Studies of covariance and nonlocality in Bohmian mechanics

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Motivation

- The principle of general covariance: *coordinates are mere event markers upon which the ultimate laws of physics should not rely.*
- *Entanglement is not invariant upon changes of quantum reference frames*[1].
  - For pure states, nonlocality (NL) is a frame-dependent quantity.
- Our proposal:
  1. Explore a broader notion of NL within Bohmian mechanics
  2. Quantify NL by analysing the particle's motion.

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1. R. M. Angelo, N. Brunner, S. Popescu, A. Short, and P. Skrzypczyk, J. Phys. A: Math. Theor. 44, 145304 (2011),
2. R. M. Angelo and A. D. Ribeiro, J. Phys. A: Math. Theor. 45, 465306 (2012),
3. F. Giacomini, E. Castro-Ruiz, and C . Brukner, Nat. Commun.10, 494 (2019).

Framework

Bohmian mechanics (BM):

- The universe consists of particles with well-defined trajectories; Generalized state $(\mathbf{x}, \psi)$.
- Two-particle system (a and b): The motion of b is given by

$$m_b \ddot{x}_b = F_b^{[\psi]}(x_a, x_b, t) = -\partial_{x_b}\left(\mathcal{V} + Q^{[\psi]}\right),$$

$$\underbrace{Q^{[\psi]} = -\frac{\hbar^2}{2|\psi|}\left(\frac{\partial^2_{x_a}}{m_a} + \frac{\partial^2_{x_b}}{m_b}\right)|\psi|}_{\text{Quantum potential}}$$

- Physical state at t: $(\underbrace{x_a, x_b}_{\text{System configuration}}, \underbrace{\psi}_{\text{Wave function}})$
- $Q^{[\psi]}$ is one of the ingredients needed to quantify NL.

In BM one needs to consider two types of NL.

- Einstein's principle of local action (LA): *externally influencing A has no immediate influence on B*
- In Newton's mechanics $\mathbf{x}$: *disturbances in A's position cannot instantaneously alter B's acceleration*
- In quantum mechanics ψ: *a measurement result on B is unaffected by operations on A*
- The LA is formulated according to a given definition of state
- In BM $(\mathbf{x}, \psi)$: the system's motion can violate LA for reasons concerning its particle and wave aspects.

Next: How to assess this violation?

- The conditional average of $F_b^{[\psi]}$ includes the systems preparation.

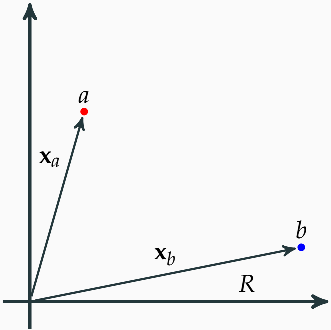
- For a system with two particles, we proposed

$$\left\langle F_b^{[\psi]} \right\rangle_{x_b} := \int F_b^{[\psi]} \rho_{x_a|x_b} \, \mathrm{d}x_a$$
$$= \int F_b^{[\psi]}(x_a, x_b) \, \rho_{x_a} \mathrm{d}x_a + \underbrace{\int F_b^{[\psi]}(x_a, x_b) \left(\rho_{x_a|x_b} - \rho_{x_a} \right) \mathrm{d}x_a}_{\eta},$$

  with

  - $\rho_{x_a, x_b} = |\psi|$ is the joint probability;
  - $\rho_{x_{a(b)}} = \int \mathrm{d}x_{b(a)} \rho_{x_a, x_b}$ are the marginals probabilities;
  - $\rho_{x_a|x_b} := \rho_{x_a, x_b} / \rho_{x_b}$ the conditional probabilities.
- η truly includes the system's preparation into the dynamics.
- it explicitly depends on the correlation between the variables: if x_a is independent, then $\rho_{x_a|x_b} = \rho_{x_a}$ and $\eta = 0$.

