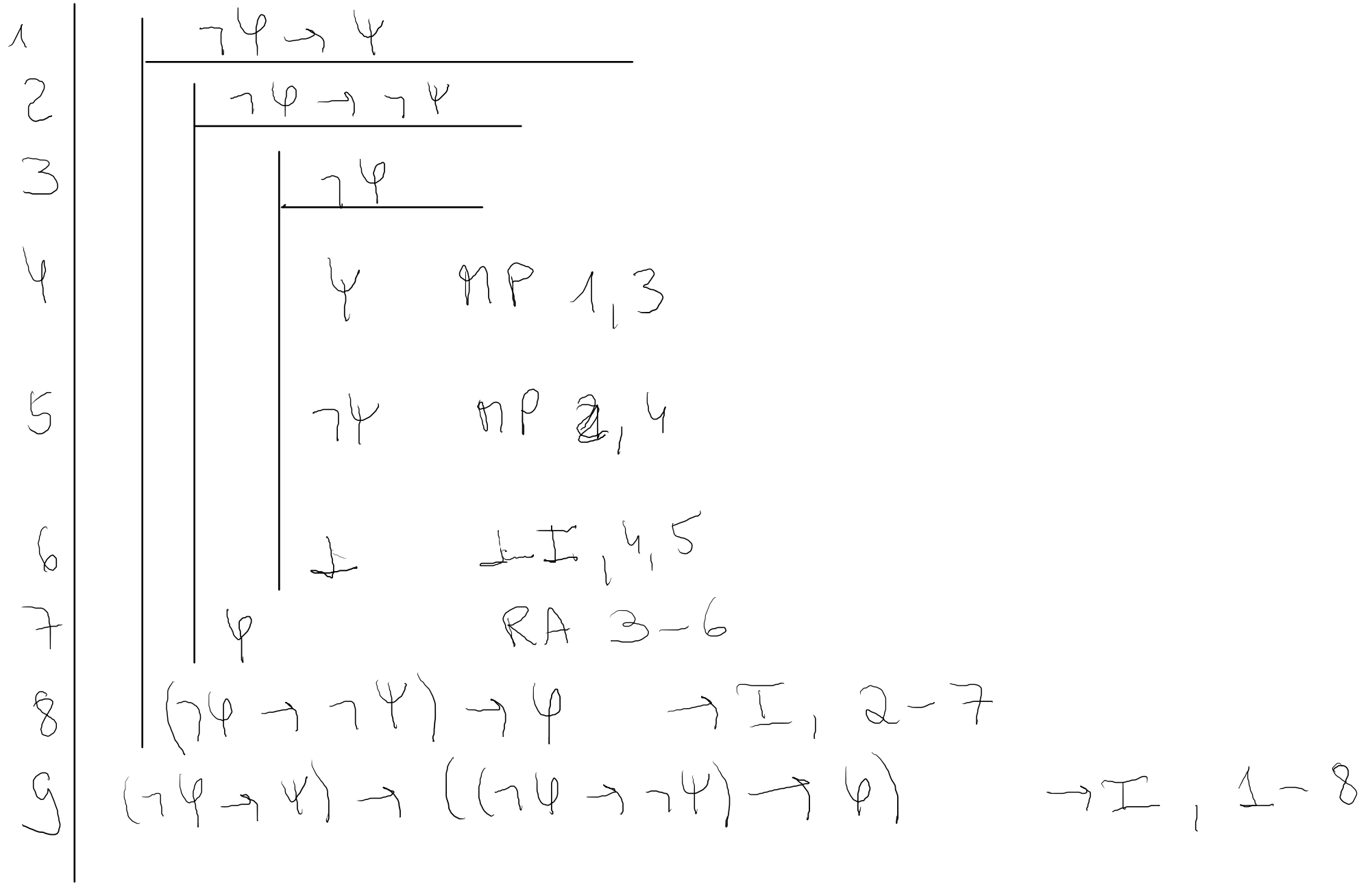
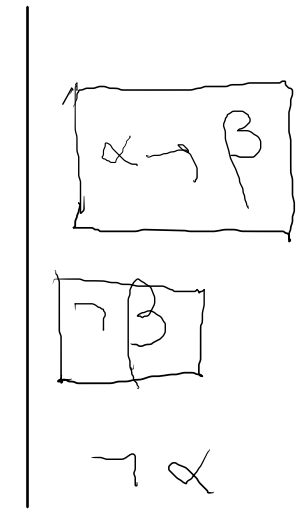
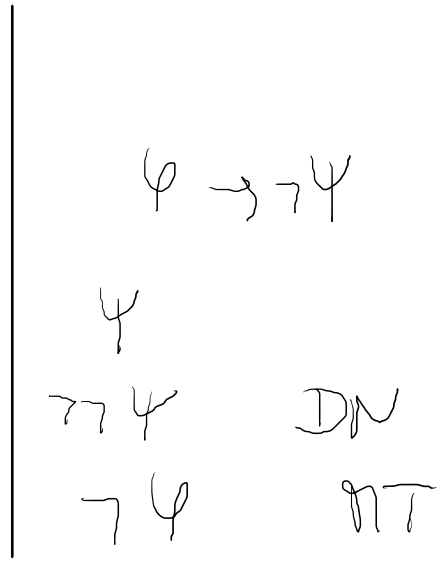
