Mach's Principle in Bohmian Mechanics

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Abstract

Mach's principle states that the inertial mass of a body is related to the distribution of other distant bodies. This implies that the mass of a single particle in the universe is zero and that its motion is indeterminate. To detail this issue and relate it to Bohmian Mechanics, I bring, in this round of ArXiv Review, the summary of the following articles:

1. Mach's Principle (https://arxiv.org/abs/physics/0407078v2)
2. On the reality of space-time geometry and the wavefunction (https://link.springer.com/content/pdf/10.1007/BF02055212.pdf)
3. Bohmian Mechanics and the Meaning of the Wave Function (https://arxiv.org/abs/quant-ph/9512031v1)
4. Some Clarifications on the Relation Between Bohmian Quantum Potential and Mach's Principle (https://link.springer.com/article/10.1007/s10773-017-3476-6)

Outline

Absolute (A) and Relative (R) Space

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- A: Considers space as a container for material bodies. Its existence is a priori.
- R: Space is just an abstraction of the contents of bodies. Existing from these.

Newton's Bucket

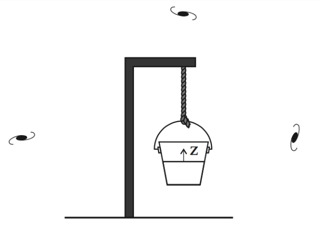

Figure 1: Stage 1

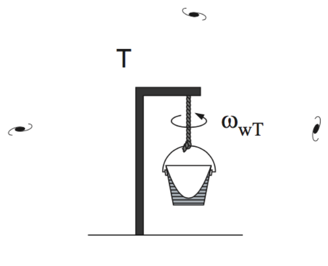

Figure 2: Stage 2

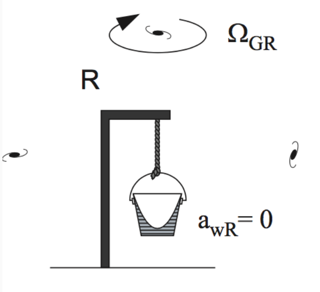

Figure 3: Stage 3

- Evidence of absolute space ?
- Implicit assumptions
  1. Universe is empty;
  2. Intrinsic properties exist even when the system is isolated ($m = F/a$).
- Mach: The concavity of the surface of the water is caused by the mass of the rest of the universe (including the *distant stars*)

The principle of Action and Reaction (AR)

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AR is more fundamental than

1. The conservation of total momentum (CM)
   If $\dot{P} \neq 0$ then $\dot{P} = F^e$ but $\sum_i \sum_j F_{ij} = 0$

$$\sum_i \sum_j F_{ij} + F^e = \sum_i m_i \ddot{r}_i = \dot{P} \tag{1}$$

2. Mach's Principle (PM)

Dynamically Complete Theory

- It is the one whose physical entities postulated in the theory satisfy the AR among themselves. (i.g. General Relativity)
- Bohmian Mechanics (MB) is *incomplete*.

Replica: Ψ is a physical law

Replica: Ψ is a physical *law*

Analogy between Ψ and $H_c\,(\mathbf{q}_1, \ldots, \mathbf{q}_N, \mathbf{p}_1, \ldots, \mathbf{p}_N) \equiv H_c(\xi)$:

1. Define *paths* (phase space and configuration);

$$\frac{d\xi}{dt} = \mathrm{Der}\, H_c \longleftrightarrow \frac{dQ}{dt} = \mathrm{Der}(\log \Psi) \tag{2}$$

2. Define distributions

$$\rho_{\text{class}}\; e^{\text{const. } H_c} \longleftrightarrow \rho_{\text{quant}} \;\sim \left| e^{\text{const. log } \Psi} \right| \tag{3}$$

item Stationary

But what about the Schrondinger Equation?

- Explicitly: Universe with particles that do not interact with each other.

