Laplacetransform med MATLAB

Til bruk ved kretsanalyse i ELE142

Dette er en rask gjennomgang av funksjoner til bruk for å regne Laplacetransformer i MATLAB med hovedvekt på følgende funksjoner

- `laplace` Laplacetransform
- `ilaplace` Invers Laplacetransform
- `partfrac` Delbrøkoppspalting
- `simplify` Forenkle uttrykk

Alle eksempler er i gule tekstbokser. For lesbarhetens skyld er unødvendige linjeskift og mellomrom fjernet (med mindre annet er spesifisert). Alle gule tekstbokser forutsetter at vi starter med blanke ark, altså at følgende kommandorekke er utført:

```
>> clear          % Sletter gamle variabler
>> clc            % Sletter skjermen for gamle utregninger
```

Videre er kommentarer skrevet med grønt, feilmeldinger i rødt og alle svar med blått. Også dette for å øke lesbarheten.

1 Laplacetransform og invers Laplacetransform

Laplacetransformen omformer fra tidsplanet til det komplekse frekvensplanet, bedre kjent som s-planet. Denne gjennomgangen tar utgangspunkt i læreboken til Nilsson/Riedel (Electric Circuits, 9th edition), og da spesielt tabell 12.1 samt oppgavene underveis i kapittel 12 og 13, kalt "Assessment Problems".

TABLE 12.1 An Abbreviated List of Laplace Transform Pairs

Type	$f(t)\ (t > 0-)$	$F(s)$
(impulse)	$\delta(t)$	1
(step)	$u(t)$	$\frac{1}{s}$
(ramp)	t	$\frac{1}{s^2}$
(exponential)	e^{-at}	$\frac{1}{s+a}$
(sine)	$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
(cosine)	$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
(damped ramp)	te^{-at}	$\frac{1}{(s+a)^2}$
(damped sine)	$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
(damped cosine)	$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$

1.1 Impulsfunksjonen (dirac-funksjonen)

$$\boxed{f(t)=\delta(t)\leftrightarrow F(s)=1}$$

```
>> syms t s             % Deklarerer først de nødvendige variablene
>>
>> f_av_t = dirac(t); % MATLAB har innebygget impuls/dirac-funksjon
>> laplace(f_av_t)     % MATLAB har innebygget Laplace-funksjon
  ans = 1
>>
>> ilaplace(1)         % MATLAB har innebygget invers Laplacefunksjon
Undefined function 'ilaplace' for input arguments of type 'double'.
>>                     % Ikke mulig å bruke ilaplace på "1".
>> ilaplace(s^0)       % Må «lure» MATLAB siden s^0 er lik 1
  ans = dirac(t)
```

1.2 Step-funsjonen (Heaviside)

$$\boxed{f(t)=u(t)\leftrightarrow F(s)=\frac{1}{s}}$$

```
>> syms t s                % Deklarerer først de nødvendige variablene
>>
>> laplace(heaviside(t)) % Heaviside er det samme som u(t)
  ans = 1/s
>>
>> ilaplace(1/s)
  ans = 1
>>                         % Legg merkle til at MATLAB ikke skriver u(t)
```

Her må vi selv anta at dette kun gjelder for t > 0 for den inverse Laplacetransformasjonen

1.3 Rampefunksjonen

$$f(t) = t \leftrightarrow F(s) = \frac{1}{s^2}$$

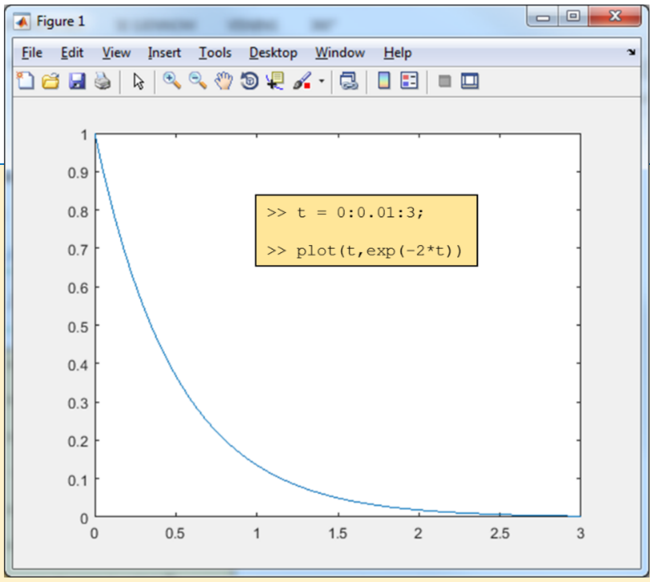
```
>> syms t s

>>

>> laplace(t)

  ans = 1/s^2

>>

>> ilaplace(1/s^2)

  ans = t
```

Både Laplacetransform og invers Laplacetransform gir korrekt svar.

1.4 Eksponentialfunksjonen

$$f(t) = e^{-at} \leftrightarrow F(s) = \frac{1}{s+a}$$

```
>> syms t a s

>>

>> laplace(exp(-a*t))

  ans = 1/(a + s)

>>

>> ilaplace(1/(s + a))

  ans = exp(-a*t)
```



Både Laplacetransform og invers Laplacetransform gir korrekt svar.

1.5 Sinusfunksjonen

$$f(t) = \sin(\omega t) \leftrightarrow F(s) = \frac{\omega}{s^2 + \omega^2}$$

```
>> syms t s w                                   % Skriver "w" for omega

>>

>> laplace(sin(w*t))

  ans = w/(w^2 + s^2)

>>

>> ilaplace(w/(w^2 + s^2))

  ans = sin(w*t)
```

Både Laplacetransform og invers Laplacetransform gir korrekt svar.

