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Equivariance Allows Handling Multiple Nuisance Variables When Analyzing Pooled Neuroimaging Datasets

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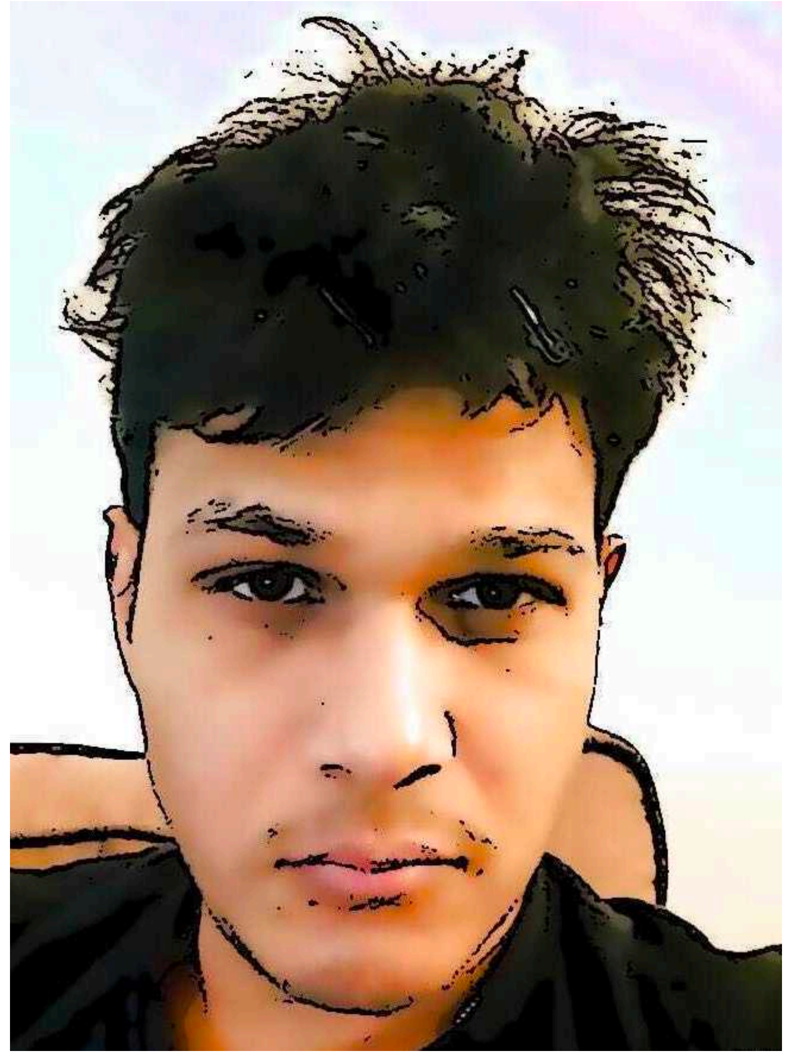

Rudra Chakraborty

Sathya Ravi

Vikas Singh

Dataset Pooling in Neuroimaging studies

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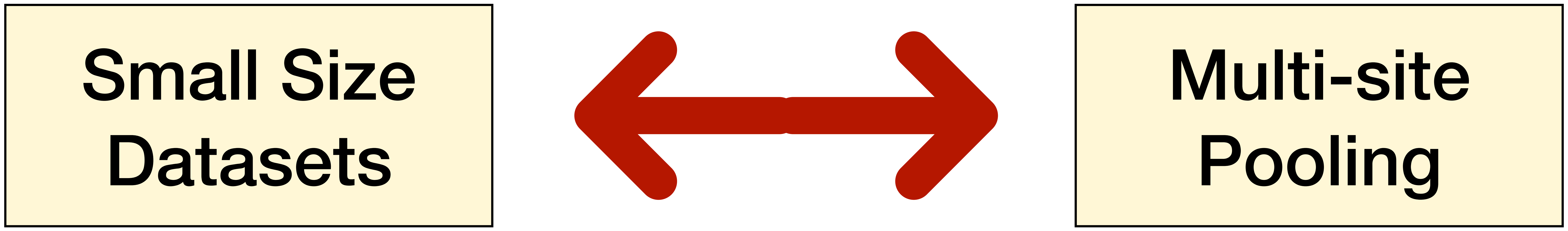

Dataset Pooling in Neuroimaging studies

Small Size Datasets

Multi-site Pooling

Improves statistical power: Better disease-pattern identification

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Models impacted with site-specific nuisance attributes

