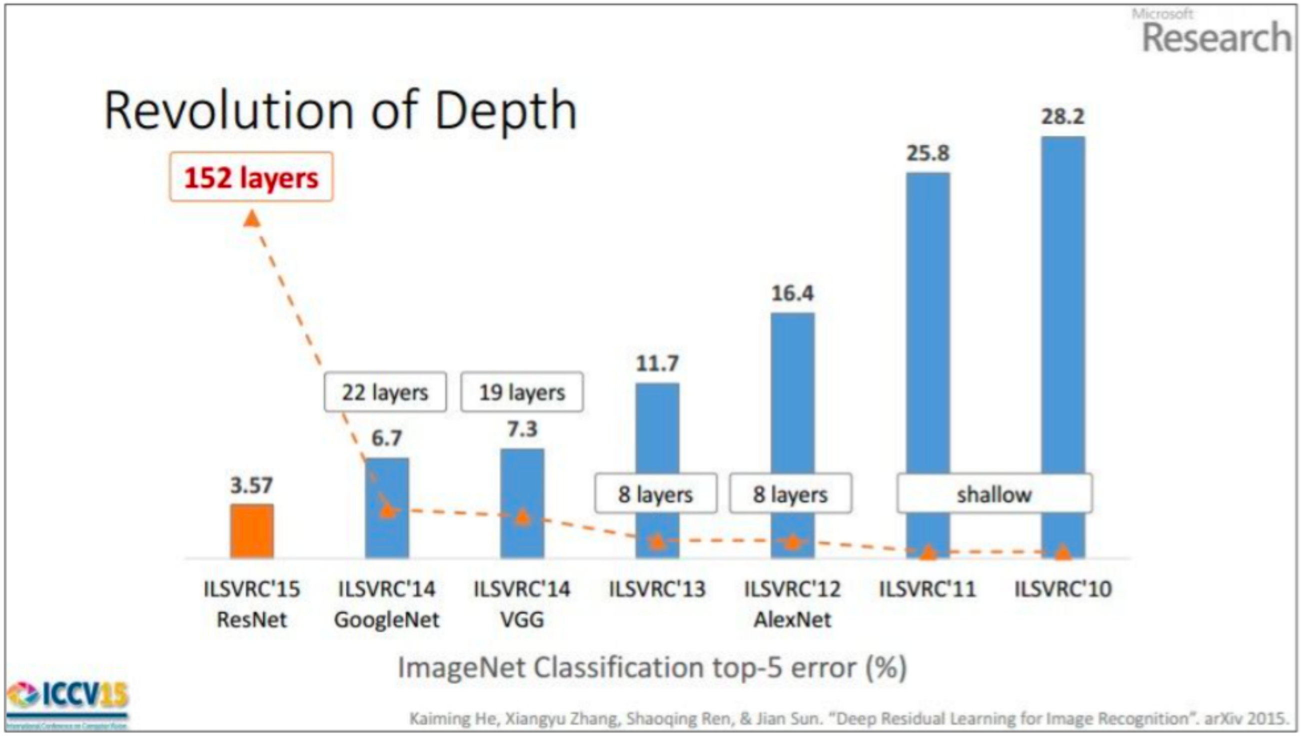
TTIC 31230, Fundamentals of Deep Learning

David McAllester, Winter 2020

Convolutional Neural Networks (CNNs)

Imagenet Classification

1000 kinds of objects.



