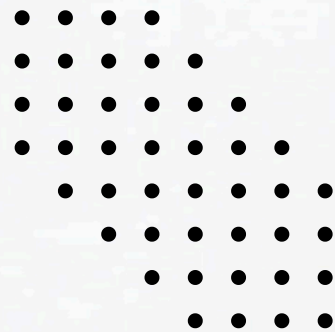




# **Inferential Statistics Report**

**Presented by :**  
Sanjay Rajan J



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# PROBLEM 1

## QUESTION:

A physiotherapist with a male football team is interested in studying the relationship between foot injuries and the positions at which the players play from the data collected.

TABLE 1: Team data

	Striker	Forward	Attacking Midfielder	Winger	<b>Total</b>
Players Injured	45	56	24	20	<b>145</b>
Players Not Injured	32	38	11	9	<b>90</b>
<b>Total</b>	<b>77</b>	<b>94</b>	<b>38</b>	<b>29</b>	<b>235</b>

Based on the above data, answer the following questions.

1.1 What is the probability that a randomly chosen player would suffer an injury?

1.2 What is the probability that a player is a forward or a winger?

1.3 What is the probability that a randomly chosen player plays in a striker position and has a foot injury?

1.4 What is the probability that a randomly chosen injured player is a striker?

### **1.1 What is the probability that a randomly chosen player would suffer an injury?**

#### **Solution:**

Let  $P(I)$  be the Probability of a randomly chosen player would suffer from an injury.

$$P(I) = \text{Total no. of Players injured} / \text{Total no. of players}$$

$$\text{Total no. of Players injured} = 145$$

$$\text{Total no. of players} = 235$$

$$P(I) = 145/235 = 0.617$$

**Probability of a randomly chosen player would suffer from an injury  
= 0.617 (61.7% chance)**

### **1.2 What is the probability that a player is a forward or a winger?**

#### **Solution:**

Let  $P(FW)$  be the Probability of a randomly chosen player is a forward or a winger.

$$P(FW) = (\text{Total Forward Players} + \text{Total Wingers}) / \text{Total no. of players}$$

$$\text{Total no. of Forward Players} = 94$$

$$\text{Total no. of Wingers} = 29$$

$$\text{Total Forward Players} + \text{Total Wingers} = 123$$

$$\text{Total no. of players} = 235$$

$$P(FW) = 123/235 = 0.523$$

**Probability of a randomly chosen player is a forward or a winger  
= 0.523 (52.3% chance)**

### **1.3 What is the probability that a randomly chosen player plays in a striker position and has a foot injury?**

#### **Solution:**

Let  $P(SI)$  be the Probability of a randomly chosen player plays in a striker position and has a foot injury.

$$P(SI) = \text{Total no. of injured Strikers} / \text{Total no. of players}$$

$$\text{Total no. of injured Strikers} = 45$$

$$\text{Total no. of players} = 235$$

$$P(SI) = 45/235 = 0.191$$

**Probability of a randomly chosen player plays in a striker position and has a foot injury = 0.191 (19.1% chance)**

### **1.4 What is the probability that a randomly chosen injured player is a striker?**

#### **Solution:**

Let  $P(IS)$  be the probability of a randomly chosen injured player is a striker.

$$P(IS) = \text{Total no. of injured Strikers} / \text{Total no. of Injured players}$$

$$\text{Total no. of injured Strikers} = 45$$

$$\text{Total no. of Injured players} = 145$$

$$P(SI) = 45/145 = 0.310$$

**Probability of a randomly chosen injured player is a striker = 0.310 (31% chance)**



## PROBLEM 2

### QUESTION:

The breaking strength of gunny bags used for packaging cement is normally distributed with a mean of 5 kg per sq. centimeter and a standard deviation of 1.5 kg per sq. centimeter. The quality team of the cement company wants to know the following about the packaging material to better understand wastage or pilferage within the supply chain; Answer the questions below based on the given information; **(Provide an appropriate visual representation of your answers, without which marks will be deducted)**