Nuisance Attribute types

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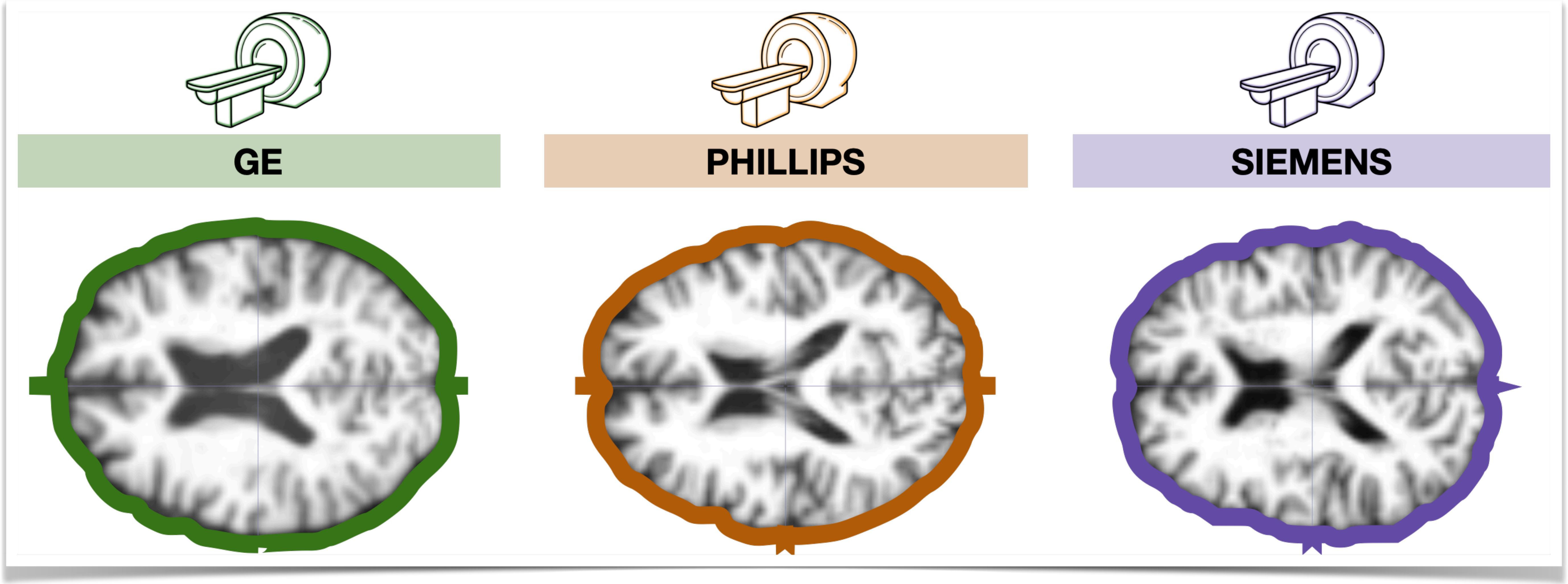
Data Acquisition Bias



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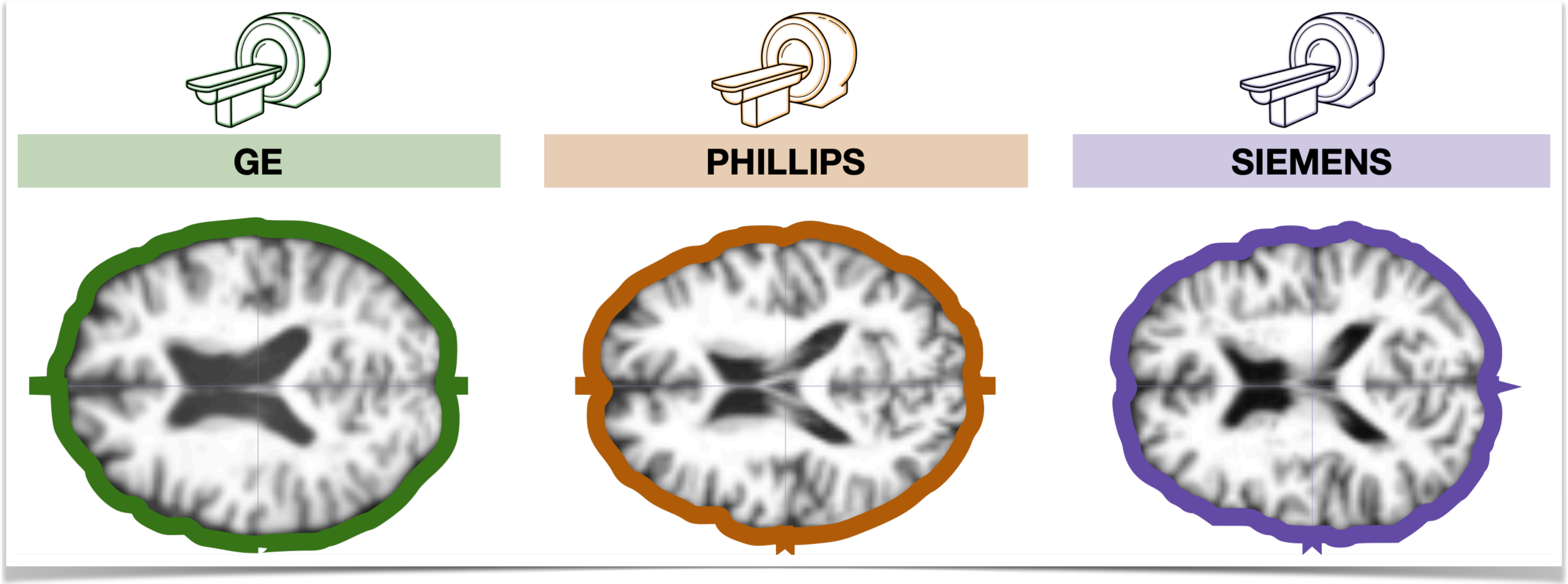

