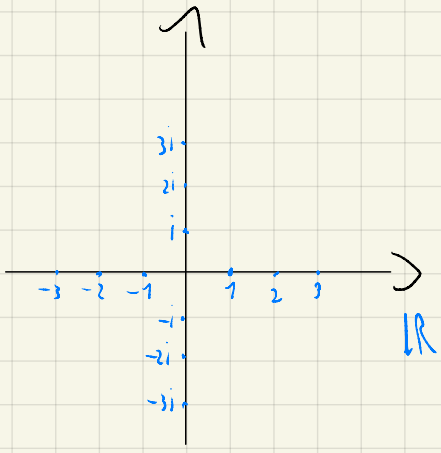
Numeri complessi $z \in \mathbb{C}$

Forma algebrica:

$z = a + i \cdot b$ $\quad i^2 = -1$ $\quad \sqrt{-1} = i$

$\bar{z} = a - i \cdot b$

$|z| = \sqrt{a^2 + b^2}$ $\quad \Big\} \; |z|^2 = z \cdot \bar{z}$



$Re(z) = a$ $\quad Im(z) = b$ $\quad$ Piano di Gauss

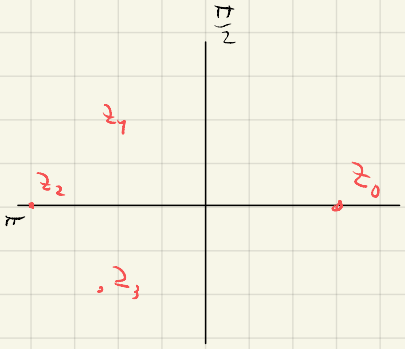
Forma goniometrica (1ª Formula di De Moivre)

$z = \rho \cdot (\cos(\theta) + i \cdot \sin(\theta))$

$\rho = |z|$

θ = angolo al quale si trova il numero sul piano di Gauss

$Arg(z) = \theta$



$Arg(z_0) = 0$

$Arg(z_1) = \tan^{-1}\left(\frac{|b|}{|a|}\right) + \frac{\pi}{2}$

$Arg(z_2) = \pi$

$Arg(z_3) = \tan^{-1}\left(\frac{|b|}{|a|}\right) + \pi$

Estrazione radici n-esime (2ª formula di De Moivre)

$$z = \sqrt[n]{w} \qquad z, w \in \mathbb{C}$$

1) Scrivo w in forma goniometrica

$$w = \rho \cdot (\cos(\theta) + i \cdot \sin(\theta)) \qquad \theta = \text{Arg}(w)$$

2) Calcolo θ_0, θ_k

$$\theta_0 = \frac{\theta}{n} \qquad \theta_k = \frac{\theta}{n} + \frac{2k\pi}{n}$$

3) Riscrivo tutto per z_k

$$z_k = \sqrt[n]{\rho} \cdot (\cos(\theta_k), + i \cdot \sin(\theta_k)) \qquad k := [0, n-1]$$

Formula risolutiva per eq. di 2° grado in $\mathbb{C}$

$$a \cdot z^2 + b \cdot z + c = 0$$

$$z = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \qquad w_{1,2} = \sqrt{b^2 - 4ac}$$

$$z_1 = \frac{-b + w_1}{2a} \qquad z_2 = \frac{-b + w_2}{2a}$$

Rappresentazione esponenziale

$$z = \rho \cdot (\cos(\theta) + i \cdot \sin(\theta)) = \rho \cdot e^{i \cdot \theta}$$