Computer Vision Assignment 1

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Honor Code : We have not refered or shared our solutions with any other team.

1 Orthocenter Property

$$OS \cdot OR = 0 \tag{1}$$

$$OS \cdot OQ = 0 \tag{2}$$

subtracting (2) from (1)

$$OS \cdot (OR - OS) = 0 \tag{3}$$

Which means **OS is orthogonal to (OR-OQ)**.

Now we know that
Q, R and S lie in the image plane $\implies$ RQ is parallel to the image plane $\implies$ $OR - OQ$ is parallel to the image plane.

Also, Oo is normal to the image plane.

Hence **Oo is orthogonal to (OR-OQ)**.

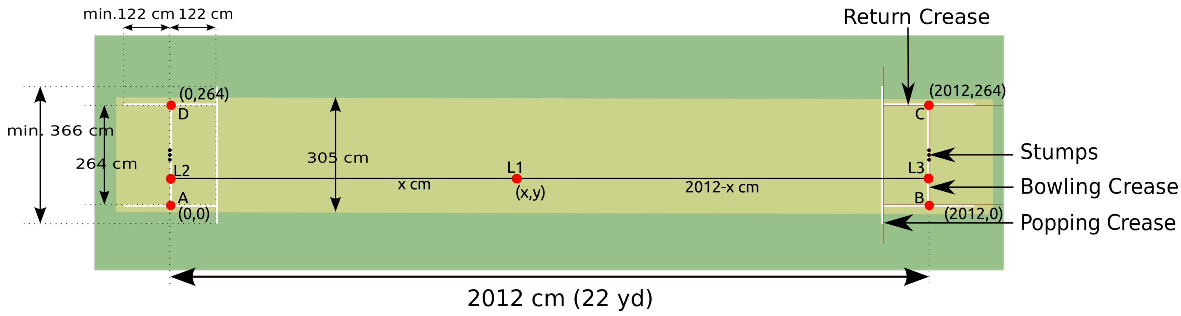
Now because (OR-OQ) is normal to the plane OSo, it is orthogonal to any line parallel to OSo
$\implies$ **oS**, being parallel to the plane, **is orthogonal to QR**.

If the three lines did not pass through the O, their intersection points with the image plane would not have represented vanishing points. So the orthocenter (which could still be obtained by a similar technique) would be of no practical use.

2 Distance of Ball on Pitch

Given : Image of cricket pitch at the time instant that ball landed on the pitch at point L1

1. Method 1 : Using **Homography**
   The cricket pitch is a planar surface with known dimensions. We can mark on image the points corresponding to points A, B, C and D as shown in the figure. For these points, we know both the actual and image coordinates. We can determine the homography matrix H which converts points from image (perspective) to corresponding points in given figure. The x coordinate of point L1 is the required answer.
   (Note: The above figure can be considered as image of cricket pitch with a camera for which image plane is parallel to the pitch.)



2. Method 2: Using **Cross-ratios**
   In this method we first find the Vanishing point for the lines along the length of the pitch. Next we join the vanishing point to the point where the ball drops on the pitch. Since these two lines share the same vanishing points, they must be parallel. Lets assume that the points of intersection for this second line are L2 and L3. Now, using the cross ratio formula we get.

$$CrossRatio = \frac{L3L2 * L1\infty}{L3\infty * L1L2} = \frac{L3L2}{L1L2} = \frac{l3l2 * l1v}{l3v * l1l2}$$

   where v (vanishing point of lines parallel to AB), l1, l2, l3 are points on image. L1, L2 and L3 are actual points.
   So, we can easily find the value of L1L2 from the above equation, as all other values are known.

3 Camera Calibration using Vanishing Points

Given :
$l1, l2, l3$: three mutually perpendicular directions in world coordinate system
$(p_{1x}, p_{1y}), (p_{2x}, p_{2y}), (p_{3x}, p_{3y})$: vanishing points in I_p
$(q_{1x}, q_{1y}), (q_{2x}, q_{2y}), (q_{3x}, q_{3y})$: vanishing points in I_q
Let
$(cp_{1x}, cp_{1y}, -f_p), (cp_{2x}, cp_{2y}, -f_p), (cp_{3x}, cp_{3y}, -f_p)$: vanishing points in camera coordinate system for camera p
$(cq_{1x}, cq_{1y}, -f_q), (cq_{2x}, cq_{2y}, -f_q), (cq_{3x}, cq_{3y}, -f_q)$: vanishing points in camera coordinate system for camera q

- Finding **Extrinsic Parameters**
  Assuming we know intrinsic parameters
  $\implies$ we know $(cp_{1x}, cp_{1y}, -f_p), (cp_{2x}, cp_{2y}, -f_p), (cp_{3x}, cp_{3y}, -f_p)$ and $(cq_{1x}, cq_{1y}, -f_q), (cq_{2x}, cq_{2y}, -f_q), (cq_{3x}, cq_{3y}, -$
  $i.e.$ we know directions $dp_i = cp_i/||cp_i||$ and $dq_i = cq_i/||cq_i||$ where $i = 1, 2, 3$
  (dp_i and dq_i are 3d vectors)

