TTIC 31230, Fundamentals of Deep Learning

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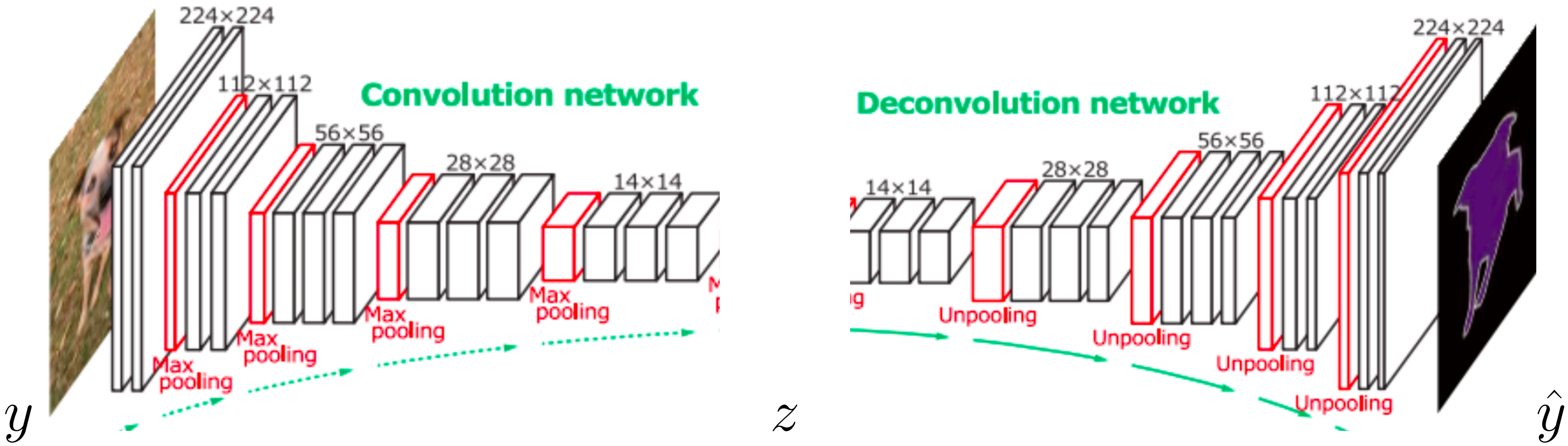
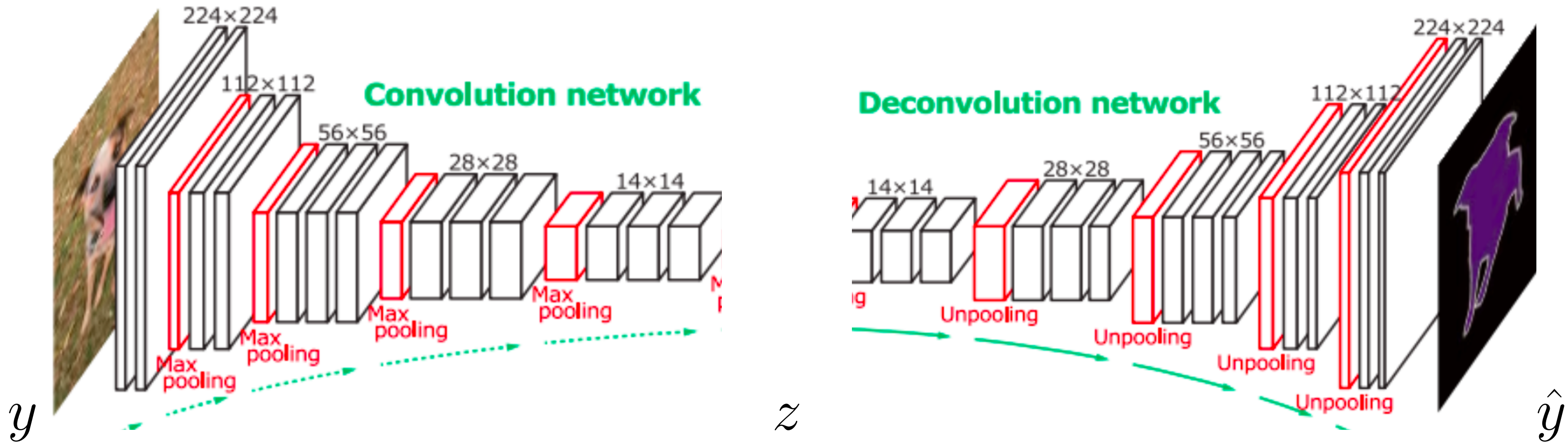
Gaussian VAEs

A General Autoencoder



In generall we have either $P_\Phi(z)$ for z discrete or $\hat{p}_\Phi(z)$ for z continuous.

A General Autoencoder



For the continuous case with $p_\Phi(z|y)$ and $\hat{p}_\Phi(z)$ both Gaussian we can assume without loss of generality that

$$\hat{p}_\Phi(z) = \mathcal{N}(0, I)$$

Gaussian VAEs: The Reparameterization Trick

$$- \ln p_\Phi(y) \;\leq\; E_{z \sim \hat{p}_\Phi(z|y)} \;-\ln \frac{p_\Phi(z) p_\Phi(y|z)}{\hat{p}_\Phi(z|y)}$$

$$=\; E_\epsilon \;-\ln \frac{p_\Phi(z) p_\Phi(y|z)}{\hat{p}_\Phi(z|y)} \quad z := f_\Phi(y, \epsilon)$$

ϵ is parameter-independent noise.

This supports SGD: $\nabla_\Phi \; E_{y,\epsilon} \; [\ldots] = E_{y,\epsilon} \; \nabla_\Phi \; [\ldots]$

Gaussian VAEs

$$\Phi^* = \operatorname*{argmin}_{\Phi} E_{y,\epsilon} \; - \ln \frac{p_\Phi(z)p_\Phi(y|z)}{\hat{p}_\Phi(z|y)}$$

$$z \;=\; z_\Phi(y) + \sigma_\Phi(y) \odot \epsilon \quad \epsilon \sim \mathcal{N}(0, I)$$

$$\hat{p}_\Phi(z[i]|y) \;=\; \mathcal{N}(z_\Phi(y)[i], \sigma_\Phi(y)[i])$$

$$p_\Phi(z[i]) \;=\; \mathcal{N}(\mu_p, \sigma_p[i]) \;\; \text{WLOG} = \mathcal{N}(0, 1)$$

$$p_\Phi(y|z) \;=\; \mathcal{N}(y_\Phi(z), \; \sigma^2 I)$$

END