1.6 Cosinusfunksjonen

$$f(t) = \cos(\omega t) \leftrightarrow F(s) = \frac{s}{s^2 + \omega^2}$$

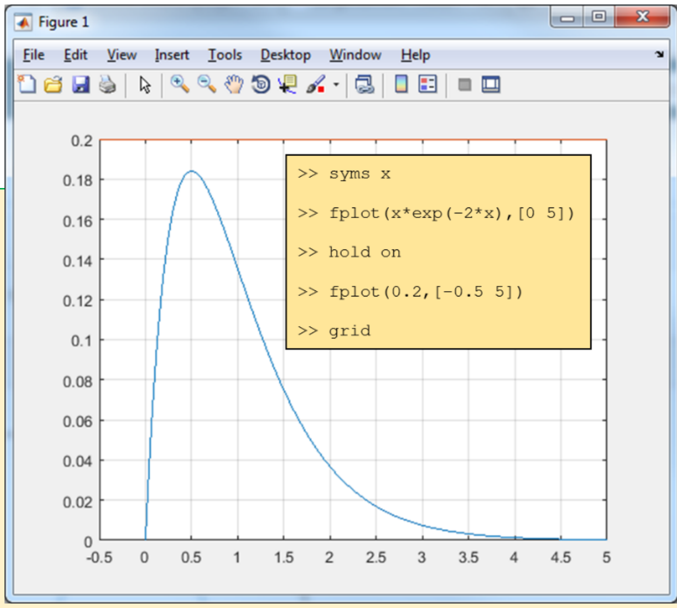
```
>> syms t s w

>>

>> laplace(cos(w*t))

  ans = s/(w^2 + s^2)

>>

>> ilaplace(s/(w^2 + s^2))

  ans = cos(w*t)
```

Både Laplacetransform og invers Laplacetransform gir korrekt svar.

1.7 Dempet rampe

$$f(t) = t \cdot e^{-at} \leftrightarrow F(s) = \frac{1}{(s+a)^2}$$

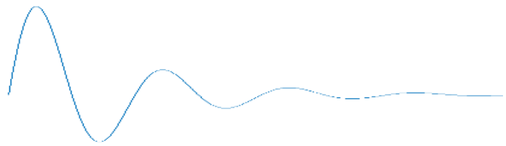
```
>> syms t s a
>>
>> laplace(t*exp(-a*t))
  ans = 1/(a + s)^2
>>
>> ilaplace(1/(a + s)^2)
  ans = t*exp(-a*t)
```



Både Laplacetransform og invers Laplacetransform gir korrekt svar.

1.8 Dempet sinus

$$f(t) = e^{-at} \cdot \sin(t) \leftrightarrow F(s) = \frac{\omega}{(s+a)^2 + \omega^2}$$

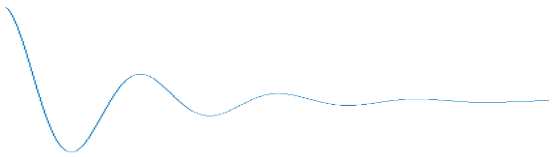


```
>> syms t s a w
>>
>> f_av_t = exp(-a*t)*sin(w*t);
>> laplace(f_av_t)          % Kunne også skrevet inn formelen direkte
  ans = w/((a + s)^2 + w^2)
>>
>> f_av_s = w/((a + s)^2 + w^2);
>> ilaplace(f_av_s)         % Kunne også skrevet inn formelen direkte
  ans = exp(-a*t)*sin(t*w)
```

Både Laplacetransform og invers Laplacetransform gir korrekt svar.

1.9 Dempet cosinus

$$f(t) = e^{-at} \cdot \cos(t) \leftrightarrow F(s) = \frac{s+a}{(s+a)^2 + \omega^2}$$



```
>> syms t s a w
>>
>> f_av_t = exp(-a*t)*cos(w*t);
>> laplace(f_av_t)
  ans = (a + s)/((a + s)^2 + w^2)
>>
>> f_av_s = (s + a)/((s + a)^2 + w^2);
>> ilaplace(f_av_s)
  ans = exp(-a*t)*cosh(t*w*1i)
>>                          % MATLAB gir et svar med cosh-funksjonen
>> simplify(ans)            % Prøver å forenkle med simplify
  ans = exp(-a*t)*cos(t*w)
```

Her ga MATLAB et svar med hyperbolisk cosinus med imaginær operand. Det var nødvendig med en «simplify»-operasjon for å få ønsket svar.

2 Oppgaver med Laplacetransformasjon og invers Laplacetransformasjon

✓ASSESSMENT PROBLEM

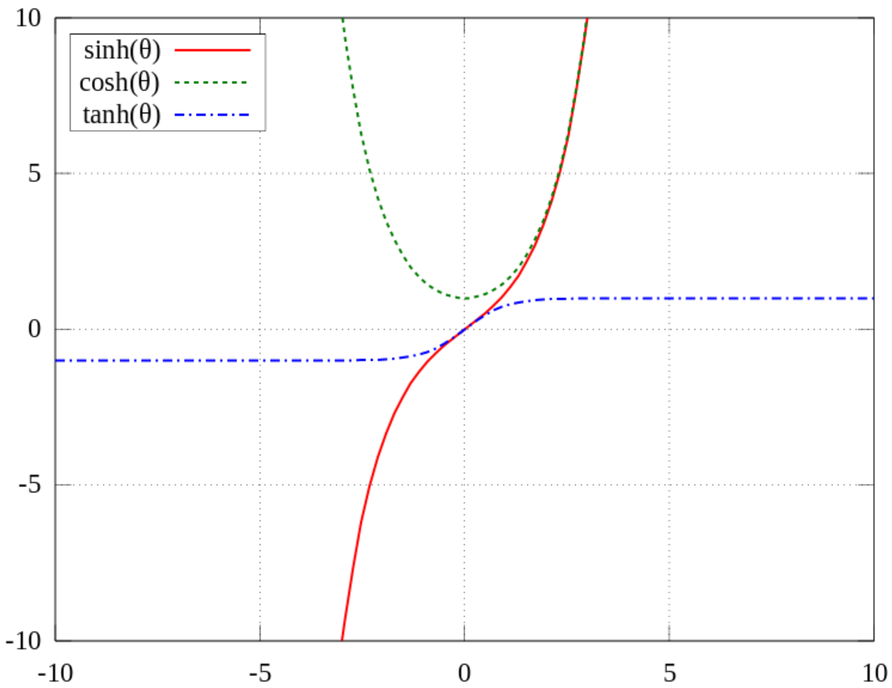
Objective 1—Be able to calculate the Laplace transform of a function using the definition of Laplace transform

12.1 Use the defining integral to
a) find the Laplace transform of cosh βt;
b) find the Laplace transform of sinh βt.

Answer: (a) $s/(s^2 - \beta^2)$;
(b) $\beta/(s^2 - \beta^2)$.

NOTE: Also try Chapter Problem 12.17.