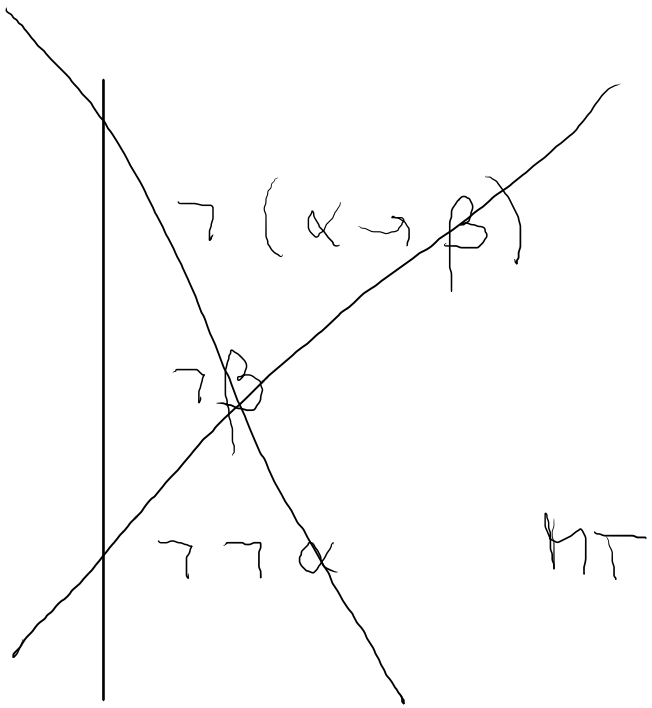
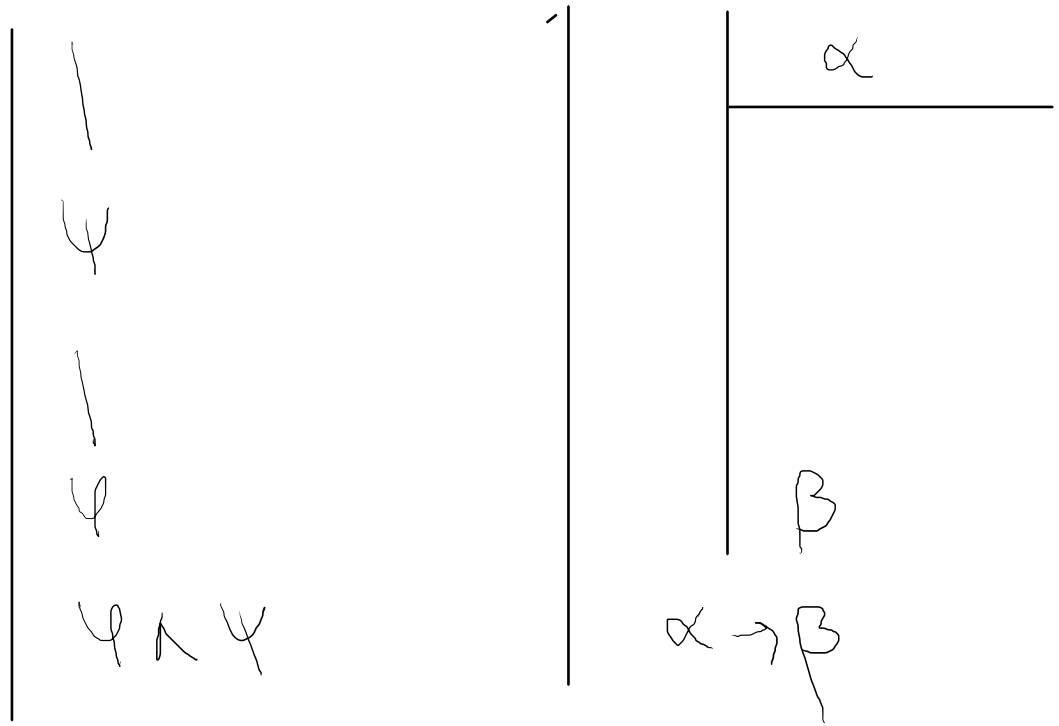
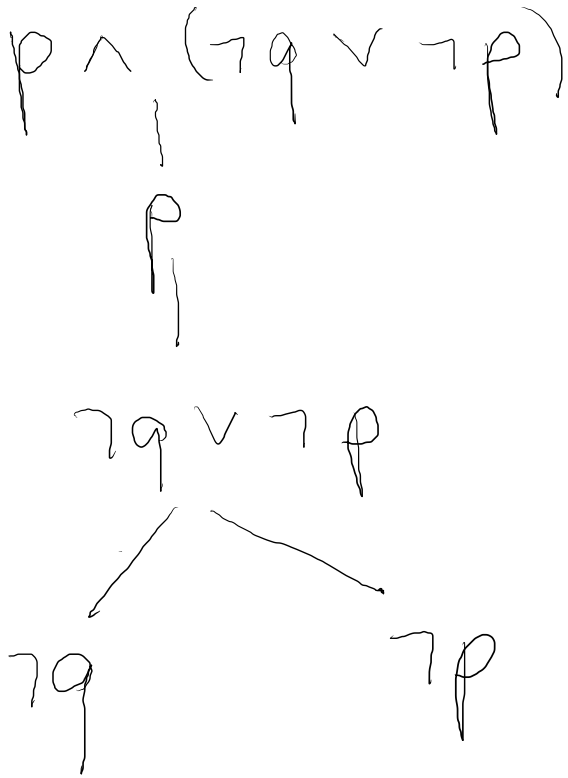

