**MODELING AND ANALYSIS OF A WARRANTY POLICY USING NEW AND RECONDITIONED PARTS**

**GROUP - F22\_AdvMEM\_Prj 3 (TEAM WARRANTERS)**

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# Introduction & Scope

Environmental protection has always been a hot topic of discussion in contemporary society. As technology advances and more things are invented and used, the environmental impact is becoming more severe. Waste is one of the main factors affecting the health of the environment. Recycling products and reducing waste are encouraged in society. Not only it can protect the health of the environment, but also it can generate a high amount of revenue. In 2013, the annual profit from the electronic waste recycling industry was about 90 million dollars.[1] Many companies choose to recycle their products nowadays. For example, Tesla has battery recycling. They design their batteries that are recoverable and recyclable. "Battery materials are refined and placed into a cell that still remain in the cell till the end of their lives and it can be recycled to recover its valuable materials to be reused over and over again." [2] This policy is established to manage the products at the end of their lives. Also, companies use recycled products to do the reparation. In some products, broken components are replaced by refurbished components. Companies like Apple have already listed this practice in their repair terms and conditions agreement.[3] From the customers' point of view, the quality of the product with "mixed components" should be satisfactory. As mentioned in the reference paper [4], the companies need to provide a significant price reduction and generous warranty coverage to showcase the quality of the product. From the company's point of view, it is more important to focus on making the most profit with a lower price and generous warranty. This problem is discussed in the reference paper. It mainly focuses on determining the optimal warranty policy when a component is randomly chosen from the mixture of new and reconditioned components for replacing a product that failed under the warranty period. In contrast, failed products are generally replaced by new ones or repaired using new components. Authors have built a mathematical model to maximize the manufacturer's expected profit. There are four variables considered in the paper: the warranty length, the sale price, the age of reconditioned components and the mixture ratio of reconditioned components to be used. The scope of this project is based on the reference paper to understand the mathematical formulas to build the model in Maple. We have also plotted the results for an optimal solution in 3D and contour plots. A detailed analysis of the experiments and numerical results are inferred in this project. Furthermore, we carried out some experiments to make various other inferences from the model. And, those results are then plotted on graphs in Excel.

# Mathematical Formulation

The formulas and the definitions are from the reference paper.

Decision Variables:

w: warranty period

p: price per unit product sold

𝜏: age of reconditioned components

𝜂: proportion of reconditioned components

Parameters:

𝛽: Weibull distribution shape parameter

𝜃: Weibull distribution scale parameter

ai: age coefficients for acquiring reconditioned components

A0: unit cost of product (not including warranty)

d1: price coefficient

d2: warranty coefficient

D1: demand amplitude factor

D2: warranty displacement constant

S0: cost base for component recovery

S1: ideal volume of recovered components to minimize fixed recovery costs

Functions:

(t): lifetime probability density function of new components

F(t): lifetime cumulative distribution function (cdf) of new components

F𝜏 (t, 𝜏): lifetime cdf of reconditioned components

Fm(t, 𝜏, 𝜂): lifetime cdf of mixed components

R(t): reliability of new components

R𝜏 (t, 𝜏): reliability of reconditioned components

Rm(t, 𝜏, 𝜂): reliability of the mixture of components

A(𝜏, 𝜂): acquisition cost for each component

N(w, 𝜏, 𝜂): expected number of failures during warranty for each unit sold

Cr(w, 𝜏, 𝜂): reserve warranty cost per unit product

Cu(w, 𝜏, 𝜂): cost per unit product

D(w, p): total demand

S(w, p, 𝜏, 𝜂): recovery cost

𝜋(w, p, 𝜏, 𝜂): profit per unit product sold

П (w, p, 𝜏, 𝜂): total profit

## Formulas:

Reliability function for a mixed population, when 𝜏=0 means the component is considered as new:

|  |  |  |
| --- | --- | --- |
|  | | (1) |
|  | (2) | |
|  | () | |
|  | () | |
|  | () | |
|  |  | |
|  | (6) | |

where,

The probability that the initial (new) product failed once with probability F(w) was repaired with one component from the mixture that did not fail during a renews warranty length w:

|  |  |
| --- | --- |
|  | (7) |

The probability of exactly k replacements/failures performed during the warranty period is given according to the following geometric distribution:

