TTIC 31230, Fundamentals of Deep Learning

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Deep Graphical Models Summary and Review

Deep Graphical Models

We are given a set of pairs $\langle x_1, \mathcal{Y}_1 \rangle, \ldots, \; \langle x_N, \; \mathcal{Y}_T \rangle$ where each $\mathcal{Y}_t$ is a "colored graph" — a graph with a label (color) assigned to each node. Semantic segmentation is an example where pixels of an image x are labeled with semantic categories.

We have a scoring function on colorings of a fixed graph.

$$s_{\Phi,x}(\hat{\mathcal{Y}}) = \sum_{n \in \mathrm{Nodes}} s^N_{\Phi,x}[n, \hat{\mathcal{Y}}[n]] \; + \sum_{\langle n,\, m \rangle \in \mathrm{Edges}} s^E_{\Phi,x}[\langle n, m \rangle, \hat{\mathcal{Y}}[n], \; \hat{\mathcal{Y}}[m]]$$

Deep Graphical Models

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$$P_{\Phi,x}(\hat{\mathcal{Y}}) = \operatorname*{softmax}_{\hat{\mathcal{Y}}} \; s_{\Phi,x}(\hat{\mathcal{Y}})$$

$$\mathcal{L}_{\Phi,x}(\mathcal{Y}) = -\ln P_{\Phi,x}(\mathcal{Y})$$

Deep Graphical Models

$$P_{\Phi,x}(\hat{\mathcal{Y}}) = \operatorname*{softmax}_{\hat{\mathcal{Y}}} \; s_{\Phi,x}(\hat{\mathcal{Y}})$$

To back-propagate from the loss $-\ln P_{\Phi,x}(\mathcal{Y})$ through the exponential softmax to the tensors $S^N_{\Phi,x}$ and $s^E_{\Phi,x}$ we need to compute or estimate

$$P_{\Phi,x}(\hat{\mathcal{Y}}[n] = y) \;\; \text{and} \;\; P_{\Phi,x}(\hat{\mathcal{Y}}[\langle n, m\rangle] = \langle y, y'\rangle)$$

for each node n and value y and each edge $\langle n, m\rangle$ and value $\langle y, y'\rangle$.

END