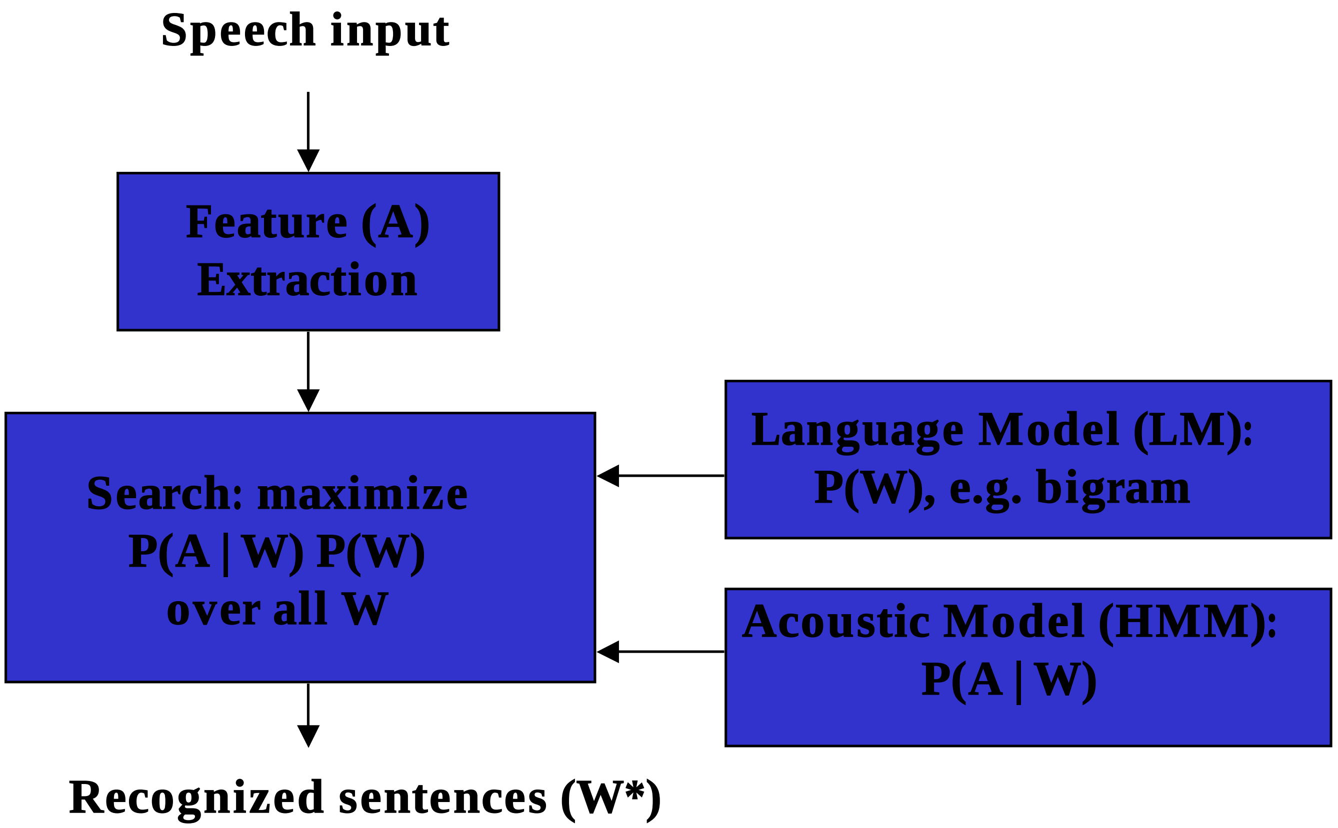
# Feature Extraction

# Architecture of a ASR System

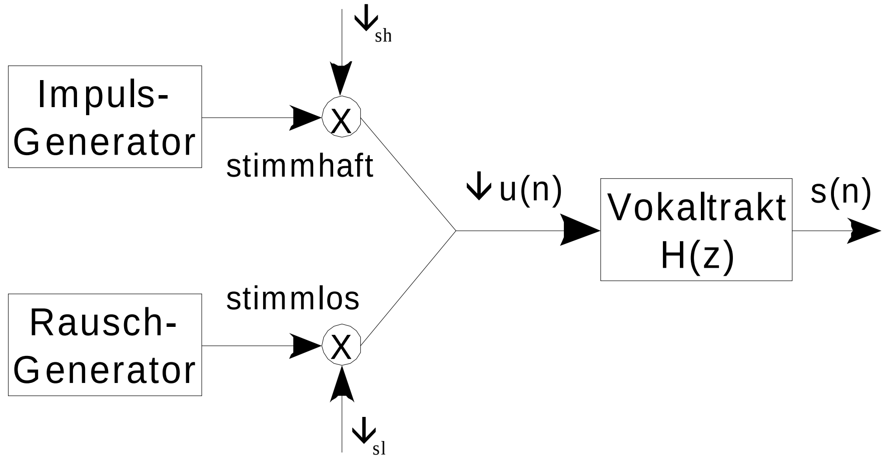
**W** = a word sequence (e.g. word/ sentence/ whole dictation)

**A** = an acoustic feature vector sequence (the input for the recognizer)



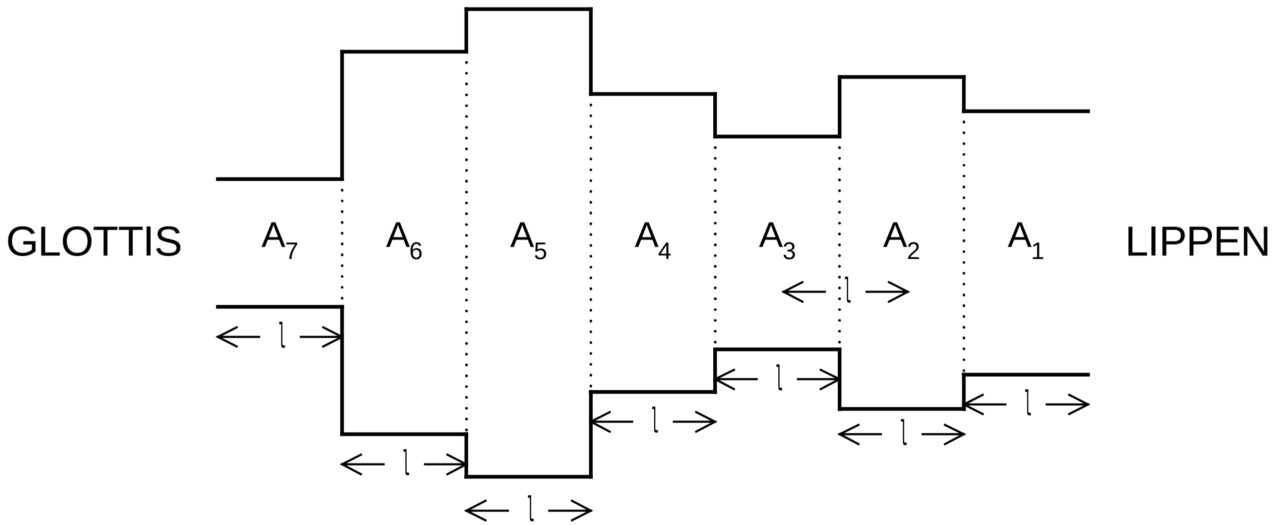
# Source Filter Model

- Assumption:
  - Speech is produced by an excitation **u** (impulses or white noise) and filtered by the vocal tract filter with the impulse function **h**.
  - So the resulting speech **s** results from the convolution: **s** = **u** * **h**.