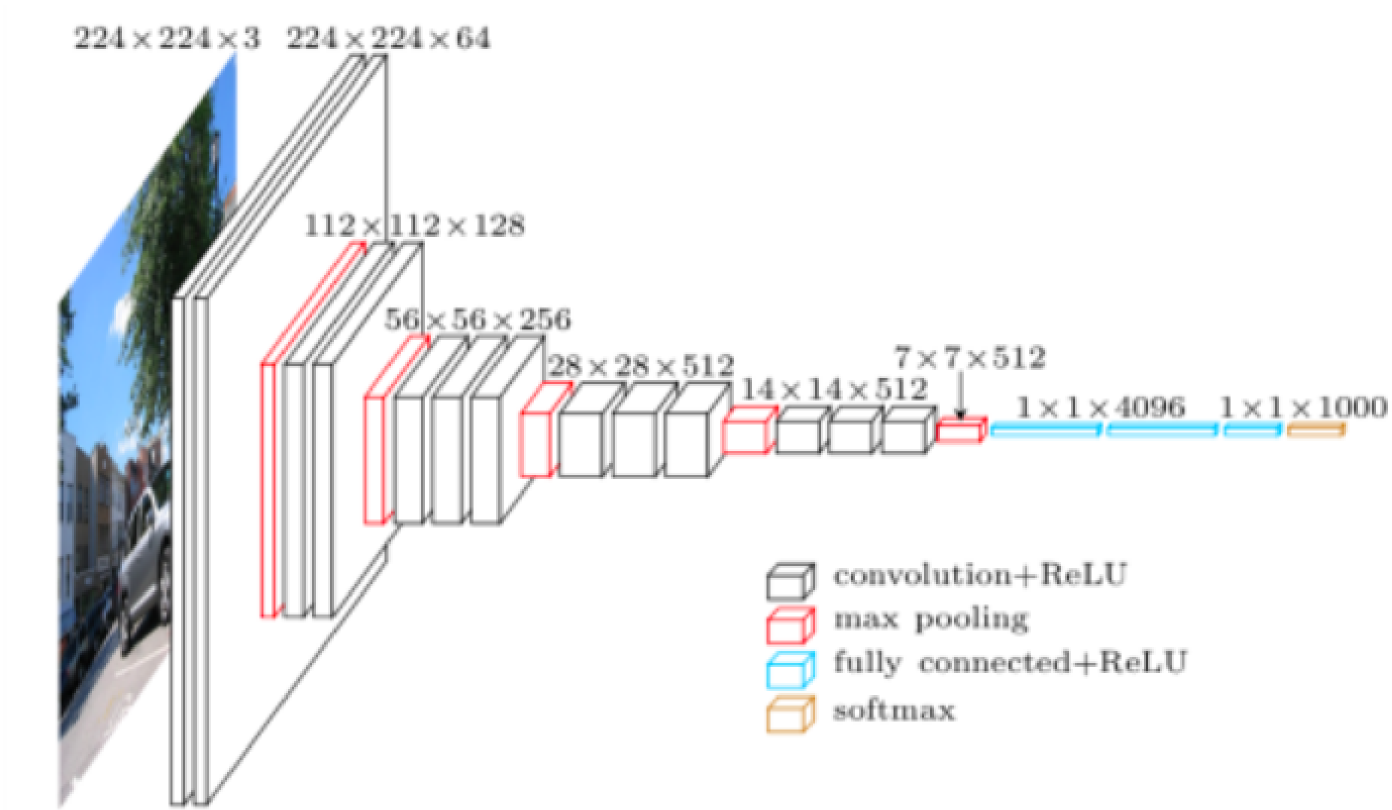
(slide from Kaiming He's recent presentation)

2016 is 3.0%, is 2017 2.25%

SOTA as of January 2020 is 1.3%

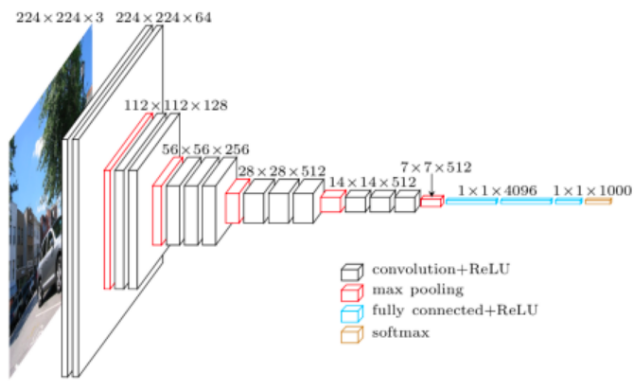
What is a CNN?

VGG, Zisserman, 2014



Davi Frossard

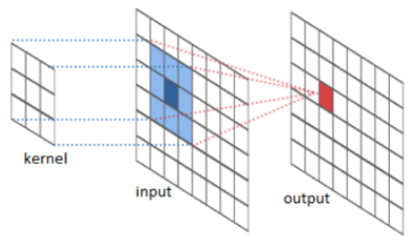
A Convolution Layer



Each box is a tensor $L_\ell[b, x, y, i]$

Each value $L_\ell[b, x, y, i]$ (for $\ell > 0$) is the output of a single linear threshold unit.

A Convolution Layer



$$W[\Delta x, \Delta y, i, j] \qquad L_\ell[b, x, y, i] \qquad L_{\ell+1}[b, x, y, j]$$

River Trail Documentation

$$L_{\ell+1}[b, x, y, j]$$

$$= \sigma\left(\left(\sum_{\Delta x, \Delta y, i} W[\Delta x, \Delta y, i, j]\; L_\ell[b, x + \Delta x, y + \Delta y, i]\right) - B[j]\right)$$

Convolution Layer in Einstein Notation

$$L_{\ell+1}[b, x, y, j]$$

$$= \sigma\left(\left(\sum_{\Delta x, \Delta y, i} W[\Delta x, \Delta y, i, j]\; L_\ell[b, x + \Delta x, y + \Delta y, i]\right) - B[j]\right)$$

$$= \sigma\left(W[\Delta X, \Delta Y, I, j]\; L_\ell[b, x + \Delta X, y + \Delta Y, I] - B[j]\right)$$

Many "Neurons" (Linear Threshold Units)

Each $L_{\ell+1}[b,x,y,j]$ is the output of a single linear threshold unit.

$$L_{\ell+1}[b,x,y,j]$$

$$= \sigma\left(W[\Delta X, \Delta Y, I, j]\; L_\ell[b, x+\Delta X, y+\Delta Y, I] - B[j]\right)$$

In this equation the input to the activation function σ is a scalar value — this is a single linear threshold unit.

