

# TTIC 31230, Fundamentals of Deep Learning

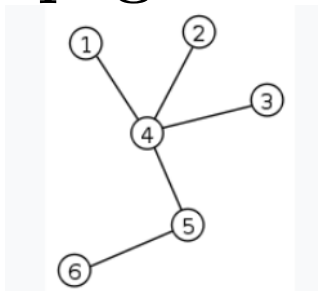
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## Loopy Belief Propagation (Loopy BP)

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We design an algorithm that is correct for tree graphs and use it on non-tree (loopy) graphs.

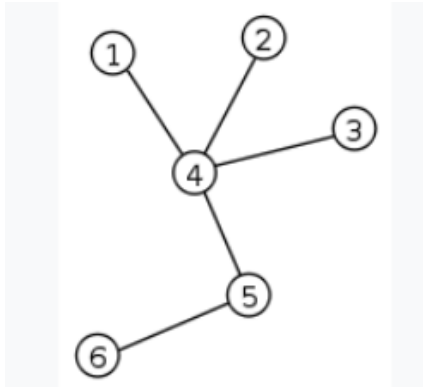
## Belief Propagation on Trees



Belief Propagation is a message passing procedure (actually dynamic programming).

For each edge  $\{n, m\}$  and possible value  $y$  for node  $n$  we define the message  $Z_{m \rightarrow n}[y]$  from  $m$  to  $n$  to be the partition function for the subtree attached to  $n$  through  $m$  and with  $\hat{\mathcal{Y}}[n]$  restricted to  $y$ .

# Dynamic Programming Computes the Messages



$$Z_{m \rightarrow n}[y] = \sum_{y'} e^{s^N[m, y'] + s^E[\langle m, n \rangle, y', y]} \left( \prod_{k \in N(m), k \neq n} Z_{k \rightarrow m}[y'] \right)$$

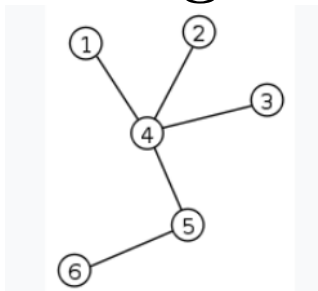
## Loopy BP

In a Loopy Graph we can initialize all message  $Z_{n \rightarrow m}[y] = 1$  and then repeating (until convergence) the updates

$$\tilde{Z}_{m \rightarrow n}[y] = \frac{1}{Z_{m \rightarrow n}} Z_{m \rightarrow n}[y] \quad Z_{m \rightarrow n} = \sum_y Z_{m \rightarrow n}[y]$$

$$Z_{m \rightarrow n}[y] = \sum_{y'} e^{s^N[m, y'] + s^E[m, n, y', y]} \left( \prod_{k \in N(m), k \neq n} \tilde{Z}_{k \rightarrow m}[y'] \right)$$

# Computing Node Marginals from Messages



$$\begin{aligned} Z^N(y) &= \sum_{\hat{\mathcal{Y}}: \hat{\mathcal{Y}}[n]=y} e^{s(\hat{\mathcal{Y}})} \\ &= e^{s^N[y]} \left( \prod_{m \in N(n)} Z_{m \rightarrow n}[y] \right) \\ \textcolor{red}{P}^N(\textcolor{red}{y}) &= Z^N(y)/Z, \quad Z = \sum_y Z^N(y) \end{aligned}$$

## Computing Edge Marginals from Messages

$$\begin{aligned}
 Z_{n,m}(y, y') &\doteq \sum_{\hat{\mathcal{Y}}: \hat{\mathcal{Y}}[n]=c, \hat{\mathcal{Y}}[m]=y'} e^{s(\hat{\mathcal{Y}})} \\
 &= e^{s^N[n,y] + s^N[m,y'] + s^E[n,m,y,y']} \\
 &\quad \prod_{m \in N(n), k \neq m} Z_{m \rightarrow n}[y] \\
 &\quad \prod_{m \in N(m), k \neq n} Z_{m \rightarrow n}[y']
 \end{aligned}$$

$$P_{n,m}(y, y') = Z_{n,m}(y, y') / Z \quad Z = \sum_{y, y'} Z_{n,m}(y, y')$$

**END**