Fra Wikipedia:



The hyperbolic functions are:

- Hyperbolic sine:

$$\sinh x = \frac{e^x - e^{-x}}{2} = \frac{e^{2x} - 1}{2e^x} = \frac{1 - e^{-2x}}{2e^{-x}}$$

- Hyperbolic cosine:

$$\cosh x = \frac{e^x + e^{-x}}{2} = \frac{e^{2x} + 1}{2e^x} = \frac{1 + e^{-2x}}{2e^{-x}}$$

- Hyperbolic tangent:

$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{e^{2x} - 1}{e^{2x} + 1} = \frac{1 - e^{-2x}}{1 + e^{-2x}}$$

```
>> syms B s t                                   % B skal symbolisere beta
>>
>> laplace(cosh(B*t))                           % Prøver med cosh direkte
  ans = -s/(B^2 - s^2)
>>                                              % OK i henhold til fasit
>> laplace((exp(B*t)+exp(-B*t))/2)              % Prøver utvidet variant
  ans = 1/(2*(B + s)) - 1/(2*(B - s))
>>                                              % Samme som ovenfor?
>> simplify(ans)                                % Prøver forenkling
  ans = -s/(B^2 - s^2)
>>                                              % OK i henhold til fasit
>>
>> laplace(sinh(B*t))
  ans = -B/(B^2 - s^2)
>>                                              % OK i henhold til fasit
>> laplace((exp(B*t)-exp(-B*t))/2)              % Prøver utvidet variant
  ans = - 1/(2*(B + s)) - 1/(2*(B - s))
>>                                              % Samme som ovenfor?
>> simplify(ans)                                % Prøver forenkling
  ans = -B/(B^2 - s^2)
>>                                              % OK i henhold til fasit
```

Legg merke til at det noen ganger er nødvendig å bruke `simplify`-funksjonen

✓ ASSESSMENT PROBLEM

Objective 1—Be able to calculate the Laplace transform of a function using the Laplace transform table or a table of operational transforms

12.2 Use the appropriate operational transform from Table 12.2 to find the Laplace transform of each function:

a) t^2e^{-at};

b) $\frac{d}{dt}(e^{-at}\sinh \beta t)$;

c) $t \cos \omega t$.

Answer: (a) $\frac{2}{(s+a)^3}$;

(b) $\frac{\beta s}{(s+a)^2 - \beta^2}$;

(c) $\frac{s^2 - \omega^2}{(s^2 + \omega^2)^2}$.

NOTE: Also try Chapter Problems 11.14 and 11.22.

```
>> % Oppgave 12.2.a

>> syms t a B w s

>> laplace(t^2*exp(-a*t))

  ans = 2/(a + s)^3

>> % Denne stemmer rett fram med fasit

>>

>> % Oppgave 12.2.b

>> laplace(diff(exp(-a*t)*sinh(B*t)))

  ans = (B*(a + s))/((a + s)^2 - B^2) - (B*a)/((a + s)^2 - B^2)

>> simplify(ans)

  ans = (B*s)/(- B^2 + a^2 + 2*a*s + s^2)

>> % Denne måtte ryddes med "simplyfy".

>> % Svaret er ikke helt i mål, første kvadratsetning gir rett svar

>>

>> % Oppgave 12.2.c

>> laplace(t*cos(w*t))

  ans = (2*s^2)/(s^2 + w^2)^2 - 1/(s^2 + w^2)

>> simplify(ans)

  ans = (s^2 - w^2)/(s^2 + w^2)^2

>> % Denne stemmer rett fram med fasit
```

ASSESSMENT PROBLEMS

Objective 2—Be able to calculate the inverse Laplace transform using partial fraction expansion and the Laplace transform table

12.3 Find $f(t)$ if

$$F(s) = \frac{6s^2 + 26s + 26}{(s+1)(s+2)(s+3)}.$$

Answer: $f(t) = (3e^{-t} + 2e^{-2t} + e^{-3t})u(t)$.

```
>> syms s t

>> ilaplace((6*s^2+26*s+26)/((s+1)*(s+2)*(s+3)))

  ans = 3*exp(-t) + 2*exp(-2*t) + exp(-3*t)
```

Dette ser ut til å stemme ganske bra med fasit, men ikke glem *u(t)*. Dette gjelder altså når t > 0.

Vi kan se mer på detaljene ved å delbrøkoppspalte:

```
>> syms s t

>> funksjon = (6*s^2+26*s+26)/((s+1)*(s+2)*(s+3));

>> partfrac(funksjon)

  ans = 3/(s + 1) + 2/(s + 2) + 1/(s + 3)

>> ilaplace(ans)

  ans = 3*exp(-t) + 2*exp(-2*t) + exp(-3*t)
```

Altså: $F(s) = \frac{6s^2 + 26s + 26}{(s+1)(s+2)(s+3)} = \frac{3}{s+1} + \frac{2}{s+2} + \frac{1}{s+3} \leftrightarrow f(t) = \left[3e^{-t} + 2e^{-2t} + e^{-3t}\right] \cdot u(t)$

Legg nok en gang merke til at `ilaplace`-funksjonen ikke tar med *u(t)*.

ASSESSMENT PROBLEMS

Objective 2—Be able to calculate the inverse Laplace transform using partial fraction expansion and the Laplace transform table

12.4 Find $f(t)$ if

$$F(s) = \frac{7s^2 + 63s + 134}{(s+3)(s+4)(s+5)}.$$

Answer: $f(t) = (4e^{-3t} + 6e^{-4t} - 3e^{-5t})u(t).$

```
>> syms s t
>> ilaplace((7*s^2+63*s+134)/((s+3)*(s+4)*(s+5)))
  ans = 4*exp(-3*t) + 6*exp(-4*t) 3 exp(-5*t)
>>
>> % Går veien om "partfrac" for å vise detaljene
>> funksjon = (7*s^2+63*s+134)/((s+3)*(s+4)*(s+5));
>> partfrac(funksjon)
  ans = 4/(s + 3) + 6/(s + 4) - 3/(s + 5)
>> ilaplace(ans)
  ans = 4*exp(-3*t) + 6*exp(-4*t) - 3*exp(-5*t)
```

Stemmer bra med fasit.