2D CNN in PyTorch

conv2d(**input, weight, bias, stride, padding, dilation, groups**)

input tensor (minibatch,in-channels,iH,iW)

weight filters (out-channels, in-channels/groups,kH,kW)

bias tensor (out-channels) . Default: None

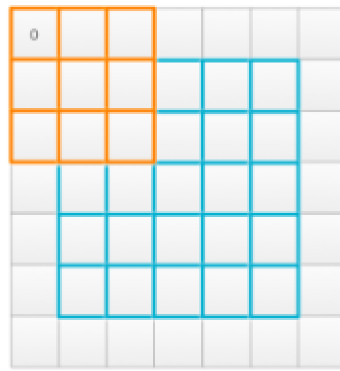
stride Single number or (sH, sW). Default: 1

padding Single number or (padH, padW). Default: 0

dilation Single number or (dH, dW). Default: 1

groups split input into groups. Default: 1

Padding



Jonathan Hui

If we pad the input with zeros then the input and output can have the same spatial dimensions.

Zero Padding in NumPy

In NumPy we can add a zero padding of width p to an image as follows:

```
padded = np.zeros(W + 2*p,  H + 2*p)

padded[p:W+p, p:H+p] = x
```

Padding

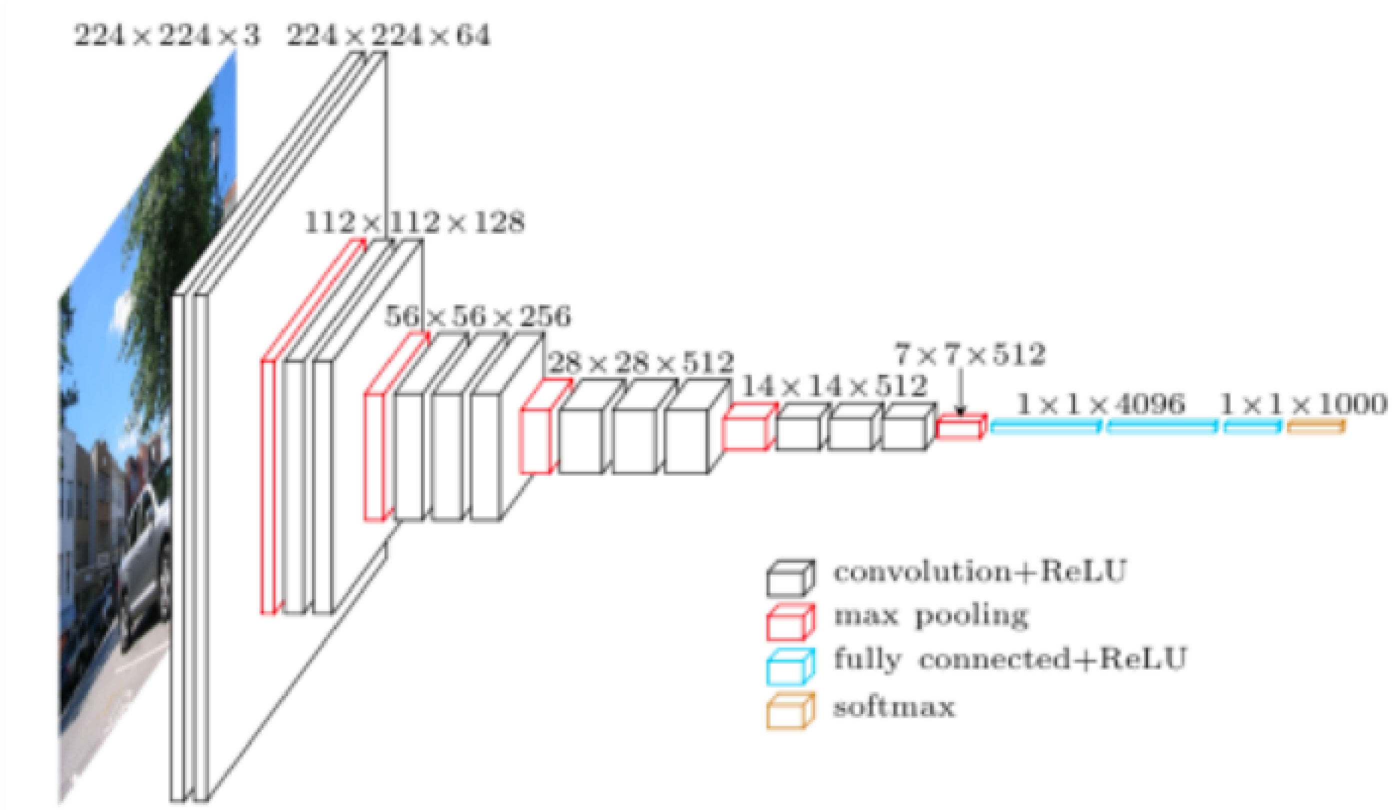
$$L'_\ell = \mathrm{Padd}(L_\ell,\ p)$$

$$L_{\ell+1}[b,x,y,j] =$$

