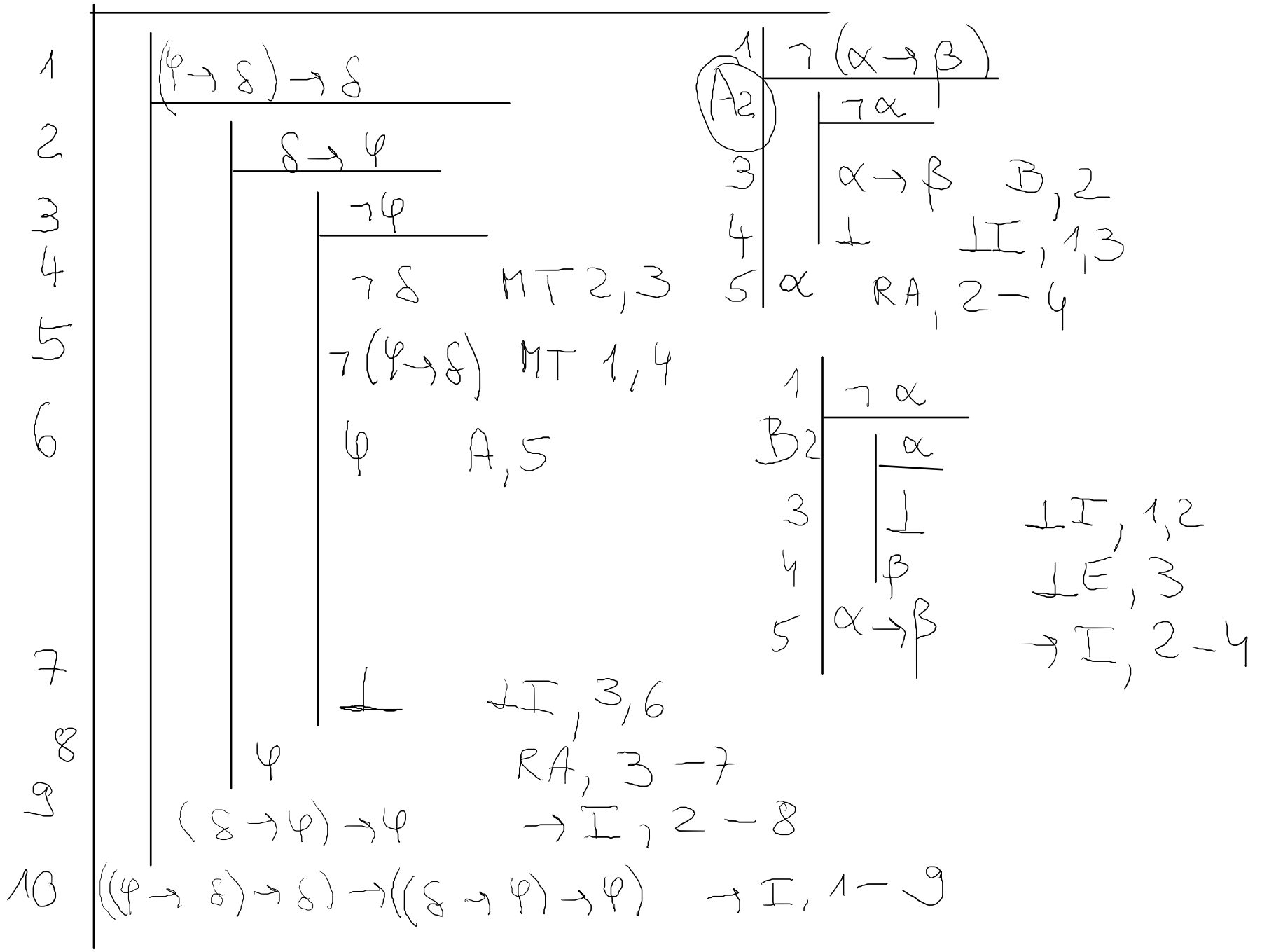

1 (φ → δ) → δ
2 δ → φ
3 ¬φ
4 ¬δ MT 2,3
5 ¬(φ → δ) MT 1,4
6 φ A,5
7 ⊥ ⊥I, 3,6
8 φ RA, 3–7
9 (δ → φ) → φ →I, 2–8
10 ((φ → δ) → δ) → ((δ → φ) → φ) →I, 1–9
1 ¬(α → β)
A2 ¬α
3 α → β B, 2
4 ⊥ ⊥I, 1,3
5 α RA, 2–4
1 ¬α
B2 α
3 ⊥ ⊥I, 1,2
4 β ⊥E, 3
5 α → β →I, 2–4