ASSESSMENT PROBLEM

Objective 2—Be able to calculate the inverse Laplace transform using partial fraction expansion and the Laplace transform table

12.5 Find $f(t)$ if

$$F(s) = \frac{10(s^2 + 119)}{(s+5)(s^2 + 10s + 169)}.$$

Answer: $f(t) = (10e^{-5t} - 8.33e^{-5t}\sin 12t)u(t).$

```
>> syms s t
>> funksjon = 10*(s^2 + 119)/((s + 5)*(s^2 + 10*s + 169));
>> partfrac(funksjon)
  ans = 10/(s + 5) - 100/(s^2 + 10*s + 169)
>> ilaplace(ans)
  ans = 10*exp(-5*t) - (25*sin(12*t)*exp(-5*t))/3
```

✓ ASSESSMENT PROBLEM

Objective 2—Be able to calculate the inverse Laplace transform using partial fraction expansion and the Laplace transform table

12.6 Find $f(t)$ if

$$F(s) = \frac{(4s^2 + 7s + 1)}{s(s + 1)^2}.$$

Answer: $f(t) = (1 + 2te^{-t} + 3e^{-t})u(t)$.

```
>> syms s t

>> funksjon = (4*s^2 + 7*s + 1)/(s*(s + 1)^2);

>> partfrac(funksjon)

  ans = 3/(s + 1) + 2/(s + 1)^2 + 1/s

>> ilaplace(ans)

  ans = 3*exp(-t) + 2*t*exp(-t) + 1
```

Stemmer bra med fasit.

✓ ASSESSMENT PROBLEM

Objective 2—Be able to calculate the inverse Laplace transform using partial fraction expansion and the Laplace transform table

12.7 Find $f(t)$ if

$$F(s) = \frac{40}{(s^2 + 4s + 5)^2}.$$

Answer: $f(t) = (-20te^{-2t}\cos t + 20e^{-2t}\sin t)u(t)$.

```
>> syms s t

>> funksjon = 40/(s^2+4*s+5)^2;

>> ilaplace(funksjon)

  ans = 20*exp(-2*t)*sin(t) - 10*t*(exp(-2*t)*cos(t) + exp(-
2*t)*sin(t)*1i) - 10*t*(exp(-2*t)*cos(t) - exp(-2*t)*sin(t)*1i)

>> % Dette så ikke så pent ut. Prøver å rydde

>> simplify(ans)

  ans = 20*exp(-2*t)*(sin(t) - t*cos(t))
```

Stemmer bra med fasit.

12.8 Find $f(t)$ if

$$F(s) = \frac{(5s^2 + 29s + 32)}{(s+2)(s+4)}.$$

Answer: $f(t) = 5\delta(t) - (3e^{-2t} - 2e^{-4t})u(t)$.

```
>> syms t s

>> funksjon = (5*s^2 + 29*s + 32)/((s + 2)*(s + 4));

>> partfrac(funksjon)

  ans = 2/(s + 4) - 3/(s + 2) + 5

>> ilaplace(funksjon)

  ans = 2*exp(-4*t) - 3*exp(-2*t) + 5*dirac(t)
```

Stemmer bra med fasit

12.9 Find $f(t)$ if

$$F(s) = \frac{(2s^3 + 8s^2 + 2s - 4)}{(s^2 + 5s + 4)}.$$

Answer: $f(t) = 2\frac{d\delta(t)}{dt} - 2\delta(t) + 4e^{-4t}u(t).$

```
>> syms t s
>> funksjon = (2*s^3 + 8*s^2 + 2*s - 4)/(s^2 + 5*s + 4);
>> partfrac(funksjon)
  ans = 2*s + 4/(s + 4) - 2
>> ilaplace(ans)
  ans = 4*exp(-4*t) - 2*dirac(t) + 2*dirac(1, t)
```

Fra hjelpesiden til dirac-funksjonen:

Description

`dirac(x)` represents the Dirac delta function of `x`.

`dirac(n,x)` represents the `n`th derivative of the Dirac delta function at `x`.

Det betyr at 2*dirac(1, t) er det samme som $2 \cdot \frac{d\delta(t)}{dt}$. Tester dette:

```
>> syms s t
>> funksjon = 4*exp(-4*t) - 2*dirac(t) + 2*diff(dirac(t));
>> laplace(funksjon)
  ans = 2*s + 4/(s + 4) - 2
```

Dette er det samme som etter delbrøkoppspaltinga ovenfor.

✓ASSESSMENT PROBLEM

Objective 3—Understand and know how to use the initial value theorem and the final value theorem

12.10 Use the initial- and final-value theorems to find the initial and final values of $f(t)$ in Assessment Problems 12.4, 12.6, and 12.7.

Answer: 7, 0; 4, 1; and 0, 0.

12.4 Find $f(t)$ if

$$F(s) = \frac{7s^2 + 63s + 134}{(s+3)(s+4)(s+5)}.$$

12.6 Find $f(t)$ if

$$F(s) = \frac{(4s^2 + 7s + 1)}{s(s+1)^2}.$$

12.7 Find $f(t)$ if

$$F(s) = \frac{40}{(s^2 + 4s + 5)^2}.$$

Start- og sluttverditeoremet sier at:

$$\lim_{t \to 0^+} f(t) = \lim_{s \to \infty} sF(s), \qquad (12.93)$$ ◀ **Initial value theorem**

$$\lim_{t \to \infty} f(t) = \lim_{s \to 0} sF(s). \qquad (12.94)$$ ◀ **Final value theorem**

```
>> % Fra oppgave 12.4

>> syms s t

>> oppg_12_4 = (7*s^2 + 63*s + 134)/((s + 3)*(s + 4)*(s + 5));

>>

>> limit(s*oppg_12_4,s,0)                   % s*F(s) når s -> null

  ans = 0

>> limit(ilaplace(oppg_12_4),t,Inf)         % f(t) når t -> uendelig

  ans = 0

>>                                          % Samme svar, alt OK

>>

>> limit(s*oppg_12_4,s,Inf)                 % s*F(s) når s -> uendelig

  ans = 7

>> limit(ilaplace(oppg_12_4),t,0)           % f(t) når t -> null

  ans = 7

>>                                          % Samme svar, alt OK
```

```
>> % Fra oppgave 12.6

>> syms s t;

>> oppg_12_6 = (4*s^2 + 7*s + 1)/(s*(s + 1)^2);

>>

>> limit(s*oppg_12_6,s,Inf)            % Startverdi i s-planet

  ans = 4

>> limit(ilaplace(oppg_12_6),t,0)      % Startverdi i tidsplanet

  ans = 4

>>

>> limit(s*oppg_12_6,s,0) )            % Sluttverdi i s-planet

  ans = 1

>> limit(ilaplace(oppg_12_6),t,Inf)    % Sluttverdi i tidsplanet

  ans = 1

>>

>> % Fra oppgave 12.7

>> oppg_12_7 = 40/(s^2+4*s+5)^2;

>>

>> limit(s*oppg_12_7,s,Inf)            % Startverdi i s-planet

  ans = 0

>> limit(ilaplace(oppg_12_7),t,0)      % Startverdi i tidsplanet

  ans = 0

>>

>> limit(s*oppg_12_7,s,0)              % Sluttverdi i s-planet

  ans = 0

>> limit(ilaplace(oppg_12_7),t,Inf)    % Sluttverdi i tidsplanet

  ans = 0
```