|  |  |
| --- | --- |
|  | (8) |

The quantity function:

|  |  |
| --- | --- |
|  | (9) |

Substituting Eq.(5) in the result yields the following:

|  |  |
| --- | --- |
|  | (10) |

Acquisition cost of component of age is defined as follows:

|  |  |
| --- | --- |
|  | (11) |

The average acquisition of a component, randomly picked from the pool of replacement components, is then a function of its age and the mixture ratio and evaluated to the following:

|  |  |
| --- | --- |
|  | (12) |

The expected reserve fund per unit is as follows:

|  |  |
| --- | --- |
|  | (13) |

Combine Eqs (10) and (12) yields the following:

|  |  |
| --- | --- |
|  | (14) |

Frist case (Eq(14) reduces to the result obtained by [14,36]:

|  |  |
| --- | --- |
|  | (15) |

In the case where (), Eq.(14) reduces to the result in [26]:

|  |  |
| --- | --- |
|  | (16) |

Total unit cost becomes the following:

|  |  |
| --- | --- |
|  | (17) |

The profit per unit of each product sold with a piece p is given by the following:

|  |  |
| --- | --- |
|  | (18) |

The expected forecast sales volume/ total demand:

|  |  |
| --- | --- |
|  | (19) |

Recovery cost:

|  |  |
| --- | --- |
|  | (20) |

The optimal model:

|  |  |
| --- | --- |
|  | (21) |

When,

|  |  |
| --- | --- |
|  |  |

|  |  |
| --- | --- |
|  | (22) |
|  |  |
|  |  |

# Experiments and Results

The above equations are framed in Maple in the attached Maple file. We have reached the optimal solution for the total net profit, as given in the reference paper, using a non-linear programming solver (NLPSolve). The values of all the parameters used in the equation was obtained from a Canadian aerospace remanufacturer for an electromechanical system: -

𝛽 = 0.95; 𝜃 = 2.0; A0 = 5.0; a1= 3.3; a2 = 0.7; d1 = 2.6; d2 = 1.9; D1 = 10^5; D2 = 2.0; S0 = 2.3; S1 = 91

The solution obtained from our code in the attached file is as follows: -

5363.85257287296281

= 0.409087089329392

p\* = 9.45058502784974

= 1.12211000872754

w\* = 0.273647411064561

The plots of total profit as a function of and for w\* = 0.27 and p\* = 9.45 are also drawn in Maple as follows: -

Chart

Description automatically generated

Figure Total profit contour plot

Chart, surface chart

Description automatically generated

Figure 3D plot

Further, we have experimented changing the following parameters to observe the behaviour of optimal solution obtained: -

1. The scale parameter, θ
2. The shape parameter, β
3. Discount rate, a1
4. Warranty period, w
5. Price coefficient, d1
6. Warranty coefficient, d2

## The impact of scale parameter θ:

The impact on the optimal solution is shown in the below table and plot made in the attached Excel file: -

Table Optimal Solutions for changes in θ values

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| θ | П\* | η\* | p\* | τ\* | w\* | N\* |
| 0.1 | 4912.994792 | 1 | 8.149032088 | 1.099438408 | 0.00480029 | 0.056584011 |
| 0.2 | 4935.805811 | 1 | 8.173133296 | 1.099862036 | 0.009623623 | 0.056795793 |
| 0.3 | 4958.619644 | 1 | 8.197388164 | 1.100297374 | 0.014478632 | 0.057008713 |
| 0.5 | 5004.287062 | 1 | 8.246431277 | 1.101191901 | 0.024291653 | 0.057438954 |
| 0.7 | 5050.028816 | 1 | 8.2962447 | 1.102111001 | 0.034249199 | 0.057875806 |
| 1 | 5118.807539 | 1 | 8.372508129 | 1.103527697 | 0.04946996 | 0.058544646 |
| 1.2 | 5164.780706 | 1 | 8.4244324 | 1.104493256 | 0.059814507 | 0.059000161 |
| 1.5 | 5233.932637 | 1 | 8.504022895 | 1.105973435 | 0.075640077 | 0.059698703 |
| 1.8 | 5303.324918 | 1 | 8.585755015 | 1.107489144 | 0.091851514 | 0.060416543 |
| 1.9 | 5326.510662 | 1 | 8.613493963 | 1.108002133 | 0.097344136 | 0.0606603 |
| 1.95 | 5339.551342 | 0.617819261 | 8.961791882 | 1.114149436 | 0.169153686 | 0.102126889 |
| 2 | 5363.852573 | 0.409086981 | 9.450584918 | 1.122109856 | 0.273647461 | 0.16193954 |
| 2.1 | 5434.5293 | 0.281732086 | 10.16592001 | 1.132357168 | 0.434974831 | 0.249563044 |
| 2.3 | 5634.809626 | 0.192695084 | 11.32053363 | 1.145705182 | 0.719799325 | 0.39116819 |
| 2.5 | 5891.096331 | 0.152806401 | 12.29911533 | 1.154349236 | 0.989519273 | 0.511329637 |
| 3 | 6701.021121 | 0.104819123 | 14.28845407 | 1.16579078 | 1.643311282 | 0.755964828 |
| 4 | 8375.38837 | 0.095592452 | 12.97825973 | 1.151988503 | 1.8 | 0.593589314 |