1 ¬¬φ → φ
2 ¬φ → ¬φ
3 ¬φ
4 φ MP 1,3
5 ¬φ MP 2,4
6 ⊥ ⊥I 1,4,5
7 φ RA 3-6
8 (¬φ → ¬φ) → φ →I 2-7
9 (¬¬φ → φ) → ((¬φ → ¬φ) → φ) →I 1-8


α → β
¬β
¬α
MT


φ → ¬ψ
ψ
¬¬ψ
DN
¬φ
MT


¬(α → β)
¬β
¬¬α
MT


ψ
ψ
ψ ∧ ψ
α
β
α → β


p ∧ (¬q ∨ ¬p)
p
¬q ∨ ¬p
¬q
¬p

$\neg\neg((p \vee q) \wedge (\neg p \wedge \neg q))$

$(p \vee q) \wedge (\neg p \wedge \neg q)$

$p \vee q$

$\neg p \wedge \neg q$

p

$\neg p$

$\neg q$

q

$\neg p$

$\neg q$

$\neg(\alpha \wedge \beta)$

$\neg\alpha \vee \neg\beta$

$\neg(\alpha \vee \beta)$

$\neg\alpha \wedge \neg\beta$

$$\neg\left[\left((p \to q) \wedge (q \to R)\right) \to \neg(\neg R \wedge p)\right]$$

