



ON THE EFFECT OF POSSIBLE REFLECTIONS AT THE RETURN STROKE WAVEFRONT ON RADIATED FIELDS FROM LIGHTNING STRIKES TO TALL STRUCTURES

Abbas Mosaddeghi¹, Farhad Rachidi¹, Marcos Rubinstein², Davide Pavanello³,
Volodymyr Shostak⁴, Wasyl Janischewskyj⁵, Emanuel Petrache⁶

¹ Swiss Federal Institute of Technology (EPFL), Lausanne, Switzerland

² Western University of Applied Sciences, Yverdon, Switzerland

³ ALSTOM Power, Baden, Switzerland

⁴ National Tech. Univ. of Ukraine "Kyiv Polytechnic Inst."

⁵ University of Toronto, Toronto, Canada

⁶ Kinectrics, Toronto, Canada

Abstract - An extension of the engineering return-stroke models for lightning strikes to tall structures that takes into account the presence of possible reflections at the return-stroke (RS) wavefront and the presence of an upward leader is presented. Closed-form, iterative solutions for the current distribution along the channel and the strike object are derived. By setting the value of the reflection coefficient to -1 at the return-stroke wavefront, the current at that location is forced to zero, avoiding thereby a discontinuity. Simulation results for the electromagnetic fields generated by RS current of lightning strikes to the CN Tower (553 m) are presented. It is shown that taking into account the reflections at the return-stroke wavefront results in fine structure in the field's late-time response. Moreover, the obtained results are in better agreement with experimental observations, reproducing both the early narrow undershoot and the far-field zero crossing.

1 INTRODUCTION

The problem of electromagnetic fields due to lightning strikes to a tall tower has been the subject of several recent studies (e.g. [1]). The presence of an elevated strike object has been included in two classes of return stroke models [2], namely, the engineering models and the electromagnetic or Antenna-Theory (AT) models. Recently, Pavanello et al. [3] presented measurements of the electric and magnetic fields at three distances from the return stroke current associated with lightning strikes to the Toronto CN Tower (553 m). The vertical component of the electric field and the azimuthal component of the magnetic field were measured simultaneously at distances of 2.0 km, 16.8 km, and 50.9 km from the CN Tower. The waveforms of the electric and magnetic fields at 16.8 km

and 50.9 km exhibited a narrow undershoot and a first zero crossing about 5 microseconds after the onset of the return stroke. For fields at 50.9 km, the expected zero crossing at about 40 microseconds was also observed. Pavanello et al. [3, 4] presented also calculations of the electric and magnetic fields using six engineering models (TL, MTLL, MTLE, BG, TCS, extended to take into account the presence of a tall structure using a distributed-source approach [5], and the model proposed by Baba and Rakov [6], based on a lumped-source approach). It was shown that the six considered engineering models produce very similar results, especially as far as the initial peak of the fields is concerned. While a reasonable agreement between simulations and measurements was found for the magnetic fields, the measured electric field peaks were found to be significantly larger than the theoretical predictions, most probably due to the enhancement effect introduced by the building on which the electric field sensors were located [7, 8]. The model proposed by Baba and Rakov reproduced better than the others the narrow undershoot that can be observed right after the first peak because its formulation does not imply the presence of the 'turn-on' term, which must be taken into account for the other five models, in which a current discontinuity appears at the return stroke wavefront [9]. None of the six considered models was able to reproduce the typical zero-crossing of the far field.

In this paper, we propose an extension of the engineering models for return strokes to tall structures that takes into account the presence of possible reflections at the return stroke wavefront. To do this, we applied the procedure proposed by Shostak et al. [10], for which we propose a closed-form iterative solution.