Quantum potential and the PM

Mach:

- *The inertial mass of an object is related to the existence of mass elsewhere;*
- Force in configuration space;
- $v\rho = m = cte.$

Newton:

- Mass is an intrinsic property of matter;
- Force in Real Space

in a universe with a single (free) particle?

- $m\mathbf{a} = 0$

Rest mass of a spinless particle, according to MB:

$$\mathcal{M} = m_0 \sqrt{1 + \frac{\hbar^2}{m_0^2} \frac{\partial_\mu \partial^\mu \sqrt{\rho}}{\sqrt{\rho}}}, \tag{4}$$

where $\psi = \sqrt{\rho} e^{\frac{iS}{\hbar}}$.

- In general it is not cancelled. (not MACH);
- What is the meaning of the second term?
- Objective: Is it possible to obtain such a relationship only from Machian considerations?

1. Set ρ in absolute space;
2. Machian effects are small;
3. $\mathcal{Q}(\rho) = \mathcal{Q}(\gamma\rho)$.

- In Relative space

$$\mathcal{M}_i = \mathcal{M}_i \left[\rho_{\mathrm{rel}} \left(x_1^\mu(\lambda), x_2^\mu(\lambda), \cdots, \hat{x}_i^\mu(\lambda), \cdots, x_n^\mu(\lambda)\right)\right] \tag{5}$$

Note: $\mathcal{M}_i \left[\rho_{\mathrm{rel}} = 0\right] = 0$.

- absolute space

$$\mathcal{M}_i = \mathcal{M}_i \left[\varrho \left(x_1^\mu(\lambda), x_2^\mu(\lambda), \cdots, x_i^\mu(\lambda), \cdots, x_n^\mu(\lambda)\right)\right] \tag{6}$$

For a single particle:

$$\mathcal{M} = \mathcal{M}(x(\lambda)) \tag{7}$$

Most general:

$$\mathcal{M} = \mathcal{M} \left[\varrho(x(\lambda)), \partial_\mu \varrho(x(\lambda)), \partial_\mu \partial_{\ nu} \varrho(x(\lambda)), \cdots\right] \tag{8}$$

What is the effect of this mass on the particle's geodesy?

Own time range:

$$\mathcal{A}_{\text{Machian}} = \int \frac{1}{2}\mathcal{M}(x(\lambda))\frac{dx^\mu}{d\lambda}\frac{dx_\mu}{d\lambda}d\lambda \tag{9}$$

whose end is

$$\frac{d^2x^\mu}{d\lambda^2} + \Gamma^\mu_{\nu\sigma}\frac{dx^\nu}{d\lambda}\frac{dx^\sigma}{d\lambda} = \frac{1}{2}\left(g^{\mu\nu} - \frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda}\right)\frac{\nabla_\nu\mathcal{M}}{\mathcal{M}} \tag{10}$$

- No Machian effects: r.h.s. =0

Classic Limit:

$$\frac{d^2\mathbf{x}}{dt^2} - \nabla\left(\frac{GM}{r}\right) = -\frac{1}{2}\frac{\nabla\mathcal{M}(\mathbf{x})}{\mathcal{M}(\mathbf{x})} \tag{11}$$

A single particle:

$$\mathbf{a} = -\frac{1}{2}\frac{\nabla\mathcal{M}(\mathbf{x})}{\mathcal{M}(\mathbf{x})} \tag{12}$$

Solution of the above equation ($\mathbf{a} = \mathbf{a}(\mathbf{x})$)

$$
\begin{aligned}
-\left(\frac{m_0}{2}\right) \int \frac{\nabla \mathcal{M}(\mathbf{r}) \cdot d\mathbf{r}}{\mathcal{M}(\mathbf{r})} &= m_0 \int \mathbf{a} \cdot d\mathbf{r} \\
-\left(\frac{m_0}{2}\right) \int \frac{d\mathcal{M}}{\mathcal{M}} &= W_{\mathrm{M}} \\
\mathcal{M}(\mathbf{r}) &= \alpha \exp\left(-\frac{2W_{\mathrm{M}}}{m_0}\right) \\
&= \alpha \exp\left(\frac{2Q_{nr}}{m_0}\right)
\end{aligned}
\tag{13}
$$

this is the non-relativistic result.