$$(p \to q) \wedge (q \to R)$$

$$\neg\neg(\neg R \wedge p)$$

$$\neg R \wedge p$$

$$p \to q$$

$$q \to R$$

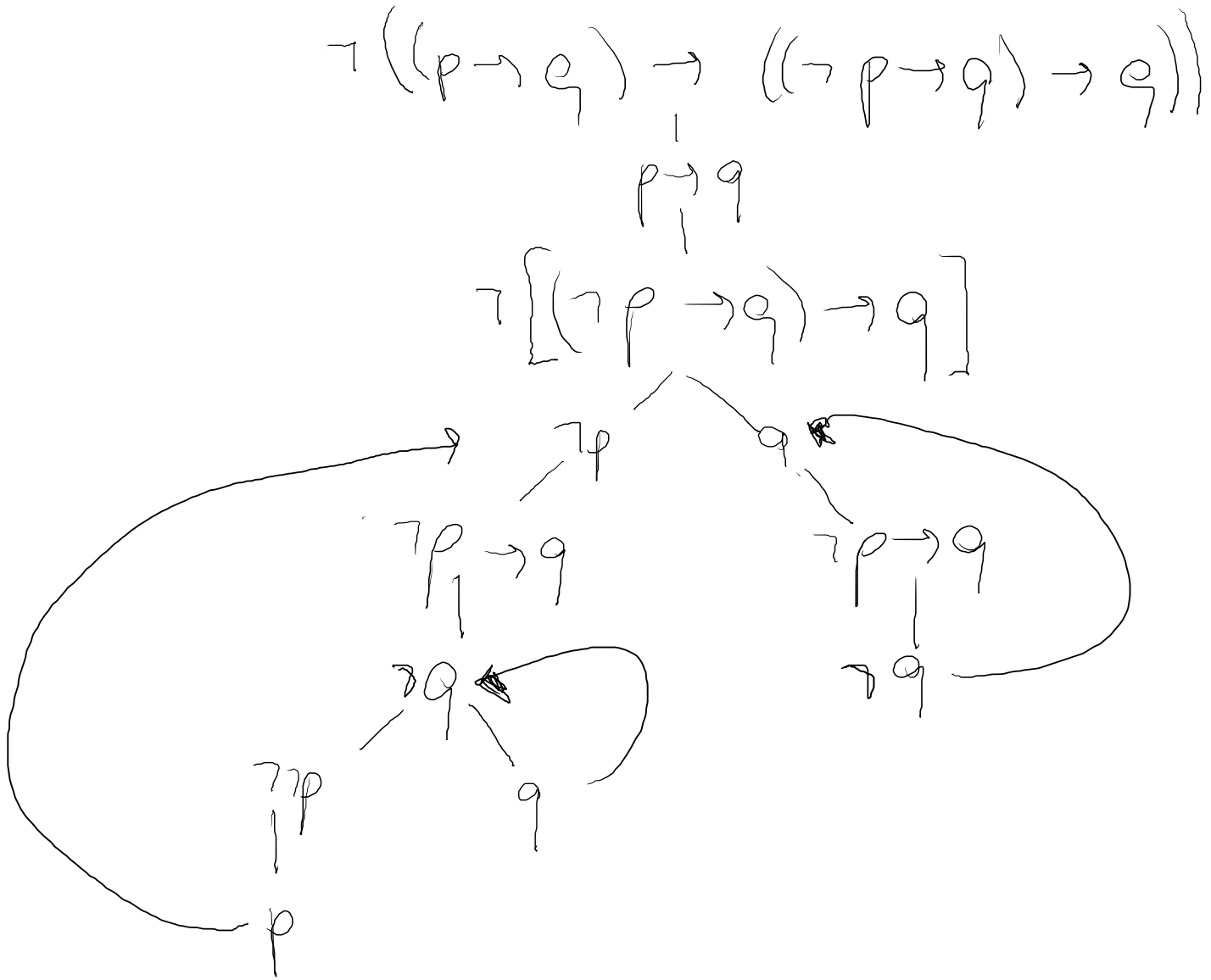
$$\neg R$$

p

$\alpha \to \beta$

$\neg(\alpha \to \beta)$

$\alpha \wedge \neg\beta$


¬((p→q)→((¬p→q)→q))
p→q
¬[(¬p→q)→q]
¬p
q
¬p→q
¬p→q
¬q
¬q
¬¬p
q
p

