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Robotics Mini Project

Kinematics Analysis of a Robot Arm

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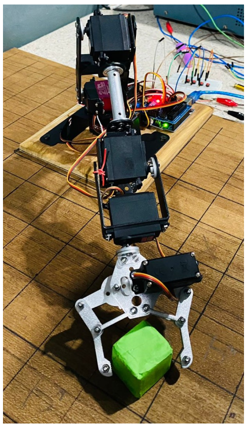
Abstract

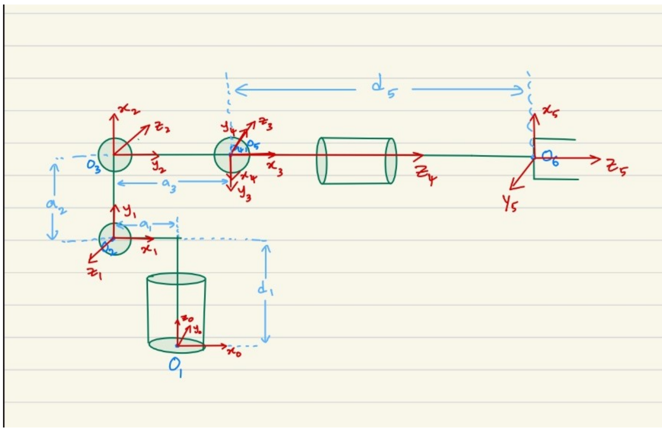
This project analyzes the kinematics of a 5-DoF robot arm designed for a pick-and-place task. The study involves constructing a Denavit-Hartenberg (DH) table, deriving forward and inverse kinematics, and calculating the manipulator Jacobian. Validation of theoretical results was performed using sample joint configurations and end-effector positions. The results demonstrate accurate execution of the specified task.

1 Introduction

The goal of this project is to analyze the kinematics of a robot arm for a pick-and-place task. The arm, with a 5-DoF configuration, was constructed using servo motors. Kinematic analysis, including forward and inverse kinematics, was used to understand and control the robot's motion.



(a) Robot arm



(b) Frame assignment

2 DH Table for the 5-DOF Robot Arm

The following table represents the Denavit-Hartenberg (DH) parameters for the 5-DOF robot arm:

Joint	d **(m)**	a **(m)**	α **(rad)**	θ **(rad)**
1	0.07	0.02	$\pi/2$	$\theta_1 - 7^\circ$
2	0	0.105	π	$\theta_2 + 85^\circ$
3	0	0.148	0	$\theta_3 + 81^\circ$
4	0	0	$\pi/2$	$\theta_4 + 90^\circ$
5	0.11	0	0	$\theta_5 + 180^\circ$

Table 1: Denavit-Hartenberg (DH) Parameters for the 5-DOF Robot Arm

The Denavit-Hartenberg (DH) table defines the kinematic parameters of the robot arm. These parameters are used to describe the relative positions and orientations of each link and joint in the robot arm.

3 Forward Kinematics for a 5-DOF Robot Arm

Forward kinematics is used to compute the position and orientation of the end-effector relative to the base frame. The transformation matrices derived from the Denavit-Hartenberg (DH) parameters are multiplied to find the overall transformation.

Transformation Matrix

The transformation matrix for each joint is given by:

$$T_i = \begin{bmatrix} \cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i & a_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Individual Transformation Matrices

Using the given DH parameters:

$$T_1 = \begin{bmatrix} \cos\theta_1 & 0 & \sin\theta_1 & 0 \\ \sin\theta_1 & 0 & -\cos\theta_1 & 0 \\ 0 & 1 & 0 & 0.07 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad T_2 = \begin{bmatrix} -\cos\theta_2 & \sin\theta_2 & 0 & -0.105\cos\theta_2 \\ -\sin\theta_2 & -\cos\theta_2 & 0 & -0.105\sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3 = \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 & 0.148\cos\theta_3 \\ \sin\theta_3 & \cos\theta_3 & 0 & 0.148\sin\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad T_4 = \begin{bmatrix} \cos\theta_4 & 0 & \sin\theta_4 & 0 \\ \sin\theta_4 & 0 & -\cos\theta_4 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_5 = \begin{bmatrix} \cos\theta_5 & -\sin\theta_5 & 0 & 0 \\ \sin\theta_5 & \cos\theta_5 & 0 & 0 \\ 0 & 0 & 1 & 0.11 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The overall transformation matrix T_0^5 can be computed as:

