

Linearized Quadrotor Dynamics: Derivation of A and B Matrices

This document derives the continuous-time linearized state-space model for a quadrotor around hover and presents the full A and B matrices explicitly.

STATE VECTOR

$$x = [x, y, z, \varphi, \theta, \psi, \dot{x}, \dot{y}, \dot{z}, p, q, r]^T$$

INPUT VECTOR

$$u = [U_1, U_2, U_3, U_4]^T$$

$U_1 = \text{collective thrust}$

$U_2, U_3, U_4 = \text{roll, pitch, yaw torques}$

EQUILIBRIUM (HOVER)

$$\varphi = \theta = \psi = 0$$

$$\dot{x} = \dot{y} = \dot{z} = 0$$

$$p = q = r = 0$$

$$U_1 = m g$$

SMALL-ANGLE APPROXIMATIONS

$$\sin(\varphi) \approx \varphi$$

$$\sin(\theta) \approx \theta$$

$$\cos(\varphi) \approx 1$$

$$\cos(\theta) \approx 1$$

TRANSLATION DYNAMICS (LINEARIZED)

$$\dot{x} = g \theta$$

$$\ddot{y} = -g \varphi$$

$$\ddot{z} = (1/m) U_1 - g$$

ROTATION DYNAMICS (LINEARIZED)

$$\dot{p} = \frac{U^2}{I_x}$$

$$\dot{q} = \frac{U^3}{I_y}$$

$$\dot{r} = \frac{U^4}{I_z}$$

EULER KINEMATICS

$$\dot{\phi} = p$$

$$\theta = q$$

$$\psi = r$$

This concludes the full explicit derivation and presentation of A and B.