2.1 What proportion of the gunny bags have a breaking strength of less than 3.17 kg per sq cm?

2.2 What proportion of the gunny bags have a breaking strength of at least 3.6 kg per sq cm.?

2.3 What proportion of the gunny bags have a breaking strength between 5 and 5.5 kg per sq cm.?

2.4 What proportion of the gunny bags have a breaking strength NOT between 3 and 7.5 kg per sq cm.?

**Given:**

Mean ( $\mu$ ) = 5

Standard Deviation ( $\sigma$ ) = 1.5

**2.1 What proportion of the gunny bags have a breaking strength of less than 3.17 kg per sq.cm?**

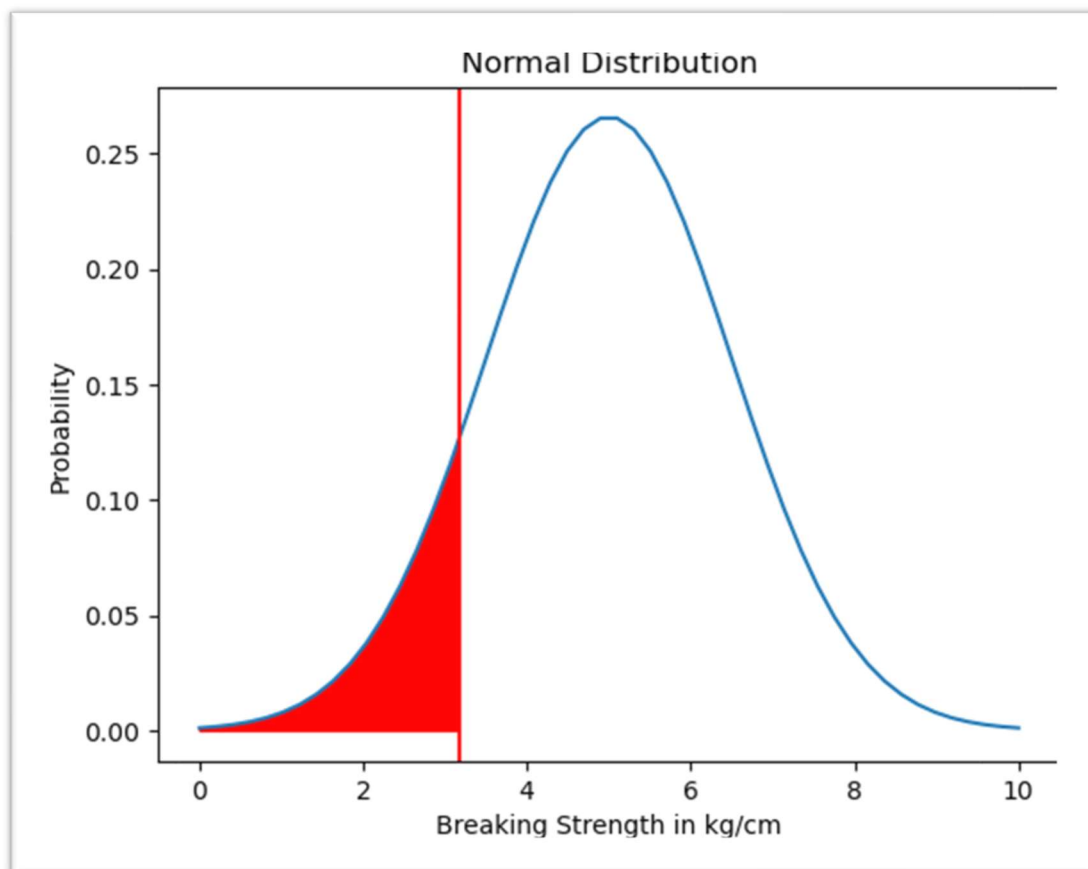
**Solution:**

To calculate the probability density, we can use the norm.cdf function of scipy.stats in python.

$$P(X < 3.17) = \text{norm.cdf}(3.17, \mu, \sigma) = 0.1112$$

**Proportion of the gunny bags have a breaking strength of less than 3.17 kg per sq.cm = 11.12%**

**Fig.1 Graph 2.1**



**2.2 What proportion of the gunny bags have a breaking strength of at least 3.6 kg per sq cm.?**

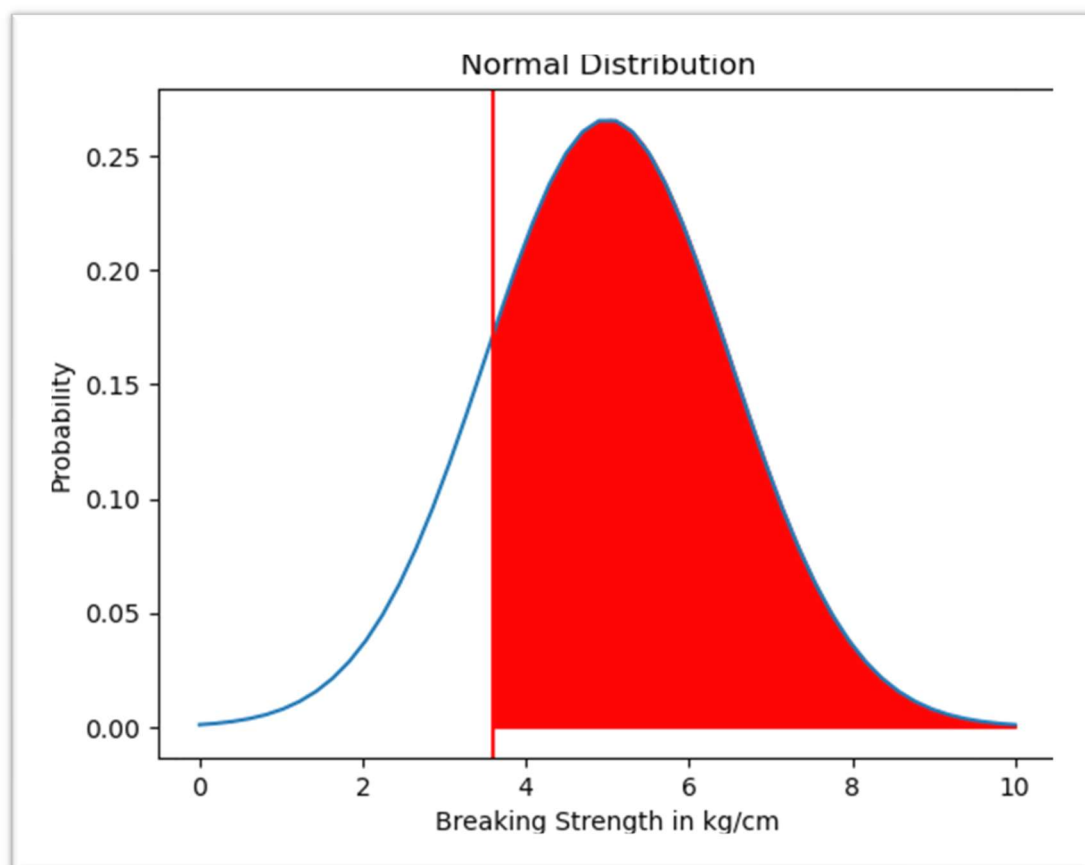
**Solution:**

To calculate the probability density, we can use the norm.cdf function of scipy.stats in python