2 EXTENDED MODEL

In this study, we make use of the engineering models extended to take into account the presence of an elevated strike object [5]. The tall object is represented as an ideal, uniform transmission line characterized by constant and frequency independent current reflection coefficients at its top and bottom (see Fig. 1). The expression for the current distribution along the lightning channel, $h < z < H$ reads

$$i(z, t) = P(z - h) i_o \left(h, t - \frac{z - h}{v^*} \right) - \rho_t i_o \left(h, t - \frac{z - h}{c} \right) + \left[(1 - \rho_t)(1 + \rho_t) \sum_{n=0}^{\infty} \rho_g^n \rho_t^n i_o \left(h, t - \frac{h + z}{c} - \frac{2nh}{c} \right) \right] \cdot u \left(t - \frac{z - h}{v} \right) \quad (1)$$

and, for the current distribution along the strike object, $0 < z \leq h$,

$$i(z, t) = (1 - \rho_t) \sum_{n=0}^{\infty} \left[\rho_t^n \rho_g^n i_o \left(h, t - \frac{h - z}{c} - \frac{2nh}{c} \right) + \rho_t^n \rho_g^{n+1} i_o \left(h, t - \frac{h + z}{c} - \frac{2nh}{c} \right) \right] u \left(t - \frac{h + z}{c} - \frac{2nh}{c} \right) \quad (2)$$

where h is the height of the tower, ρ_t and ρ_g are the top and bottom current reflection coefficients for upward and downward propagating waves, respectively, c is the speed of light, $P(z)$ is a model-dependent function, $u(t)$ the Heaviside unit-step function, v is the return-stroke front speed, and v^* is the current-wave speed [2].

In the proposed model we suppose that the RS channel is initiated at a height of h_0 above the tower. The time dependent wavefront level $H(t)$ and channel length $h_c(t)$ are simply given as

$$H(t) = h + h_0 + h_c(t) \quad (3)$$

$$h_c(t) = vt$$

In Fig. 1, the multiple reflections are illustrated using a lattice diagram. Starting at the level of h_0 where the downward and upward connecting leaders meet, the return stroke channel is assumed to be initiated, moving vertically in opposite directions at the speed of v .

The current injected into the tower from its top is reflected back and forth at its ends, and portions of it are transmitted into the channel; these transmitted pulses, which are assumed to travel at the speed of light c , catch up with the return stroke wavefront traveling at a lower speed v . Assuming that the current vanishes abruptly at the return stroke wavefront results in a discontinuity at the wavefront [9].

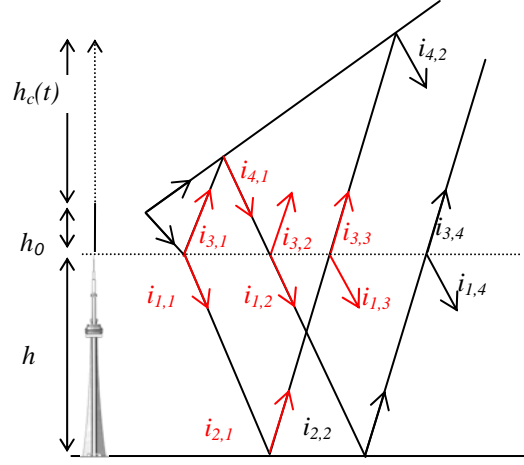


Fig. 1 – Lattice diagram of the return stroke current multiple reflections along the tower and the channel

One possible solution to avoid the presence of such discontinuity is to assume a reflection coefficient at the return stroke wavefront ρ_c set to -1. In this case the value of the current at the return stroke wavefront is forced to zero, avoiding therefore a discontinuity.

In this study, we developed an iterative approach to obtain closed-form expressions for the current distribution along the tower and along the channel, taking into account the reflection coefficient at the return stroke wavefront and the initiation of the RS above the tower ($h + h_0$).

The new expressions read

$$i(z, t) = P(z - h - h_0) i_o \left(h + h_0, t - \frac{z - h - h_0}{v} \right) + i_o \left(h + h_0, t - \frac{h + h_0 - z}{v} \right) \quad (4)$$

$$+ \sum_{n=1}^{\infty} i_{3,n} i_o \left(h + h_0, t - \frac{z - h}{c} - t_{3,n} \right) \quad (z > h)$$

$$+ \sum_{n=1}^{\infty} i_{4,n} i_o \left(h + h_0, t - \frac{h_c(t_{4,n}) - z}{c} - t_{4,n} \right)$$

$$i(z, t) = \sum_{n=1}^{\infty} i_{1,n} i_o \left(h + h_0, t - \frac{h - z}{c} - t_{1,n} \right) + \sum_{n=1}^{\infty} i_{2,n} i_o \left(h + h_0, t - \frac{z}{c} - t_{2,n} \right) \quad (0 < z < h) \quad (5)$$

The times and magnitudes of successive current terms in (4) and (5) have been determined and they are given in the Appendix.

