Appendix: Mathematical Derivations for Recursive Kernel Model (Ilianne's Law)

1. Recursive Kernel Formalism

Let the field $\psi(x, t)$ evolve under a memory-integrated operator:

$$\psi(t) = \psi_0 + \int_0^t K(t - t') \psi(t') dt'$$

Where $K(t - t')$ is the memory kernel representing recursive feedback. The system becomes non-Markovian and history-dependent.

2. Laplace Domain Transformation

Transform to Laplace domain:

$$\mathcal{L}\{ \phi (t)\} = \psi(s) = \psi_0 / (1 - K(s))$$

Where:
$K(s) = \mathcal{L}\{K(t)\} = \int_0^\infty e^{-st} K(t) dt$

Resonance condition:
Instability when $1 - K(s) = 0 \Rightarrow$ pole in $\psi(s)$

This defines conditions for resonance or recursive blow-up.

3. Recursive Coupling in Angular Harmonics

Assume harmonic expansion of source field:
$S(k, \tau) = \Sigma_{\ell,m} a_{\ell m}(\tau) Y_{\ell m}(\hat{k})$

Recursive evolution with memory kernel:
$S_{Ilianne}(k, \tau) = S(k, \tau) + \int_0^\tau K(\tau - \tau') S(k, \tau') d\tau'$

Now consider recursive coupling of harmonic modes:
$a_{\ell m}(\tau) = \Sigma_{\ell 1,\ell 2,m1,m2} \langle \ell 1\ m1\ \ell 2\ m2 \mid \ell\ m\rangle a_{\ell 1\ m1}(\tau)\ a_{\ell 2\ m2}(\tau)$

$Y_{\ell 1\ m1} Y_{\ell 2\ m2} = \Sigma_{\ell,m} \langle \ell 1\ m1\ \ell 2\ m2 \mid \ell\ m\rangle Y_{\ell\ m}$

4. Recursive Modulation of Power Spectrum

Angular power spectrum:
$C_\ell = \langle |a_{\ell m}|^2\rangle$

Under recursive modulation:
$C_\ell(\tau) = C_\ell(0) + \int_0^\tau K(\tau - \tau') C_\ell(\tau') d\tau'$

In Clebsch-weighted form:
$C_\ell(\tau) = \Sigma_{\ell 1,\ell 2} \langle \ell 1\ \ell 2 \mid \ell\rangle^2 \int K(\tau - \tau') C_{\ell 1}(\tau') C_{\ell 2}(\tau') d\tau'$

5. Isochoric Instability Threshold

Under volume constraint:
$dV_\psi/dt = 0 \Rightarrow$ no energy dissipation

Recursive accumulation:
$\psi(x, t) = \psi_0(x) + \int_0^t K(t - t')\, \psi(x, t')\, dt'$

Singularity emerges when:
$\lim_{t \to t_c} \partial^2\psi/\partial t^2 \to \infty$

This defines a blow-up condition in recursive systems, analogous to gravitational collapse.

6. Recursive Multipole Modulation (CMB)

Define modulated spectrum:
$C_\ell^{Ilianne} = C_\ell^{\Lambda CDM} \cdot [1 + \varepsilon_\ell \cos(\omega_\ell B + \delta_\ell)]$

Where:
ε_ℓ: modulation amplitude
ω_ℓ: lattice frequency
δ_ℓ: phase offset
B: global curvature constraint

This reproduces observed low-ℓ anomalies (multipole suppression, Axis of Evil).

Summary of Derivation Tools

Domain	Equation Component	Role
Time Evolution	$\psi(s) = \psi_0 / [1 - K(s)]$	Memory-stability criterion (Laplace domain)
Angular Coupling	CG-weighted recursion	Multipole transitions in CMB harmonics
Spectral Modulation	$C_\ell = C_\ell^0 + \int K\, C_\ell$	Recursive evolution of angular power spectrum
Collapse Condition	$\partial^2\psi / \partial t^2 \to \infty$	Isochoric singularity (runaway feedback)
Physical Kernel	$K(\tau - \tau')$: exp, power-law, etc.	Kernel form determines feedback signature