# Vocaltract Model

- concatenation of ideal cylindrical tubes
- same width but different surfaces $A_n$




$$H(z) = \frac{\prod_{j=0}^{M}(1+k_j)}{1-\sum_{j=1}^{M} a_j z^{-j}} \qquad \text{with} \quad k_j = \frac{A_j - A_{j+1}}{A_j + A_{j+1}}$$

# Sampling

- Shannon theorem: $f_a \geq 2f_g$
  - signal can be exactly reproduced
- sampling frequencies for ASR:
  - telephone speech: $f_a$ = 8 kHz
  - computer dictation: $f_a$ = 16 kHz

- from the time signal s(t) follows:

$$s[n] = s(t - nT) \quad \text{with } \mathrm{T} = 1/\mathrm{f_a}$$

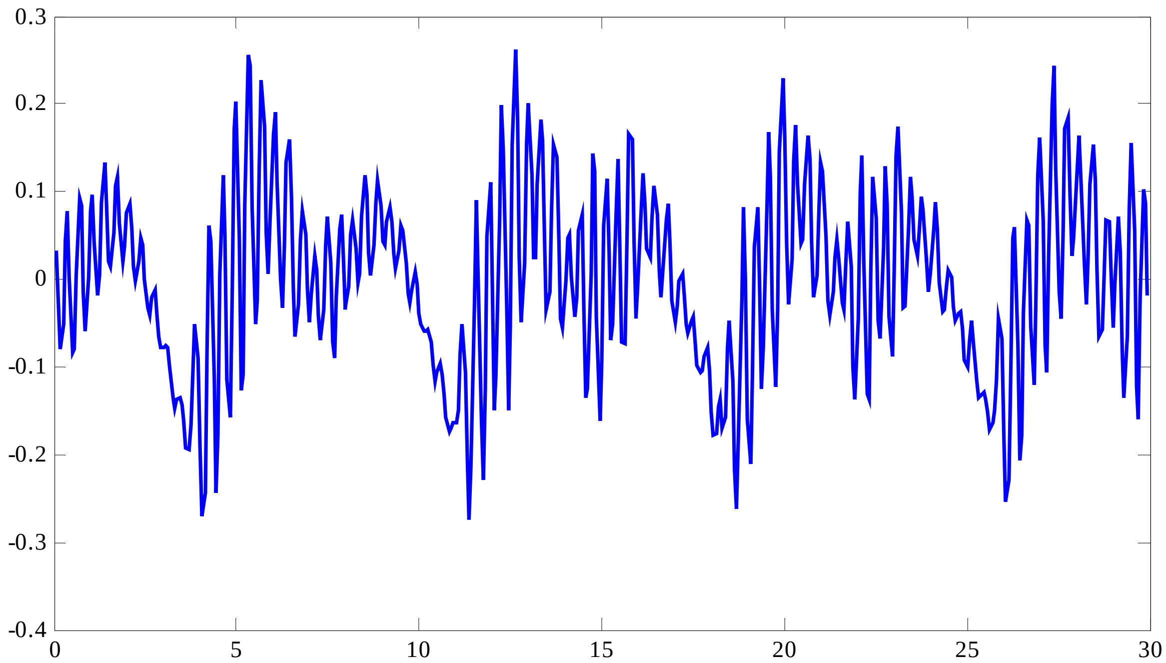
# Short-Term Spectral Analysis

## Why Short-Term Analysis?

- The frequency distribution over an entire utterance doesn't help much for recognition.
- Most acoustic events (e.g. phonemes) have durations in the range of 10 to 100 ms.
- Many acoustic events are not static and need more detailed analysis.

# Short-Term Spectral Analysis

- We assume the speech-signal is short-time stationary!



# Windowing

- For Short-term analysis the signal must be zero outside of a defined range
- this is performed by multiplying the signal with a window
- Normally we choose a **window-width** of 20 – 30 ms
- The **window-shift** is usually 10 ms