$$P(X \geq 3.6) = 1 - P(X < 3.6) = 0.8247$$

**Proportion of the gunny bags have a breaking strength of at least 3.6 kg per sq.cm = 82.47%**

**Fig.2 Graph 2.2**



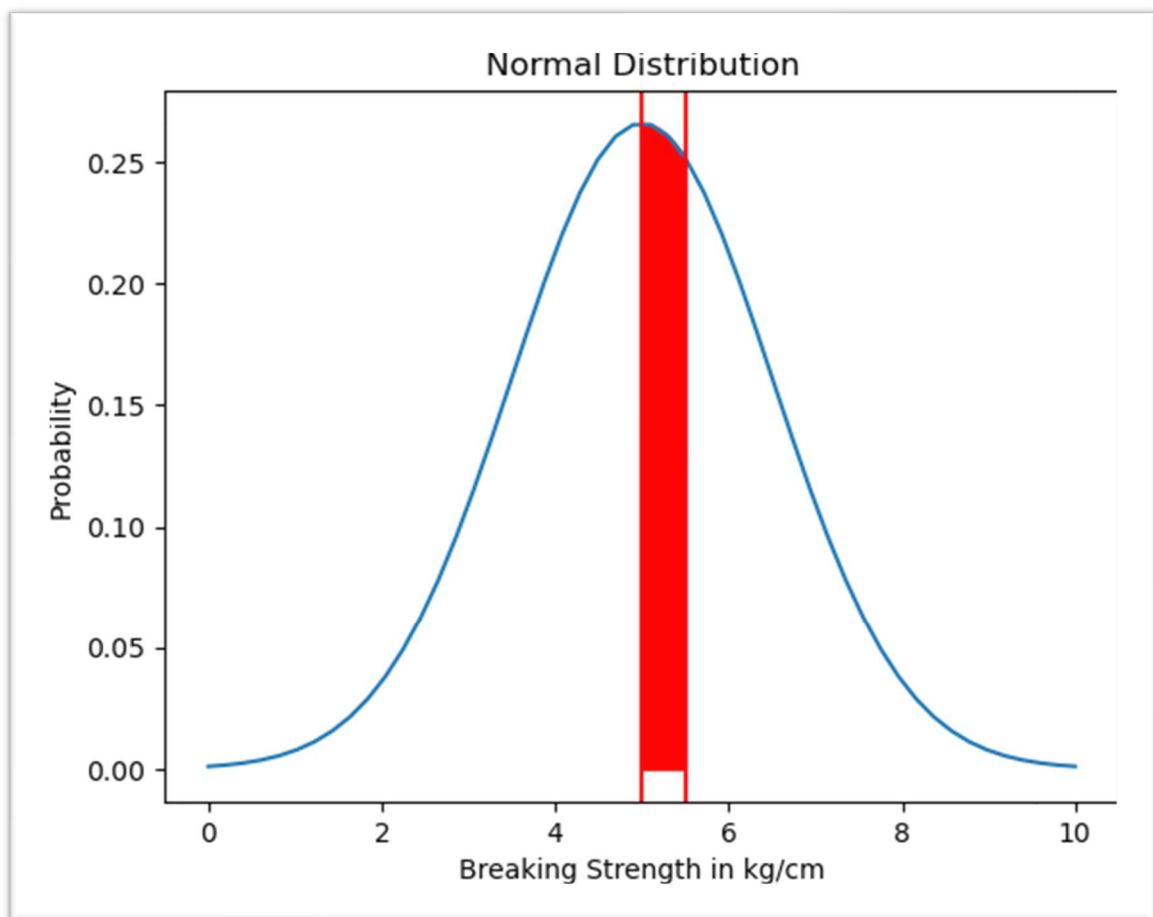
### 2.3 What proportion of the gunny bags have a breaking strength between 5 and 5.5 kg per sq cm.?

To calculate the probability density, we can use the norm.cdf function of scipy.stats in python

$$P(5 < X < 5.5) = P(X < 5.5) - P(X < 5) = 0.1306$$

**Proportion of the gunny bags have a breaking strength between 5 and 5.5 kg per sq.cm = 13.06%**

**Fig.3 Graph 2.3**



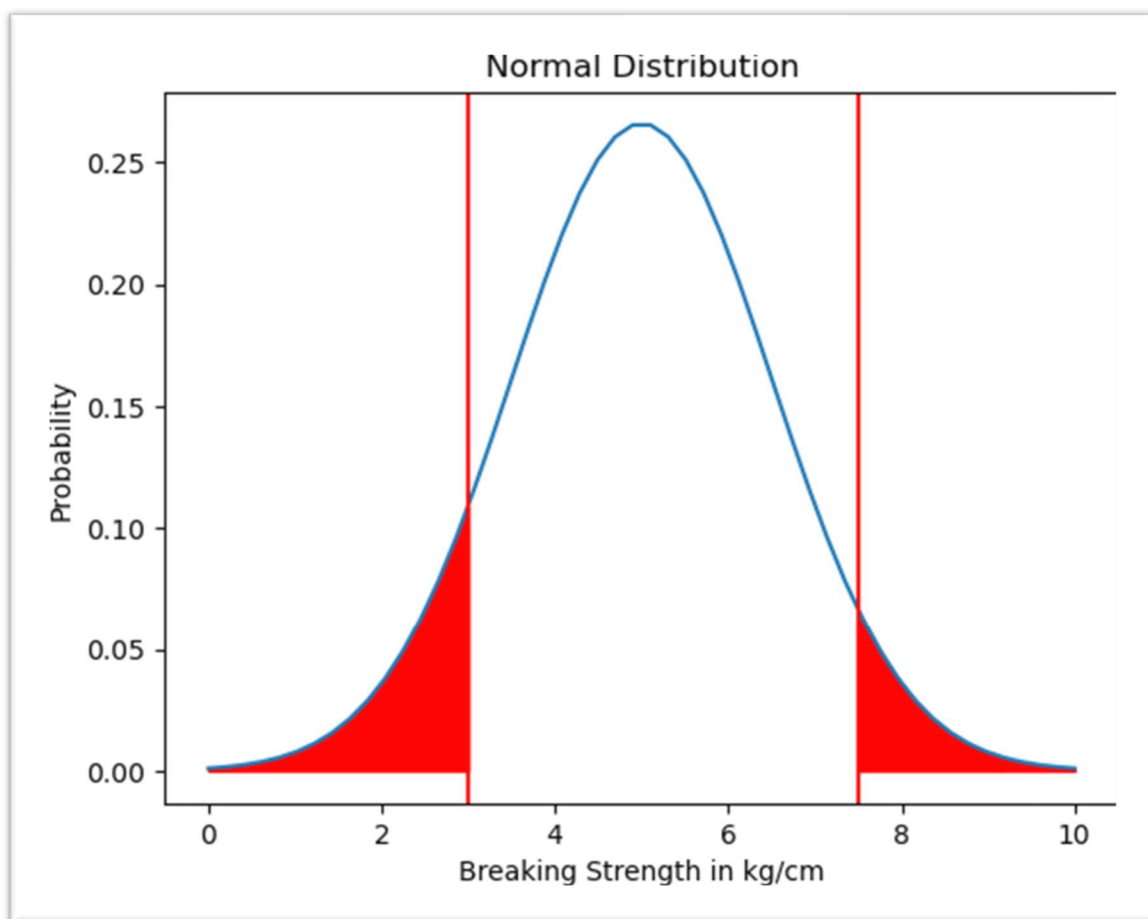
## 2.4 What proportion of the gunny bags have a breaking strength NOT between 3 and 7.5 kg per sq cm.?

