

BOARD QUESTION PAPER : MARCH 2018

Note:

- All questions are compulsory.
- Figures to the right indicate full marks.
- Graph of L.P.P. should be drawn on graph paper only.
- Use of logarithmic table is allowed.
- Answers to the questions of Section - I and Section - II should be written in only one answer book.
- Answer to every new question must be written on a new page.

SECTION – I

Q.1. (A) Select and write the appropriate answer from the given alternatives in each of the following sub-questions:

(6) [12]

- i. If $A = \begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix}$, then adjoint of matrix A is _____.

(A) $\begin{bmatrix} 1 & 3 \\ -4 & 2 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & -3 \\ -4 & 2 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 3 \\ 4 & -2 \end{bmatrix}$

(D) $\begin{bmatrix} -1 & -3 \\ -4 & 2 \end{bmatrix}$

- ii. The principal solutions of $\sec x = \frac{2}{\sqrt{3}}$ are

(A) $\frac{\pi}{3}, \frac{11\pi}{6}$

(B) $\frac{\pi}{6}, \frac{11\pi}{6}$

(C) $\frac{\pi}{4}, \frac{11\pi}{4}$

(D) $\frac{\pi}{6}, \frac{11\pi}{4}$

- iii. The measure of acute angle between the lines whose direction ratios are 3, 2, 6 and -2, 1, 2 is _____.

(A) $\cos^{-1}\left(\frac{1}{7}\right)$

(B) $\cos^{-1}\left(\frac{8}{15}\right)$

(C) $\cos^{-1}\left(\frac{1}{3}\right)$

(D) $\cos^{-1}\left(\frac{8}{21}\right)$

(B) Attempt any THREE of the following:

(6)

- Write the negations of the following statements:
 - All students of this college live in the hostel.
 - 6 is an even number or 36 is a perfect square.
- If a line makes angles α, β, γ with the co-ordinate axes, prove that $\cos 2\alpha + \cos 2\beta + \cos 2\gamma + 1 = 0$.
- Find the distance of the point (1, 2, -1) from the plane $x - 2y + 4z - 10 = 0$.
- Find the vector equation of the line which passes through the point with position vector $4\hat{i} - \hat{j} + 2\hat{k}$ and is in the direction of $-2\hat{i} + \hat{j} + \hat{k}$.
- If $\vec{a} = 3\hat{i} - 2\hat{j} + 7\hat{k}$, $\vec{b} = 5\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{c} = \hat{i} + \hat{j} - \hat{k}$, then find $\vec{a} \cdot (\vec{b} \times \vec{c})$.

Q.2. (A) Attempt any TWO of the following:**(6)[14]**

- i. Using vector method prove that the medians of a triangle are concurrent.
- ii. Using the truth table, prove the following logical equivalence:
 $p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$.
- iii. If the origin is the centroid of the triangle whose vertices are $A(2, p, -3)$, $B(q, -2, 5)$ and $R(-5, 1, r)$, then find the values of p, q, r .

(B) Attempt any TWO of the following:**(8)**

- i. Show that a homogeneous equation of degree two in x and y , i.e. $ax^2 + 2hxy + by^2 = 0$ represents a pair of lines passing through the origin if $h^2 - ab \geq 0$.
- ii. In $\triangle ABC$, prove that $\tan\left(\frac{C-A}{2}\right) = \left(\frac{c-a}{c+a}\right) \cot \frac{B}{2}$
- iii. Find the inverse of the matrix, $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ using elementary row transformations.

Q.3. (A) Attempt any TWO of the following:**(6)[14]**

- i. Find the joint equation of the pair of lines passing through the origin, which are perpendicular to the lines represented by $5x^2 + 2xy - 3y^2 = 0$.
- ii. Find the angle between the lines $\frac{x-1}{4} = \frac{y-3}{1} = \frac{z}{8}$ and $\frac{x-2}{2} = \frac{y+1}{2} = \frac{z-4}{1}$
- iii. Write converse, inverse and contrapositive of the following conditional statement:
If an angle is a right angle then its measure is 90° .