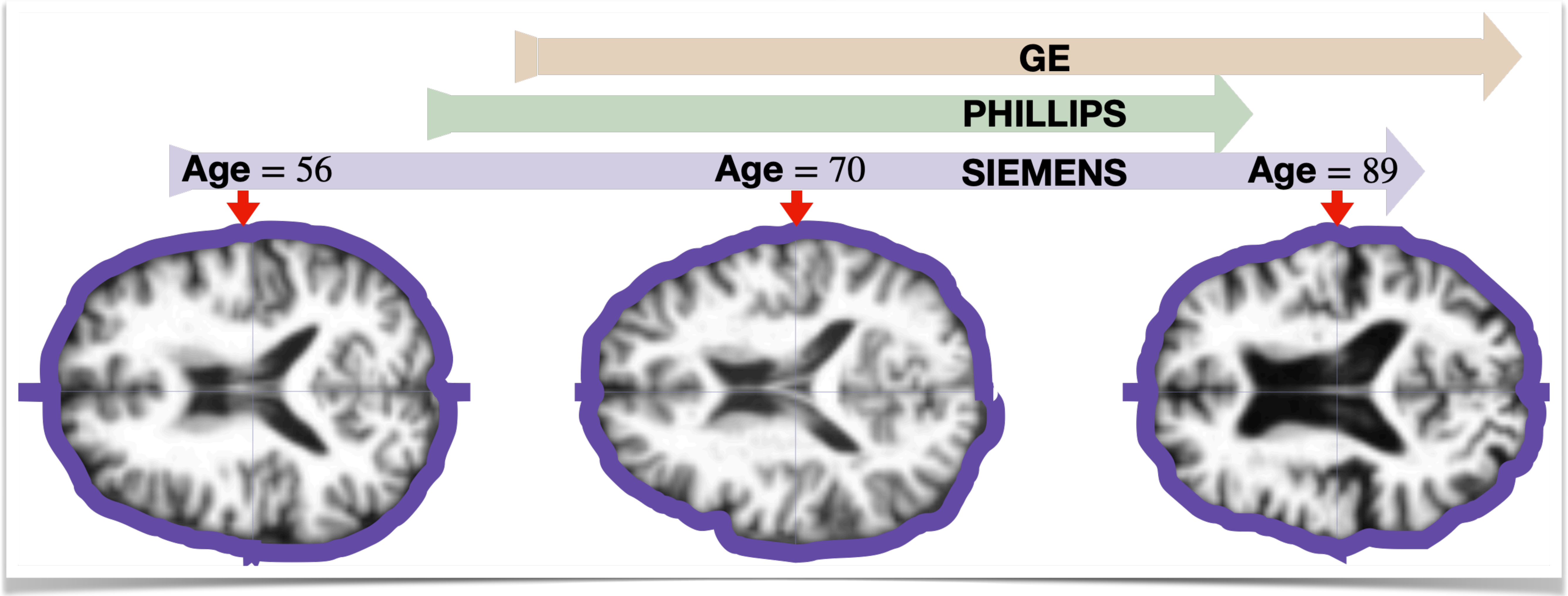

Population Bias

Research Question

Can we pool datasets in the presence of more than one nuisance attributes ?

Generic Invariant Representation Learning Framework

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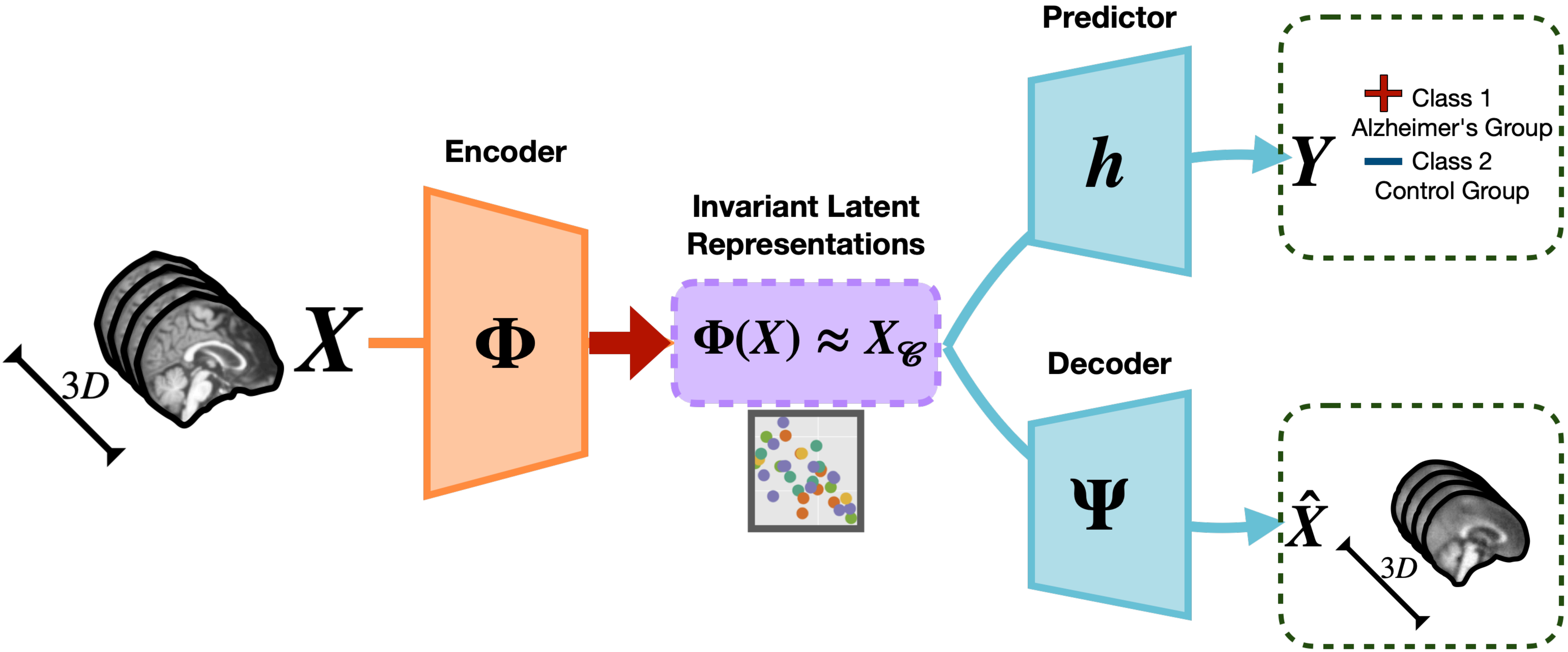