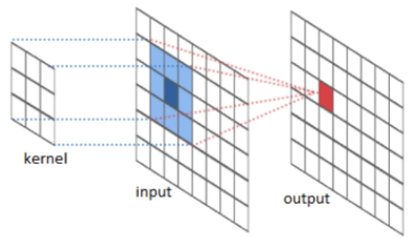
$$\sigma\left(W[\Delta X, \Delta Y, I, j]\ L'_\ell[b, x+\Delta X, y+\Delta Y, I] - B[j]\right)$$

If the input is padded but the output is not padded then Δx and Δy are non-negative.

Reducing Spatial Dimention



Reducing Spatial Dimensions: Max Pooling



$$L_{\ell+1}[b,x,y,i] = \max_{\Delta x,\Delta y} L_\ell[b,\ s*x+\Delta x,\ s*y+\Delta y,\ i]$$

This is typically done with a stride greater than one so that the image dimension is reduced.

Reducing Spatial Dimensions: Strided Convolution

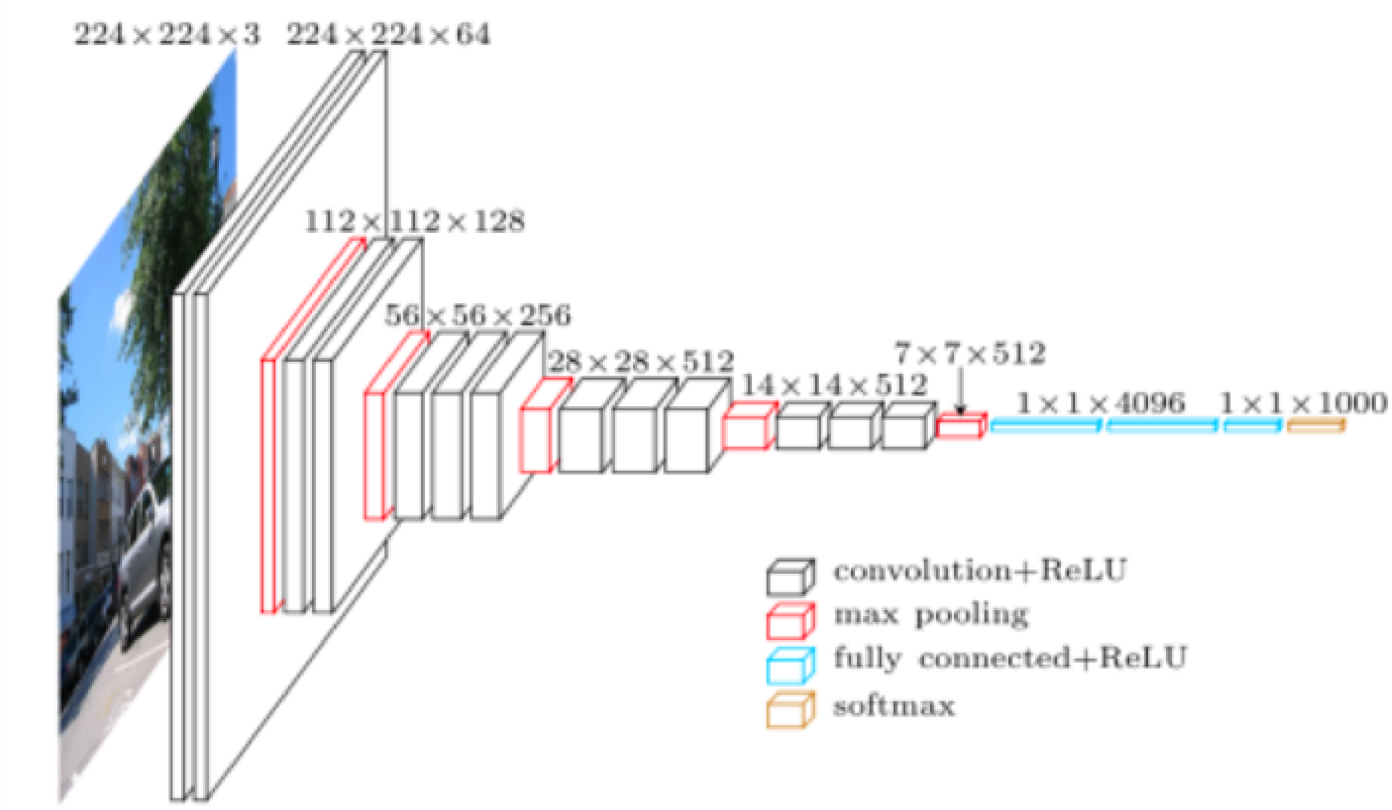
We can move the filter by a "stride" s for each spatial step.

$$L_{\ell+1}[b, x, y, j] =$$

$$\sigma\left(W[\Delta X, \Delta Y, I, j] L_\ell[b, s * x + \Delta X, s * y + \Delta Y, I] - B[j]\right)$$

For strides greater than 1 the spatial dimention is reduced.