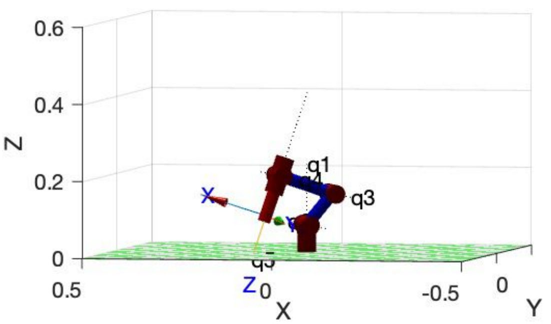
$$T_0^5 = T_1 \cdot T_2 \cdot T_3 \cdot T_4 \cdot T_5$$

Example Calculation

Using sample joint angles:

$$\theta_1 = 30^\circ,\ \theta_2 = 45^\circ,\ \theta_3 = 30^\circ,\ \theta_4 = 90^\circ,\ \theta_5 = 30^\circ$$

From the calculated T_0^5, the position and orientation of the end-effector are determined:



Position: (x, y, z) = (10 cm, 4 cm, 9 cm)

The forward kinematics with sample joint angles were verified to correctly compute the end-effector's position.

4 Inverse Kinematics and Jacobian for a 5-DOF Robot Arm

Inverse kinematics calculates the joint angles $(\theta_1, \theta_2, \theta_3, \theta_4, \theta_5)$ required to position the end-effector at a specified location (x, y, z) and orientation.

The Jacobian matrix relates joint velocities to the end-effector's linear and angular velocities:

$$\dot{x} = J \cdot \dot{\theta}$$

where J is the Jacobian matrix, $\dot{x}$ is the velocity vector of the end-effector, and $\dot{\theta}$ is the joint velocity vector.

The inverse kinematics and jacobian for the task of picking up the green cube at the position (12.5, -22.5, 0.08) from the initial position are given below.

Individual Transformation Matrices

$$T_{0\_1} = \begin{bmatrix} 0.4856 & 5.353e-17 & -0.8742 & 0 \\ -0.8742 & 2.974e-17 & -0.4856 & 0 \\ 0 & 1.6123e-17 & 1 & 0.07 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad T_{1\_2} = \begin{bmatrix} 0.198 & 0.9802 & 1.2e-16 & 0.02079 \\ 0.9802 & -0.198 & -2.424e-17 & 0.1029 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{2\_3} = \begin{bmatrix} -0.3173 & -0.9483 & 0 & -0.04697 \\ 0.9483 & -0.3173 & 0 & 0.1404 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad T_{3\_4} = \begin{bmatrix} 0.3407 & -5.757e-17 & 0.9402 & 0 \\ 0.9402 & 2.086e-17 & -0.3407 & 0 \\ 0 & 1.6123e-17 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{4\_5} = \begin{bmatrix} -1 & 7.981e-14 & 0 & 0 \\ -7.981e-14 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0.11 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Final Homogeneous Transformation Matrix (T)

$$T = \begin{bmatrix} 0.08433 & -0.8742 & 0.4783 & 0.125 \\ -0.1518 & -0.4856 & -0.8609 & -0.225 \\ 0.9848 & -7.852e-14 & -0.1736 & 0.08 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Joint Angles

Radians: $[-0.9415, -0.1120, 0.4800, -0.3477, 0.0000]$

This configuration is within joint limits.

Jacobian Matrix (Base Frame)

$$J = \begin{bmatrix} 0.2250 & -0.0049 & -0.0451 & -0.0093 & 0 \\ 0.1250 & 0.0087 & 0.0812 & 0.0167 & 0 \\ 0.0000 & 0.2574 & -0.2366 & -0.1083 & 0 \\ -0.0000 & -0.8742 & 0.8742 & 0.8742 & 0.4783 \\ 0.0000 & -0.4856 & 0.4856 & 0.4856 & -0.8609 \\ 1.0000 & 0.0000 & -0.0000 & -0.0000 & -0.1736 \end{bmatrix}$$