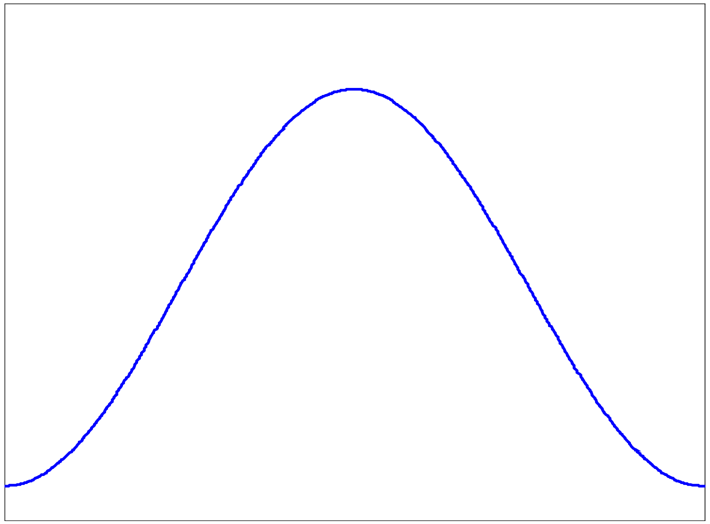
# Window Shapes

Rectangular Window

$W_n=1$

Hamming Window

$W_n= 0.54-0.46 \cos(2\pi n / (N-1))$

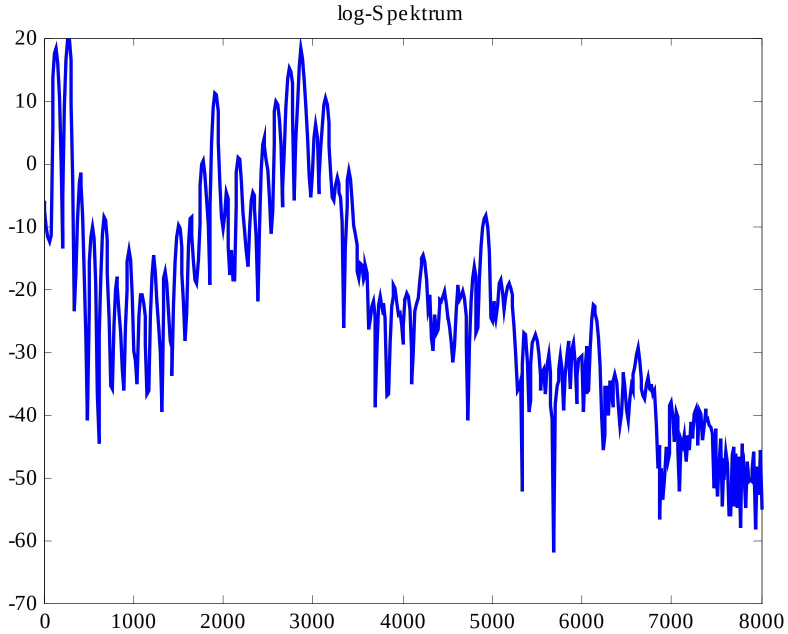


Other common windows: Gauss-, Hann-, Blackmann-Window

# Short-Term Spectral Analysis

- Discrete Fourier Transformation (DFT)