$$\neg\left[(p \to q) \to ((\neg p \to q) \to q)\right]$$

$$\equiv \neg\left[(\neg p \vee q) \to ((p \vee q) \to q)\right]$$

$$\equiv \neg\left[\neg(\neg p \vee q) \vee (\neg(p \vee q) \vee q)\right]$$

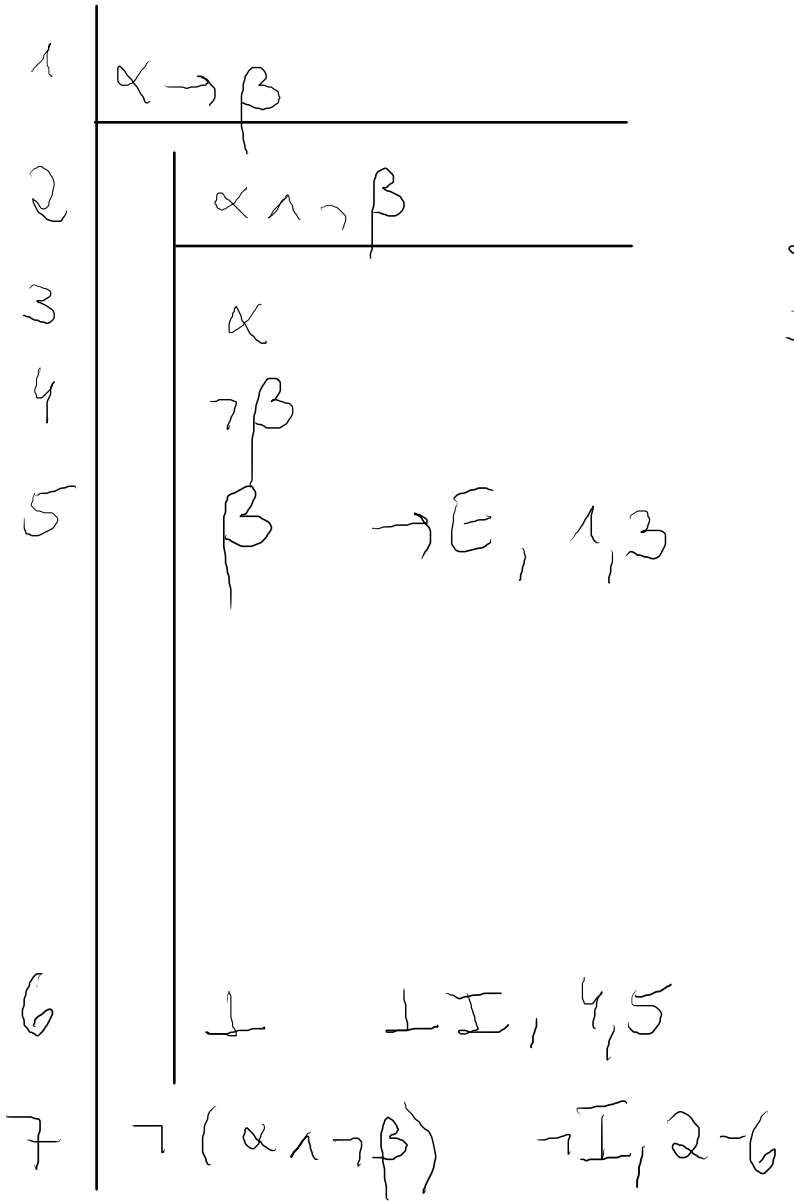
$$\equiv \neg\left[(p \wedge \neg q) \vee ((\neg p \wedge \neg q) \vee q)\right]$$