Conservation of the four-moment:

$$\mathcal{A} = \int \varrho \left(\partial_\mu S \partial^\mu S - m_0^2 \right) d^4x \tag{14}$$

$$\begin{aligned} \mathcal{A}_{\text{Machian}} &= \int \varrho \left(\partial_\mu S \partial^\mu S - \mathcal{M}^2 \right) d^4x \\ &= \int \varrho \left(\partial_\mu S \partial^\mu S - \mathcal{F}\left[\varrho(x), \partial_\mu \varrho(x), \partial_\mu \partial_\nu \varrho(x), \cdots \right] \right) d^4x \end{aligned} \tag{15}$$

Ansatz ($\mathcal{M}(\mathbf{r}) = \alpha \exp\left(\frac{2Q_{nr}}{m_0}\right)$):

$$\mathcal{M}^2 = \mathcal{F} = \beta \exp(\mathcal{Q}) \tag{16}$$

Second condition

$$\mathcal{M}^2 = \mathcal{F} \cong m_0^2 (1 + \mathcal{Q}) \quad , \quad \frac{Q}{m_0^2} \ll 1 \tag{17}$$

Action extremes:

$$\partial_\mu S \partial^\mu S - m_0^2 - \mathfrak{Q}\left[\varrho(x), \partial_\mu \varrho(x), \partial_\mu \partial_\nu \varrho(x), \cdots\right] = 0 \tag{18}$$

and

$$\varrho \frac{\partial \mathfrak{Q}}{\partial \varrho} - \partial_\mu \left(\varrho \frac{\partial \mathfrak{Q}}{\partial \left(\partial_\mu \varrho\right)} \right) + \partial_\mu \partial^\mu \left(\varrho \frac{\partial \mathfrak{Q}}{\partial \left(\, partial_\mu \partial^\mu \varrho\right)} \right) = 0 \tag{19}$$

Solution:

$$\mathfrak{Q} = C \left((\varrho)^r\right)^m \left(\partial_\mu (\varrho)^r\right)^n \; left(\partial_\mu \partial^\mu (\varrho)^{rp} \tag{20}$$

of the third condition

$$\mathfrak{Q}\left[\varrho' = \gamma \varrho\right] = \mathfrak{Q}[\varrho] \Longrightarrow \gamma^{r(m+n+p)} = 1, \quad r \neq 0 \tag{21}$$

That is,

$$m + n + p = 0$$

. Simplest choice is:

$$m = -1, \quad n = 0 \quad \text{and} \quad p = 1$$

. And with $r = 1/2$ the eq.(20) becomes:

$$\mathcal{Q} = \frac{\mathfrak{Q}}{m_0^2} = \frac{C}{m_0^2} \frac{\partial^\mu \partial_\mu \sqrt{\varrho}}{\sqrt{\varrho}} \tag{22}$$

with

$$\mathcal{Q} = \frac{\mathfrak{Q}}{m_0^2} = \frac{C}{m_0^2}\frac{\partial^\mu \partial_\mu \sqrt{\varrho}}{\sqrt{\varrho}} \tag{23}$$

in $\mathcal{M}^2 = \mathcal{F} = \beta \exp(\mathcal{Q})$, we find:

$$\mathcal{M} = m_0 \sqrt{1 + \frac{\hbar^2}{m_0^2}\frac{\partial_\mu \partial^\mu \sqrt{\rho}}{\sqrt{\rho}}}, \tag{24}$$

where $\psi = \sqrt{\rho} e^{\frac{iS}{\hbar}}$.