$$f(x) = \log(x^2+5x-6)$$

1) DOMINIO:

$x^2+5x-6>0 \rightarrow (x+6)(x-1)>0 \rightarrow$

$\rightarrow x<-6 \vee x>1 \rightarrow \mathbb{D} =]-\infty;-6[\cup]1;+\infty[$

2) INTERSEZIONI CON GLI ASSI:

$x=0 \notin \mathbb{D}$

$y=0 \rightarrow \log(x^2+5x-6)=0 \rightarrow \log(x^2+5x-6)=\log 1$

$x^2+5x-6=1 \rightarrow x^2+5x-7=0$

$x_{1,2} = \dfrac{-5\pm\sqrt{53}}{2} \rightarrow \left(\dfrac{-5\pm\sqrt{53}}{2}, 0\right) \in f(x)$

3) SIMMETRIE:

$f(-x) = \log(x^2-5x-6)$ $\neq f(x)$ NO PARI

$\neq -f(x)$ NO DISPARI

4) SEGNO:

$f(x)>0 \rightarrow \log(x^2+5x-6)>0 \rightarrow x^2+5x-6>1$

$x^2+5x-7>0 \rightarrow x<\dfrac{-5-\sqrt{53}}{2} \vee x>\dfrac{-5+\sqrt{53}}{2}$

5) LIMITI:

$$\lim_{x \to \pm\infty} f(x) = +\infty$$

DelHopital

$$\lim_{x \to \pm\infty} \frac{f(x)}{x} = \left[\frac{\infty}{\infty}\right] \stackrel{H}{=} \lim_{x \to \pm\infty} \frac{2x+5}{x^2+5x-6} = 0 \to$$ NO ASINTOTI OBLIQUI

$$\lim_{x \to -6^-} f(x) = \log 0^+ = -\infty \to x = -6$$ ASINTOTO VERTICALE

$$\lim_{x \to 1^+} f(x) = \log 0^+ = -\infty \to x = 1$$ ASINTOTO VERTICALE

6) DERIVATE:

$$f'(x) = \frac{2x+5}{x^2+5x-6}$$

$N \geq 0 \to x \geq -\frac{5}{2}$

$D > 0 \quad x < -6 \vee x > 1$

$f'(x) \geq 0$

		-6	$-\frac{5}{2}$	1	
N	$-$				$+$
D	$+$	$\circ$		$\circ$	$+$
$f'(x)$	$-$	$\circ$		$\circ$	$+$
$f(x)$	$\searrow$				$\nearrow$

$$f''(x) = \frac{2x^2+10x-12-4x^2-25-20x}{(x^2+5x-6)^2}$$

$f''(x) \geq 0 \to +2x^2+10x+37 \leq 0$

$\Delta < 0 \to \cup \to \emptyset \to$ No FLESSI

$f''(x) < 0 \quad \forall x \in D$

$$f(x) = \frac{1 - 2\log x}{x^2}$$

1) DOMINIO: $x > 0 \rightarrow \boxed{\mathbb{D} = \mathbb{R}^+}$

2) SIMMETRIE: $f(-x) \begin{cases} \neq f(x) & \text{NO PARI} \\ \neq -f(x) & \text{NO DISPARI} \end{cases}$

3) INTERSEZIONI CON GLI ASSI:

$x = 0 \notin \mathbb{D}$

$y = 0 \rightarrow 2\log x = 1 \rightarrow \log x = \frac{1}{2} \rightarrow x = \sqrt{e}$ $\quad (\sqrt{e}, 0) \in f(x)$

4) SEGNO:

$f(x) > 0 \rightarrow 1 - 2\log x > 0 \rightarrow \log x < \frac{1}{2} \rightarrow x < \sqrt{e}$

5) LIMITI:

$\lim_{x \to 0^+} f(x) = \frac{+\infty}{0^+} = +\infty \rightarrow x = 0$ ASINTOTO VERTICALE