To calculate the probability density, we can use the norm.cdf function of scipy.stats in python

$$P(X < 3 \text{ and } X > 7.5) = P(X < 3) + (1 - P(X < 7.5)) = 0.139$$

**Proportion of the gunny bags have a breaking strength NOT between 3 and 7.5 kg per sq.cm = 13.9%**

**Fig.4 Graph 2.4**



## PROBLEM 3

### QUESTION:

Zingaro stone printing is a company that specializes in printing images or patterns on polished or unpolished stones. However, for the optimum level of printing of the image, the stone surface has to have a Brinell's hardness index of at least 150. Recently, Zingaro has received a batch of polished and unpolished stones from its clients. Use the data provided to answer the following (assuming a 5% significance level);

3.1 Zingaro has reason to believe that the unpolished stones may not be suitable for printing. Do you think Zingaro is justified in thinking so?

3.2 Is the mean hardness of the polished and unpolished stones the same?

### Data Overview:

Preview of the Dataset:

**Fig.5 First 5 rows of the dataset1**

	Unpolished	Treated and Polished
0	164.481713	133.209393
1	154.307045	138.482771
2	129.861048	159.665201
3	159.096184	145.663528
4	135.256748	136.789227

Shape:

- There are 75 rows and 2 columns
- There are no null values in this dataset.

Basic Information:

**Fig.6 Basic Info1**

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 75 entries, 0 to 74
Data columns (total 2 columns):
#   Column                                Non-Null Count  Dtype
---  -
0   Unpolished                            75 non-null     float64
1   Treated and Polished                  75 non-null     float64
dtypes: float64(2)
memory usage: 1.3 KB
```

Numerical Statistics:

**Fig.7 Numerical Statistics1**

	count	mean	std	min	25%	50%	75%	max
Unpolished	75.0	134.110527	33.041804	48.406838	115.329753	135.597121	158.215098	200.161313
Treated and Polished	75.0	147.788117	15.587355	107.524167	138.268300	145.721322	157.373318	192.272856

**3.1 Zingaro has reason to believe that the unpolished stones may not be suitable for printing. Do you think Zingaro is justified in thinking so?**

**Null Hypothesis** : The Mean hardness index of unpolished stones is greater than or equal to 150. ( $\mu \geq 150$ )

**Alternate Hypothesis** : The Mean hardness index of unpolished stones is lesser than 150. ( $\mu < 150$ )

- Level of Significance  $\alpha = 0.05$
- Normally distributed population and Sample size =  $75 > 30$
- Population standard deviation is known - No

- We can use 1-sample T-test for this problem.
- The `ttest_1samp()` function of Scipy will be used to compute the test statistic and p-value.
- By performing the 1-Sample T-test, the values of test-statistic and p-values are obtained.

**Fig.8 T-Test 3.1**

```
1-Sample T-test
The test_stat is  -4.164629601426757
The p-value is  4.171286997419652e-05
```

- As the p-value( $4.171286e-05$ ) is much less than the level of significance( $\alpha = 0.05$ ), we can reject the null hypothesis at 95% confidence.
- We have enough statistical evidence to say, that mean hardness index of unpolished stones is less than 150. So, Unpolished stones are not suitable for printing. Hence, Zingaro stone printing company's assumption is correct.

### **3.2 Is the mean hardness of the polished and unpolished stones the same?**

**Null Hypothesis** : The Mean hardness index of unpolished stones and polished stones are equal. ( $\mu_1 = \mu_2$ )

**Alternate Hypothesis** : The Mean hardness index of unpolished stones and polished stones are unequal. ( $\mu_1 \neq \mu_2$ )

- Level of Significance  $\alpha = 0.05$
- Normally distributed population and Sample size =  $75 > 30$ .



- Independent populations - As we are taking random samples for two different stones, the two samples are from two independent populations.
- We can use 2 Independent-Sample T-test for this problem.
- The `ttest_ind()` function of Scipy will be used to compute the test statistic and p-value.
- By performing the 2 Independent-Sample T-test, the values of test-statistic and p-values are obtained.

**Fig.9 2samp T-Test 3.1**

```
2 Independent-Sample T-test
The test_stat is  -3.2422320501414053
The p-value is  0.0014655150194628353
```

- As the p-value(0.00146) is much less than the level of significance( $\alpha = 0.05$ ), we can reject the null hypothesis at 95% confidence.
- We have enough statistical evidence to say, that **mean hardness index of unpolished and polished stones are unequal and significantly differs.** ( $\mu_1 \neq \mu_2$ )

## PROBLEM 4

### QUESTION:

Dental implant data: The hardness of metal implants in dental cavities depends on multiple factors, such as the method of implant, the temperature at which the metal is treated, the alloy used as well as the dentists who may favor one method above another and may work better in his/her favorite method. The response is the variable of interest.

4.1 How does the hardness of implants vary depending on dentists?

4.2 How does the hardness of implants vary depending on methods?

4.3 What is the interaction effect between the dentist and method on the hardness of dental implants for each type of alloy?

4.4 How does the hardness of implants vary depending on dentists and methods together?

## Data Overview:

Preview of the Dataset:

**Fig.10 First 5 rows of the dataset2**

	Dentist	Method	Alloy	Temp	Response
0	1	1	1	1500	813
1	1	1	1	1600	792
2	1	1	1	1700	792
3	1	1	2	1500	907
4	1	1	2	1600	792

Shape:

There are 90 rows and 5 columns

Basic Information:

**Fig.11 Basic Info2**

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 90 entries, 0 to 89
Data columns (total 5 columns):
#   Column      Non-Null Count  Dtype
---  -
0   Dentist     90 non-null    int64
1   Method      90 non-null    int64
2   Alloy       90 non-null    int64
3   Temp        90 non-null    int64
4   Response    90 non-null    int64
dtypes: int64(5)
memory usage: 3.6 KB
```

The datatype of these columns – Dentist, Method, Alloy is int64.

