The measurement problem in Bohmian mechanics

Candidate: Leonardo Oleynik
Advisor: Prof. Dr. Renato Moreira Angelo

July of 2021

Group: Foundations of Quantum Mechanics

Abstract

According to MB, we only see the particle and not its pilot wave. However, in order to predict the particle's trajectories, we need to know the wave. So, how to know it or determine it if we only have access to the particle? The answer seems to be in the description of the measurement process according to MB, which is very similar to von Neumann's pre-measurement, but which, unlike this one, does not postulate any reduction of the state. We will also see that, according to Carlo Rovelli[1], this description lacks only one ingredient: decoherence.

[1] C. Rovelli, Preparation in Bohmian Mechanics. arXiv:2104.11339.

Outline

Motivation

Motivation

- C. Rovelli: *According to MB, we see the particle and not its pilot wave. But in order to make predictions, we need to know the wave. How to know it, if only see the particle?*
- By preparation? But what is preparation at MQ (and especially at MB)?
- Preparation and the measurement problem.
  - MQ + postulated from the collapse;
  - MB does not support such a postulate.
- C. Rovelli: *The measurement problem is completely solved in the MB when considering the decoherence effects produced by the environment.*

A brief review of MB

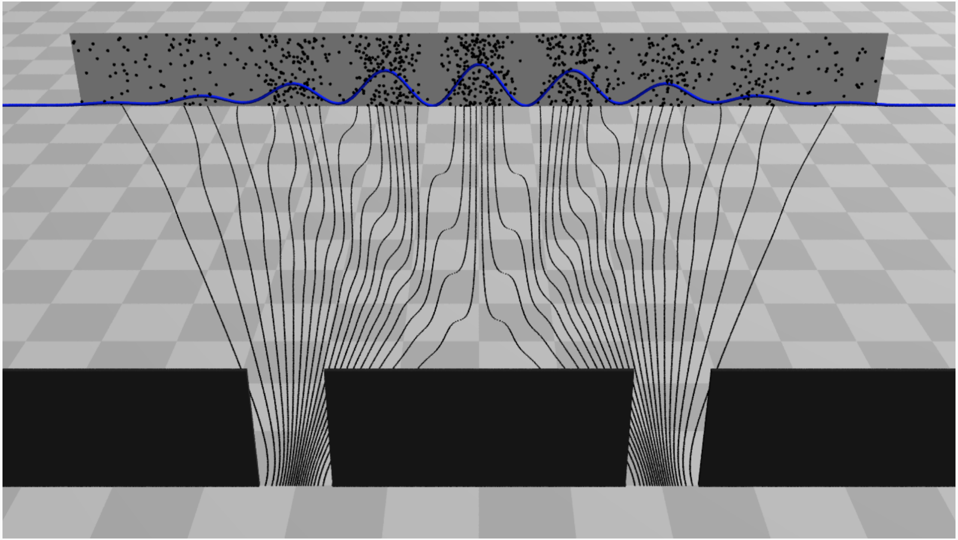
Bohmian ontology

- State of a system with 2 degrees of freedom: (Ψ, X_a, X_b)
    - $\Psi = \Psi(x_a, x_b, t)$ represents a physical field; and
    - X_k represents the trajectories ($k = a, b$).
- Dynamics:

$$i\hbar\partial_t \Psi = H\Psi \qquad \dot{X}_k = v^{[\Psi]}(X_a, X_b) = \frac{\hbar}{m_k}\,\mathrm{Im}\left\{\frac{\partial_k \Psi}{\Psi}\right\}_{\substack{x_a = X_a \\ x_b = X_b}}$$

- Postulate: *the ensemble of trajectories is distributed according to Born's rule for some t.*

Double Slit Experiment: Wave and Particle Duality



Double slit experiment: effective dynamics

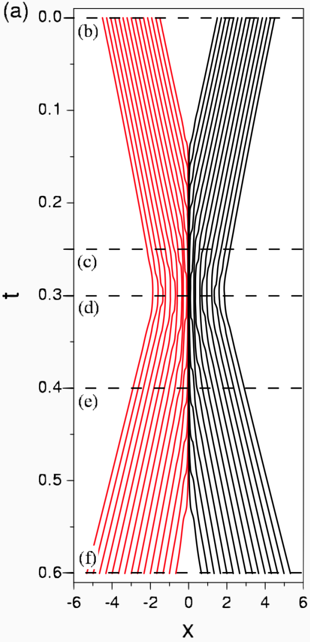
- $\Psi(x) = [\psi_1(x) + \psi_2(x)]/\sqrt{2}$;
- Localized packages and effective dynamics;
- If $supp(\psi_1) \bigcap supp(\psi_2) = 0$ and $X \in supp(\psi_1)$, then

$$\Psi(X) = \psi_1(X),$$

since $\psi_2(X) = 0$. AND,

$$\dot{X} = v^{[\psi]}(X) = v^{[\psi_1]}(X),$$

i.e., the dynamics depend only on the values of Ψ in the vicinity of X.