$\neg p \vee (q \to p) \not\vdash \neg p \wedge q$

$\neg p \vee (q \to p) \not\vdash \neg p \wedge q$

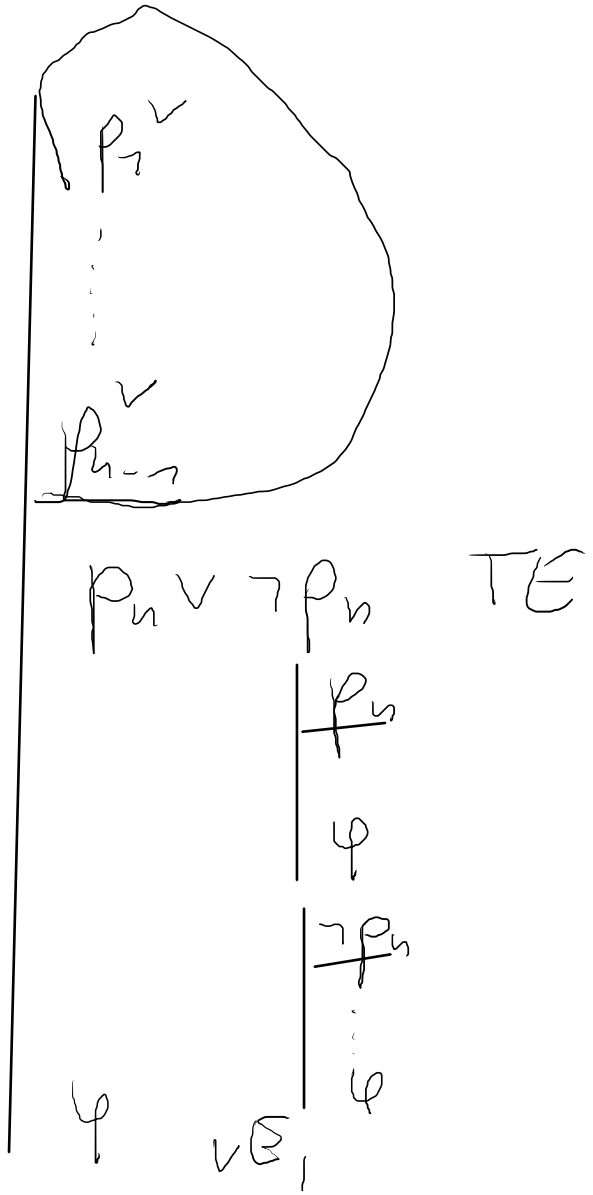
existe a val. v com

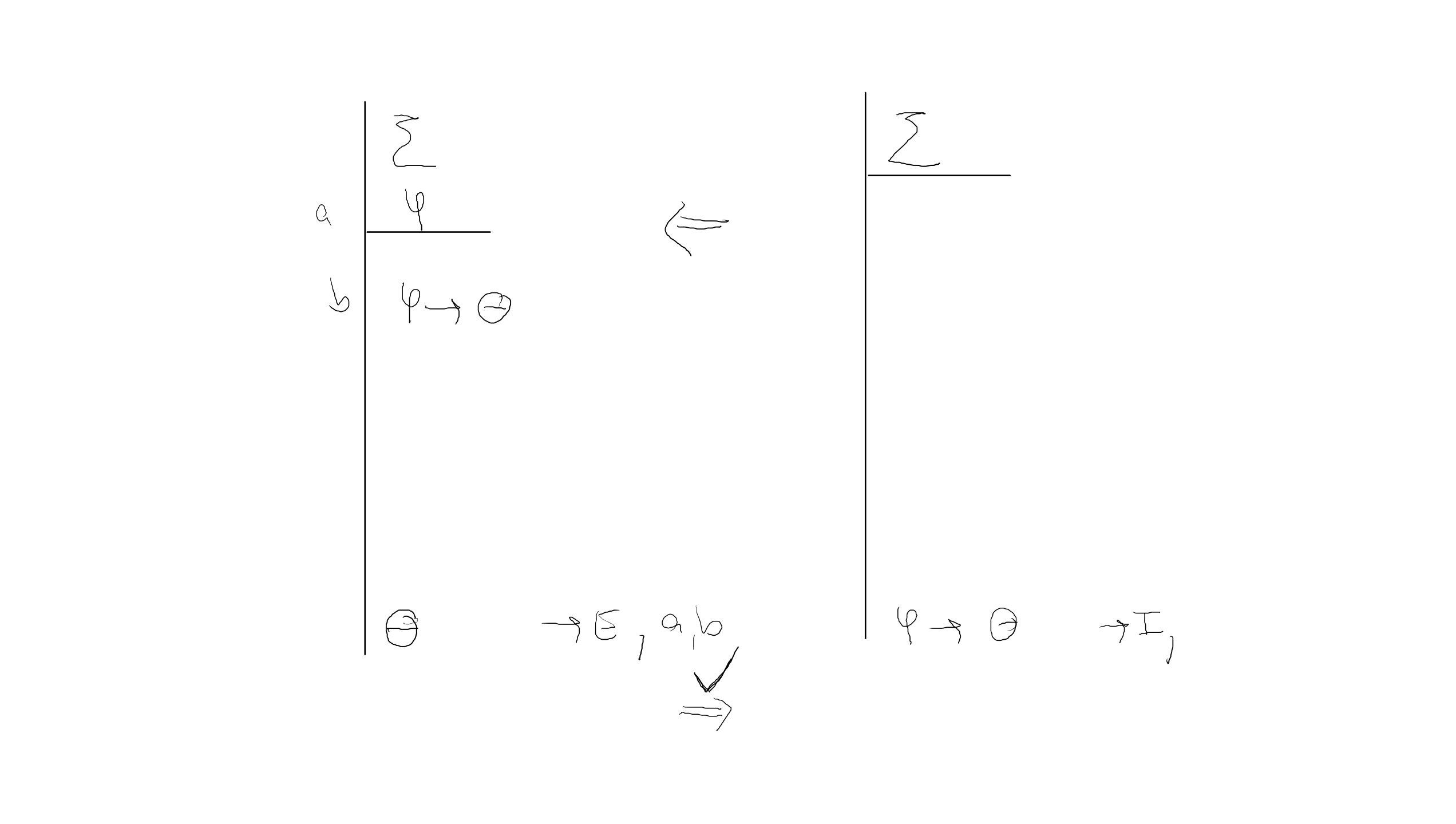
$v(p) = f$ tal que

$v(q) = f$ $v(\neg p \vee (q \to p)) = v$

mas $v(\neg p \wedge q) = f$.

p_1^v
p_{n-1}^v
$\neg p_n$
φ


p_1^v
p_{n-1}^v
$p_n \vee \neg p_n$
TE
p_n
φ
$\neg p_n$
φ
φ
$\vee E_1$



	Σ		Σ
a	φ	$\Leftarrow$	
b	$\varphi \to \Theta$		
	Θ	$\to E, a, b$	$\varphi \to \Theta$ $\to I,$

