Studies of covariance and nonlocality in Bohmian mechanics

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Abstract

The principle of general covariance motivates a relational prescription of the world, implying that reference frames must be conceived as physical systems rather than coordinate systems. With this prescription, it has been shown that entanglement is not invariant upon changes of quantum reference frames, meaning that, at least for pure states, nonlocality is a frame-dependent quantity. Something similar happens in classical mechanics. When one applies it to non-inertial frames, fictitious forces induce non-local effects, identified by Newton's second law. Here, we explore a broader notion of this phenomenon within Bohmian mechanics, a causal interpretation of quantum mechanics given in terms of particles and their trajectories. Our analysis shows that the nonlocality prescribed by classical and quantum mechanics is intimately associated with each theory's definition of physical state. With that in mind, we propose a way to quantify entanglement by analysing a distant particle's equation of motion.

Motivation

- The principle of general covariance: *coordinates are mere event markers upon which the ultimate laws of physics should not rely.*
- *Entanglement is not invariant upon changes of quantum reference frames*[1].
    - For pure states, nonlocality (NL) is a frame-dependent quantity.
- Our proposal:
    1. Explore a broader notion of NL within Bohmian mechanics
    2. Quantify NL by analysing the particle's motion.

[1]

1. R. M. Angelo, N. Brunner, S. Popescu, A. Short, and P. Skrzypczyk, J. Phys. A: Math. Theor. 44, 145304 (2011),
2. R. M. Angelo and A. D. Ribeiro, J. Phys. A: Math. Theor. 45, 465306 (2012),
3. F. Giacomini, E. Castro-Ruiz, and C . Brukner, Nat. Commun.10, 494 (2019).

Framework

Bohmian mechanics (BM):

- The universe consists of particles with well-defined trajectories; Generalized state $(\mathbf{x}, \psi)$.
- Two-particle system (a and b): The motion of b is given by

$$m_b \ddot{x}_b = F_b^{[\psi]}(x_a, x_b, t) = -\partial_{x_b}\left(\mathcal{V} + Q^{[\psi]}\right),$$

$$\underbrace{Q^{[\psi]} = -\frac{\hbar^2}{2|\psi|}\left(\frac{\partial^2_{x_a}}{m_a} + \frac{\partial^2_{x_b}}{m_b}\right)|\psi|}_{\text{Quantum potential}}$$

- Physical state at t: $\Big(\underbrace{x_a, x_b}_{\text{System configuration}}, \underbrace{\psi}_{\text{Wave function}}\Big)$
- $Q^{[\psi]}$ is one of the ingredients needed to quantify NL.

In BM one needs to consider two types of NL.

- Einstein's principle of local action (LA): *externally influencing A has no immediate influence on B*
- In Newton's mechanics **x**: *disturbances in A's position cannot instantaneously alter B's acceleration*
- In quantum mechanics ψ: *a measurement result on B is unaffected by operations on A*
- The LA is formulated according to a given definition of state
- In BM $(\mathbf{x}, \psi)$: the system's motion can violate LA for reasons concerning its particle and wave aspects.

Next: How to assess this violation?

- The conditional average of $F_b^{[\psi]}$ includes the system's preparation.

The local average

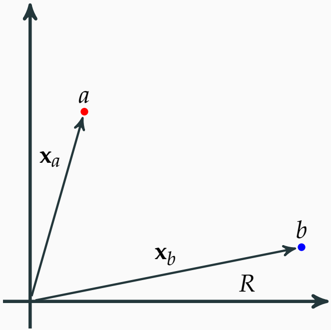
- For a system with two particles, we proposed

$$\left\langle F_b^{[\psi]} \right\rangle_{x_b} := \int F_b^{[\psi]} \rho_{x_a|x_b} \, \mathrm{d}x_a$$
$$= \int F_b^{[\psi]}(x_a, x_b) \, \rho_{x_a} \mathrm{d}x_a + \underbrace{\int F_b^{[\psi]}(x_a, x_b) \left(\rho_{x_a|x_b} - \rho_{x_a} \right) \mathrm{d}x_a,}_{\eta}$$

  with

  - $\rho_{x_a, x_b} = |\psi|$ is the joint probability;
  - $\rho_{x_{a(b)}} = \int \mathrm{d}x_{b(a)} \rho_{x_a, x_b}$ are the marginals probabilities;
  - $\rho_{x_a|x_b} := \rho_{x_a, x_b} / \rho_{x_b}$ the conditional probabilities.
- η truly includes the system's preparation into the dynamics.
- it explicitly depends on the correlation between the variables: if x_a is independent, then $\rho_{x_a|x_b} = \rho_{x_a}$ and $\eta = 0$.