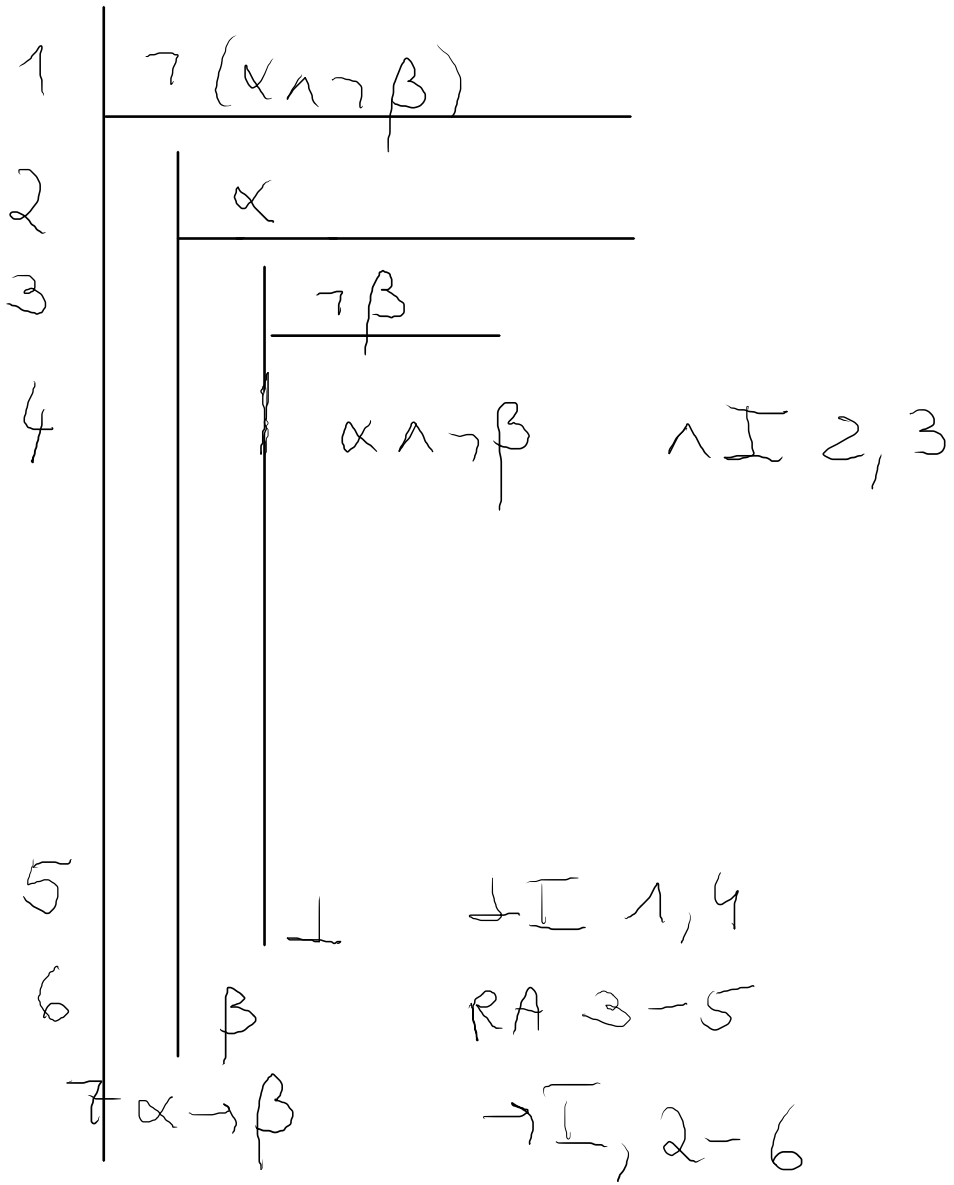
$$\equiv \neg(p \wedge \neg q) \wedge \neg((\neg p \wedge \neg q) \vee q)$$