3 Kretseksempler med Laplace

✓ASSESSMENT PROBLEMS

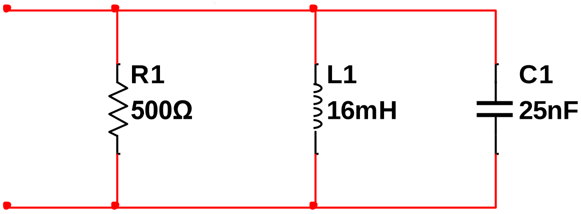
Objective 1—Be able to transform a circuit into the *s* domain using Laplace transforms

13.1 A 500 Ω resistor, a 16 mH inductor, and a 25 nF capacitor are connected in parallel.

a) Express the admittance of this parallel combination of elements as a rational function of *s*.

b) Compute the numerical values of the zeros and poles.

Answer: (a) $25 \times 10^{-9}(s^2 + 80{,}000s + 25 \times 10^8)/s$;
(b) $-z_1 = -40{,}000 - j30{,}000$;
$-z_2 = -40{,}000 + j30{,}000$; $p_1 = 0$.



$$Y = \frac{1}{R} + \frac{1}{sL} + sC = \frac{1}{500} + \frac{1}{s \cdot 16m} + s \cdot 25n$$

```
>> syms s
>> R = 500;
>> L = 16e-3;
>> C = 25e-9;
>> Y = 1/R + 1/(s*L) + s*C

  Y = (944473296573929*s)/37778931862957161709568 + 125/(2*s) +
1/500

>>                        % MATLAB får avrundingsfeil på heltallsform
>> Y = vpa(Y,10)          % Variable Precision Arithmetic = avrunding

  Y = 0.000000025*s + 62.5/s + 0.002

>> Y = collect(Y)         % Collect = få uttrykket på felles nevner

  Y = (49999999999999997737405591294313.0*s^2 +
4.0000000000000000195156391047939e36*s + 1.25e41)/(2.0e39*s)

>> Y = vpa(Y,10)

  Y = (5.0e-40*(5.0e31*s^2 + 4.0e36*s + 1.25e41))/s

>> Y = combine(Y)         % Slå sammen parenteser

  Y = (0.0000000249999999999999988685287721112507*s^2 +
0.002000000000000000002302575992465*s +
62.500000000000000041462313663375)/s

>> Y = vpa(Y,10)

Y = (0.000000025*s^2 + 0.002*s + 62.5)/s
```

Eksempelet ovenfor ble ikke særlig vakkert på grunn av avrundingsproblematikk. Det er bedre om vi «hjelper» MATLAB og setter opp uttrykket med kun positive tierpotenser (neste side)

$$Y = \frac{1}{R} + \frac{1}{sL} + sC = \frac{1}{500} + \frac{1}{s \cdot 16m} + s \cdot 25n = \frac{1}{500} + \frac{62{,}5}{s} + \frac{s}{40 \times 10^6} = \frac{1}{500} + \frac{125}{2s} + \frac{s}{40 \times 10^6}$$

```
>> syms s

>> Y = 1/500 + 125/(2*s) + s/40e6

  Y = s/40000000 + 125/(2*s) + 1/500

>> Y = collect(Y)            % For å få uttrykket på felles nevner

  Y = (s^2 + 80000*s + 2500000000)/(40000000*s)

>> Y = combine(Y)            % For å få rydde i uttrykket

  Y = (s^2/40000000 + s/500 + 125/2)/s

>> pretty(Y)                 % For å få vise Y på en vakker måte

    2
   s         s    125
-------- + --- + ---
40000000   500    2
--------------------
          s
>>

>>

>> % Oppgave b er å finne nullpunkter og poler til impedansen

>> Z = inv(Y);

>> poler = solve(1/Z, s)

  poler =  - 40000 - 30000i

           - 40000 + 30000i

>> nullpunkter = solve(Z)

  nullpunkter = 0
```

Dette stemmer bra med fasit.

✓ASSESSMENT PROBLEMS

Objective 1—Be able to transform a circuit into the *s* domain using Laplace transforms

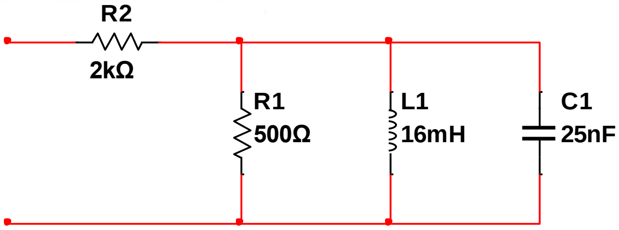
13.2 The parallel circuit in Assessment Problem 13.1 is placed in series with a 2000 Ω resistor.

a) Express the impedance of this series combination as a rational function of s.

b) Compute the numerical values of the zeros and poles.