Two free particles: analysis from a classical inertial reference frame R



Assume:

- a separable gaussian wave function $\psi = \psi_a \psi_b$;
  $\psi_k = \psi_k(x_k, t) = \sqrt{G_{\delta_k}(x_k - \bar{x}_k)}$,
  with mean value $\bar{x}_k$ and deviation δ_k and $k = a, b$.

Then:

- The accelaretion of b is

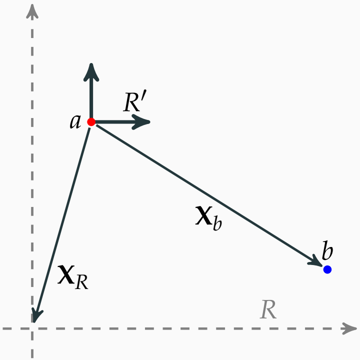
$$\ddot{x}_b = \frac{x_b - \bar{x}_b}{t_b^2},$$

  with $t_b := t_{b,0}\sqrt{1 + (t/t_{b,0})^2}$ and $t_{b,0} = 2m_b\delta_{b,0}^2/\hbar$ the Ehrenfest's scale of time for b.
- The motion is entirely local: no entanglement + inertial R. Thus, our formalism is innocuous,

$$\langle \ddot{x}_b \rangle_{x_b} = \ddot{x}_b;$$

there is no NL to be detected.

Two free particles: analysis from a quantum reference frame R'



Consider:

- Particle b is entangled with R:

$$\Psi(X_R, X_b) = \psi_a(-X_R)\psi_b(X_b - X_R) \neq \Psi_R(X_R)\Psi_b(X_b)$$

- Acceleration of b

$$\ddot{X}_b = \frac{X_b - \bar{X}_b}{t_b^2} - \frac{X_R - \bar{X}_R}{t_b^2} + \frac{X_R - \bar{X}_R}{t_a^2},$$

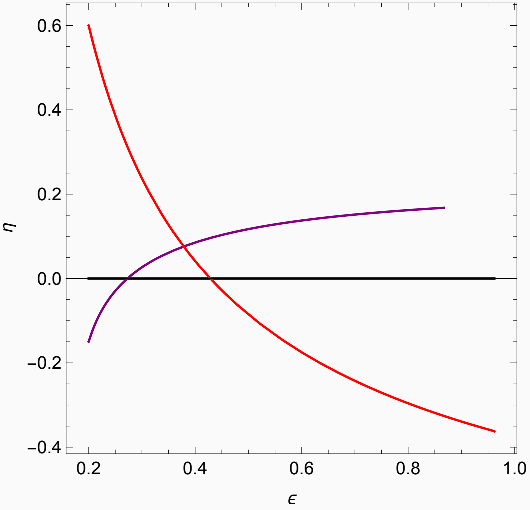
  is in general non-local (except for the special case $t_a = t_b$).

- Therefore, we must identify a non-local influence caused by entanglement in the averaged motion.
- Local average of b

$$\langle \ddot{X}_b \rangle_{X_b} = \underbrace{\frac{X_b - \bar{X}_b}{t_b^2}}_{\text{Local acceleration}} \left\{ 1 + \underbrace{\left[\left(\frac{t_b}{t_a} \right)^2 - 1 \right] \lambda^2}_{\eta} \right\},$$

  which λ is the statistical correlation between X_b and X_R.

Results and conclusion



- It indicates a monotonic and a counter monotonic relation between η and entanglement (ϵ).
- It is monotonic whenever Ehrenfest's scale associated with the quantum reference frame is bigger than the scale associated with the system: $t_{a,0} > t_{b,0}$. And counter monotonic, otherwise.
- Conclusion
  1. Broader notion of nonlocality (NL) within Bohmian mechanics;
  2. NL is intimately related to the physical state definition;
  3. Our proposal: NL quantifier based on the averaged motion;
  4. Case study: 2 free particles. η is a promising NL quantifier.