6}, \frac{7\pi}{6}$   
(C)  $\frac{5\pi}{6}, \frac{11\pi}{6}$  (D)  $\frac{\pi}{6}, \frac{11\pi}{6}$
2. The acute angle between the two planes  $x + y + 2z = 3$  and  $3x - 2y + 2z = 7$  is \_\_\_\_\_. [1]  
(A)  $\sin^{-1}\left(\frac{5}{\sqrt{102}}\right)$  (B)  $\cos^{-1}\left(\frac{5}{\sqrt{102}}\right)$   
(C)  $\sin^{-1}\left(\frac{15}{\sqrt{102}}\right)$  (D)  $\cos^{-1}\left(\frac{15}{\sqrt{102}}\right)$
3. The direction ratios of the line which is perpendicular to the lines with direction ratios  $-1, 2, 2$  and  $0, 2, 1$  are \_\_\_\_\_. [1]  
(A)  $-2, -1, -2$  (B)  $2, 1, 2$   
(C)  $2, -1, -2$  (D)  $-2, 1, -2$
4. If  $f(x) = (1 + 2x)^{\frac{1}{x}}$ , for  $x \neq 0$  is continuous at  $x = 0$ , then  $f(0) =$  \_\_\_\_\_. [1]  
(A)  $e$  (B)  $e^2$   
(C)  $0$  (D)  $2$
5.  $\int \frac{dx}{9x^2 + 1} =$  \_\_\_\_\_. [1]  
(A)  $\frac{1}{3} \tan^{-1}(2x) + c$  (B)  $\frac{1}{3} \tan^{-1} x + c$   
(C)  $\frac{1}{3} \tan^{-1}(3x) + c$  (D)  $\frac{1}{3} \tan^{-1}(6x) + c$

6. If  $y = ae^{5x} + be^{-5x}$ , then the differential equation is \_\_\_\_\_. [1]

(A)  $\frac{d^2y}{dx^2} = 25y$  (B)  $\frac{d^2y}{dx^2} = -25y$   
 (C)  $\frac{d^2y}{dx^2} = -5y$  (D)  $\frac{d^2y}{dx^2} = 5y$

### SECTION B

7. Write the truth values of the following statements: [2]

i. 2 is a rational number and  $\sqrt{2}$  is an irrational number.

ii.  $2 + 3 = 5$  or  $\sqrt{2} + \sqrt{3} = \sqrt{5}$

8. Find the volume of the parallelepiped, if the coterminus edges are given by the vectors  $2\hat{i} + 5\hat{j} - 4\hat{k}$ ,  $5\hat{i} + 7\hat{j} + 5\hat{k}$ ,  $4\hat{i} + 5\hat{j} - 2\hat{k}$ . [2]

OR

Find the value of p, if the vectors  $\hat{i} - 2\hat{j} + \hat{k}$ ,  $2\hat{i} - 5\hat{j} + p\hat{k}$  and  $5\hat{i} - 9\hat{j} + 4\hat{k}$  are coplanar.

9. Show that the points A(-7, 4, -2), B(-2, 1, 0) and C (3, -2, 2) are collinear. [2]

10. Write the equation of the plane  $3x + 4y - 2z = 5$  in the vector form [2]

11. If  $y = x^x$ , find  $\frac{dy}{dx}$ . [2]

12. Find the equation of tangent to the curve  $y = x^2 + 4x + 1$  at (-1, -2). [2]

13. Evaluate:  $\int \frac{e^x(1+x)}{\cos^2(xe^x)} dx$  [2]

14. Evaluate:  $\int_0^{\frac{\pi}{2}} \sin^2 x dx$  [2]

### SECTION C

15. In  $\triangle ABC$ , prove that

$$\sin\left(\frac{B-C}{2}\right) = \left(\frac{b-c}{a}\right) \cos\left(\frac{A}{2}\right) \quad [3]$$

OR

Show that  $\sin^{-1}\left(\frac{5}{13}\right) + \cos^{-1}\left(\frac{3}{5}\right) = \tan^{-1}\left(\frac{63}{16}\right)$