Answer: (a) $2000(s + 50{,}000)^2/(s^2 + 80{,}000s + 25 \times 10^8)$;

(b) $-z_1 = -z_2 = -50{,}000$;
$-p_1 = -40{,}000 - j30{,}000$,
$-p_2 = -40{,}000 + j30{,}000$.



$$Z_{gammel} = \frac{s}{\frac{s^2}{40\times10^6} + \frac{s}{500} + \frac{125}{2}}$$

```
>> syms s

>> Z_gammel=s/(s^2/40e6 + s/500 + 125/2);

>> Z_ny = Z_gammel + 2000;

>> Z_ny = collect(Z_ny);            % Felles nevner

>> pretty(Z_ny)

      2
2000 s  + 200000000 s + 5000000000000
-------------------------------------
       2
      s  + 80000 s + 2500000000
>> Z_ny = simplify(Z_ny);           % Forenkle uttrykket

>> pretty(Z_ny)

                    2
    2000 (s + 50000)
-------------------------
 2
s  + 80000 s + 2500000000
>>

>> % Oppgave b, finn poler og nullpunkter

>> nullpunkter = solve(Z_ny, s)

  nullpunkter = -50000

                -50000

>> poler = solve(1/Z_ny, s)

  poler = - 40000 - 30000i

          - 40000 + 30000i
```

✓ ASSESSMENT PROBLEM

Objective 2—Know how to analyze a circuit in the s domain and be able to transform an s domain solution to the time domain

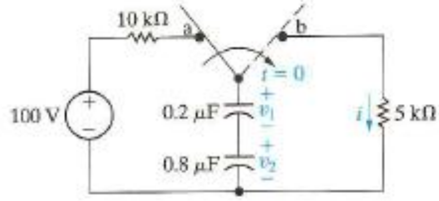
13.3 The switch in the circuit shown has been in position a for a long time. At $t = 0$, the switch is thrown to position b.

a) Find I, V_1, and V_2 as rational functions of s.

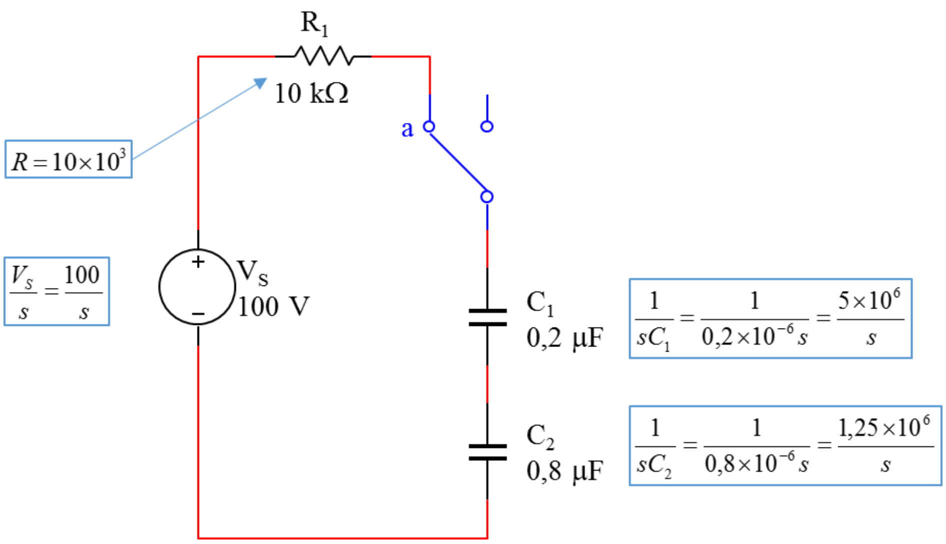
b) Find the time-domain expressions for i, v_1, and v_2.

Answer: (a) $I = 0.02/(s + 1250)$, $V_1 = 80/(s + 1250)$, $V_2 = 20/(s + 1250)$;

(b) $i = 20e^{-1250t}u(t)$ mA, $v_1 = 80e^{-1250t}u(t)$ V, $v_2 = 20e^{-1250t}u(t)$ V.

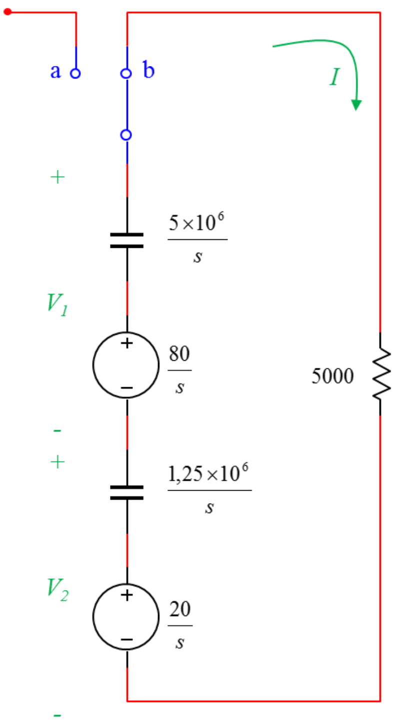


Må først finne spenninga til kondensatorene når de er fullt oppladet. Etter lang tid i posisjon a:




```
>> syms s
>> Z_R1 = 10e3;
>> Z_C1 = 5e6/s;
>> Z_C2 = 1.25e6/s;
>> VS = 100/s;
>> V1 = VS*(Z_C1/(Z_R1 + Z_C1 + Z_C2));     % Spenningsdeling
>> V1_oppladet = limit(s*V1,s,0)            % Sluttverditeoremet
  V1_oppladet = 80
>> V2 = VS*(Z_C2/(Z_R1 + Z_C1 + Z_C2));     % Spenningsdeling
>> V2_oppladet = limit(s*V2,s,0)            % Sluttverditeoremet
  V2_oppladet = 20
```

Etter at bryteren svitsjer over:



MATLAB-koden blir:

```
>> syms s

>> I = (80/s + 20/s)/(1.25e6/s + 5e6/s + 5000)

  I = 100/(s*(6250000/s + 5000))

>> I = simplify(I)

  I = 1/(50*(s + 1250))

>> V1 = 80/s - I*5e6/s

  V1 = 80/s - 100000/(s*(s + 1250))

>> V1 = collect(V1)

  V1 = 80/(s + 1250)

>> V2 = 20/s - I*1.25e6/s

  V2 = 20/s - 25000/(s*(s + 1250))

>> V2 = collect(V2)

  V2 = 20/(s + 1250)
```

Dette stemmer bra med fasit.

✓ASSESSMENT PROBLEM

Objective 2—Know how to analyze a circuit in the s domain and be able to transform an s domain solution to the time domain

13.4 The energy stored in the circuit shown is zero at the time when the switch is closed.

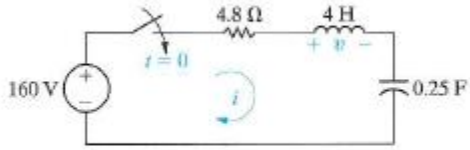
a) Find the *s*-domain expression for I.

b) Find the time-domain expression for i when $t > 0$.

c) Find the *s*-domain expression for V.

d) Find the time-domain expression for v when $t > 0$.