## The impact of price coefficient d1:

The impact on the optimal solution is shown in the below table and plot made in the attached Excel file: -

Table 2 Optimal Solutions for changes of d1 values

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| d1 | П\* | η\* | p\* | τ\* | w\* | D\* |
| 2 | 25964.66611 | 0.028375575 | 27.20828424 | 1.216806383 | 2.001971418 | 2483.496747 |
| 2.1 | 18984.55066 | 0.040466119 | 21.41620114 | 1.199167678 | 1.598140702 | 1828.055396 |
| 2.2 | 14208.08951 | 0.058181675 | 17.46930424 | 1.18288398 | 1.260304558 | 1746.562722 |
| 2.5 | 6675.83541 | 0.208491466 | 10.85883941 | 1.138086433 | 0.493662379 | 1460.60578 |
| 2.6 | 5363.852573 | 0.409087202 | 9.450584332 | 1.122109205 | 0.273647311 | 1385.419719 |
| 2.61 | 5252.365241 | 0.450554006 | 9.310925744 | 1.120263787 | 0.249883293 | 1380.487917 |
| 2.63 | 5038.892677 | 0.572505243 | 9.016327113 | 1.116144945 | 0.198111874 | 1374.569175 |
| 2.69 | 4466.434897 | 1 | 8.506618525 | 1.10928 | 0.118435439 | 1313.3568 |

## The impact of warranty coefficient d2:

The impact on the optimal solution is shown in the below table and plot made in the attached Excel file: -

Table 3 Optimal Solutions for changes of d2 values

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| d2 | П\* | η\* | p\* | τ\* | w\* |
| 0.5 | 1884.405671 | 1 | 8.428674333 | 1.104300719 | 0.26358546 |
| 1 | 2795.979965 | 1 | 8.597425405 | 1.107483226 | 0.191238419 |
| 1.2 | 3246.117879 | 1 | 8.631414625 | 1.108171957 | 0.167345626 |
| 1.4 | 3755.690766 | 1 | 8.649466326 | 1.10856442 | 0.146042599 |
| 1.5 | 4035.412307 | 1 | 8.653301449 | 1.108662328 | 0.13630393 |
| 1.6 | 4333.333028 | 1 | 8.654105675 | 1.10870065 | 0.127142449 |
| 1.7 | 4650.747097 | 1 | 8.652198766 | 1.108686003 | 0.118534367 |
| 1.9 | 5363.852573 | 0.409087168 | 9.450584992 | 1.122109664 | 0.273647368 |
| 1.91 | 5405.601794 | 0.38200583 | 9.547120498 | 1.123598954 | 0.293051105 |
| 1.92 | 5448.12066 | 0.358758293 | 9.641380079 | 1.125026666 | 0.311870428 |
| 1.95 | 5580.291056 | 0.304560912 | 9.91529532 | 1.129038447 | 0.365835325 |
| 2 | 5816.172259 | 0.244096969 | 10.35727194 | 1.135107525 | 0.450625977 |
| 2.1 | 6349.807098 | 0.173172165 | 11.23355577 | 1.14587113 | 0.61060068 |
| 2.3 | 7705.260492 | 0.103226242 | 13.07485493 | 1.164345692 | 0.91500634 |
| 2.5 | 9553.591836 | 0.0674872 | 15.1210911 | 1.180394436 | 1.211541412 |
| 3 | 17751.60597 | 0.026534075 | 21.49917184 | 1.214351202 | 1.943343033 |

## The impact of shape parameter (β), discount rate (a1) and warranty period (w):

Those experiments are already done in the reference paper. We have done it in our attached Maple file to verify our model and their results and graphs are plotted in the attached Excel file.