The validity of the proposed expressions has been carefully checked through analysis and numerical

simulations. It can be easily seen that (4) and (5) reduce to (1) and (2), simply by substituting $h_0=0$ and $\rho_c=0$.

3 SIMULATIONS

We considered the same configuration of Pavanello et al. [3, 4], namely the 553-m-tall CN Tower. The values adopted for the reflection coefficients are: $\rho_t = -0.35$ at tower top, $\rho_g = 0.4$ at bottom of the tower (ground), and $\rho_c = -1$ at the return stroke wavefront. The undisturbed current is represented using the sum of two Heidler's functions:

$$i_0(t) = \frac{I_{o1}}{\eta_1} \frac{(t/\tau_{11})^2}{1+(t/\tau_{11})^2} \exp(-t/\tau_{11}) + \frac{I_{o2}}{\eta_2} \frac{(t/\tau_{12})^2}{1+(t/\tau_{12})^2} \exp(-t/\tau_{22}) \quad (6)$$

The values of the parameters are: $I_{o1} = 5.35$ kA, $\eta_1 = 0.639$, $\tau_{11} = 0.25$ μ s, $\tau_{21} = 2.5$ μ s, $I_{o2} = 3.25$ kA, $\eta_2 = 0.876$, $\tau_{12} = 2$ μ s, $\tau_{22} = 230$ μ s. The adopted return stroke model is the MTLE model [11] and the return stroke speed is assumed to be $v = 120$ m/ μ s. The RS initiation is assumed to take place at 14 m above the top of the tower ($h_0=14$ m).

Figure 2 presents the vertical component of the electric field and the azimuthal component of the magnetic field computed at the three distances considered in the work of Pavanello et al. [3, 4]. For each case, the results obtained using (i) the original MTLE model, in which no reflection is considered at the return stroke wavefront [5], and (ii) the extended MTLE model, in which a reflection coefficient equal to -1 is considered at the return stroke wavefront. This condition allowed us to eliminate the discontinuity at the return stroke wavefront, which results in the turn-on term [9].

It can be seen that, for the considered structure, taking into account the reflections at the return stroke wavefront results in fine structure in the field's late-time response. Additionally, the obtained results are in better agreement with experimental observations than those obtained disregarding possible reflections at the return-stroke wavefront. Namely, the early narrow undershoot typical of measured fields from lightning strikes to the CN Tower [3], and the far-field zero crossing are now well reproduced by the extended model. Note also that the taking into account of the negative current reflections at the return stroke wavefront results in a reduction of electric and magnetic field magnitudes (for the same undisturbed current).

4 CONCLUSIONS

In this paper, we proposed an extension of the engineering models for return-strokes to tall structures that takes into account the presence of possible reflections at the return stroke wavefront and RS initiation above the structure. Based on the approach proposed by Shostak et al. [10], we derived closed-form iterative solutions for the current distribution along the channel and the strike object. By setting the value for the reflection coefficient at the return stroke wavefront ρ_c to -1, the current at the return stroke wavefront is forced to zero, avoiding thereby a discontinuity.

Simulation results for the electromagnetic fields generated by lightning strikes to the CN Tower showed that taking into account the reflections at the return stroke wavefront results in fine structure in the field's late-time response. Moreover, the obtained results are in better agreement with experimental observations, than results obtained disregarding the presence of possible reflections at the wavefront. Specifically, the model is able to reproduce both the early narrow undershoot and the far-field zero crossing.

