TTIC 31230, Fundamentals of Deep Learning

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Optimal Discrimination

and Jensen-Shannon Divergence

GANs

The generator tries to fool the discriminator.

$$\Phi^* = \operatorname*{argmax}_{\Phi} \min_{\Psi} E_{\langle i, y \rangle \sim \tilde{p}_\Phi} - \ln P_\Psi(i|y)$$

Assuming universality of both the generator p_Φ and the discriminator P_Ψ we have $p_{\Phi^*} = \text{pop}$.

Assuming Universality of Ψ Only

$$\Psi^*(\Phi) = \operatorname*{argmin}_{\Psi} E_{\langle i, y\rangle \sim \tilde{p}_\Phi} - \ln P_\Psi(i|y)$$

$$P_{\Psi^*(\Phi)}(1|y) = \tilde{p}_\Phi(1|y) = \frac{\tilde{p}_\Phi(1,y)}{\tilde{p}_\Phi(y)} = \frac{\frac{1}{2}\mathrm{pop}(y)}{\frac{1}{2}\mathrm{pop}(y) + \frac{1}{2}p_\Phi(y)}$$

$$P_{\Psi^*(\Phi)}(-1|y) = \tilde{p}_\Phi(-1|y) = \frac{\tilde{p}_\Phi(-1,y)}{\tilde{p}_\Phi(y)} = \frac{\frac{1}{2}p_\Phi(y)}{\frac{1}{2}\mathrm{pop}(y) + \frac{1}{2}p_\Phi(y)}$$

Assuming Universality of Ψ Only

$$
\begin{aligned}
\Phi^* &= \operatorname*{argmax}_{\Phi} \ \min_{\Psi} \ E_{\langle i,\, y\rangle \sim \tilde{p}_\Phi} \ - \ln P_\Psi(i|y) \\
&= \operatorname*{argmax}_{\Phi} E_{(i,y)\sim\tilde{p}_\Phi} \ - \ln \tilde{p}_\Phi(i|y) \\
&= \operatorname*{argmin}_{\Phi} E_{(i,y)\sim\tilde{p}_\Phi} \ \ln \tilde{p}_\Phi(i|y) \\
&= \operatorname*{argmin}_{\Phi} \frac{1}{2} \ln \tilde{p}_\Phi(1|y) + \frac{1}{2} \ln \tilde{p}_\Phi(-1|y)
\end{aligned}
$$

Assuming Universality of Ψ Only

$$= \frac{1}{2}\ln \tilde{p}_\Phi(1|y) + \frac{1}{2}\ln \tilde{p}_\Phi(-1|y)$$

$$= \frac{1}{2}E_{y\sim \text{pop}}\ \ln \frac{\frac{1}{2}\text{pop}(y)}{\frac{1}{2}\text{pop}(y)+\frac{1}{2}p_\Phi(y)} + \frac{1}{2}E_{y\sim p_\Phi}\ \ln \frac{\frac{1}{2}p_\Phi(y)}{\frac{1}{2}\text{pop}(y)+\frac{1}{2}p_\Phi(y)}$$

$$= \frac{1}{2}\left(\ KL\left(\text{pop}, \frac{\text{pop}+p_\Phi}{2}\right) +\ \ KL\left(p_\Phi, \frac{\text{pop}+p_\Phi}{2}\right)\right) - \ln 2$$

$$\Phi^* = \underset{\Phi}{\text{argmin}}\, \text{JSD}(\text{pop}, p_\Phi)$$

END