Two free particles: analysis from a classical inertial reference frame R



Assume:

- a separable gaussian wave function $\psi = \psi_a \psi_b$;
  $\psi_k = \psi_k(x_k, t) = \sqrt{G_{\delta_k}(x_k - \bar{x}_k)}$,
  with mean value $\bar{x}_k$ and deviation δ_k and $k = a, b$.

Then:

- The accelaretion of b is

$$\ddot{x}_b = \frac{x_b - \bar{x}_b}{t_b^2},$$

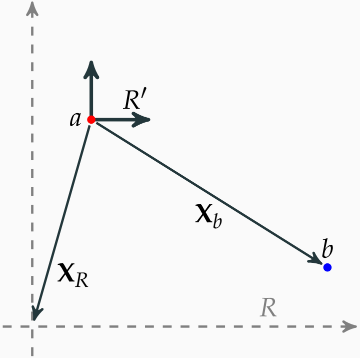
  with $t_b := t_{b,0}\sqrt{1 + (t/t_{b,0})^2}$ and $t_{b,0} = 2m_b\delta_{b,0}^2/\hbar$ the Ehrenfest's scale of time for b.

- The motion is entirely local: no entanglement + inertial R. Thus, our formalism is innocuous,

$$\langle \ddot{x}_b \rangle_{x_b} = \ddot{x}_b;$$

  there is no NL to be detected.

Two free particles: analysis from a quantum reference frame R'



Consider:

- Particle b is entangled with R:

$$\Psi(X_R, X_b) = \psi_a(-X_R)\psi_b(X_b - X_R) \neq \Psi_R(X_R)\Psi_b(X_b)$$

- Acceleration of b

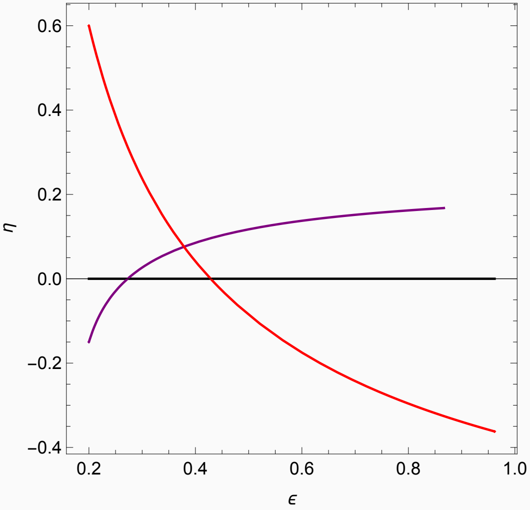
$$\ddot{X}_b = \frac{X_b - \bar{X}_b}{t_b^2} - \frac{X_R - \bar{X}_R}{t_b^2} + \frac{X_R - \bar{X}_R}{t_a^2},$$

is in general non-local (except for the special case $t_a = t_b$).

- Therefore, we must identify a non-local influence caused by entanglement in the averaged motion.
- Local average of b

$$\langle \ddot{X}_b \rangle_{X_b} = \underbrace{\frac{X_b - \bar{X}_b}{t_b^2}}_{\text{Local acceleration}} \left\{ 1 + \underbrace{\left[\left(\frac{t_b}{t_a} \right)^2 - 1 \right] \lambda^2}_{\eta} \right\},$$

which λ is the statistical correlation between X_b and X_R.



- It indicates a monotonic and a counter monotonic relation between η and entanglement (ϵ).
- It is monotonic whenever Ehrenfest's scale associated with the quantum reference frame is bigger than the scale associated with the system: $t_{a,0} > t_{b,0}$. And counter monotonic, otherwise.
- Conclusion
  1. Broader notion of nonlocality (NL) within Bohmian mechanics;
  2. NL is intimately related to the physical state definition;
  3. Our proposal: NL quantifier based on the averaged motion;
  4. Case study: 2 free particles. η is a promising NL quantifier.