5 APPENDIX

The times and magnitudes of the current terms used in (7) and (8) read

$$\begin{aligned} t_{1,1} &= \frac{h_0}{v}, \quad i_{1,1} = (1 - \rho_t) \\ t_{3,1} &= \frac{h_0}{v}, \quad i_{3,1} = -\rho_t \\ h_c(t_{4,n}) &= h + h_0 + vt_{4,n} \end{aligned} \quad (7)$$

And, for $n=1,2, \dots$

$$\begin{aligned} t_{2,n} &= \frac{h}{c} + t_{1,n}, \quad i_{2,n} = \rho_g i_{1,n} \\ t_{1,2n} &= \frac{h}{c} + t_{2,n}, \quad i_{1,2n} = \rho_t i_{2,n} \\ t_{3,2n} &= \frac{h}{c} + t_{2,n}, \quad i_{3,2n} = (1 + \rho_t) i_{2,n} \\ t_{4,n} &= \left(t_{3,n} + \frac{h_0}{c} \right) / \left(1 - \frac{v}{c} \right), \quad i_{4,n} = \rho_c i_{3,n} \\ t_{3,2n+1} &= 2t_{4,n} - t_{3,2n}, \quad i_{3,2n+1} = -\rho_t i_{4,n} \\ t_{1,2n+1} &= 2t_{4,n} - t_{1,2n}, \quad i_{1,2n+1} = (1 - \rho_t) i_{4,n} \end{aligned} \quad (8)$$