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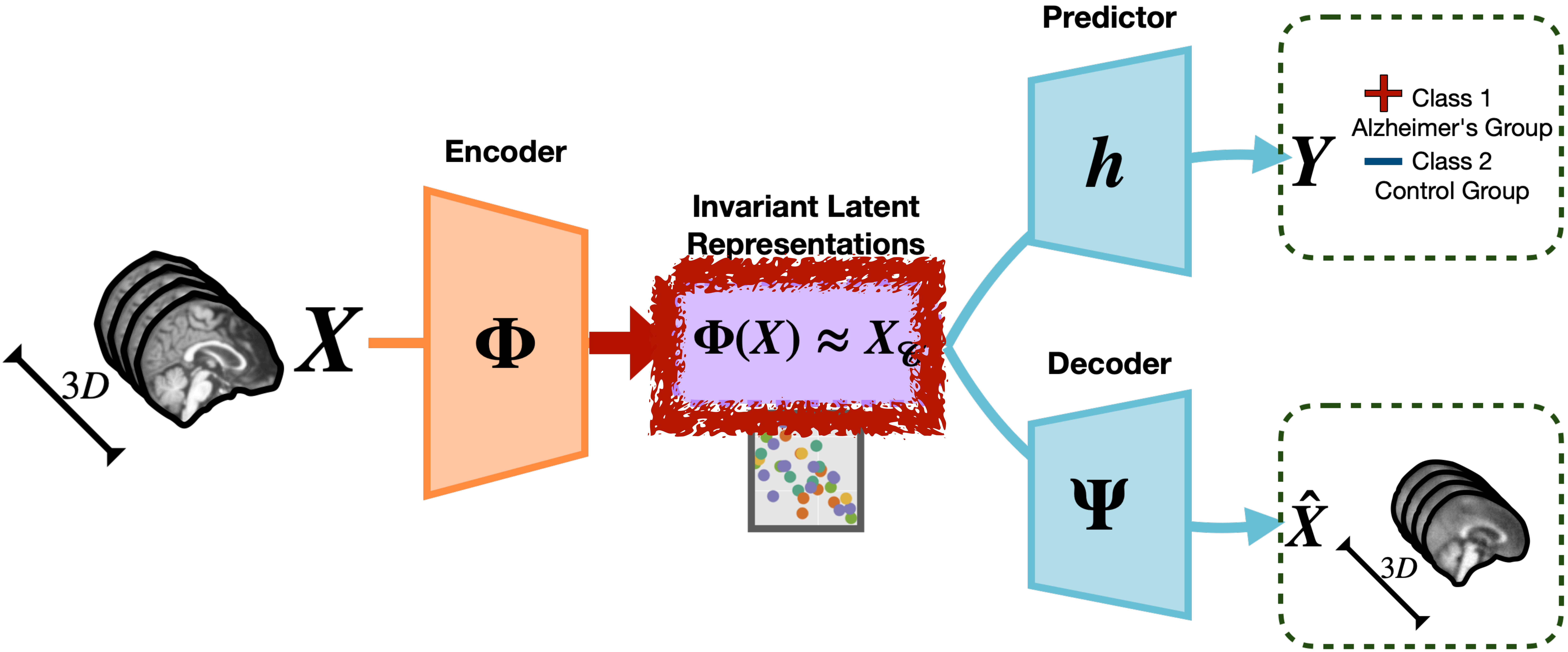

Problems with Multiple Nuisance Attributes

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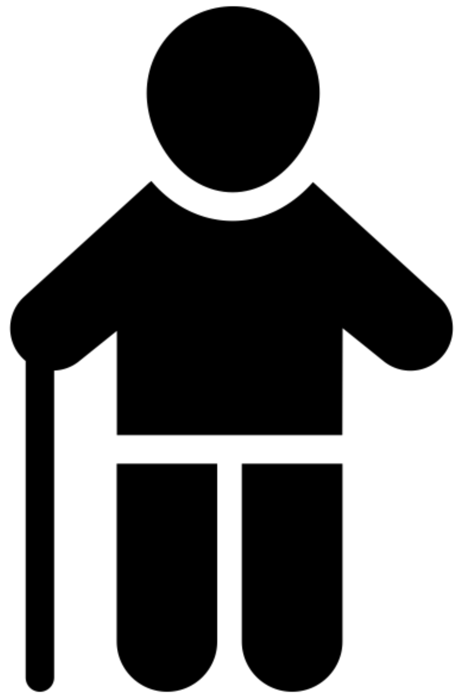
Nuisance 1:
Site (Scanner)



Nuisance 2:
Covariates (Age)

