# Appendix: Mathematical Derivations for Recursive Kernel Model (Ilianne’s Law) – v2

## 1. Recursive Kernel Formalism

Let the field ψ(x, t) evolve under a memory-integrated operator:

ψ(t) = ψ₀ + ∫₀ᵗ K(t - t') ψ(t') dt'

Where K(t - t') is the memory kernel representing recursive feedback. The system becomes non-Markovian and history-dependent.

## 1.1 Dimensional Analysis Table

Term | Expression | Dimensional Unit

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Field (ψ) | Scalar recursive potential | Energy density (J/m³)

Memory Kernel (K) | Weighting over time difference | s⁻¹ or 1/time

Harmonic Mode (a\_{ℓm}) | Coefficient in Y\_{ℓm} expansion | Unitless (normalized)

Power Spectrum (C\_ℓ) | ⟨|a\_{ℓm}|²⟩ | Unitless (normalized power)

## 2. Laplace Domain Transformation

Laplace transform:

ψ(s) = ψ₀ / (1 - K(s)

Where K(s) = ∫₀^∞ e^(-st) K(t) dt

Instability when 1 - K(s) = 0 → Pole in ψ(s)

## 3. Recursive Coupling in Angular Harmonics

S(k, τ) = Σ\_{ℓ,m} a\_{ℓm}(τ) Y\_{ℓm}(k̂)

S\_Ilianne(k, τ) = S(k, τ) + ∫₀^τ K(τ - τ') S(k, τ') dτ'

a\_{ℓm}(τ) = Σ ⟨ℓ₁m₁ ℓ₂m₂|ℓm⟩ a\_{ℓ₁m₁} a\_{ℓ₂m₂}

Y\_{ℓ₁m₁} Y\_{ℓ₂m₂} = Σ ⟨ℓ₁m₁ ℓ₂m₂|ℓm⟩ Y\_{ℓm}

Truncation: ℓ\_max = 6 for practical simulation

## 4. Recursive Modulation of Power Spectrum

C\_ℓ = ⟨|a\_{ℓm}|²⟩

C\_ℓ(τ) = C\_ℓ(0) + ∫₀^τ K(τ - τ') C\_ℓ(τ') dτ'

C\_ℓ(τ) = Σ ⟨ℓ₁ ℓ₂|ℓ⟩² ∫ K(τ - τ') C\_{ℓ₁} C\_{ℓ₂} dτ'

Example: K(τ) = e^{-τ/τ₀}, τ₀ = 1

## 5. Isochoric Instability Threshold

Constraint: dV\_ψ/dt = 0 (no dissipation)

ψ(x, t) = ψ₀(x) + ∫₀^t K(t - t') ψ(x, t') dt'

Singularity: ∂²ψ/∂t² → ∞ as t → t\_c

## 6. Recursive Multipole Modulation (CMB)

C\_ℓ^Ilianne = C\_ℓ^ΛCDM [1 + ε\_ℓ cos(ω\_ℓ B + δ\_ℓ)]

ε\_ℓ: amplitude, ω\_ℓ: frequency, δ\_ℓ: phase offset, B: curvature constraint

## 7. Kernel Function Examples & Graphs

K(τ) forms:

- Exponential: K(τ) = e^{-τ/τ₀}

- Power-law: K(τ) = (τ + ε)^{-n}

- Oscillatory: K(τ) = cos(ωτ) e^{-τ/τ₀}

Graphs: To be added in Phase 2

## 8. Physical Interpretation Overview

- Recursive feedback → phase-locking in CMB

- Isochoric constraint → field pressure trap

- Departure from inflation → memory-based emergence

## Summary of Derivation Tools

Domain | Equation Component | Role

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Time Evolution | ψ(s) = ψ₀ / [1 - K(s)] | Memory-stability criterion

Angular Coupling | CG-weighted recursion | CMB multipole transitions

Spectral Modulation | C\_ℓ = C\_ℓ⁰ + ∫ K C\_ℓ | Recursive power evolution

Collapse Condition | ∂²ψ / ∂t² → ∞ | Isochoric singularity

Physical Kernel | K(τ - τ'): exp, power-law | Feedback signature type