Fully Connected (FC) Layers



Fully Connected (FC) Layers

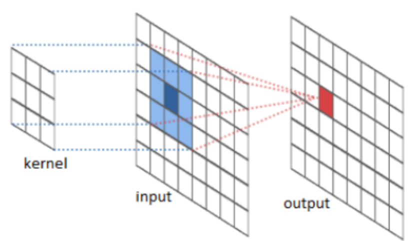
We reshape $L_\ell[b, x, y, i]$ to $L_\ell[b, i']$ and then

$$L_{\ell+1}[b, j] = \sigma \left(W[j, I] \; L_\ell[b, I] - B[j] \right)$$

Dilation

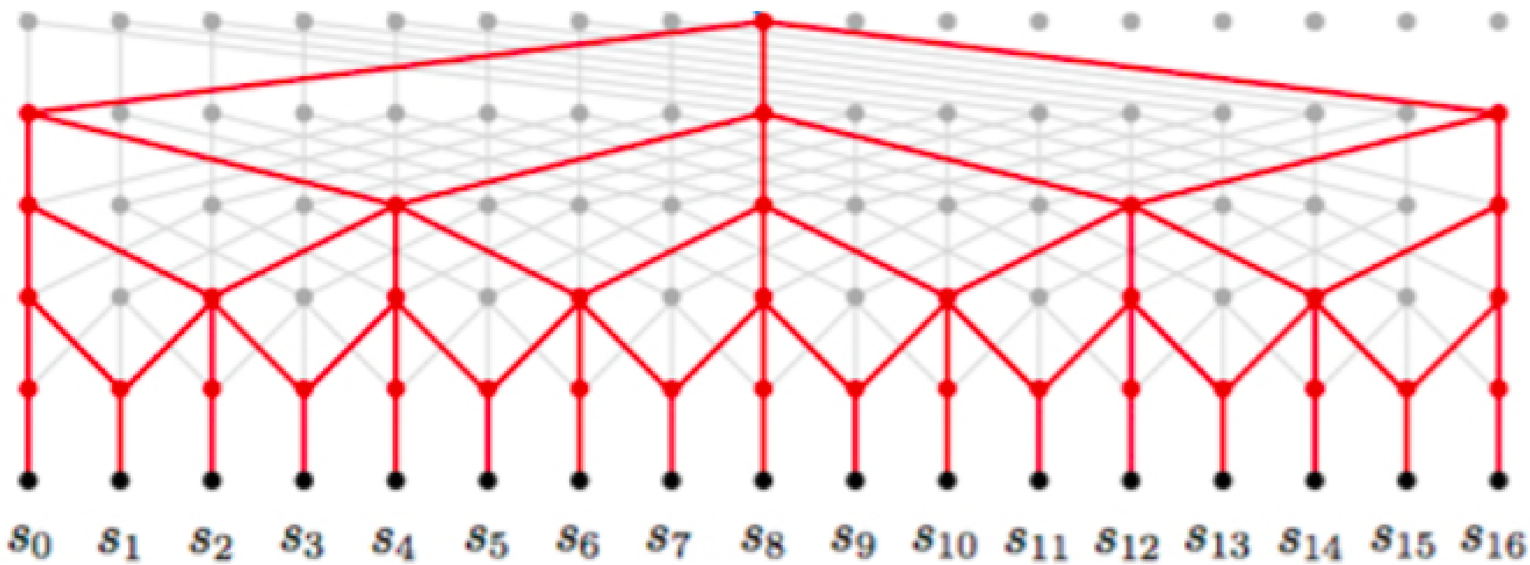
A CNN for image classification typically reduces an $N \times N$ image to a single feature vector.

Dilation is a trick for treating the whole CNN as a "filter" that can be passed over an $M \times M$ image with $M > N$.



An output tensor with full spatial dimension can be useful in, for example, image segmentation.

Dilation



This is called a "fully convolutional" CNN.