  R : rotation matrix from camera p to q

$$\begin{bmatrix} dq_1 & dq_2 & dq_3 \end{bmatrix} = \mathbf{R} \begin{bmatrix} dp_1 & dp_2 & dp_3 \end{bmatrix}$$

  Directions dp_i are orthonormal.

$$\mathbf{R} = \begin{bmatrix} dq_1 & dq_2 & dq_3 \end{bmatrix} \begin{bmatrix} dp_1 \\ dp_2 \\ dp_3 \end{bmatrix}$$

Hence, we can infer **R** given vanishing points in camera coordinate systems.
Vanishing Points are not affected by translation of camera. Hence, we **cannot find** t, translation between two camera coordinate systems.

- Finding **Intrinsic Parameters**
  We can get o_{px}, o_{py} and o_{qx}, o_{qy} using the orthocenter property of vanishing points

$$cp_i = (-(p_{ix} - o_{px})s_p, -(p_{iy} - o_{py})s_p, -f_p)$$

$$cq_i = (-(q_{ix} - o_{qx})s_q, -(q_{iy} - o_{qy})s_q, -f_q)$$

also these vectors are orthogonal

$$((p_{1x} - o_{px})(p_{2x} - o_{px})s_p^2 + (p_{1y} - o_{py})(p_{2y} - o_{py})s_p^2 + f_p^2) = 0$$

$$\frac{f_p}{s_p} = \sqrt{-((p_{1x} - o_{px})(p_{2x} - o_{px}) + (p_{1y} - o_{py})(p_{2y} - o_{py}))}$$

similarly, we can find

$$\frac{f_q}{s_q} = \sqrt{-((q_{1x} - o_{qx})(q_{2x} - o_{qx}) + (q_{1y} - o_{qy})(q_{2y} - o_{qy}))}$$

4 Orthographic Image

P : Coordinates of point in world coordinate system
$r1, r2, r3$: X, Y, Z axes of camera's coordinate system in world coordinate system
$\therefore r3$ is normal to projection or image plane
t : center of projection in world coordinate system
$\therefore$ projection plane passes through t
Hence, the equation of image plane in world coordinate system is

$$(Q - t) \cdot r3 = 0$$

If Q is image of point P, then $(Q - P) \parallel r3$.

$$\therefore Q = P + \lambda r3$$

$$\therefore (P + \lambda r3 - t) \cdot r3 = 0$$

$$\therefore P \cdot r3 - t \cdot r3 + \lambda ||r3||^2 = 0$$

$$\therefore \lambda = \frac{t \cdot r3 - P \cdot r3}{||r3||^2}$$

$$\boxed{Q = P + \frac{t \cdot r3 - P \cdot r3}{||r3||^2} r3}$$

5 Projection Matrix

For the **dataset 1**, the camera calibration matrix obtained is:

$$M = \begin{bmatrix} 0.2905 & 0.0532 & -0.1866 & -0.6283 \\ -0.0881 & 0.3264 & -0.0881 & -0.6010 \\ 0.0002 & 0.0002 & 0.0002 & -0.0021 \end{bmatrix}$$

For the **dataset 2**, the camera calibration matrix obtained is:

$$M = \begin{bmatrix} -0.0087 & -0.0011 & 0.0039 & -0.9986 \\ -0.0001 & -0.0092 & -0.0005 & 0.0520 \\ -0.0000 & -0.0000 & -0.0000 & -0.0027 \end{bmatrix}$$

Code used for validating the Calibration matrix using the input image and world coordinates. We use max error and mean error between individual coordinates obtained after using M on World coordinated with Image coordinates:

```
function [] = findError( f3D, f2D, M )
%findError Outputs maximum error

%Using the obtained 'M' matrix on the given 3D points
f2 = M*f3D;
%Homogenous coordinates
f2 = [f2(1,:)./f2(3,:); f2(2,:)./f2(3,:); f2(3,:)./f2(3,:)];

% Using given image coordinates to validate
K = abs((f2 - f2D)./f2D);
disp('Maximum error:');
disp(max(max(K)));

disp('Mean error:');
disp(mean(mean(K)));
end
```

Error obtained for dataset 1:

```
Validating M
Maximum error:
   1.7246e-13

Mean error:
   3.0109e-14
```

Error obtained for dataset 2:

```
Validating M
Maximum error:
    0.1084

Mean error:
    0.0023
```

When we introduce noise to the data used to calculate M(err), the max error increases to 0.205 which is many orders of magnitude higher than the dataset without noise. This illustrates that even a noise corresponding to 0.05 times max value can greatly impact M.

6 Size of paper

The points should be selected in the following order:

- First select the points of a4 paper such that the ordering is in anti clockwise order and first two points correspond to shorter edge of paper. The fourth point should be selected while pressing shift.
- Similarly select 4 points on the other page.