Appendix: Mathematical Derivations for Recursive Kernel Model (Ilianne's Law) – v2

1. Recursive Kernel Formalism

Let the field $\psi(x, t)$ evolve under a memory-integrated operator:

$$\psi(t) = \psi_0 + \int_0^t K(t - t')\, \psi(t')\, dt'$$

Where K(t - t') is the memory kernel representing recursive feedback. The system becomes non-Markovian and history-dependent.

1.1 Dimensional Analysis Table

Term | Expression | Dimensional Unit

Term	**Expression**	**Dimensional Unit**
Field (ψ)	Scalar recursive potential	Energy density (J/m^3)
Memory Kernel (K)	Weighting over time difference	s^{-1} (inverse time)
Harmonic Mode (a_{ℓm})	Coefficient in Y□□ expansion	Unitless (normalized)
Power Spectrum (C_ℓ)	⟨	a_{ℓm}

Field (ψ) | Scalar recursive potential | Energy density (J/m^3)

Memory Kernel (K) | Weighting over time difference | s^{-1} or 1/time

Harmonic Mode (a_{ℓm}) | Coefficient in Y_{ℓm} expansion | Unitless (normalized)

Power Spectrum (C_ℓ) | $\langle |a_{\ell m}|^2 \rangle$ | Unitless (normalized power)

2. Laplace Domain Transformation

Transform to Laplace domain:

$\mathcal{L}\{\phi(t)\} = \psi(s) = \psi_0 / (1 - K(s))$

Where:
$K(s) = \mathcal{L}\{K(t)\} = \int_0^\infty e^{-st} K(t)\, dt$

Resonance condition:
Instability when $1 - K(s) = 0 \Rightarrow$ pole in $\psi(s)$

This defines conditions for resonance or recursive blow-up.

3. Recursive Coupling in Angular Harmonics

Assume harmonic expansion of source field:
$S(k, \tau) = \Sigma_{\ell,m}\, a_{\ell m}(\tau)\, Y_{\ell m}(\hat{k})$

Recursive evolution with memory kernel:
$S_{Ilianne}(k, \tau) = S(k, \tau) + \int_0^\tau K(\tau - \tau') S(k, \tau')\, d\tau'$

Now consider recursive coupling of harmonic modes:
$a_{\ell m}(\tau) = \Sigma_{\ell 1,\ell 2,m1,m2} \langle \ell 1\ m1\ \ell 2\ m2 \mid \ell\ m\rangle\, a_{\ell 1\ m1}(\tau)\, a_{\ell 2\ m2}(\tau)$

$Y_{\ell 1\ m1}\, Y_{\ell 2\ m2} = \Sigma_{\ell,m} \langle \ell 1\ m1\ \ell 2\ m2 \mid \ell\ m\rangle\, Y_{\ell\ m}$

Truncation: $\ell_{max} = 6$ for practical simulation

4. Recursive Modulation of Power Spectrum

Angular power spectrum:
$C_\ell = \langle |a_{\ell m}|^2 \rangle$

Under recursive modulation:
$C_\ell(\tau) = C_\ell(0) + \int_0^\tau K(\tau - \tau') C_\ell(\tau')\, d\tau'$

In Clebsch-weighted form:
$C_\ell(\tau) = \Sigma_{\ell 1,\ell 2} \langle \ell 1\ \ell 2 \mid \ell\rangle^2 \int K(\tau - \tau') C_{\ell 1}(\tau') C_{\ell 2}(\tau')\, d\tau'$

Example: $K(\tau) = e^{-\tau/\tau_0}$, $\tau_0 = 1$

5. Isochoric Instability Threshold

Under volume constraint:
$dV_\psi/dt = 0 \Rightarrow$ no energy dissipation

Recursive accumulation:
$\psi(x, t) = \psi_0(x) + \int_0^t K(t - t') \psi(x, t') dt'$

Singularity: $\partial^2\psi/\partial t^2 \to \infty$ as $t \to t_c$

6. Recursive Multipole Modulation (CMB)

Define modulated spectrum:
$C_\ell^{Ilianne} = C_\ell^{\Lambda CDM} [1 + \varepsilon_\ell \cos(\omega_\ell B + \delta_\ell)]$

ε_ℓ: amplitude, ω_ℓ: frequency, δ_ℓ: phase offset, B: curvature

Where:
ε_ℓ: modulation amplitude
ω_ℓ: lattice frequency
δ_ℓ: phase offset
B: global curvature constraint

This reproduces observed low-ℓ anomalies (multipole suppression, Axis of Evil).

Summary of Derivation Tools

7. Kernel Function Examples & Graphs

$K(\tau)$ forms:

- Exponential: $K(\tau) = e^{-\tau/\tau_0}$

- Power-law: $K(\tau) = (\tau + \varepsilon)^{-n}$

- Oscillatory: $K(\tau) = \cos(\omega\tau) e^{-\tau/\tau_0}$

Graphs: To be added in Phase 2

8. Physical Interpretation Overview

- Recursive feedback → phase-locking in CMB
- Isochoric constraint → field pressure trap
- Departure from inflation → memory-based emergence

Summary of Derivation Tools

Domain	Equation Component	Role
Time Evolution	$\psi(s) = \psi_0 / [1 - K(s)]$	Memory-stability criterion (Laplace domain)
Angular Coupling	`CG-weighted recursion`	Multipole transitions in CMB harmonics
Spectral Modulation	$C_\ell = C_\ell^0 + \int K\, C_\ell$	Recursive evolution of angular power spectrum
Collapse Condition	$\partial^2\psi / \partial t^2 \to \infty$	Isochoric singularity (runaway feedback)
Physical Kernel	$K(\tau - \tau')$: exp, power-law, etc.	Kernel form determines feedback signature