**Fig.12 Datatype before correction**

```
1    18
2    18
3    18
4    18
5    18
Name: Dentist, dtype: int64

1    30
2    30
3    30
Name: Method, dtype: int64

1    45
2    45
Name: Alloy, dtype: int64
```

After converting these columns into categorical columns,

**Fig.13 Basic info after correction**

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 90 entries, 0 to 89
Data columns (total 5 columns):
#   Column      Non-Null Count  Dtype
---  -
0   Dentist     90 non-null    category
1   Method      90 non-null    category
2   Alloy       90 non-null    category
3   Temp        90 non-null    int64
4   Response    90 non-null    int64
dtypes: category(3), int64(2)
memory usage: 2.3 KB
```

Numerical Statistics:

**Fig.14 Numerical Statistics2**

	count	mean	std	min	25%	50%	75%	max
Dentist	90.0	3.000000	1.422136	1.0	2.0	3.0	4.0	5.0
Method	90.0	2.000000	0.821071	1.0	1.0	2.0	3.0	3.0
Alloy	90.0	1.500000	0.502801	1.0	1.0	1.5	2.0	2.0
Temp	90.0	1600.000000	82.107083	1500.0	1500.0	1600.0	1700.0	1700.0
Response	90.0	741.777778	145.767845	289.0	698.0	767.0	824.0	1115.0

## 4.1 How does the hardness of implants vary depending on dentists?

**Solution:**

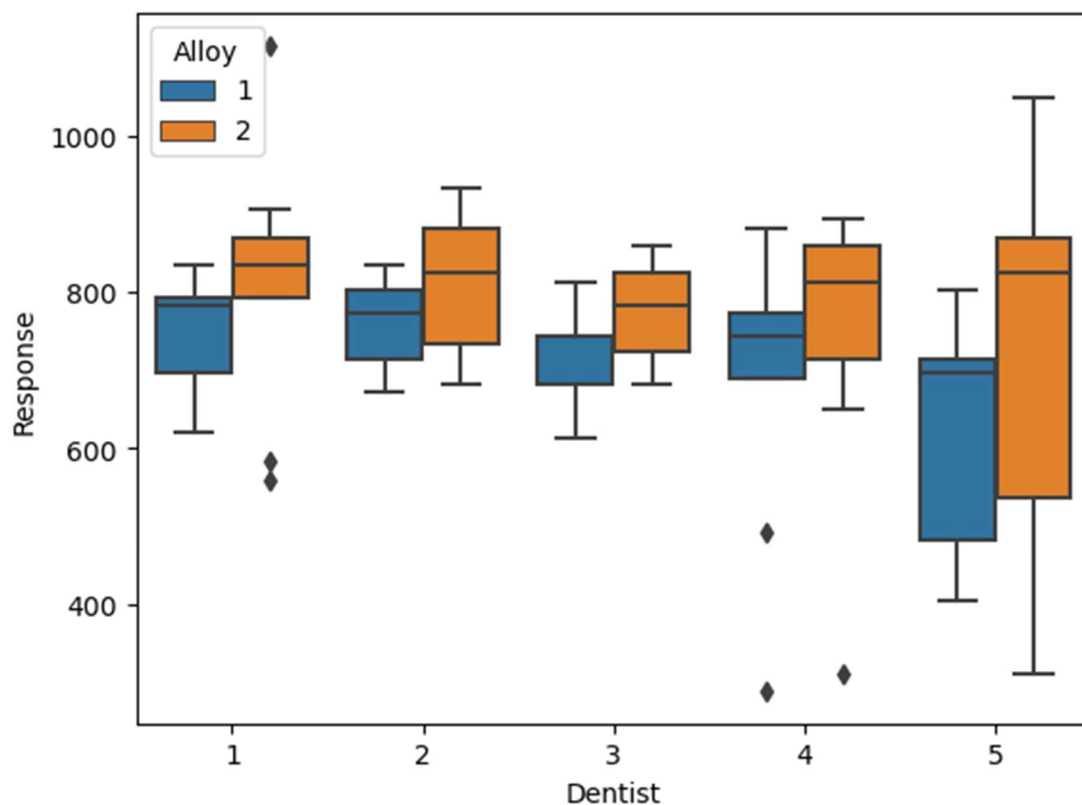
Visual Representation:

**Fig.15 Alloy-Dentist Groupby**

Alloy	Dentist	
1	1	749.888889
	2	761.222222
	3	717.555556
	4	681.111111
	5	627.666667
2	1	816.222222
	2	812.111111
	3	779.666667
	4	746.222222
	5	726.111111

Name: Response, dtype: float64

**Fig.16 Dentist vs Response vs Alloy**



Assumptions of the test:

### **Shapiro-Wilk's test**

For Alloy 1:

Null Hypothesis (H0) : The Mean Implant hardness for Alloy 1 follows normal distribution.

Alternate Hypothesis (Ha) : The Mean Implant hardness for Alloy 1 does not follows normal distribution.

Since p-value( $1.19e-05$ ) of the test is smaller than the significance level(0.05), we can reject the null hypothesis and the response doesn't follows the normal distribution.

For Alloy 2:

Null Hypothesis (H0) : The Mean Implant hardness for Alloy 2 follows normal distribution.

Alternate Hypothesis (Ha) : The Mean Implant hardness for Alloy 2 does not follows normal distribution.

Since p-value(0.0004) of the test is smaller than the significance level(0.05), we can reject the null hypothesis and the response doesn't follows the normal distribution.

### **Levene's test:**

Null Hypothesis (H0) : All the population variances are equal

Alternate Hypothesis (Ha) : At least one variance is different from the rest

Since p-value(0.0078) of the test is smaller than the significance level(0.05), we can reject the null hypothesis.

Case 1 (Alloy 1):

Null Hypothesis (H<sub>0</sub>) : The Mean Implant hardness is same across all the dentists for Alloy 1.

Alternate Hypothesis (H<sub>a</sub>) : At least one pair of dentists is different from the rest for Alloy 1.

By performing One way ANOVA for Response variable for Alloy 1,

**Fig.17 Dentist\_Alloy1**

-----Alloy 1-----					
	df	sum_sq	mean_sq	F	PR(>F)
C(Dentist)	4.0	106683.688889	26670.922222	1.977112	0.116567
Residual	40.0	539593.555556	13489.838889	NaN	NaN

The p-value (0.116567) is greater than the significance level ( $\alpha = 0.05$ ). So, we fail to reject the null hypothesis.

The Mean implant hardness is same across all the dentists for Alloy 1.

Case 2 (Alloy 2):

Null Hypothesis (H<sub>0</sub>) : The Mean Implant hardness is same across all the dentists for Alloy 2.

Alternate Hypothesis (H<sub>a</sub>) : At least one pair of dentists is different from the rest for Alloy 2.

By performing One way ANOVA for Response variable for Alloy 2,

**Fig.18 Dentist\_Alloy2**

-----Alloy 2-----					
	df	sum_sq	mean_sq	F	PR(>F)
C(Dentist)	4.0	5.679791e+04	14199.477778	0.524835	0.718031
Residual	40.0	1.082205e+06	27055.122222	NaN	NaN

The p-value (0.718031) is greater than the significance level ( $\alpha = 0.05$ ). So, we fail to reject the null hypothesis.

The Mean implant hardness is same across all the dentists for Alloy 2.

## INFERENCES:

The Mean implant hardness is same across all the dentists for Alloy 1 and Alloy 2.



## 4.2 How does the hardness of implants vary depending on methods?

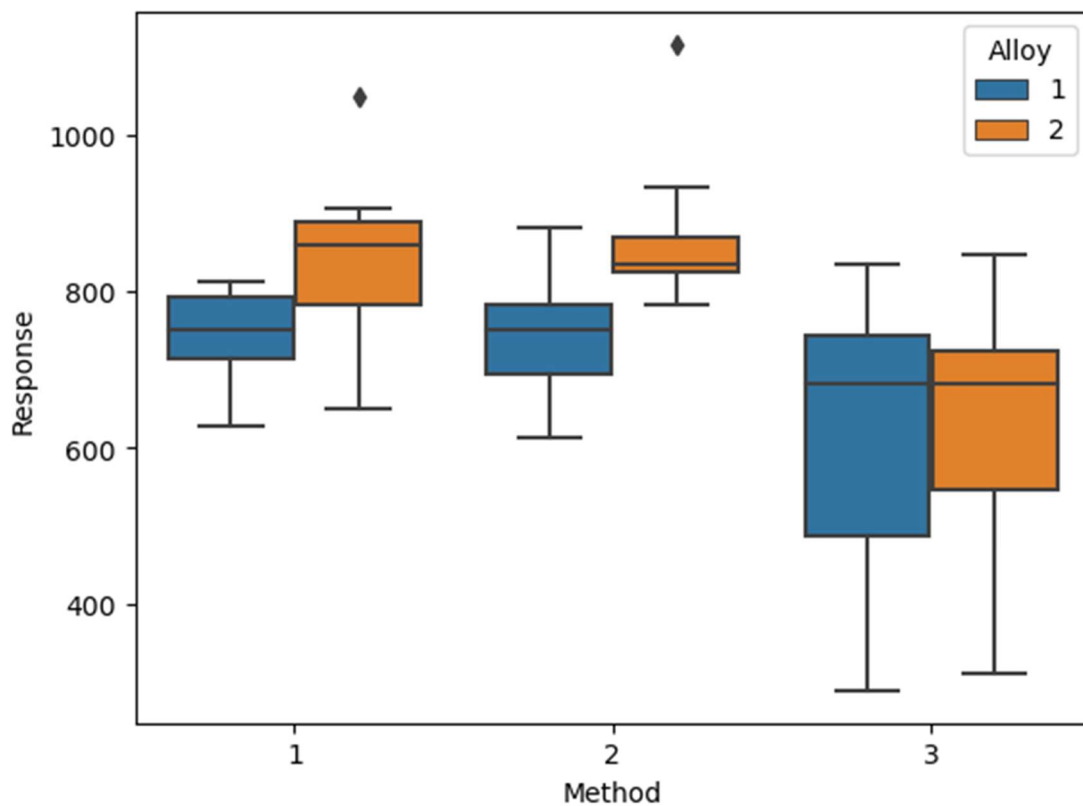
**Solution:**

Visual Representation:

**Fig.19 Alloy-Methods Groupby**