(B) Attempt any TWO of the following:**(8)**

- i. Prove that: $\sin^{-1}\left(\frac{3}{5}\right) + \cos^{-1}\left(\frac{12}{13}\right) = \sin^{-1}\left(\frac{56}{65}\right)$
- ii. Find the vector equation of the plane passing through the points $A(1, 0, 1)$, $B(1, -1, 1)$ and $C(4, -3, 2)$.
- iii. Minimize $Z = 7x + y$ subject to
 $5x + y \geq 5, x + y \geq 3, x \geq 0, y \geq 0$

BOARD QUESTION PAPER : MARCH 2018

Note:

- i. All questions are compulsory.
- ii. Figures to the right indicate full marks.
- iii. Graph of L.P.P. should be drawn on graph paper only.
- iv. Use of logarithmic table is allowed.
- v. Answers to the questions of Section - I and Section - II should be written in only one answer book.
- vi. Answer to every new question must be written on a new page.

SECTION – II

Q.4. (A) Select and write the appropriate answer from the given alternatives in each of the following sub-questions: (6)[12]

- i. Let the p.m.f. of a random variable X be –

$$P(x) = \frac{3-x}{10} \text{ for } x = -1, 0, 1, 2$$
$$= 0 \text{ otherwise}$$

Then E(X) is _____.

- (A) 1 (B) 2
(C) 0 (D) -1

- ii. If $\int_0^k \frac{1}{2+8x^2} dx = \frac{\pi}{16}$, then the value of k is _____.

- (A) $\frac{1}{2}$ (B) $\frac{1}{3}$
(C) $\frac{1}{4}$ (D) $\frac{1}{5}$

- iii. Integrating factor of the linear differential equation $x \frac{dy}{dx} + 2y = x^2 \log x$ is _____.

- (A) $\frac{1}{x^2}$ (B) $\frac{1}{x}$
(C) x (D) x^2

(B) Attempt any THREE of the following: (6)

- i. Evaluate: $\int e^x \left[\frac{\cos x - \sin x}{\sin^2 x} \right] dx$

- ii. If $y = \tan^2(\log x^3)$, find $\frac{dy}{dx}$.

- iii. Find the area of ellipse $\frac{x^2}{1} + \frac{y^2}{4} = 1$.

- iv. Obtain the differential equation by eliminating the arbitrary constants from the following equation: $y = c_1 e^{2x} + c_2 e^{-2x}$

- v. Given $X \sim B(n, p)$
If $n = 10$ and $p = 0.4$, find E(X) and Var. (X).

Q.5. (A) Attempt any TWO of the following:**(6)[14]**

- i. Evaluate: $\int \frac{1}{3 + 2\sin x + \cos x} dx$
- ii. If $x = a \cos^3 t$, $y = a \sin^3 t$,
show that $\frac{dy}{dx} = -\left(\frac{y}{x}\right)^{\frac{1}{3}}$
- iii. Examine the continuity of the function:
$$f(x) = \frac{\log 100 + \log(0.01 + x)}{3x}, \text{ for } x \neq 0$$
$$= \frac{100}{3} \quad \text{for } x = 0; \text{ at } x = 0$$

(B) Attempt any TWO of the following:**(8)**

- i. Find the maximum and minimum value of the function:
 $f(x) = 2x^3 - 21x^2 + 36x - 20$.
- ii. Prove that: $\int \frac{1}{a^2 - x^2} dx = \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + c$
- iii. Show that: $\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx$, if $f(x)$ is an even function.
 $= 0$, if $f(x)$ is an odd function.

Q.6. (A) Attempt any TWO of the following:**(6)[14]**

- i. If $f(x) = \frac{x^2 - 9}{x - 3} + \alpha$, for $x > 3$
 $= 5$, for $x = 3$
 $= 2x^2 + 3x + \beta$, for $x < 3$
is continuous at $x = 3$, find α and β .
- ii. Find $\frac{dy}{dx}$ if $y = \tan^{-1} \left(\frac{5x+1}{3-x-6x^2} \right)$
- iii. A fair coin is tossed 9 times. Find the probability that it shows head exactly 5 times.

(B) Attempt any TWO of the following:**(8)**

- i. Verify Rolle's theorem for the following function:
 $f(x) = x^2 - 4x + 10$ on $[0, 4]$
- ii. Find the particular solution of the differential equation:
 $y(1 + \log x) \frac{dx}{dy} - x \log x = 0$
when $y = e^2$ and $x = e$
- iii. Find the variance and standard deviation of the random variable X whose probability distribution is given below:

x	0	1	2	3
$P(X=x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$