$\Rightarrow$

(V) $p_1^V, \ldots, p_n^V \vdash \varphi^V \Rightarrow \varphi \checkmark$

$\neg\varphi \checkmark$

$p_1^V, \ldots, p_n^V \vdash (\varphi \vee \psi)$ $\quad \varphi \vee \psi$

$\Uparrow$ $\vee I$

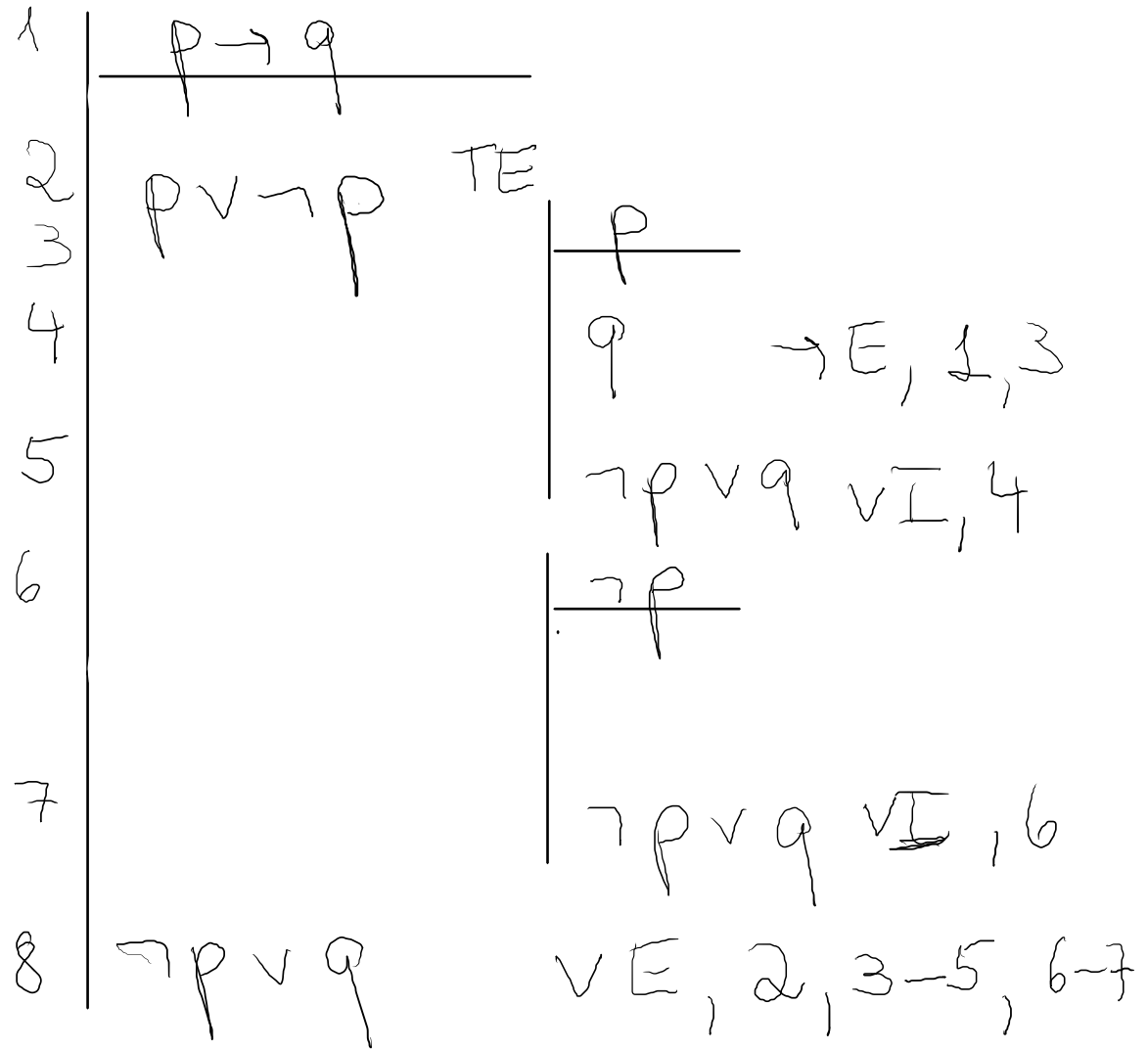
$p_1^V, \ldots, p_n^V \vdash \psi$

$p_1^V, \ldots, p_n^V \vdash \neg\varphi$

$\vdots$

$\vartheta(\varphi) = p$

$\vartheta(\psi) = p_V$


1 p → q
2 p ∨ ¬p TE
3 p
4 q →E, 1, 3
5 ¬p ∨ q ∨I, 4
6 ¬p
7 ¬p ∨ q ∨I, 6
8 ¬p ∨ q ∨E, 2, 3–5, 6–7

24. u)

1	$(((\varphi \to \psi) \to (\neg\delta \to \neg\gamma)) \to \delta) \to \theta$	
2	$\theta \to \varphi$	C:
3	γ	
4	$\neg\varphi$	
5	$\neg\theta$	MT 2,4
6	$\neg(((\varphi \to \psi) \to (\neg\delta \to \neg\gamma)) \to \delta)$	MT 1,5