```
Alloy  Method
1      1      751.133333
      2      745.000000
      3      626.333333
2      1      836.666667
      2      863.666667
      3      627.866667
Name: Response, dtype: float64
```

**Fig.20 Methods vs Response vs Alloy**



Assumptions of the test:

### **Shapiro-Wilk's test**

For Alloy 1:

Null Hypothesis ( $H_0$ ) : The Mean Implant hardness for Alloy 1 follows normal distribution.

Alternate Hypothesis ( $H_a$ ) : The Mean Implant hardness for Alloy 1 does not follows normal distribution.

Since p-value( $1.19e-05$ ) of the test is smaller than the significance level( $0.05$ ), we can reject the null hypothesis and the response doesn't follows the normal distribution.

For Alloy 2:

Null Hypothesis ( $H_0$ ) : The Mean Implant hardness for Alloy 2 follows normal distribution.

Alternate Hypothesis ( $H_a$ ) : The Mean Implant hardness for Alloy 2 does not follows normal distribution.

Since p-value( $0.0004$ ) of the test is smaller than the significance level( $0.05$ ), we can reject the null hypothesis and the response doesn't follows the normal distribution.

### **Levene's test:**

Null Hypothesis ( $H_0$ ) : All the population variances are equal

Alternate Hypothesis ( $H_a$ ) : At least one variance is different from the rest

Since p-value( $0.004$ ) of the test is smaller than the significance level( $0.05$ ), we can reject the null hypothesis.

Case 1 (Alloy 1):

Null Hypothesis (H<sub>0</sub>) : The Mean Implant hardness is same across all the methods for Alloy 1.

Alternate Hypothesis (H<sub>a</sub>) : At least one pair of methods is different from the rest for Alloy 1.

By performing One way ANOVA for Response variable for Alloy 1,

**Fig.21 Method\_Alloy1**