Answer: (a) $I = 40/(s^2 + 1.2s + 1)$;
(b) $i = (50e^{-0.6t} \sin 0.8t)u(t)$ A;
(c) $V = 160s/(s^2 + 1.2s + 1)$;
(d) $v = [200e^{-0.6t} \cos (0.8t + 36.87^\circ)]u(t)$ V.




```
>> % Oppgave a
>> syms s
>> I = (160/s)/(4.8 + 4*s + 1/(0.25*s))
  I = 160/(s*(4*s + 4/s + 24/5))
>> I = simplify(I)                  % For å forenkle uttrykket
  I = 200/(5*s^2 + 6*s + 5)
>> % Oppgave b
>> i_av_t = ilaplace(I)
  i_av_t = 50*sin((4*t)/5)*exp(-(3*t)/5)
>> vpa(i_av_t, 5)                   % Ønsker uttrykket på desimalform
  ans = 50.0*exp(-0.6*t)*sin(0.8*t)
>> % Oppgave c
>> V = I*4*s
  V = (800*s)/(5*s^2 + 6*s + 5)
>> % Oppgave d
>> v_av_t = ilaplace(V)
  v_av_t = 160*exp(-(3*t)/5)*(cos((4*t)/5) - (3*sin((4*t)/5))/4)
>> vpa(ans,5)
  ans = 160.0*exp(-0.6*t)*(cos(0.8*t) - 0.75*sin(0.8*t))
```

Svaret på oppgave d er ikke identisk med fasit, men følgende regel kan vise at svaret stemmer:

$$\boxed{\cos(a+b) = \cos(a)\cdot\cos(b) - \sin(a)\cdot\sin(b)}$$

✓ ASSESSMENT PROBLEM

Objective 2—Know how to analyze a circuit in the *s* domain and be able to transform an *s* domain solution to the time domain

13.5 The dc current and voltage sources are applied simultaneously to the circuit shown. No energy is stored in the circuit at the instant of application.

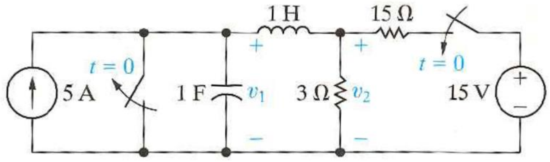
a) Derive the s-domain expressions for V_1 and V_2.

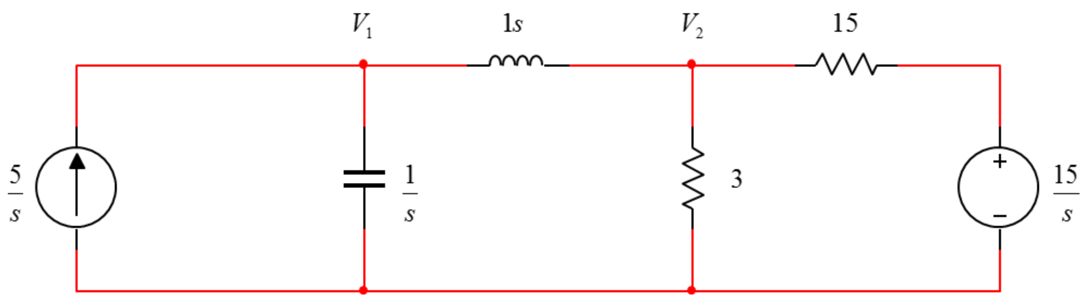
b) For $t > 0$, derive the time-domain expressions for v_1 and v_2.

c) Calculate $v_1(0^+)$ and $v_2(0^+)$.

d) Compute the steady-state values of v_1 and v_2.

Answer: (a) $V_1 = [5(s+3)]/[s(s+0.5)(s+2)]$,
$V_2 = [2.5(s^2+6)]/[s(s+0.5)(s+2)]$;
(b) $v_1 = (15 - \frac{50}{3}e^{-0.5t} + \frac{5}{3}e^{-2t})u(t)$ V,
$v_2 = (15 - \frac{125}{6}e^{-0.5t} + \frac{25}{3}e^{-2t})u(t)$ V;
(c) $v_1(0^+) = 0$, $v_2(0^+) = 2.5$ V;
(d) $v_1 = v_2 = 15$ V.






```
>> % Oppgave a

>> syms s V1 V2

>> Likn1 = 5/s - V1/(1/s) - (V1 - V2)/(1*s) == 0;

>> Likn2 = (V1 - V2)/(1*s) + (15/s - V2)/15 - V2/3 == 0;

>> svar = solve([Likn1 Likn2] , [V1 V2]);

>> V1 = svar.V1

  V1 = (10*(s + 3))/(s*(2*s^2 + 5*s + 2))

>> V2 = svar.V2

  V2 = (5*(s^2 + 6))/(s*(2*s^2 + 5*s + 2))

>> poles(V1)        % For å sjekke om fasit og MATLAB har samme nevner

   -1/2

     -2

      0

>> % Ser ut til å stemme bra! Fasit stemmer med MATLAB

>> % Fortsetter på neste side
```

Fortsetter med de samme variablene (uten å ha slettet med en CLEAR)

```
>> V1                          % Sjekker hvordan V1 ser ut i minnet

  V1 = (10*(s + 3))/(s*(2*s^2 + 5*s + 2))

>> V2                          % Sjekker hvordan V2 ser ut i minnet

  V2 = (5*(s^2 + 6))/(s*(2*s^2 + 5*s + 2))


>> % Oppgave b

>> v1 = ilaplace(V1)

  v1 = (5*exp(-2*t))/3 - (50*exp(-t/2))/3 + 15

>> v2 = ilaplace(V2)

  v2 = (25*exp(-2*t))/3 - (125*exp(-t/2))/6 + 15

>>

>> % Oppgave c

>> v1_null_pluss = limit(s*V1,s,inf)      % Bruker startverditeoremet

  v1_null_pluss = 0

>> v2_null_pluss = limit(s*V2,s,inf)      % Bruker startverditeoremet

  v2_null_pluss = 5/2

>>

>> % Oppgave d

>> v2_ss = limit(s*V2,s,0)                % Bruker sluttverditeoremet

  v2_ss = 15

>> v1_ss = limit(s*V1,s,0)                % Bruker sluttverditeoremet

  v1_ss = 15
```

Fasit og MATLAB stemmer overens

✓ ASSESSMENT PROBLEM

Objective 2—Know how to analyze a circuit in the s domain and be able to transform an s-domain solution to the time domain

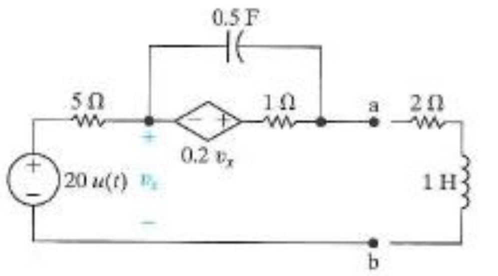
13.6 The initial charge on the capacitor in the circuit shown is zero.

a) Find the s-domain Thévenin equivalent circuit with respect to terminals a and b.

b) Find the s-domain expression for the current that the circuit delivers to a load consisting of a 1 H inductor in series with a 2 Ω resistor.

