I. Deriving $F_t$

$$x = \begin{bmatrix} X \\ Y \\ \psi_t \\ m_x^1 \\ m_y^1 \\ m_x^2 \\ m_y^2 \\ . \\ . \\ . \\ m_x^n \\ m_x^y \end{bmatrix}$$

All the $m$ states are static hence do not change with time – Hence that part of the Jacobian will be filled with $I_{2n}$ ($2n$ for both $x$ and $y$). Hence, we can separate only the changing states.

$$x_V = \begin{bmatrix} X \\ Y \\ \psi_t \end{bmatrix}$$

Hence the state transition matrix can be written as

$$F_t = \begin{bmatrix} \frac{\partial x_v}{\partial t} & 0_{3\times 2n} \\ 0_{2n\times 3} & I_{2n} \end{bmatrix}$$

Let $s_x = \delta t(\dot{x}_t cos\psi - \dot{y}_t sin\psi)$ and $s_y = \delta t(\dot{x}_t sin\psi + \dot{y}_t cos\psi)$

$$F_{t,11} = \frac{\partial x_v}{\partial t} = \begin{bmatrix} \frac{\partial(X+s_x)}{\partial X} & \frac{\partial(X+s_x)}{\partial Y} & \frac{\partial(X+s_x)}{\partial \psi} \\ \frac{\partial(Y+s_y)}{\partial X} & \frac{\partial(Y+s_y)}{\partial Y} & \frac{\partial(Y+s_y)}{\partial \psi} \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & \delta t(-\dot{x}_t \sin\psi - \dot{y}_t \cos\psi) \\ 0 & 1 & \delta t(\dot{x}_t \cos\psi - \dot{y}_t \sin\psi) \\ 0 & 0 & 1 \end{bmatrix}$$

II. Deriving $H_t$

$$r_t^j = \sqrt{\left(m_x^j - X_t\right)^2 + \left(m_y^j - Y_t\right)^2}$$

$$\beta_t^j = atan2\left(m_y^j - Y_t, m_x^j - X_t\right) - \psi_t$$

For each $r_j$ the Jacobian rows entries will be

$$\frac{\partial r_t^j}{\partial X} = -\frac{m_x^j - X}{r_t^j},$$

$$\frac{\partial r_t^j}{\partial Y} = -\frac{m_y^j - Y}{r_t^j},$$

$$\frac{\partial r_t^j}{\partial \psi} = 0,$$

$$\frac{\partial r_t^j}{\partial m_x^j} = \frac{m_x^j - X}{r_t^j},$$

$$\frac{\partial r_t^j}{\partial m_y^j} = \frac{m_y^j - Y}{r_t^j}$$

For each $\beta_j$ the Jacobian row entries will be

$$\frac{\partial \beta_t^j}{\partial X} = \frac{m_y^j - Y}{\left(m_x^j - X_t\right)^2 + \left(m_y^j - Y_t\right)^2},$$

$$\frac{\partial \beta_t^j}{\partial Y} = -\frac{m_x^j - X}{\left(m_x^j - X_t\right)^2 + \left(m_y^j - Y_t\right)^2},$$

$$\frac{\partial \beta^j}{\partial \psi} = -1,$$

$$\frac{\partial \beta_t^j}{\partial m_x^j} = -\frac{m_y^j - Y}{\left(m_x^j - X_t\right)^2 + \left(m_y^j - Y_t\right)^2},,$$

$$\frac{\partial \beta_t^j}{\partial m_y^j} = \frac{m_x^j - X}{\left(m_x^j - X_t\right)^2 + \left(m_y^j - Y_t\right)^2}$$

To simplify the matrix lets assign sub matrixes

$$A_j = \begin{bmatrix} \frac{\partial r_t^j}{\partial X} & \frac{\partial r_t^j}{\partial Y} & 0 \\ \frac{\partial \beta_t^j}{\partial X} & \frac{\partial \beta_t^j}{\partial Y} & -1 \end{bmatrix}$$

$$B_j = \begin{bmatrix} \frac{\partial r_t^j}{\partial m_x^j} & \frac{\partial r_t^j}{\partial m_y^j} \\ \frac{\partial \beta_t^j}{\partial m_x^j} & \frac{\partial \beta_t^j}{\partial m_y^j} \end{bmatrix}$$

Hence using these blocks, we can put together $H_t$

$$H_t = \begin{bmatrix} A_1 & 0 & 0 & \cdots & 0 & B_1 & 0 & \cdots & 0 \\ A_2 & 0 & 0 & \cdots & 0 & 0 & B_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ A_n & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & B_n \end{bmatrix}$$