```
-----Alloy 1-----
              df      sum_sq      mean_sq      F      PR(>F)
C(Method)    2.0    148472.177778    74236.088889    6.263327    0.004163
Residual    42.0    497805.066667    11852.501587      NaN      NaN
```

The p-value (0.004163) is smaller than the significance level ( $\alpha = 0.05$ ). So, we can reject the null hypothesis and have evidence for alternate hypothesis.

At least one pair of methods is different from the rest for Alloy 2.

Case 2 (Alloy 2):

Null Hypothesis (H<sub>0</sub>) : The Mean Implant hardness is same across all the methods for Alloy 2.

Alternate Hypothesis (H<sub>a</sub>) : At least one pair of methods is different from the rest for Alloy 2.

By performing One way ANOVA for Response variable for Alloy 2,

**Fig.22 Method\_Alloy2**

-----Alloy 2-----					
	df	sum_sq	mean_sq	F	PR(>F)
C(Method)	2.0	499640.4	249820.200000	16.4108	0.000005
Residual	42.0	639362.4	15222.914286	NaN	NaN

The p-value (0.000005) is smaller than the significance level ( $\alpha = 0.05$ ). So, we can reject the null hypothesis and have evidence for alternate hypothesis.

At least one pair of methods is different from the rest for Alloy 2.

By performing **Multiple Comparison test (Tukey HSD)**, we can clearly identify which method is different from other groups.

Null Hypothesis (H0) :  $\mu_1 = \mu_2$  (and)  $\mu_1 = \mu_3$  (and)  $\mu_1 = \mu_2$

Alternate Hypothesis (Ha) :  $\mu_1 \neq \mu_2$  (or)  $\mu_1 \neq \mu_3$  (or)  $\mu_2 \neq \mu_3$

**Fig.23 Tukey HSD**

-----Alloy 1-----						
Multiple Comparison of Means - Tukey HSD, FWER=0.05						
=====						
group1	group2	meandiff	p-adj	lower	upper	reject
-----						
1	2	-6.1333	0.987	-102.714	90.4473	False
1	3	-124.8	0.0085	-221.3807	-28.2193	True
2	3	-118.6667	0.0128	-215.2473	-22.086	True
-----						
-----Alloy 2-----						
Multiple Comparison of Means - Tukey HSD, FWER=0.05						
=====						
group1	group2	meandiff	p-adj	lower	upper	reject
-----						
1	2	27.0	0.8212	-82.4546	136.4546	False
1	3	-208.8	0.0001	-318.2546	-99.3454	True
2	3	-235.8	0.0	-345.2546	-126.3454	True
-----						

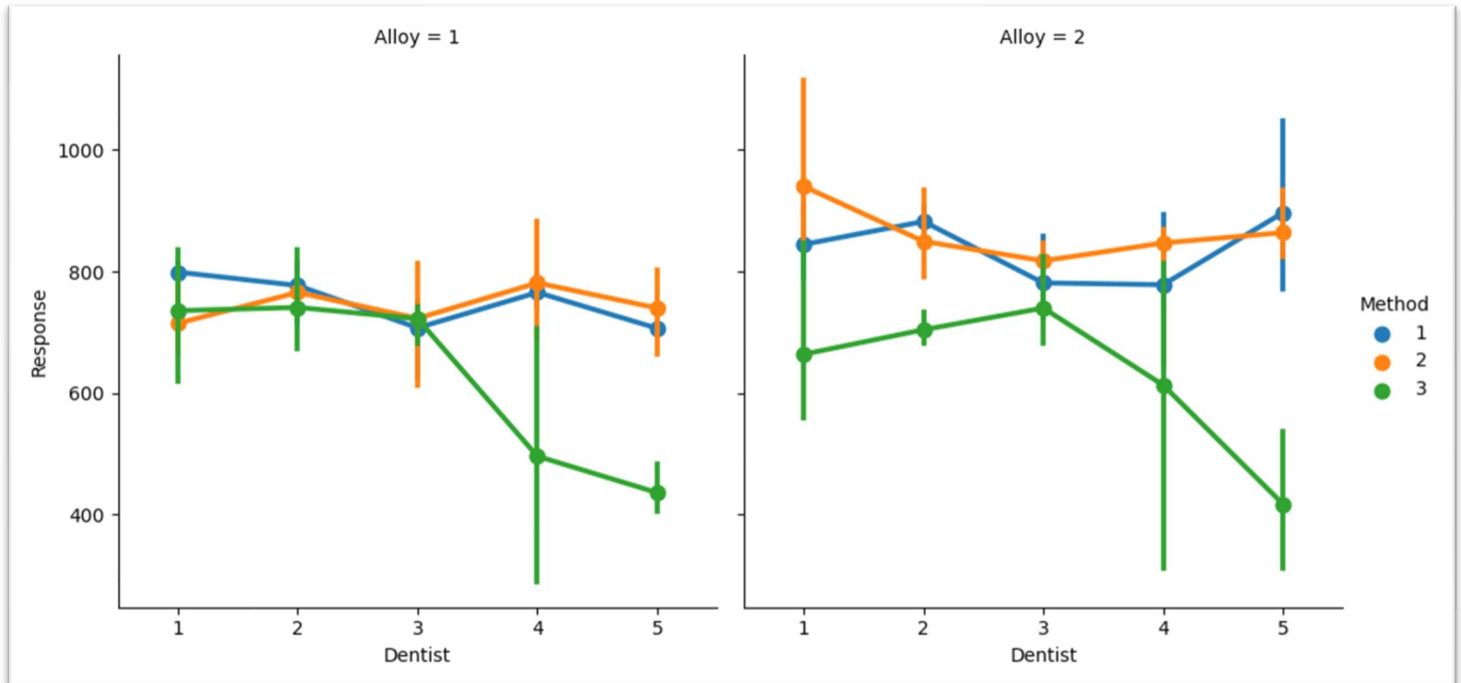
As the p-value for comparing the mean hardness for the pair Method1 - Method3 and Method2 - Method3 is less than their significance level ( $\alpha=0.05$ ), the null hypothesis can be rejected.

## INFERENCES:

Thus, we can say that the mean hardness for Method1 and Method2 is similar but Method3 is significantly different from the other two methods.

### 4.3 What is the interaction effect between the dentist and method on the hardness of dental implants for each type of alloy?

Fig.24 Interaction Plot



### INFERENCES:

- Method 3 shows a significant decline in response across dentists
- Method 1 and Method 2 share a similar effect with respect to dentists and implant hardness.
- A strong decline in implant hardness particularly in Dentists 4 and Dentist 5.
- By performing **Multiple Comparison test (Tukey HSD)**, we can clearly identify which combinations is different from other groups

#### 4.4 How does the hardness of implants vary depending on dentists and methods together?

For Alloy 1:

Null Hypothesis (H<sub>0</sub>) : The effect of ‘Dentist’ on ‘Method’ does not depend on the effect of Implant hardness (Response variable) for Alloy 1 (no interaction effect)

Alternate Hypothesis (H<sub>a</sub>) : There is an interaction effect between ‘Dentist’ and ‘Method’ on Implant hardness for Alloy 1 (Response variable)

**Fig.25 2-way ANOVA for Alloy1**

-----Alloy 1-----					
	df	sum_sq	mean_sq	F	PR(>F)
C(Dentist)	4.0	106683.688889	26670.922222	3.899638	0.011484
C(Method)	2.0	148472.177778	74236.088889	10.854287	0.000284
C(Dentist):C(Method)	8.0	185941.377778	23242.672222	3.398383	0.006793