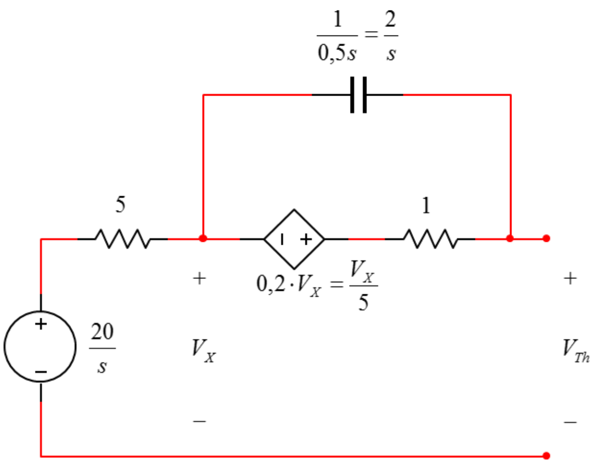
Answer: (a) $V_{Th} = V_{ab} = [20(s + 2.4)]/[s(s + 2)]$,
$Z_{Th} = 5(s + 2.8)/(s + 2)$;

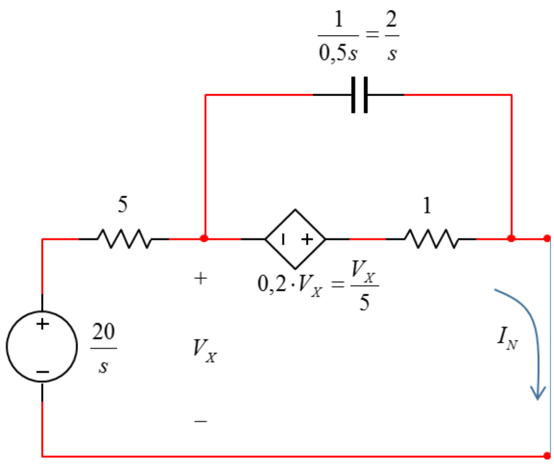
(b) $I_{ab} = [20(s + 2.4)]/[s(s + 3)(s + 6)]$.



Oppgave a

Må først tegne kretsen i s-planet med mål om å finne Thevenin-ekvivalenten:





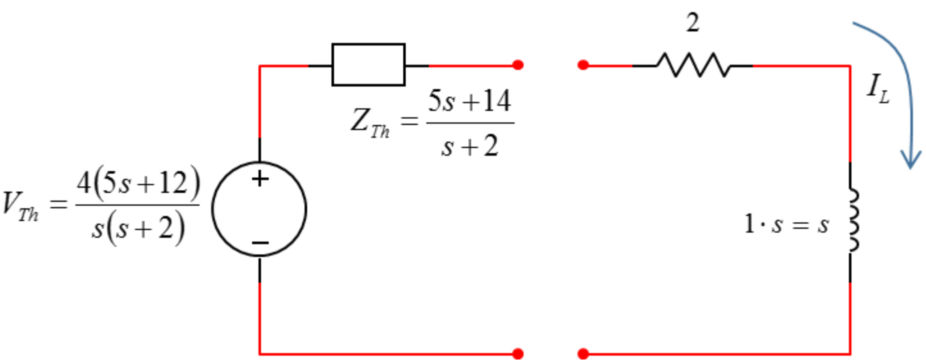

```
>> Oppgave a

>> syms s Vx Vth In Zth

>> % Først finne Theveninspenninga (venstre figur)

>> likn1 = Vx - 20/s == 0;

>> likn2 = (Vth - Vx)/(2/s) - ((Vx + Vx/5) - Vth)/1 == 0;

>> svar = solve([likn1 likn2] , [Vx Vth]);

>> Vth = svar.Vth

  Vth = (4*(5*s + 12))/(s*(s + 2))

>> % Fortsetter på neste side
```

```
>> % Så finne Norton-strømmen (høyre figur)

>> likn1 = (20/s - Vx)/5 - Vx/(2/s) - (Vx + Vx/5)/1 == 0;

>> Vx = solve(likn1,Vx)

  Vx = 40/(5*s^2 + 14*s)

>> In = (20/s - Vx)/5

  In = 4/s - 8/(5*s^2 + 14*s)

>> In = collect(In)              % Vil ha svaret på felles brøkstrek

  In = (20*s + 48)/(5*s^2 + 14*s)

>> Zth = Vth/In

  Zth = (4*(5*s + 12)*(5*s^2 + 14*s))/(s*(20*s + 48)*(s + 2))

>> Zth = simplify(Zth)           % Svaret bør kunne forenkles

  Zth = (5*s + 14)/(s + 2)
```

Oppgave b

b) Find the s-domain expression for the current that the circuit delivers to a load consisting of a 1 H inductor in series with a 2 Ω resistor.

Erstatter kretsen med Thevenin-ekvivalenten:




```
>> % Fortsetter fra tidligere
>> Vth                                    % Sjekker Theveninspenningen
  Vth = (4*(5*s + 12))/(s*(s + 2))
>> Zth                                    % Sjekker Theveninimpedansen
  Zth = (5*s + 14)/(s + 2)
>> syms IL                                % Må legge til en ny variabel
>> IL = Vth/(Zth + 2 + s)
  IL = (4*(5*s + 12))/(s*(s + 2)*(s + (5*s + 14)/(s + 2) + 2))
>> IL = simplify(IL)
  IL = (20*s + 48)/(s*(s^2 + 9*s + 18))
>> poles(IL)           % Sjekker om løsninga har samme poler som fasit
  ans = -3
        -6
         0
>> % Fasit stemmer overens med MATLAB
```