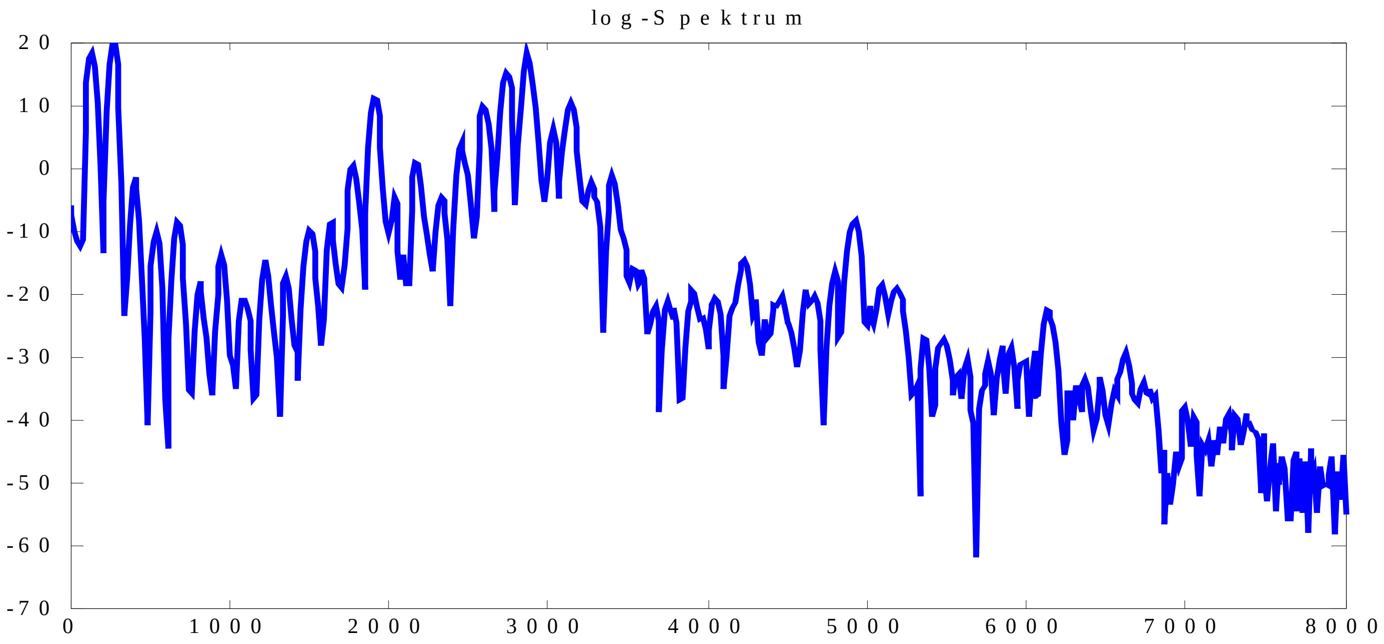
$$S(e^{j\omega}) = \sum_{n=0}^{N-1} s[n] e^{-j\omega n/N}$$




resulting short term spectrum (log-scale)

# Deconvolution

- We are interested in the formant-structure
- spectral smoothing
  - Cepstrum
  - Linear Prediction



# The Cepstrum

So if $s = u*h$ (source filter model),

then $FT\{s\} = FT\{u\} \cdot FT\{h\}$

and $\log FT\{s\} = \log FT\{u\} + \log FT\{h\}$

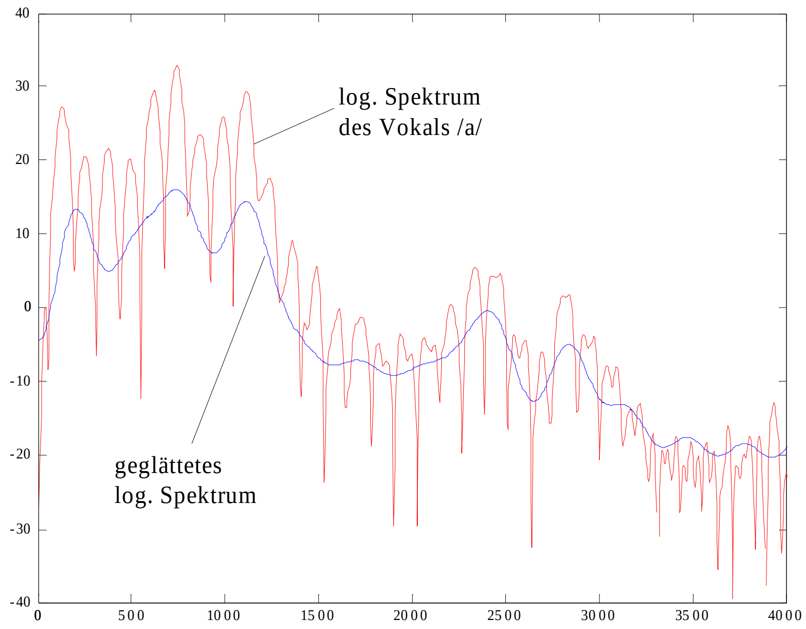
thus $FT^{-1}\{\log FT\{s\}\} = FT^{-1}\{\log FT\{u\}\} + FT^{-1}\{\log FT\{h\}\}$