Closest Match of participants across sites

Discards Samples outside the common support

Causal Diagram + Invariance Condition = Optimization objective

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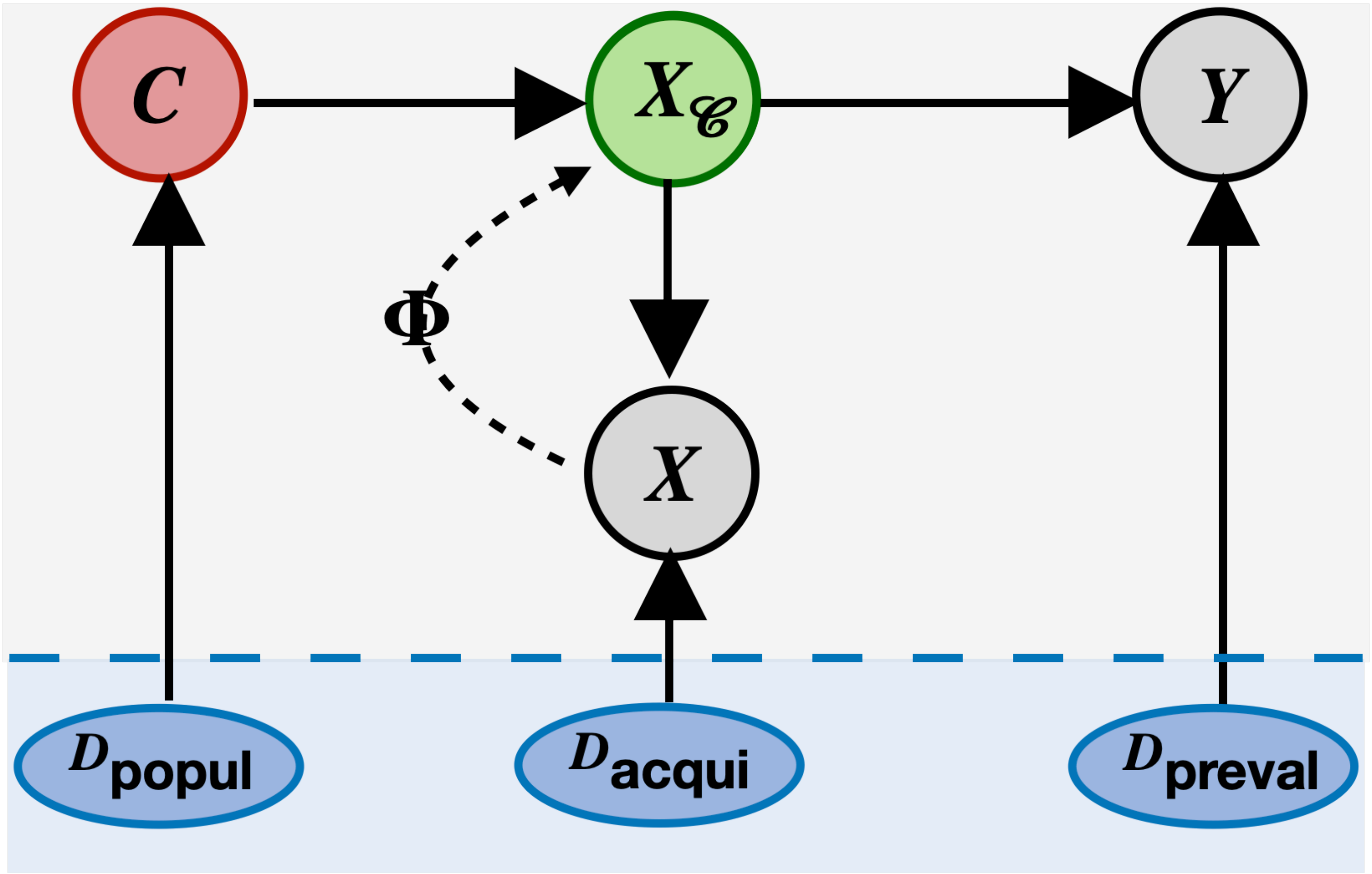
Causal Diagram