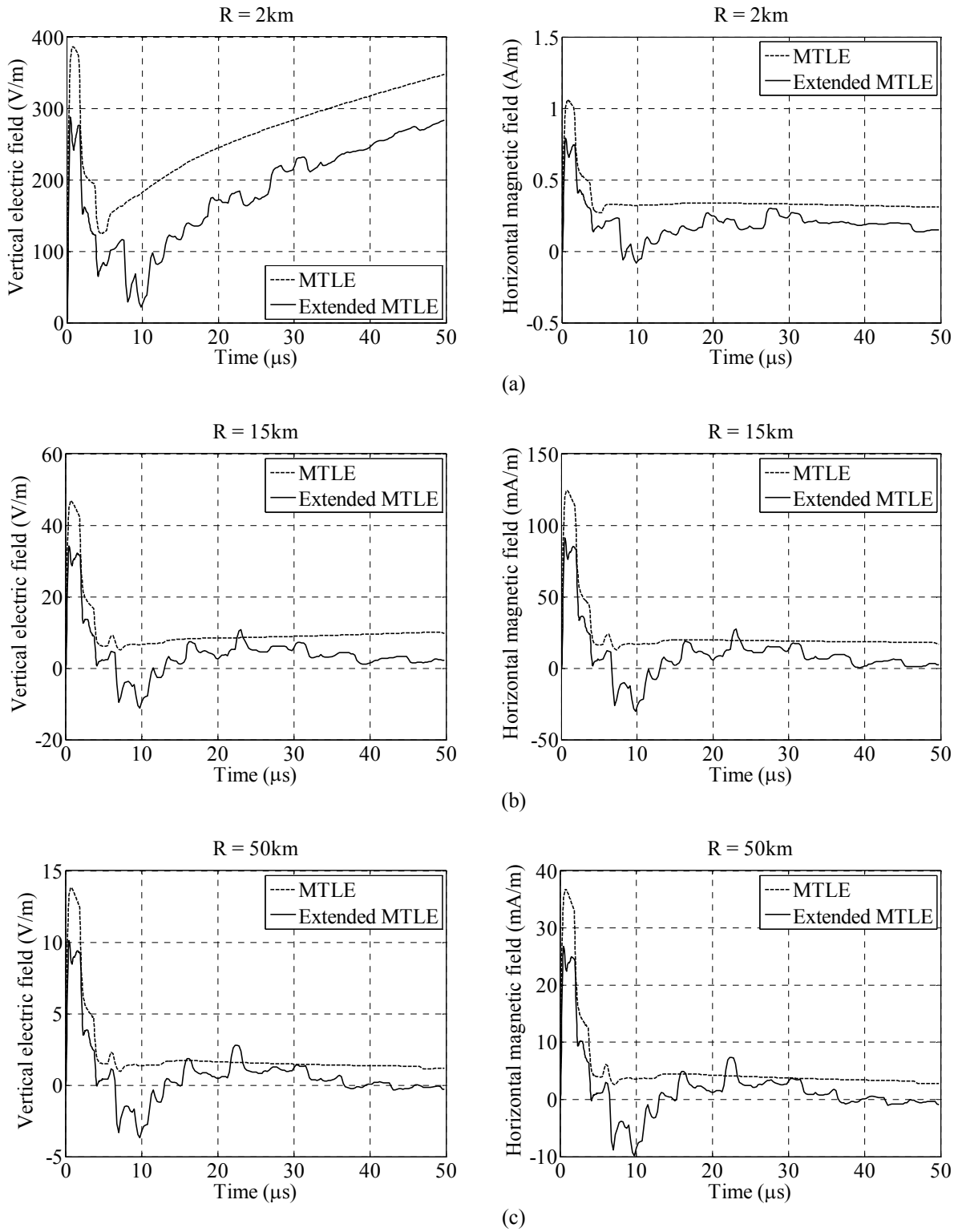


Fig. 2 - Vertical electric and azimuthal magnetic fields calculated at three distances of 2.0 km (a), 15.0 km (b) and 50 km (c) using MTLE and Extended MTLE models.

6 ACKNOWLEDGMENTS

Financial support from the Swiss Office for Education and Research SER (Grant No C05.0149) and the Swiss National Science Foundation (Project No. 200021-122457) are acknowledged.

7 REFERENCES

- [1] F. Rachidi, "Modeling Lightning Return Strokes to Tall Structures: A Review," *Journal of Lightning Research*, vol. 1, pp. 16-31, January 2007.
- [2] V. A. Rakov and M. A. Uman, "Review and evaluation of lightning return stroke models including some aspects of their application," *IEEE Transactions on Electromagnetic Compatibility*, vol. 40, pp. 403-26, 1998.
- [3] D. Pavanello, F. Rachidi, W. Janischewskyj, M. Rubinstein, A. M. Hussein, E. Petrache, V. Shostak, I. Boev, C. A. Nucci, W. A. Chisholm, M. Nyffeler, J. S. Chang, and A. Jaquier, "On Return-Stroke Currents and Remote Electromagnetic Fields Associated with Lightning Strikes to Tall Structures. Part II: Experiment and Model Validation," *Journal of Geophysical Research*, vol. 112, 2007.
- [4] D. Pavanello, A. Mosaddeghi, F. Rachidi, M. Rubinstein, W. Janischewskyj, V. Shostak, A. M. Hussein, C. A. Nucci, and J. S. Chang, "Ability of engineering models to reproduce electromagnetic fields from lightning return strokes to tall towers," in *29th International Conference on Lightning Protection (ICLP)*, Uppsala, Sweden, 2008.
- [5] F. Rachidi, V. A. Rakov, C. A. Nucci, and J. L. Bermudez, "The Effect of Vertically-Extended Strike Object on the Distribution of Current Along the Lightning Channel," *Journal of Geophysical Research*, vol. 107, p. 4699, 2002.
- [6] Y. Baba and V. A. Rakov, "On the use of lumped sources in lightning return stroke models," *Journal of Geophysical Research*, vol. 110, 2005.
- [7] A. Mosaddeghi, D. Pavanello, F. Rachidi, M. Rubinstein, and P. Zweigacker, "Distortion of electric and magnetic fields from lightning due to close-by metallic structures," in *7th International Symposium and Exhibition on Electromagnetic Compatibility and Electromagnetic Ecology*, St. Petersburg, Russia, 2007.
- [8] Y. Baba and V. A. Rakov, "Electromagnetic fields at the top of a tall building associated with nearby lightning return strokes," *IEEE Transactions on Electromagnetic Compatibility*, vol. 49, pp. 632-643, 2007.
- [9] D. Pavanello, F. Rachidi, J. L. Bermudez, M. Rubinstein, and C. A. Nucci, "Electromagnetic Field Radiated by Lightning to Tall Towers: Treatment of the Discontinuity at the Return Stroke Wavefront," *Journal of Geophysical Research*, vol. 109, 2004.
- [10] V. Shostak, W. Janischewskyj, A. Hussein, and B. Kordi, "Electromagnetic fields of lightning strikes to a tall tower: a model that accounts for upward-connecting discharges," in *25th ICLP (International Conference on Lightning Protection)*, Rhodes, Greece, 2000, pp. 60 - 65.
- [11] C. A. Nucci, C. Mazzetti, F. Rachidi, and M. Ianoz, "On lightning return stroke models for LEMP calculations," in *19th International Conference on Lightning Protection (ICLP)*, Graz, Austria, 1988.