- The coefficients of this transformation are called cepstral coefficients or simply cepstrum

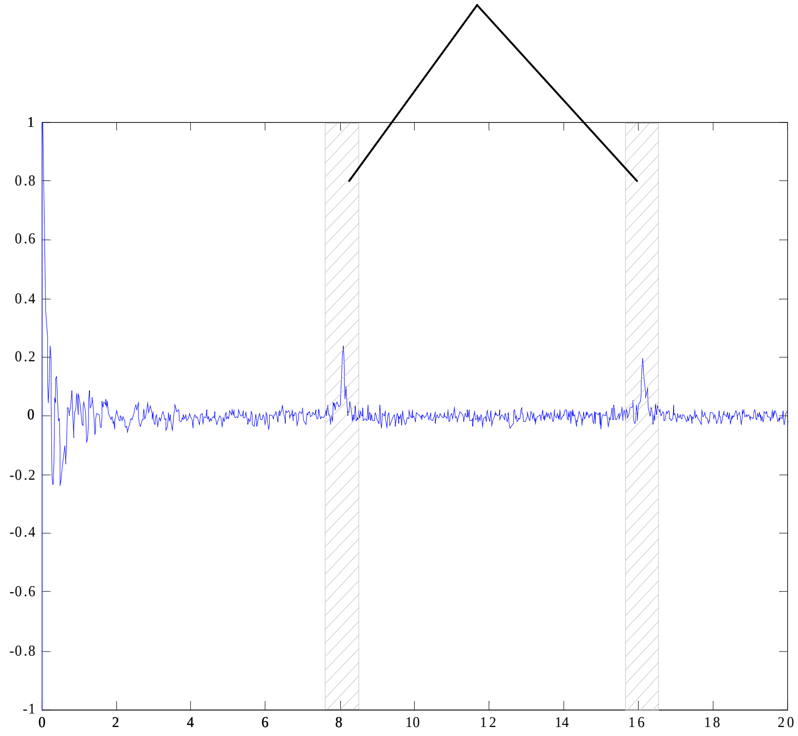
# The Cepstrum

## Example of the vowel „a“

Original and smoothed spectrum




Cepstral Peaks



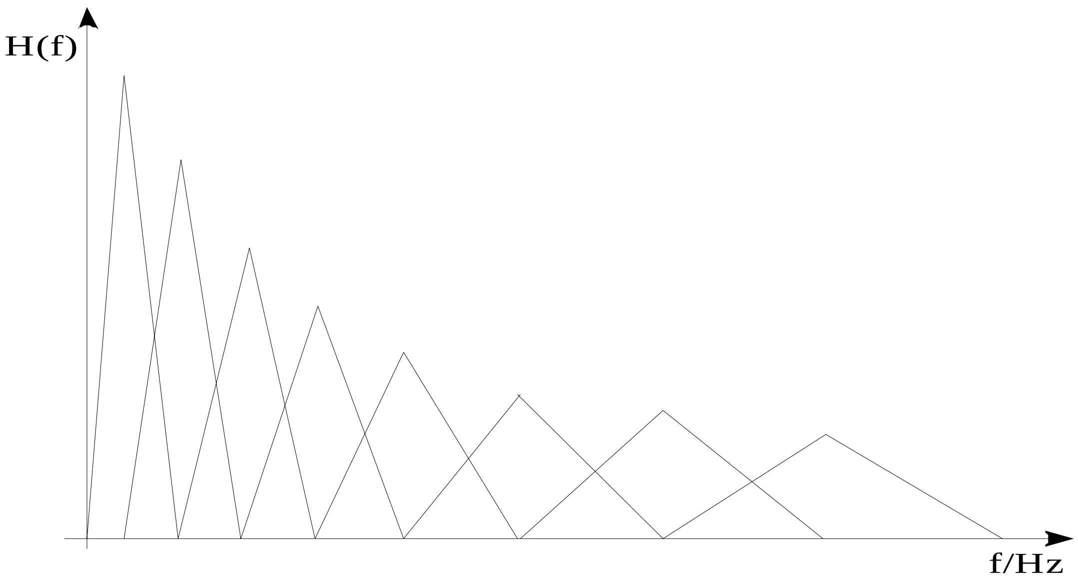
# The Cepstrum

- For speech recognition, only the lower cepstral coefficiens are used.
- When we set some of the coefficients to 0.0 then this process is called **liftering**.
- The lower coefficients reflect the macrostructure and the higher coefficients the microstructure of the spectrum.
- The 0th coefficients reflects the signal energy

