Calculus I HW12

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Since $\frac{sin(t)}{t} < 1 \ \ \forall t > 0$, subsitute t for $\frac{1}{n}$ yields

$$nsin(\frac{1}{n}) < 1 \Rightarrow 1 - nsin(\frac{1}{n}) > 0$$

Since $n > 0$ we also have

$$\frac{1 - nsin(\frac{1}{n})}{n} > 0 \ \ \forall n \in \mathbb{N}$$

Let $m = \frac{1}{n}$ Now note

$$\lim_{n\to\infty} \frac{\left|1 - nsin\left(\frac{1}{n}\right)\right|}{\left|\frac{1}{n^2}\right|} = \lim_{m\to 0} \frac{m - sin\,(m)}{m^2} = \lim_{m\to 0} \frac{1 - cos\,(m)}{2m} = \lim_{m\to 0} \frac{sin\,(m)}{2} = 0$$

hence $\exists N$ s.t. $\left|\frac{a_n}{b_n} - 0\right| < 1 \ \ \forall n \geq N \Rightarrow \exists N \, s.t. \, |a_n| < |b_n| \ \ \forall n \geq N$ Also since