Dilation

To implement a fully convolutional CNN we can "dilate" the filters by a dilation parameter d.

$$L_{\ell+1}[b, x, y, j]$$

$$= \sigma(W[\Delta X, \Delta Y, I, j] L_\ell[b, x + d * \Delta X, y + d * \Delta Y, I] + B[j])$$

Vector Concatenation

We will write

$$L[b, x, y, J_1 + J_2] = L_1[b, x, y, J_1] \;; L[b, x, y, J_2]$$

To mean that the vector $L[b, x, y, J_1 + J_2]$ is the concatenation of the vectors $L_1[b, x, y, J_1]$ and $L_2[b, x, y, J_2]$.

Hypercolumns

For a given image location $\langle x, y\rangle$ we concatenate all the feature vectors of all layers above the point $\langle x, y\rangle$.

$$
\begin{aligned}
& L\left[b, x, y, \sum_{\ell} J_\ell\right] \\
= \; & L_0\,[b, x, y, J_0] \\
& \vdots \\
& ; L_\ell\left[b, \left\lfloor x\left(\frac{X_\ell}{X_1}\right)\right\rfloor, \left\lfloor y\left(\frac{Y_\ell}{Y_0}\right)\right\rfloor, J_\ell\right] \\
& \vdots \\
& ; L_{\mathcal{L}-1}[b, J_{\mathcal{L}-1}]
\end{aligned}
$$

Grouping

The input features and the output features are each divided into G groups.

$$L_{\ell+1}[b,x,y,J] = L^0_{\ell+1}[b,x,y,J/G];\cdots;L^{G-1}_{\ell+1}[b,x,y,J/G]$$

where we have G filters $W^g[\Delta X, \Delta Y, I/G, J/G]$ with

$$L^g_{\ell+1}[b,x,y,j]$$

$$= \sigma(W^g[\Delta X, \Delta Y, I/G, j]L^g_\ell[x+\Delta X, y+\Delta Y, I/G, j] - B^g[j])$$

This uses a factor of G fewer weights.

2D CNN in PyTorch

conv2d(**input, weight, bias, stride, padding, dilation, groups**)

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weight filters (out-channels, in-channels/groups,kH,kW)

bias tensor (out-channels) . Default: None

stride Single number or (sH, sW). Default: 1

padding Single number or (padH, padW). Default: 0

dilation Single number or (dH, dW). Default: 1

groups split input into groups. Default: 1

Modern Trends

Modern Convolutions use 3X3 filters. This is faster and has fewer parameters. Expressive power is preserved by increasing depth with many stride 1 layers.

Max pooling and dilation seem to have disappeared.

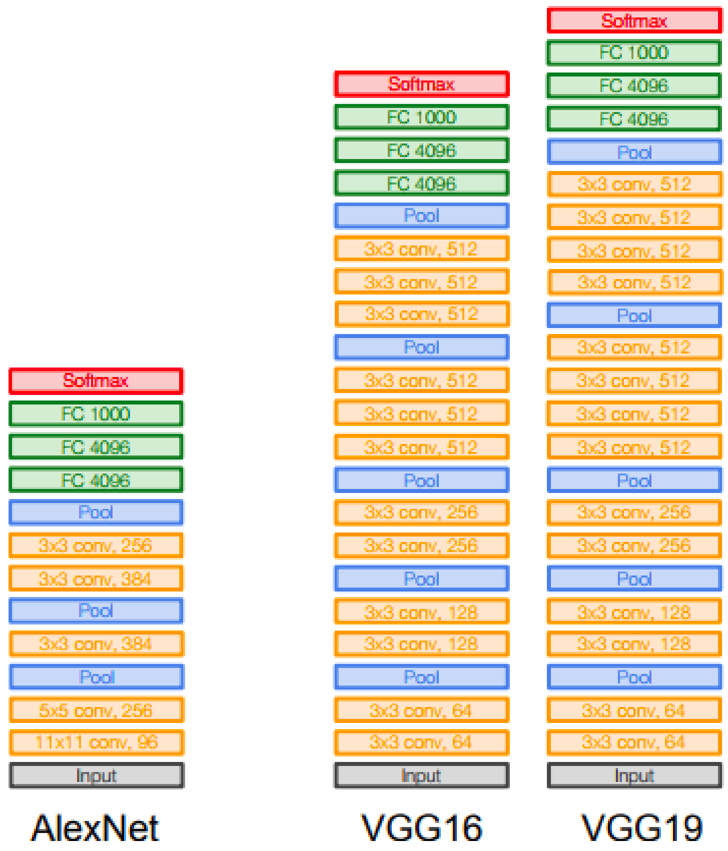
ResNet and resnet-like architectures are now dominant.