# Non-linear frequency scales

- Mel-scale:

$$f_{mel} = 1125 \log(0.0016\, f + 1)$$



# Linear Predictive Coding (LPC)

Idea:

- samples can be approximated from past samples:

$$s[n] \approx a_0 + a_1 s[n-1] + a_2 s[n-2] + \cdots + a_p s[n-p]$$

- The actual signal differs from the estimated signal:

$$s[n] = -\sum_{j=1}^{p} a_j s[n-j] + e[n] \Rightarrow e[n] = s[n] - \hat{s}[n] = \sum_{j=0}^{p} a_j s[n-j]$$

# Linear Predictive Coding (LPC)

which after a z-transformation becomes:

$$A(z) = \frac{E(z)}{S(z)}$$

using the Z-Transformation:

$$A(z) = \frac{S(z) - \sum_{j=1}^{p} \alpha_j \, S(z) z^{-j}}{S(z)}$$

$$= 1 - \sum_{j=1}^{p} \alpha_j \, z^{-j}$$

if we assume a=α, we can model the vocaltract parameters!

# Linear Predictive Coding (LPC)

- Because we want to find the LPC coefficients $\alpha_j$, we have to minimize the squared error:

$$\sum_{n=0}^{N} e_n^2 = \sum_{n=0}^{N} \left( \sum_{j=0}^{p} a_j f_{n-j} \right)^2$$

- using the autocorrelation method and the Levinson-Durbin recursion, we‘ll receive the LPC coefficients $\alpha_j$.

# Linear Predictive Coding (LPC)

- the frequency response is computed by

$$H(z) = \frac{1}{DFT\{[1, \alpha_1, \alpha_2, ..., \alpha_{\mathrm{p}}, \underbrace{0, ..., 0}_{\text{N-p-1 Zeros}}]\}}$$

# Perceptual Linear Prediction (PLP)

- The combination of Cepstrum and LPC is called Perceptual Linear Prediction
- We can show that the autorrelation of a discrete signal (but without the log scale) is equal to the real cepstrum.

$$\boxed{\mathrm{DFT}} \rightarrow \boxed{|S(f)|^2} \rightarrow \boxed{\mathrm{mel}} \rightarrow \boxed{|S(f)|^{1/3}} \rightarrow \boxed{\mathrm{DFT}^{-1}} \rightarrow \boxed{\mathrm{Durbin}}$$

# Deltas and Acceleration Coefficients

- after these signal processing methods we want to put more context into the feature vectors.
  - Deltas:

$$\Delta x_{\mu} = \frac{1}{2}(x_{\mu+1} - x_{\mu-1})$$

  - Acceleration:

$$\Delta\Delta x_{\mu} = \frac{1}{2}(\Delta x_{\mu+1} - \Delta x_{\mu-1})$$

# Concatenating feature vectors

- concatenating:

$$y(m) = \begin{pmatrix} x(m-1) \\ x(m) \\ x(m+1) \end{pmatrix}$$

- resulting dimensions:
  - HTK (typical):
    - using 13 Mel-Cepstrum Comp. + Deltas + Acc.
  - other systems:
    - using 16 Mel-Cepstrum Comp. + Deltas + Signal Energy => 33 components
    - 165 dimensions after concatenating 5 feature vectors!!!

