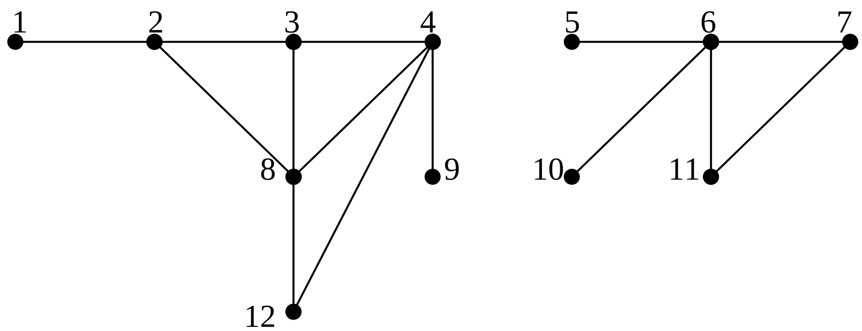
CMPS 101
Midterm 2 review
Solutions to selected problems



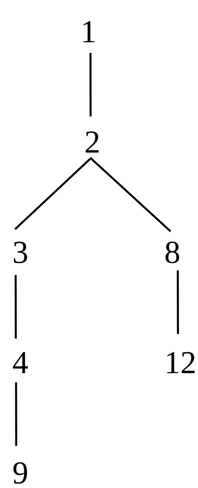
<u>Problem 1a</u>

Trace BFS on the graph of figure 1 (above) with 1 as source. For each vertex, record its color, parent, and distance fields, draw the resulting BFS tree, and determine the order in which vertices are added to the Queue. Process adjacency lists in ascending numerical order.

Solution: Queue: 1 2 3 8 4 12 9

	adj	color	distance	parent
1	2	b	0	nil
2	1 3 8	b	1	1
3	2 4 8	b	2	2
4	3 8 9 12	b	3	3
5	6	w	∞	nil
6	5 7 10 11	w	∞	nil
7	6 11	w	∞	nil
8	2 3 4 12	b	2	2
9	4	b	4	4
10	6	w	∞	nil
11	6 7	w	∞	nil
12	4 8	b	3	8

BFS Tree:



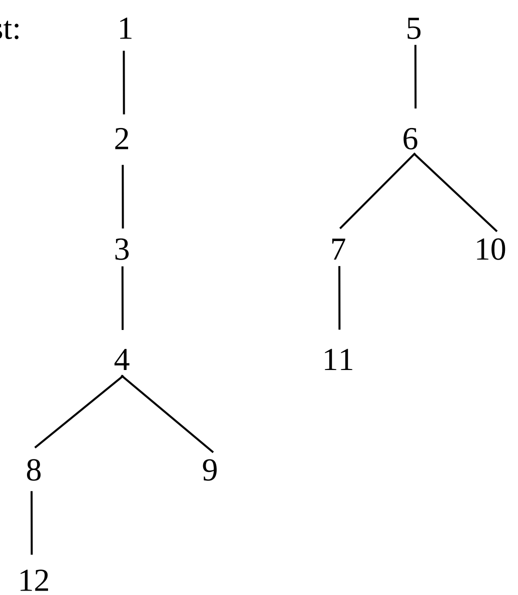
Problem 2a

Trace DFS on the The graph in figure 1 (above). For each vertex, record its color, parent, discover, and finish times, and draw the resulting DFS forest. Classify each edge as tree, back, forward, or cross. Process adjacency lists in ascending numerical order. Process vertices in the main loop of DFS in (ascending) numerical order.

Solution:

	adj	color	discover	finish	parent
1	2	b	1	14	nil
2	1 3 8	b	2	13	1
3	2 4 8	b	3	12	2
4	3 8 9 12	b	4	11	3
5	6	b	15	24	nil
6	5 7 10 11	b	16	23	5
7	6 11	b	17	20	6
8	2 3 4 12	b	5	8	4
9	4	b	9	10	4
10	6	b	21	22	6
11	6 7	b	18	19	7
12	4 8	b	6	7	8

DFS Forest:



Classification of edges:
Tree: {1, 2}, {2, 3}, {3, 4}, {4, 8}, {4, 9}, {8, 12}, {5, 6}, {6, 7}, {6, 10}, {7, 11}
Back: {2, 8}, {3, 8}, {4, 12}, {6, 11}

Problem 6

Let G be a directed graph. Determine whether, at any point during a Depth First Search of G, there can exist an edge of the following kind. (No justification is required.)

a. A tree edge which joins a white vertex to a gray vertex.
b. A back edge which joins a black vertex to a white vertex.
c. A forward edge which joins a gray vertex to a black vertex.
d. A cross edge which joins a black vertex to a gray vertex.

Answers:

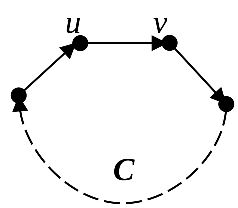
a. no
b. no
c. yes
d. no

Problem 8

Let G be a directed graph. Prove that if G contains a directed cycle, then G contains a back edge. (Hint: use the white path theorem.)

Proof:

Suppose G contains a directed cycle, call it C. Let v be the first vertex on C to be discovered by DFS, and let u be the vertex on C that precedes v.



Since no vertex on C is discovered before v, at the time of discovery of v the vertices of C form a path from v to u consisting of white vertices. By the white-path theorem, u becomes a descendent of v in some DFS tree. Therefore (u, v) is a back edge. ///

Problem 9

Let G be a connected graph, and suppose that $|E(G)| = |V(G)|$. Show that G is unicyclic, i.e. G contains exactly one cycle. (Hint: use lemma 1 and lemma 3 from the graph theory handout, and note that this is lemma 7.)

Proof:

If G contained no cycles, then being connected it would be a tree, whence $|E(G)| = |V(G)| - 1$ by lemma 1, contrary to hypothesis. Therefore G contains at least one cycle. We must show that it contains no more than one cycle. Assume, to get a contradiction, that G contains two distinct cycles, call them C_1 and C_2. Since these cycles are different, there must exist edges $e_1 \in E(C_1) - E(C_2)$ and $e_2 \in E(C_2) - E(C_1)$. Remove these two edges from G, calling the resulting graph $H = G - e_1 - e_2$. Observe that H is necessarily connected since the removal of e_1 does not disconnect G and leaves C_2 intact, and therefore the removal of e_2 does not disconnect $G - e_1$. Now $|E(H)| = |E(G)| - 2$ and $|V(H)| = |V(G)|$ by the construction of H. Our hypothesis says $|E(G)| = |V(G)|$, so that $|E(H)| = |V(H)| - 2$. Lemma 3 says $|E(H)| \geq |V(H)| - 1$ since H is connected. Thus $|V(H)| - 2 \geq |V(H)| - 1$ which is absurd. Our assumption was therefore false, and G cannot contain two cycles. ///