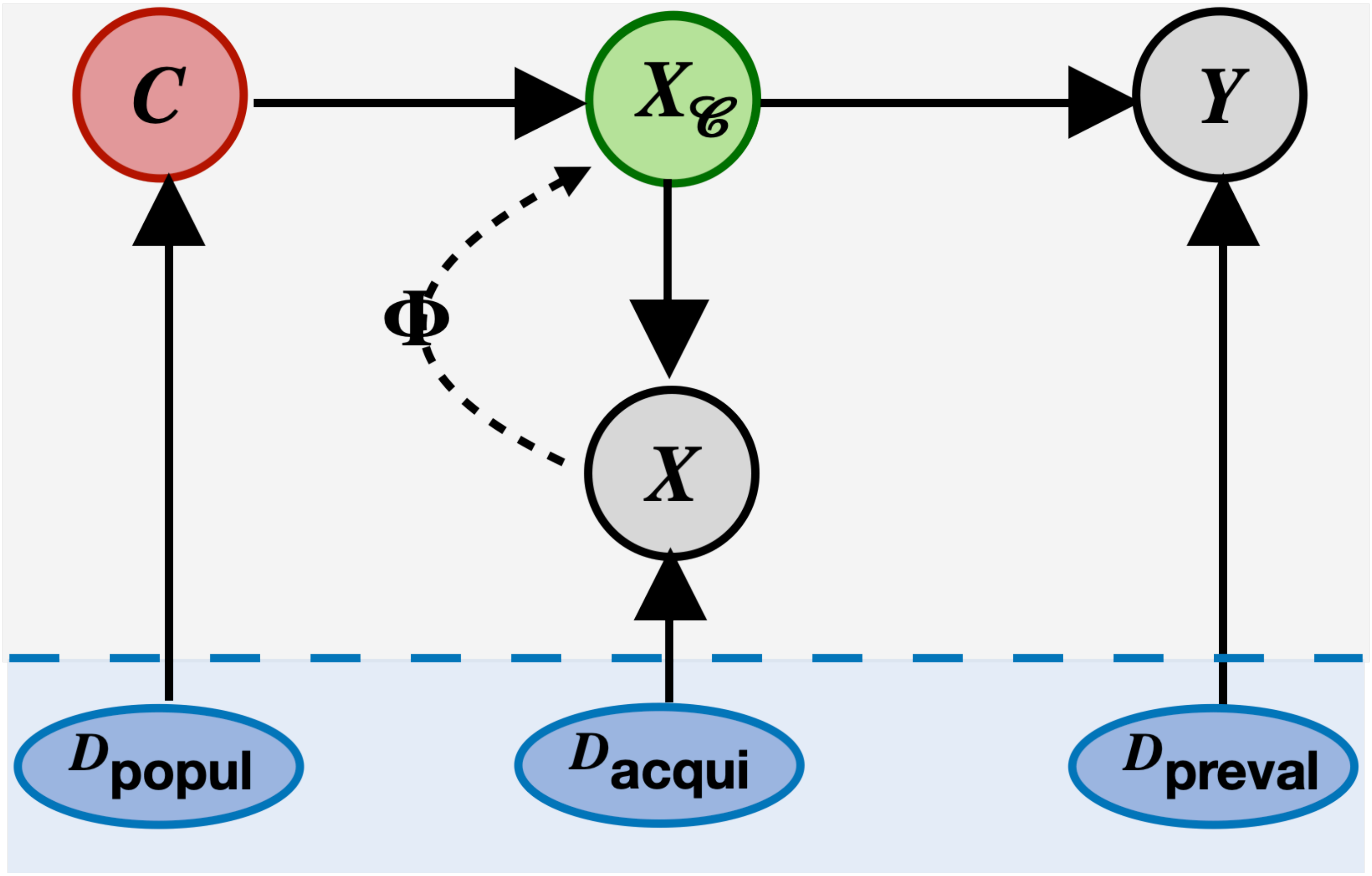
Invariance Condition

$X_{\mathscr{C}}$ does not change across the sites for the **same value** of C

$$X_{\mathscr{C}} \perp D \mid C$$

Causal Diagram + Invariance Condition = Optimization objective

Causal Diagram



Invariance Condition

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$$\min_{\Phi} \mathrm{MMD}\left(P_{\text{site}_1}\left(\Phi(X) \mid C\right), P_{\text{site}_2}\left(\Phi(X) \mid C\right)\right)$$

Improved Distribution Matching via Equivariant Mappings

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Equivariant Mappings!

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Equivariant Mappings!

Definition 1: A mapping $f: \mathscr{X} \to \mathscr{Y}$ is said to be G-equivariant iff

$$f(g \cdot x) = g \cdot f(x) \quad g \in \text{Group } G$$

Stage one - Equivariance to Age

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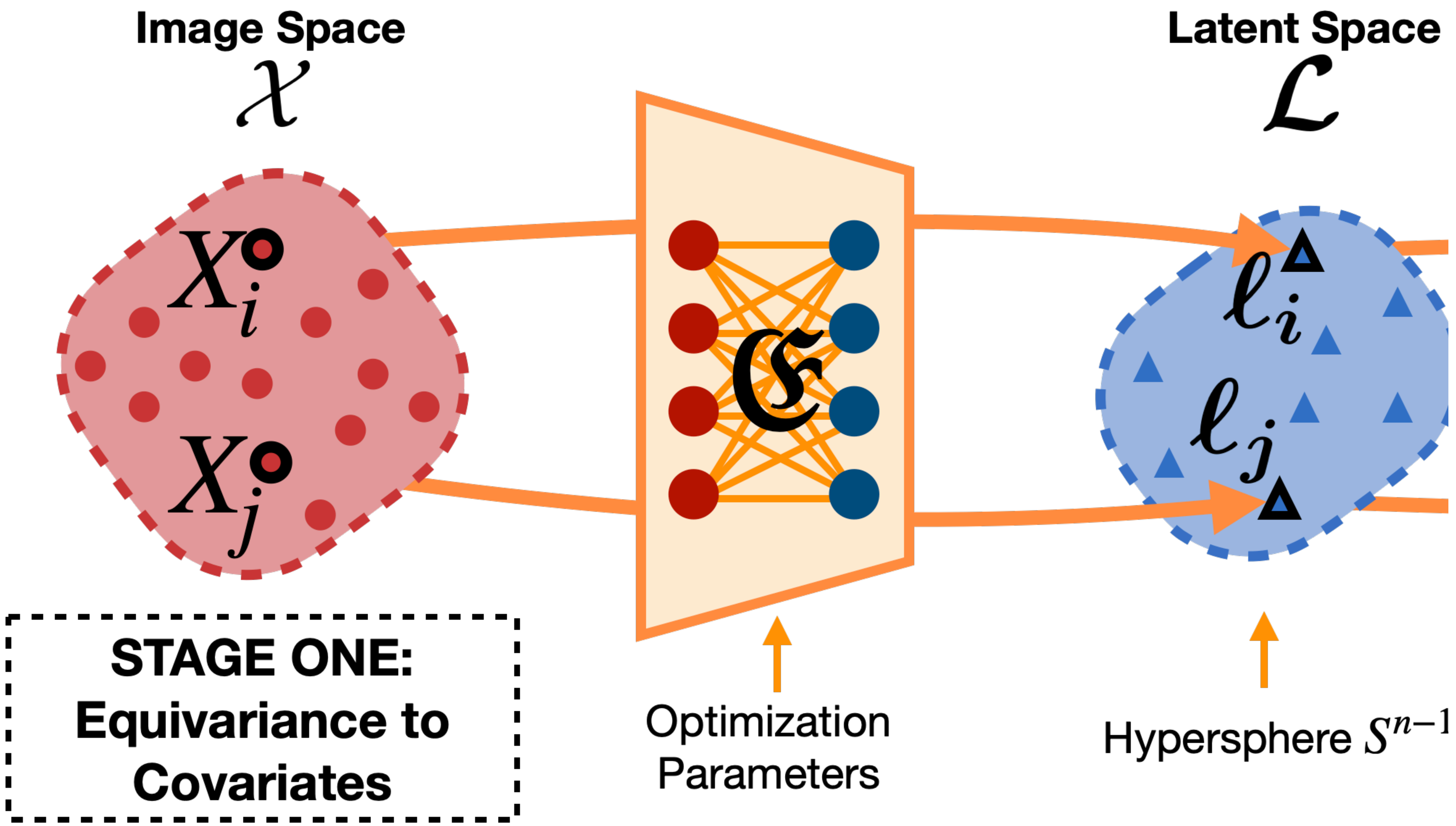