# Analysis and discussion of the results

## The impact of scale parameter θ:

In the reference paper, authors tested β to check how β values impact the solutions. In this project, we want to check how θ can impact the solutions. *Table 1* shows, the optimal values of П\*, η\*, p\*, τ\*, w\*. Based on the results, *Figure 3* shows the graph that is made in Excel. It has three cases for θ value, when θ<1.9, 1.9 <θ<3 and θ>3. When θ <1.9, the η remains at one, and the values of N do not vary much. It means the number of failures during warranty for each unit sold has not increased. However, between 1.9 <θ<3, the number of failures dramatically increased. The new components are always better than the used component. After θ>3, the number of failure rates start to decrease.

Chart, line chart

Description automatically generated

Figure 3 Plot of the impact of scale parameter θ

## The impact of price coefficient d1:

In this experiment, we want to check if d1 values impacts the solution. d1 is the rate of decrease of the sales volume with the increasing price of the product. *Table 2* shows the variations of profits, and the decision variables according to the change of d1. *Figure 4* shows the graph that is made based on the results. When the value of d1 is increasing, all decision variables decrease. It also impacts the profit. However, the total demand stays relatively high after d1>2.1. It still decreases but by a small number.

Chart, line chart

Description automatically generated

Figure 4 Plot of Impact of Price Coefficient d1

## The impact of warranty coefficient d2:

Chart, line chart

Description automatically generated In this experiment, we changed the value of d2 to check the impact on the values of П\*, η\*, p\*, τ\*, w\*. d2 is the sales volume increase rate with the increasing warranty length. *Table 3* shows the results. To make it clearer, *Figure 5* is made in Excel. This experiment can also be divided into two phases, d2<1.7 and d2>1.7. When d2 <1.7, the optimal value of η stays at 1. From the conclusion that we got from the previous experiment; it means the new component is worse than the used component. Also, the length of the warranty decreases. However, after d2>1.7 the η value starts to decrease and the warranty length becomes longer. The value of П \* keeps increasing while the value of d2 increases.

Figure 5 Plot of Impact of Warranty Coefficient d2

# Recommendations for improvement of the model

* We observed from the above analysis that changing the value of price coefficient d1 from 2.6 to 2.5, the net profit is drastically increased from 5364 to 6678. Also, the warranty length w is increased from 0.27 to 0.49. That is certainly a considerable change of values. Therefore, one of our recommendations would be taking the d1 value as 2.5.
* Also, changing the value of warranty coefficient d2 from 1.9 to 2.0, the net profit is drastically increased from 5364 to 5816. Also, the warranty length w is increased from 0.27 to 0.45. That is also a satisfactory change of values. We also recommend taking the d1 value as 2.0.
* Furthermore, increasing the scale parameter value, θ increases the net profit value significantly without affecting the other parameters much. As we can see in the impact of scale parameter experiments, from θ = 3 to 4, net profit increases from 6701 to 8375 and the price per unit is also decreased from 14.3 to 13. Moreover, the rest of the values remain almost the same. So, increasing θ would be our other recommendation.

# Conclusions

The model for optimal warranty policy was replicated from the reference paper provided. Further, it is solved using a non-linear programming solver (NLPSolve) in Maple. Furthermore, the results obtained exactly matched the solution in the paper. The plots obtained are accurate as well.

Some of the experiments are replicated from the paper to check the exactness of the model. The results found are perfectly the same as in the paper. And we did some other experiments varying the values of the scale parameter (θ), price coefficient (d1) and warranty coefficient (d2). Further, the inferences from those experiments were noted and based on the analysis, we thought of some recommendations for the improvements in the model.

# References

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