7	$(\varphi \to \psi) \to (\neg\delta \to \neg\gamma)$	A, 6
8	$\neg\delta$	C, 6
9	$\varphi \to \psi$	D, 4
10	$\neg\delta \to \neg\gamma$	$\to$E 7,9
11	$\neg\neg\gamma$	DN, 3
12	$\neg\neg\delta$	MT 10, 11
13	$\bot$	$\bot$I, 8, 12
14	φ	RA, 4 – 13

$\neg(\alpha \to \beta)$

β

α

β R, 2

$\alpha \to \beta$ $\to$I

$\bot$

$\neg\beta$ $\neg$I

D

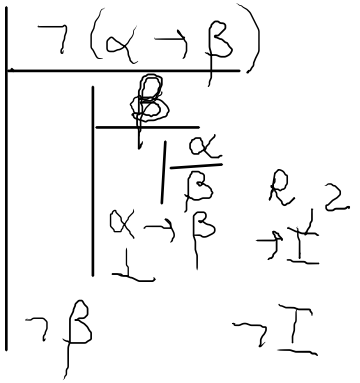
$\neg\varphi$

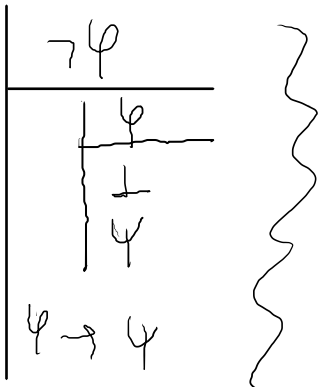
φ

$\bot$

ψ

$\varphi \to \psi$





Regras derivadas

m	$\alpha \to \beta$	
n	$\neg\beta$	
	$\neg\alpha$	MT, m, n

	$\varphi \vee \neg\varphi$	TE

m	φ	
	$\neg\neg\varphi$	DN, m ‖ $\neg\neg$I, m

m	$\neg\varphi$	
n	$\bot$	
	φ	RA, $m-n$