Quantum Algorithms for the Leader Election Problem in Anonymous Networks

Part 3: Physical implementation

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Abstract

This project is composed of two objectives (1) to implement AELQ and (2) to develop a quantum circuit simulator to test it and two complementary solutions: the first, more abstract, lies in understanding, conceiving and appropriating the logic and language behind TANI algorithm, using MQ, to do so; and the second, more pragmatic, consists of building the simulator, in which we use Python and Quiskit as language and library, respectively. Regarding the validation and testing, we will compare the unanimity of the election and the average execution time of our protocol with the theoretical results.

Tests and Results

- Functionality: *impartial* election.
- Performance: average number of *phases* required.

Performance: No noise

Number of parts	Status before measurement	Performance
2	$\|R_0\rangle = \frac{\|01\rangle+\|10\rangle}{\sqrt{2}}$	1,0
4	$\|R_0\rangle = \frac{\|0001\rangle+\|0010\rangle+\|0011\rangle+}{\sqrt{14}}$	1.84

Table 1: Performance for 2 and 4 parts. Noiseless simulation.

Performance: noisy

Number of parts	Status before measurement	Performance
2	$\|R'_0\rangle = \alpha(\|00\rangle + \|11\rangle) + \beta(\|01\rangle + \|10\rangle)$	> 1.0
4	$\|R_0\rangle = \alpha\,\|Error\rangle + \beta\,\|Correct\rangle$	> 1.84

Table 2: Performance for 2 and 4 parts. Simulation with noise.

- There is now a non-zero probability of error $|\alpha|^2$;
- However, $|\beta|^2 >> |\alpha|^2 > 0$.

Results: Functionality

Part	Theoretical	Simulation	Simulation with noise
A	1/4	0.2474	0.2712
B	1/4	0.2534	0.2467
C	1/4	0.2439	0.2417
D	1/4	0.2553	0.2404

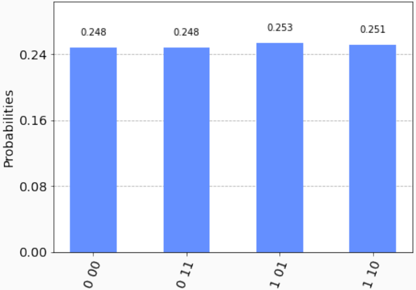
Table 3: Probability of each party winning the election according to each method.

Results: Performance

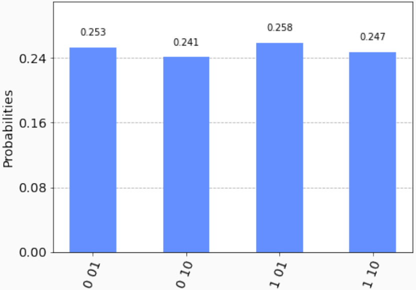
n	Theoretical	Simulation	Noise simulation	Classic
2	1	1.0000	1.1668	1.5
3	$3/2$	1.5018	1.5644	2,331
4	$46/25 \approx 1.840$	1.8403	2.0097	2.669

Table 4: Average number of phases needed to complete the election as a function of n according to each method.

Results: Algorithm for $n = 2$

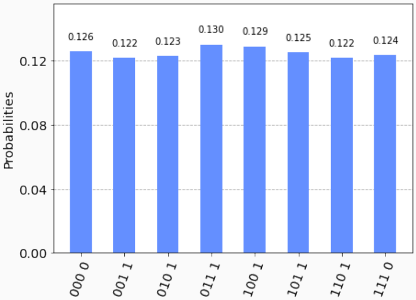


Histogram of Subroutine A for 2 parts. Registers (from bottom to top): S and R_0

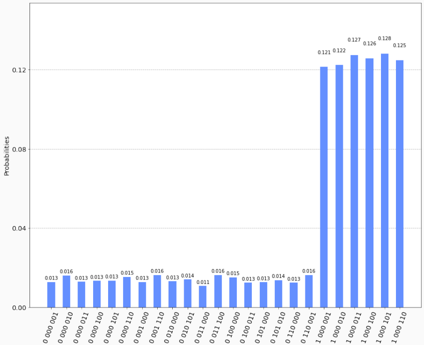


Histogram of Subroutine B for 2 shares. Registers (from bottom to top): S and R_0

Results: Algorithm for $n = 3$



Histogram of Subroutine A for 3 shares.Registers (from bottom to top): R_0 and S



Histogram of Subroutine B for 3 shares.Registers (from bottom to top): S,R_1 and R_0