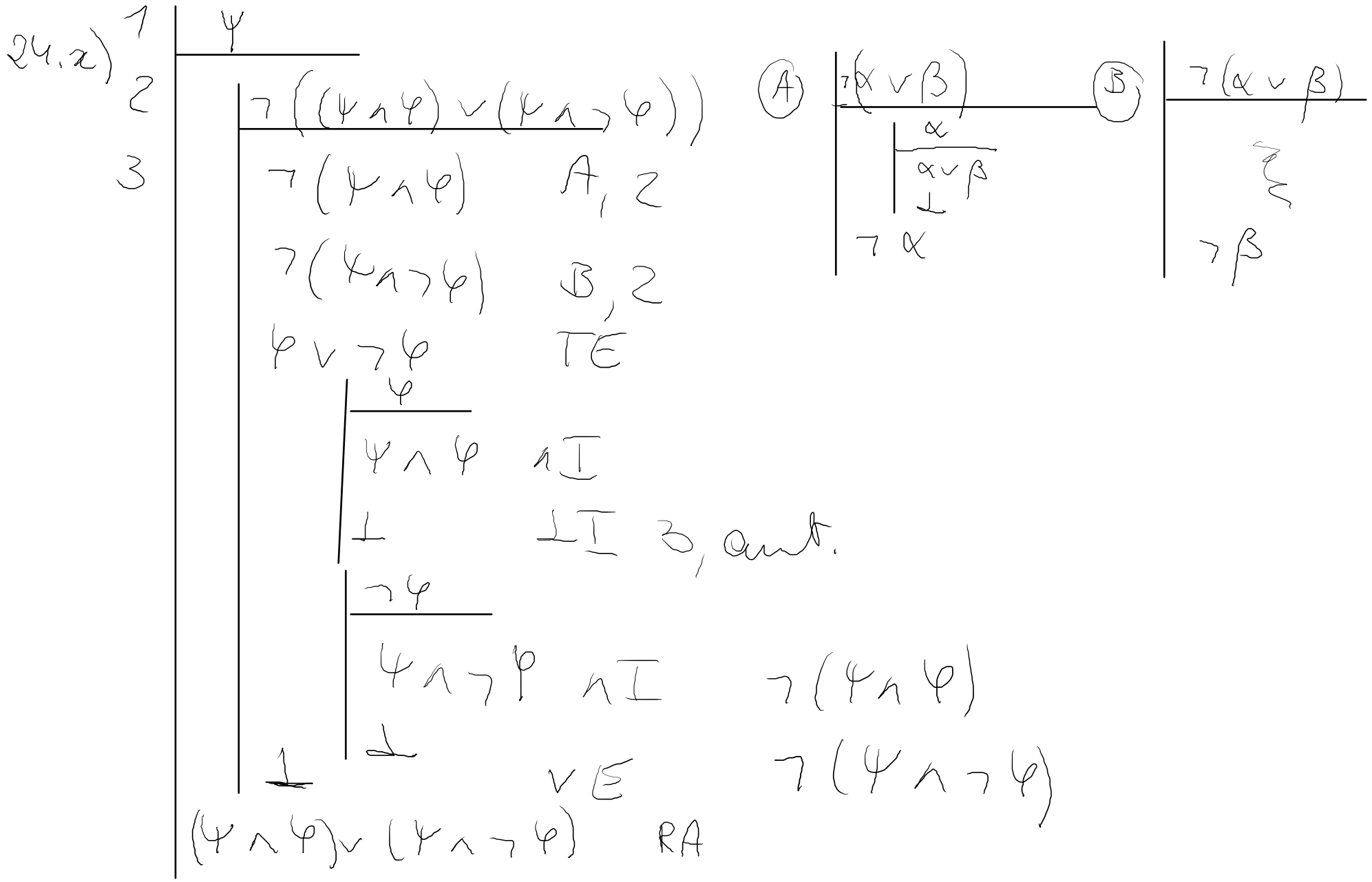
$$\equiv (\neg p \vee q) \wedge (\neg(\neg p \wedge \neg q) \wedge \neg q)$$

$$\equiv (\neg p \vee q) \wedge (p \vee q) \wedge \neg q$$

$$\equiv$$


1 α → β
2 α ∧ ¬β
3 α
4 ¬β
5 β →E, 1,3
6 ⊥ ⊥I, 4,5
7 ¬(α ∧ ¬β) ¬I, 2-6


1 ¬(α ∧ ¬β)
2 α
3 ¬β
4 α ∧ ¬β ∧I 2, 3
5 ⊥ ⊥I 1, 4
6 β RA 3-5
7 α → β →I, 2-6


(24.2)
1 ψ
2 ¬((ψ∧φ)∨(ψ∧¬φ))
3 ¬(ψ∧φ) A, 2
¬(ψ∧¬φ) B, 2
φ∨¬φ TE
φ
ψ∧φ ∧I
⊥ ⊥I 3, aus.
¬φ
ψ∧¬φ ∧I ¬(ψ∧φ)
⊥
⊥ ∨E ¬(ψ∧¬φ)
(ψ∧φ)∨(ψ∧¬φ) RA
A ¬(α∨β)
α
α∨β
⊥
¬α
B ¬(α∨β)
¬β