# Linear Discriminant Analysis (LDA)

- Lots of the used features contain no relevant informations.
- So we can reduce the dimensionality of the feature-vectors „without“ loss.
- LDA is a statistical method to transform features with respect to
  - the main components of the feature-class
  - the main components of the whole distribution

# Linear Discriminant Analysis

- Original feature vectors are multiplied by:

$$y = \Theta^T x$$

- the LDA-Matrix $\Theta$ is defined as the solution of the eingenvalue problem:

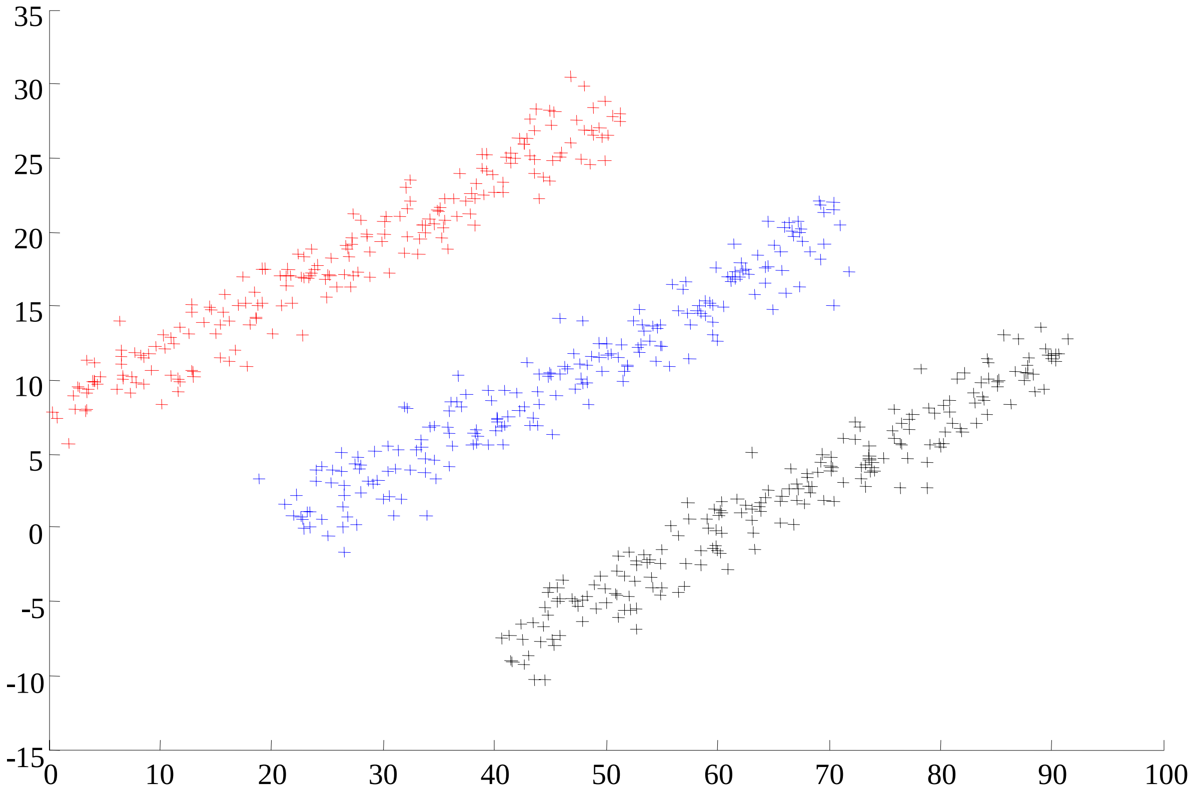
$$S_W^{-1} S_T \Theta = \Theta \Lambda$$

- with the Within-Scatter matrix $S_W$ and the Total-Scatter matrix $S_T$

$$S_W = \frac{1}{N} \sum_{k=1}^{K} \sum_{\substack{i=1 \\ g(i)=k}}^{N_k} (x - m_k)(x - m_k)^T \qquad S_T = \frac{1}{N} \sum_{i=1}^{N} (x - m_0)(x - m_0)^T$$

# Linear Discriminant Analysis

- Example: we want to discriminate the three classes in  the 1 dimensional space



# From Speech to a resulting feature-vector