Alexnet, 2012

Given Input$[227, 227, 3]$

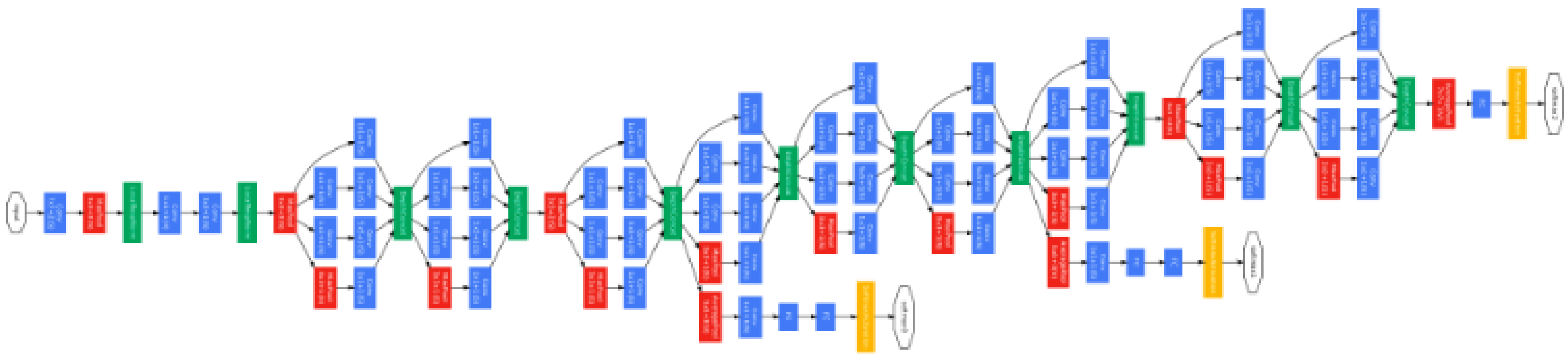
$$
\begin{aligned}
L_1[55 \times 55 \times 96] &= \mathrm{ReLU}(\mathrm{CONV}(\mathrm{Input}, \Phi_1, \text{width } 11, \text{pad } 0, \text{stride } 4)) \\
L_2[27 \times 27 \times 96] &= \mathrm{MaxPool}(L_1, \text{width } 3, \text{stride } 2)) \\
L_3[27 \times 27 \times 256] &= \mathrm{ReLU}(\mathrm{CONV}(L_2, \Phi_3, \text{width } 5, \text{pad } 2, \text{stride } 1)) \\
L_4[13 \times 13 \times 256] &= \mathrm{MaxPool}(L_3, \text{width } 3, \text{stride } 2)) \\
L_5[13 \times 13 \times 384] &= \mathrm{ReLU}(\mathrm{CONV}(L_4, \Phi_5, \text{width } 3, \text{pad } 1, \text{stride } 1)) \\
L_6[13 \times 13 \times 384] &= \mathrm{ReLU}(\mathrm{CONV}(L_5, \Phi_6, \text{width } 3, \text{pad } 1, \text{stride } 1)) \\
L_7[13 \times 13 \times 256] &= \mathrm{ReLU}(\mathrm{CONV}(L_6, \Phi_7, \text{width } 3, \text{pad } 1, \text{stride } 1)) \\
L_8[6 \times 6 \times 256] &= \mathrm{MaxPool}(L_7, \text{width } 3, \text{stride } 2)) \\
L_9[4096] &= \mathrm{ReLU}(\mathrm{FC}(L_8, \Phi_9)) \\
L_{10}[4096] &= \mathrm{ReLU}(\mathrm{FC}(L_9, \Phi_{10})) \\
s[1000] &= \mathrm{ReLU}(\mathrm{FC}(L_{10}, \Phi_s)) \;\; \text{class scores}
\end{aligned}
$$

