(w) $(\psi \to \varphi) \vee (\delta \to \gamma) \vdash (\psi \to \gamma) \vee (\delta \to \varphi)$

(x) $\psi \vdash (\psi \wedge \varphi) \vee (\psi \wedge \neg\varphi)$

Nota: Caso uses regras derivadas, terás que as mostrar separamente.

25. Justifique a veracidade ou falsidade das afirmações seguintes. Se $\Sigma \vdash \varphi$ apresente uma dedução. Caso contrário, indique uma valorização que satisfaz Σ mas não satisfaz φ.

(a) $\neg\delta, \delta \vee \varphi \vdash \varphi$

(b) $\varphi \wedge \neg\varphi \vdash \neg(\delta \to \psi) \wedge (\delta \to \psi)$

(c) $\neg(\neg\delta \vee \varphi) \vdash \delta$

(d) $\delta \vee \psi, \neg\psi \vee \varphi \vdash \delta \vee \varphi$

(e) $\neg(\delta \to \psi) \vdash \psi \to \delta$

(f) $\neg\varphi \vdash \varphi \to \psi$

(g) $\vdash (\neg\delta \to \delta) \to \delta$

(h) $\neg\delta \to \neg\psi \vdash \neg\psi \to \neg\delta$

26. Justifica a validade ou falsidade de cada uma das afirmações seguintes, apresentando demonstrações ou contra-exemplos:

(a) se $\vdash \psi$, então $\delta_1, \dots, \delta_n \vdash \psi$;

(b) se $\delta_1, \dots, \delta_n \vdash \psi$, então $\vdash \psi$;

(c) se $\delta_1, \delta_2 \vdash \psi$, então $\delta_2, \delta_1 \vdash \psi$;

(d) se $\delta_1, \delta_2 \vdash \varphi \to \psi$ e $\delta_1, \delta_2 \vdash \varphi$, então $\delta_1, \delta_2 \vdash \psi$;

(e) se $\delta_1, \delta_2 \vdash \varphi \to \psi$ e $\delta_1 \vdash \varphi$, então $\delta_1, \delta_2 \vdash \psi$;

(f) se $\delta_1, \delta_2 \vdash \varphi \to \psi$ e $\delta_1 \vdash \varphi$, então $\delta_1 \vdash \psi$;

(g) se $\delta_1 \vdash \varphi \to \psi$ e $\delta_2 \vdash \varphi$, então $\delta_1, \delta_2 \vdash \psi$;

(h) se $\delta, \delta \vdash \psi$ se e só se $\delta \vdash \psi$.

27. Mostra sem usar a completude de DN as seguintes equivalencias dedutivas. Nota que tens de mostrar que da primeita fórmula se deduz a segunda e vice-versa.

(a) $\neg(\delta_1 \wedge \delta_2) \dashv\vdash \neg\delta_1 \vee \neg\delta_2$

(b) $\neg(\delta_1 \vee \delta_2) \dashv\vdash \neg\delta_1 \wedge \neg\delta_2$

(c) $\delta_1 \to \delta_2 \dashv\vdash \neg\delta_1 \vee \delta_2$

(d) $\neg(\delta_1 \to \delta_2) \dashv\vdash \delta_1 \wedge \neg\delta_2$

(e) $\neg\delta_1 \wedge \delta_2 \vdash \delta_1 \vee \delta_2$

(f) $\neg\delta_1 \wedge \neg\delta_2 \vdash \delta_1 \to \delta_2$

(g) $\delta_1 \wedge \delta_2 \vdash \delta_1 \vee \delta_2$

(h) $\delta_1 \wedge \neg\delta_2 \vdash \neg(\delta_1 \wedge \delta_2)$

(i) $(\psi \wedge \varphi) \wedge \delta \dashv\vdash \psi \wedge (\varphi \wedge \delta)$

(j) $(\psi \vee \varphi) \vee \delta \dashv\vdash \psi \vee (\varphi \vee \delta)$

(k) $\neg(\neg\delta_1 \wedge \neg\delta_2) \dashv\vdash \delta_1 \vee \delta_2$

(l) $\delta \to \varphi \dashv\vdash (\delta \to (\delta \wedge \varphi)) \wedge ((\delta \wedge \varphi) \to \delta)$

(m) $\delta_1 \to \delta_2 \dashv\vdash \neg(\delta_1 \wedge \neg\delta_2)$

(n) $\delta \to \varphi \dashv\vdash (\varphi \to (\delta \vee \varphi)) \wedge ((\delta \vee \varphi) \to \varphi)$

(o) $\delta_1 \wedge \delta_2 \dashv\vdash \neg(\neg\delta_1 \vee \neg\delta_2)$

(p) $(\delta \to \varphi) \wedge (\varphi \to \delta) \dashv\vdash (\delta \vee \varphi) \to (\delta \wedge \varphi)$

(q) $(\delta \to \varphi) \dashv\vdash (\neg\varphi \to \neg\delta)$

(r) $(\delta \to \varphi) \wedge (\varphi \to \delta) \dashv\vdash (\delta \vee \varphi) \to (\delta \wedge \varphi)$

(s) $\varphi \wedge (\delta \vee \psi) \dashv\vdash (\varphi \wedge \delta) \vee (\varphi \wedge \psi)$

(t) $(\varphi \vee \psi) \to \delta \dashv\vdash (\varphi \to \delta) \wedge (\psi \to \delta)$

(u) $(\delta \vee \varphi) \wedge (\delta \vee \psi) \dashv\vdash \delta \vee (\varphi \wedge \psi)$

(v) $\varphi \to (\psi \wedge \delta) \dashv\vdash (\varphi \to \psi) \wedge (\varphi \to \delta)$