Measurement problem

Measurement problem

It can be formulated as the impossibility of a quantum theory simultaneously satisfying the following premises:

1. The state always evolves deterministically according to eq. Schrdinger (linear and unitary);
    - Abandonment: Collapse Postulate or GWR.
2. A measurement always reveals a physical system in a localized state;
    - Abandon: Many worlds.
3. $|\Psi\rangle$ is the complete description of the quantum state.
    - Abandon: Theories with *hidden* variables (e.g. MB).

Von Neumann premeasurement

- Characterizing the measurement in terms of entanglement dynamics: $\underbrace{\mathcal{A}}_{\text{S. of interest}} + \underbrace{\mathcal{B}}_{\text{Apparatus}}$;
- Initial state:

$$|\Psi_0\rangle = \underbrace{\sum_n c_n |\psi_n\rangle}_{\in\mathcal{H}_\mathcal{A}} \underbrace{|\phi_0\rangle}_{\in\mathcal{H}_\mathcal{B}} .$$

  where $O_\mathcal{A} |\psi_n\rangle = o_n |\psi_n\rangle$;
- Interaction: $H = g\, O_\mathcal{A} \otimes P_\mathcal{B}$ such that $\mathrm{e}^{-i\Delta x P_\mathcal{B}/\hbar} |x_\mathcal{B}\rangle = |x_\mathcal{B} + \Delta x\rangle$;
- End state:

$$\begin{aligned}|\Psi\rangle &= \mathrm{e}^{-i\tau H/\hbar} |\Psi_0\rangle \\ &= \sum_n c_n |\psi_n\rangle |\phi_0(x_b + g\tau o_n)\rangle \\ &= \sum_n c_n |\psi_n\rangle |\phi_n\rangle .\end{aligned}$$

- S. of interest: $\sum_n |c_n|^2 |\psi_n\rangle\langle\psi_n|$, i.e., in $|c_n|^2$ we often find $|\psi_n\rangle\langle\psi_n|$;
- Apparatus in a chain, postulate of the reduction $|\psi_n\rangle |\phi_n\rangle$ and arbitrary cut between the classical and quantum domains.

Measurement process according to MB

- Final state in the representation of positions:

$$\Psi(x_a, x_b) = \sum_n c_n \psi_n(x_a)\phi_n(x_b),$$

  where x_a =p. system of interest and x_b =p. of the apparatus.
- No overlapping packages ϕ_n: if $X_b \in supp(\phi_n)$ then $\phi_{m\neq n}(X_b) = 0$ and

$$\Psi(x_a, X_b) = \psi_n(x_a)\phi_n(X_b).$$

- Effective wave function:

$$\dot{X}_a = v^{[\Psi]}(X_a, X_b) = v^{[\psi_n]}(X_a)$$

  that is, the system of interest is effectively driven only by the ψ_n eigenstate.
- C. Rovelli: *How can I guarantee that initially the state is* $|\Psi_0\rangle = \sum_n c_n |\psi_n\rangle |\phi_0\rangle$*?*
  - Second preparation?
  - Inconsistency between apparatus and environment.

Demo

$$
\begin{aligned}
\dot{X}_a &= v^{[\Psi]}(X_a, X_b) \\
&= \frac{\hbar}{m_a} \operatorname{Im}\left\{ \frac{\partial_a \Psi'}{\Psi'} \right\}_{\substack{x_a = X_a \\ x_b = X_b}} \\
&= \frac{\hbar}{m_a} \operatorname{Im}\left\{ \frac{\partial_a \Psi(x_a, X_b)}{\Psi(x_a, X_b)} \right\}_{x_a = X_a} \\
&= \frac{\hbar}{m_a} \operatorname{Im}\left\{ \frac{\partial_a \psi_n(x_a)}{\psi_n(x_a)} \right\}_{x_a = X_a} \qquad = v^{[\psi_n]}(Shah)
\end{aligned}
$$