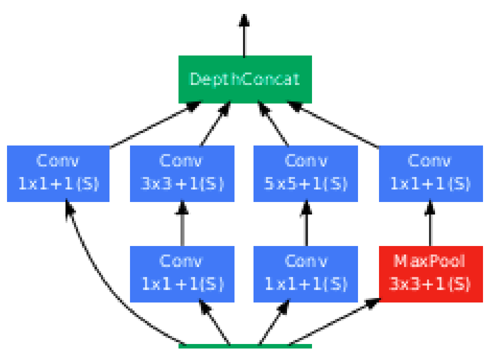
VGG, 2014



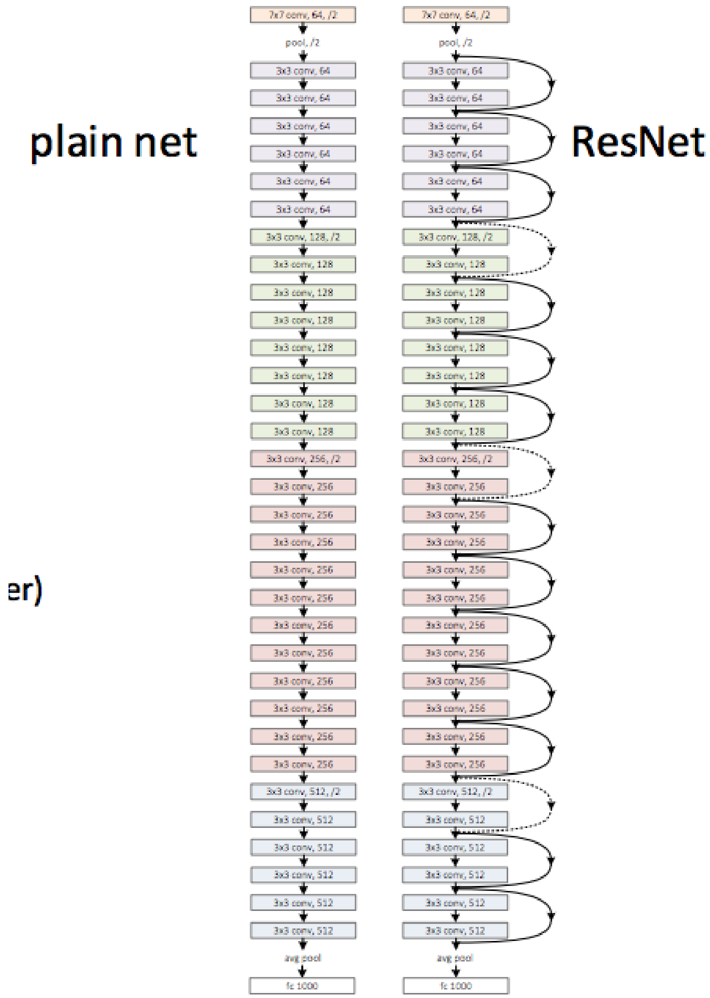
Stanford CS231

Inception, Google, 2014





ResNet, 2015



[Kaiming He]

Imagenet Classification

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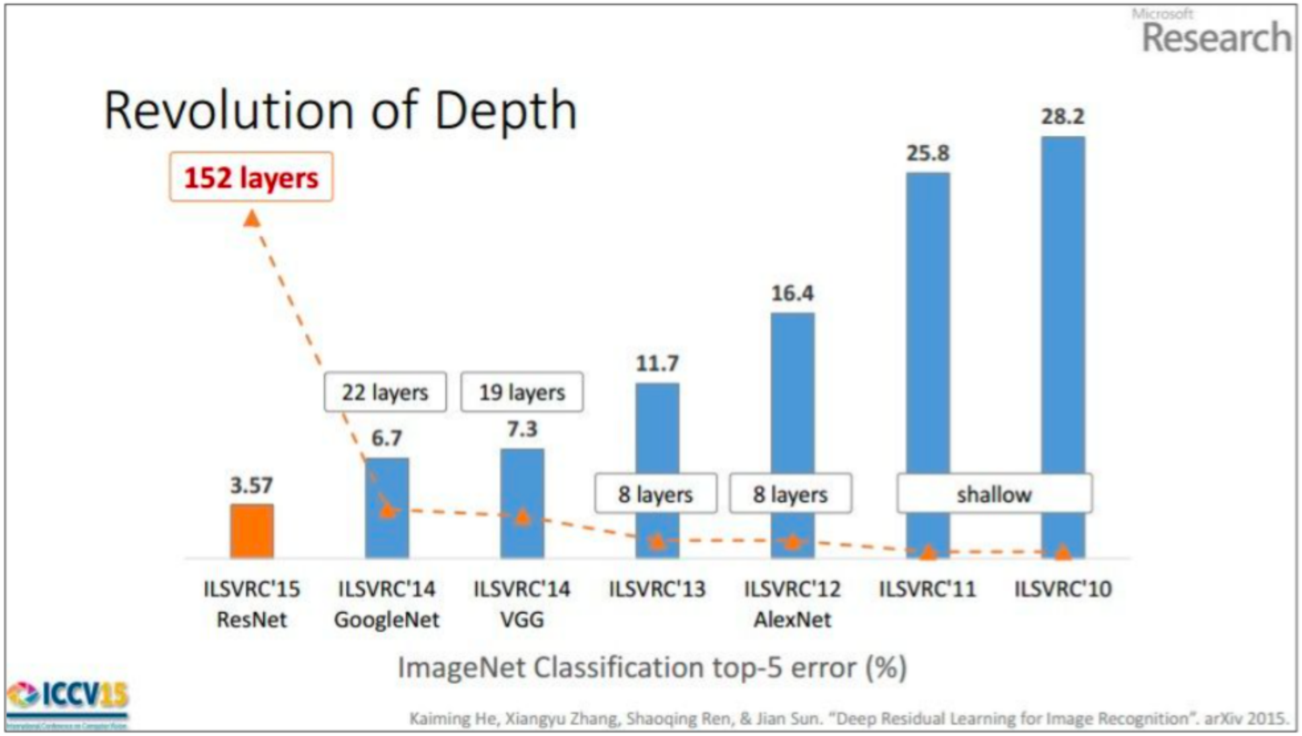

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