16. If A( $\vec{a}$ ) and B( $\vec{b}$ ) are any two points in the space and R( $\vec{r}$ ) be a point on the line segment AB

dividing it internally in the ratio m : n, then prove that  $\vec{r} = \frac{m\vec{b} + n\vec{a}}{m+n}$  [3]

17. The equation of a line is  $2x - 2 = 3y + 1 = 6z - 2$ , find its direction ratios and also find the vector equation of the line. [3]

18. Discuss the continuity of the function

$$f(x) = \frac{\log(2+x) \log(2-x)}{\tan x}, \quad \text{for } x \neq 0$$

$$= 1 \quad \text{for } x = 0$$

at the point  $x = 0$

[3]

19. The probability distribution of a random variable  $X$ , the number of defects per 10 meters of a fabric is given by

$x$	0	1	2	3	4
$P(X=x)$	0.45	0.35	0.15	0.03	0.02

Find the variance of  $X$ .

[3]

**OR**

For the following probability density function (p.d.f.) of  $X$ , find: (i)  $P(X < 1)$ , (ii)  $P(|X| < 1)$

$$\text{if } f(x) = \frac{x^2}{18}, \quad -3 < x < 3$$

$$0, \quad \text{otherwise}$$

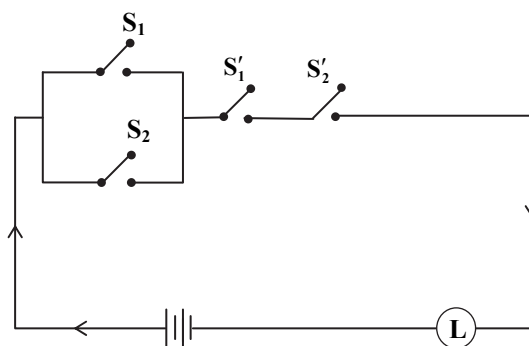
20. Given is  $X \sim B(n, p)$ .

If  $E(X) = 6$ ,  $\text{Var.}(X) = 4.2$ , find  $n$  and  $p$ .

[3]

### SECTION D

21. Find the symbolic form of the given switching circuit. Construct its switching table and interpret your result.



[4]

22. If three numbers are added, their sum is 2. If two times the second number is subtracted from the sum of first and third numbers we get 8 and if three times the first number is added to the sum of second and third numbers we get 4. Find the numbers using matrices.

[4]

23. In  $\Delta ABC$ , with usual notations prove that  $b^2 = c^2 + a^2 - 2ca \cos B$

[4]

**OR**

In  $\Delta ABC$ , with usual notations prove that

$$(a-b)^2 \cos^2\left(\frac{C}{2}\right) + (a+b)^2 \sin^2\left(\frac{C}{2}\right) = c^2.$$

24. Find 'p' and 'q' if the equation  $px^2 - 8xy + 3y^2 + 14x + 2y + q = 0$  represents a pair of perpendicular lines.

[4]

25. Maximize:  $z = 3x + 5y$  Subject to  $3x + y \leq 21,$   
 $x + 4y \leq 24,$   $x \geq 0, y \geq 0$  [4]  
 $x + y \leq 9,$

26. If  $x = f(t)$  and  $y = g(t)$  are differentiable functions of  $t$ , then prove that  $y$  is a differentiable function of  $x$  and

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}, \text{ where } \frac{dx}{dt} \neq 0$$

- Hence find  $\frac{dy}{dx}$  if  $x = a \cos^2 t$  and  $y = a \sin^2 t$ . [4]

27.  $f(x) = (x-1)(x-2)(x-3)$ ,  $x \in [0, 4]$ , find 'c' if LMVT can be applied. [4]

**OR**

A rod of 108 meters long is bent to form a rectangle. Find its dimensions if the area is maximum.

28. prove that :  $\int \frac{dx}{\sqrt{x^2 + a^2}} = \log \left| x + \sqrt{x^2 + a^2} \right| + c$  [4]

29. Show that:  $\int_0^{\frac{\pi}{4}} \log(1 + \tan x) dx = \frac{\pi}{8} \log 2$  [4]

30. Solve the differential equation:

$$\frac{dy}{dx} + y \sec x = \tan x$$
 [4]

**OR**

Solve the differential equation:

$$(x+y) \frac{dy}{dx} = 1$$