Residual	30.0	205180.000000	6839.333333	NaN	NaN

**C(Dentist):** The p-value (0.011484) is greater than the significance level ( $\alpha = 0.05$ ). So, we fail to reject the null hypothesis.

**C(Method):** The p-value (0.000284) is smaller than the significance level ( $\alpha = 0.05$ ). So, we can reject the null hypothesis.

**C(Dentist):C(Method) :** The p-value (0.006793) is smaller than the significance level ( $\alpha = 0.05$ ). So, we can reject the null hypothesis.

For Alloy 2:

Null Hypothesis (H0) : The effect of 'Dentist' on 'Method' does not depend on the effect of Implant hardness (Response variable) for Alloy 2(no interaction effect)

Alternate Hypothesis (Ha) : There is an interaction effect between 'Dentist' and 'Method' on Implant hardness for Alloy 2 (Response variable)

**Fig.26 2-way ANOVA for Alloy2**

-----Alloy 2-----					
	df	sum_sq	mean_sq	F	PR(>F)
C(Dentist)	4.0	56797.911111	14199.477778	1.106152	0.371833
C(Method)	2.0	499640.400000	249820.200000	19.461218	0.000004
C(Dentist):C(Method)	8.0	197459.822222	24682.477778	1.922787	0.093234
Residual	30.0	385104.666667	12836.822222	NaN	NaN

**C(Dentist):** The p-value (0.371833) is greater than the significance level ( $\alpha = 0.05$ ). So, we fail to reject the null hypothesis.

**C(Method):** The p-value (0.000004) is smaller than the significance level ( $\alpha = 0.05$ ). So, we can reject the null hypothesis.

**C(Dentist):C(Method) :** The p-value (0.093234) is greater than the significance level ( $\alpha = 0.05$ ). So, we fail to reject the null hypothesis.



By performing **Multiple Comparison test (Tukey HSD)**, we can clearly identify which combinations is different from other groups.

**Fig.27 Tukey HSD for Dentists**

-----Alloy 1-----						
Multiple Comparison of Means - Tukey HSD, FWER=0.05						
group1	group2	meandiff	p-adj	lower	upper	reject
1	2	11.3333	0.9996	-145.0423	167.709	False
1	3	-32.3333	0.9757	-188.709	124.0423	False
1	4	-68.7778	0.7189	-225.1535	87.5979	False
1	5	-122.2222	0.1889	-278.5979	34.1535	False
2	3	-43.6667	0.9298	-200.0423	112.709	False
2	4	-80.1111	0.5916	-236.4868	76.2646	False
2	5	-133.5556	0.1258	-289.9312	22.8201	False
3	4	-36.4444	0.9626	-192.8201	119.9312	False
3	5	-89.8889	0.4805	-246.2646	66.4868	False
4	5	-53.4444	0.8643	-209.8201	102.9312	False
-----Alloy 2-----						
Multiple Comparison of Means - Tukey HSD, FWER=0.05						
group1	group2	meandiff	p-adj	lower	upper	reject
1	2	-4.1111	1.0	-225.5687	217.3465	False
1	3	-36.5556	0.9895	-258.0131	184.902	False
1	4	-70.0	0.8941	-291.4576	151.4576	False
1	5	-90.1111	0.7724	-311.5687	131.3465	False
2	3	-32.4444	0.9933	-253.902	189.0131	False
2	4	-65.8889	0.9132	-287.3465	155.5687	False
2	5	-86.0	0.8008	-307.4576	135.4576	False
3	4	-33.4444	0.9925	-254.902	188.0131	False
3	5	-53.5556	0.9574	-275.0131	167.902	False
4	5	-20.1111	0.999	-241.5687	201.3465	False

INFERENCE:

- All the Dentists combinations are same and are not having impact on hardness levels

**Fig.28 Tukey HSD for Methods**

-----Alloy 1-----						
Multiple Comparison of Means - Tukey HSD, FWER=0.05						
group1	group2	meandiff	p-adj	lower	upper	reject
1	2	-6.1333	0.987	-102.714	90.4473	False
1	3	-124.8	0.0085	-221.3807	-28.2193	True
2	3	-118.6667	0.0128	-215.2473	-22.086	True
-----Alloy 2-----						
Multiple Comparison of Means - Tukey HSD, FWER=0.05						
group1	group2	meandiff	p-adj	lower	upper	reject
1	2	27.0	0.8212	-82.4546	136.4546	False
1	3	-208.8	0.0001	-318.2546	-99.3454	True
2	3	-235.8	0.0	-345.2546	-126.3454	True

INFERENCE:

- We can clearly see that, Method 3 is having impact on the hardness level.
- For Alloy1 and Alloy2, pairs of (Method 1 and Method 3) and (Method 2 and Method 3) are having impact.