Stage one - Equivariance to Age

Stage two - Invariance to Site

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Step 1: Preserve equivariance from stage one with an equivariant mapping

Lemma 5: For a mapping $\tau : \mathscr{L} \to G/H$ and arbitrary mapping $b : \mathscr{L} \to Z$

$\Phi(\ell) = \tau(\ell) \cdot b(\tau(\ell)^{-1} \cdot \ell)$ is G-equivariant i.e, $\Phi(g \cdot \ell) = g\Phi(\ell)$

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Step 2: Optimize MMD loss alongside reconstruction and prediction losses

$$L_{\text{stage2}} = \sum \underbrace{\boxed{\|\ell - \Psi(\Phi(\ell))\|^2}}_{\text{Reconstruction loss}} + \underbrace{\boxed{\|Y - h(\Phi(\ell))\|^2}}_{\text{Prediction loss}} + \text{MMD}$$

subject to $\Phi(\ell) = \tau(\ell) \cdot b\left(\tau(\ell)^{-1} \cdot \ell\right)$

Experimental Results

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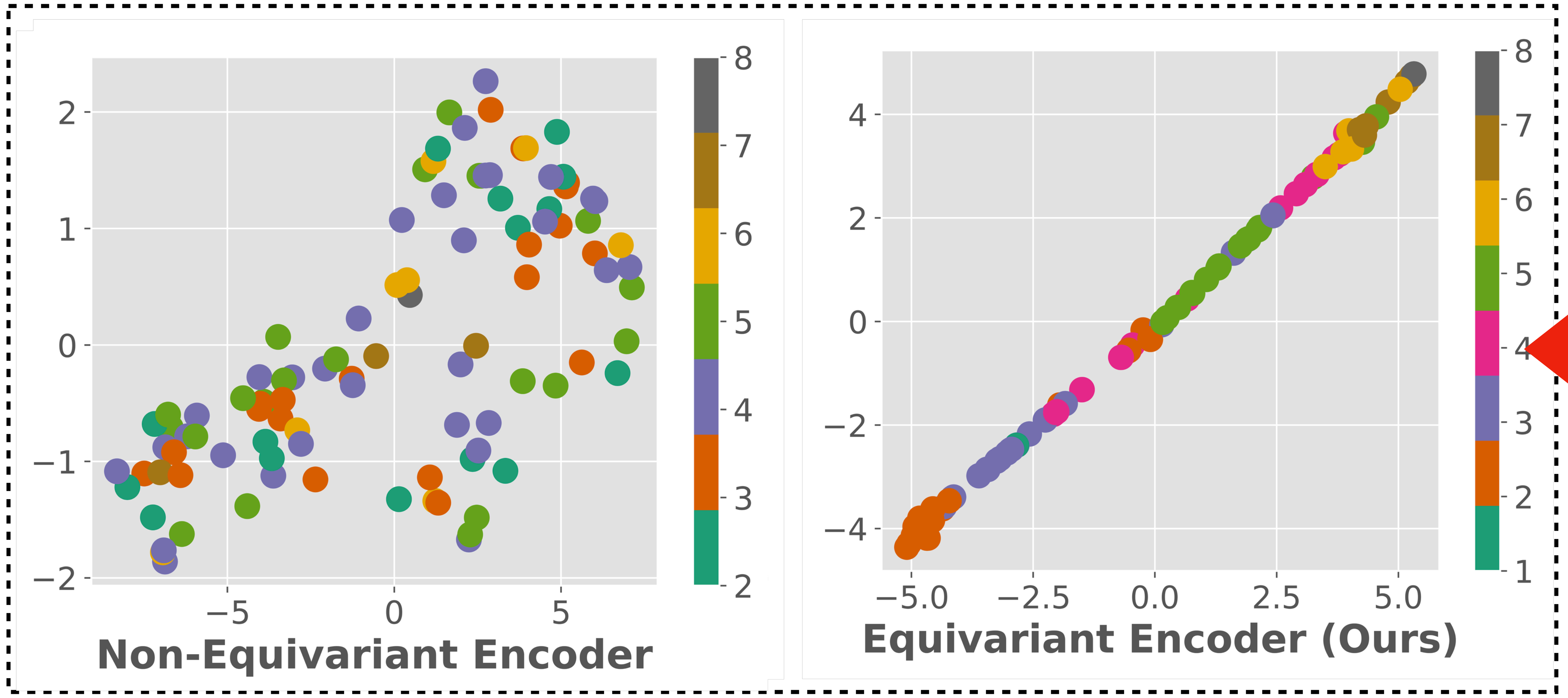

