

Algorithm Design and Analysis: Homework 3

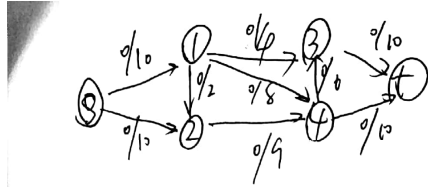
Due on Nov 2, 2017 at 9:00am

14784547 luochenqi

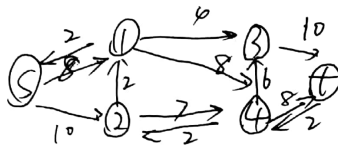
Luochenqi

Problem 1

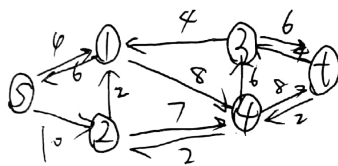
see the image



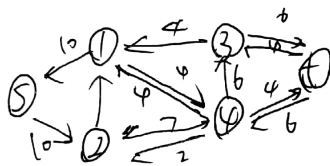
$$s \rightarrow 1 \rightarrow 2 \rightarrow 4 \rightarrow t$$



$$s \rightarrow 1 \rightarrow 3 \rightarrow t$$



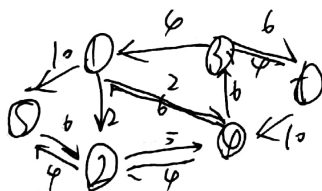
$$s \rightarrow 1 \rightarrow 4 \rightarrow t$$



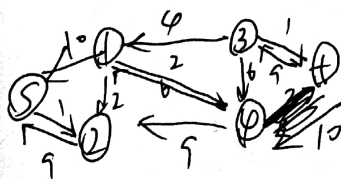
$$s \rightarrow 2 \rightarrow 1 \rightarrow 4 \rightarrow t$$



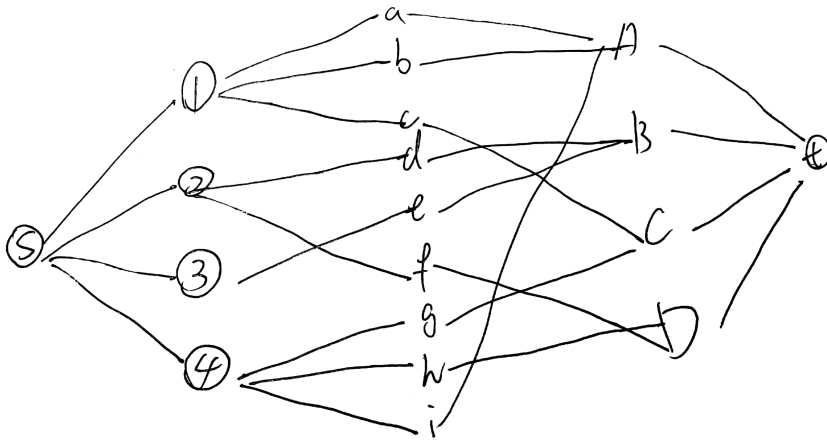
$$s \rightarrow 2 \rightarrow 4 \rightarrow t$$



$$s \rightarrow 2 \rightarrow 3 \rightarrow t$$



$$\text{max flow} = 19$$

Problem 2

first we got the 1....k department, every department choose one be the chosen staff. then, four classes ABCD, there max flow capacity is m_1, m_2, m_3, m_4 draw the network image below.

Problem 3

- a)
1. Take ~~original~~ N' be the modified network.
 2. Search augmenting path in N'
 3. if we find an augmenting path p , return $f + f_p$
otherwise, we return f .

The capacity $c(p)$ of p in N' is 1.

$f' = f + f_p$ is a max flow of N

Construct residual network takes $O(|V| + |E|)$

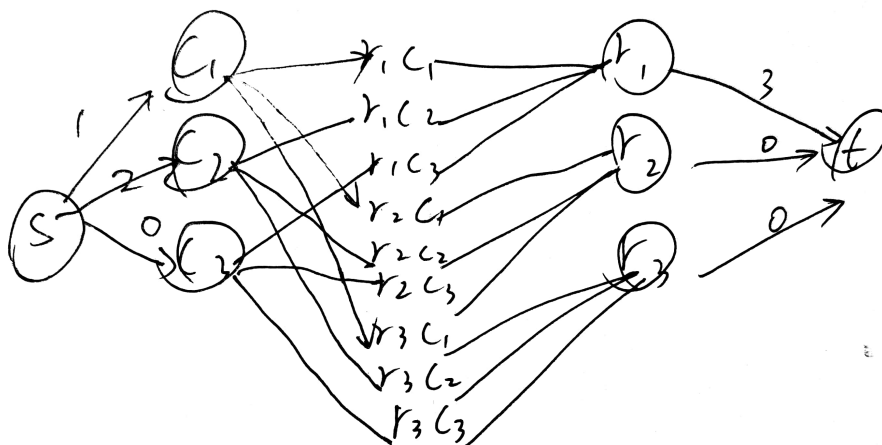
find augmenting path takes $O(|V| + |E|)$

(c. 3.5)

$$T(n) = O(|V| + |E|)$$

- b). if $f(u, v) < c(u, v)$ it has no effect on the flow. in N'
if $f(u, v) = c(u, v)$.
- 1) find a simple path p_1 in N' from u to s .
 - 2) find a simple p_2 from s to v .
 - 3) take p_2, p_1 and route 1 unit of flow t to s along this path
adding to f . we get f' of $(f+1)$ f' is a flow in N'
 - 4) perform a search for augmenting path in N'
 - 5). if we find return $f + f_p$ otherwise return f'

Problem 4



matrix can split into column and row. first of all, we set column to be the vertex of first level. $R_i C_j$ is the second level. R_i is the third level. and we can set both column and row capacity separately.

Problem 5

we should add a judgement before we do the standard maximum flow problem.

we find all the edge connecting the node u compare the $f(u, v)$ and $c(u)$ if $f(u, v) > c(u)$ return $f(u, v)$ otherwise return $c(u)$.

Problem 6

first compute a minimum s-t cut C , and define its volume by C . For each edge E_i , try to increase the capacity of e_i by 1 and compute a minimum cut in the new network. if $C_i = C$ for i , then C_i is also a minimum cut in the origin graph, if $C_i \neq C$ then the minimum is not unique. if there is a different minimum cut C' in the original graph, e_i is not in C' , so $C = C_i$. So, iff $|C| < |C_i|$ has a unique minimum cut and it takes most $m+1$ time, $T(n) = O(m)$.