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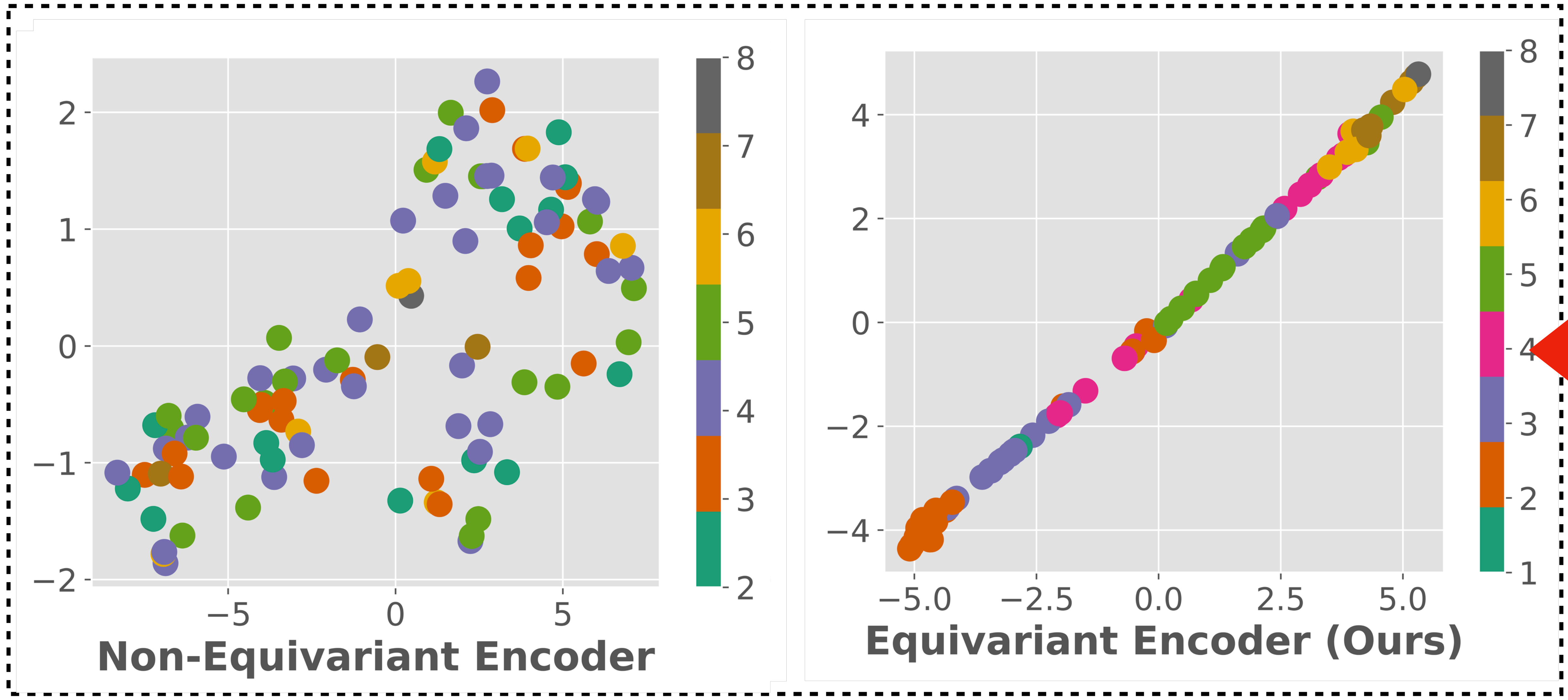
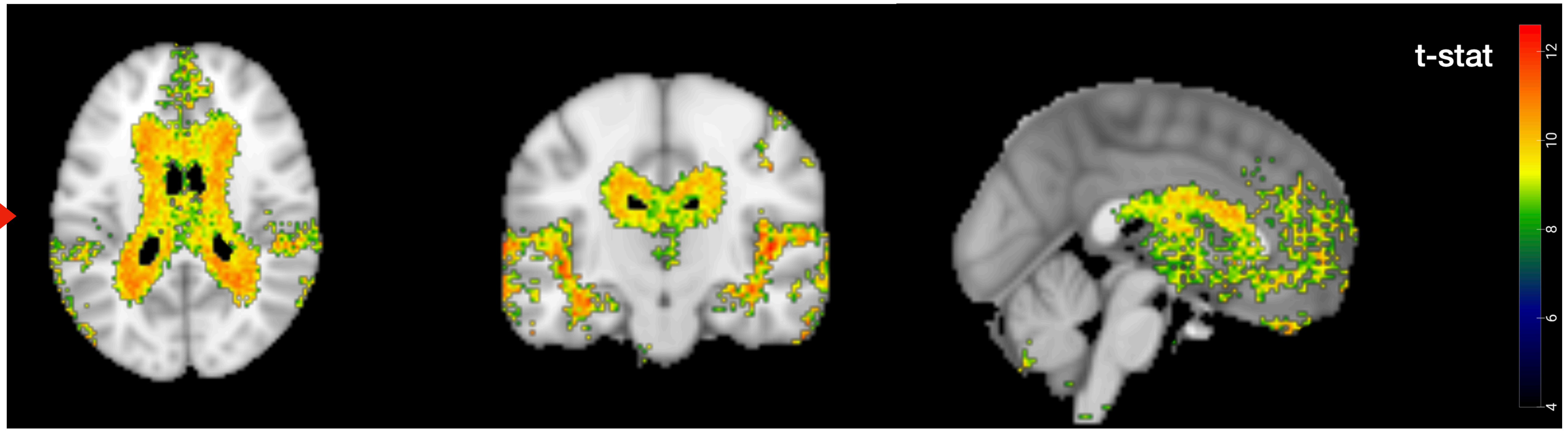


Equivariant encoder results in a monotonic trend in the latent features as covariate values are varied.

Preserves disease-specific signal as indicated by high density of significant voxels



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