$\lim_{x \to +\infty} f(x) = \left[\frac{\infty}{\infty}\right] \overset{H}{=} \lim_{x \to +\infty} \frac{-2}{2x^2} = 0^- \rightarrow y = 0$ ASINTOTO ORIZZONTALE

6) DERIVATE:

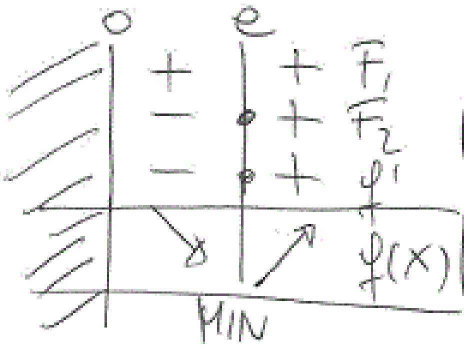
$$f'(x) = \frac{-2x - 2x(1 - 2\log x)}{x^4} = \frac{-4x + 4x\log x}{x^4}$$

$f'(x) \geq 0 \rightarrow x(-2 - 2 + 4\log x) \geq 0$

$x(4\log x - 4) \geq 0 \rightarrow x(\log x - 1) \geq 0$

$F_1 \geq 0 \rightarrow x \geq 0$

$F_2 \geq 0 \rightarrow x \geq e$



$\left(e, -\frac{1}{e^2}\right)$ MIN

$$f''(x) = \frac{(\cancel{-4} + 4\log x \cancel{+4})x^4 - 4x^3(-4x + 4x\log x)}{x^8}$$

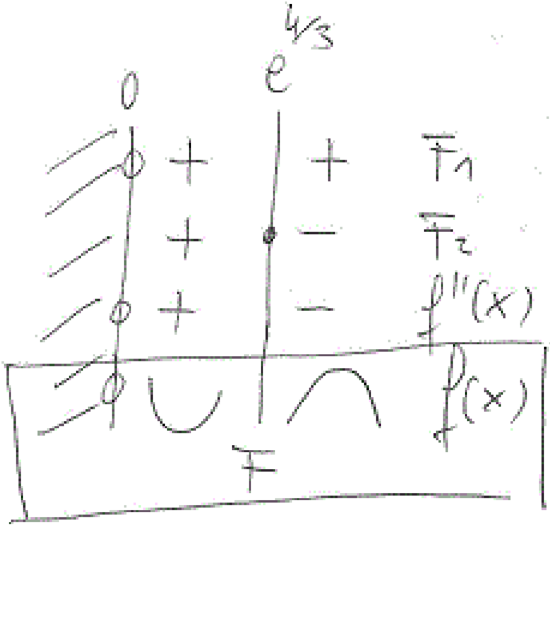
$f''(x) \geq 0 \rightarrow x^3(4x\log x + 16x - 16x\log x) \geq 0$

$\cancel{4}x^4(\log x + 4 - 4\log x) \geq 0$

$x^4(4 - 3\log x) \geq 0$

$F_1 \geq 0 \quad x^4 \geq 0 \quad \forall x$

$F_2 \geq 0 \quad \log x \leq \frac{4}{3} \rightarrow x \leq e^{4/3}$



$x = e^{4/3}$ FLESSO

$$f(x)=\frac{2x}{x^2-1}$$

1) DOMINIO: $x^2-1\neq 0 \rightarrow x\neq \pm 1$

$$\underline{\mathbb{D}=\mathbb{R}-\{\pm 1\}}$$

2) SIMMETRIE:

$$f(-x)=\frac{-2x}{x^2-1}=-f(x)\rightarrow f(x)\ \underline{\text{DISPARI}}$$

3) INTERSEZIONI CON GLI ASSI:

$$\begin{cases} x=0 \\ y=0 \end{cases} \rightarrow \underline{(0,0)\in f(x)}$$

$$\begin{cases} y=0 \\ f(x)=0 \end{cases} \quad \begin{cases} y=0 \\ 2x=0 \end{cases} \quad \begin{cases} y=0 \\ x=0 \end{cases}$$

4) SEGNO:

$$f(x)\geq 0 \rightarrow \begin{matrix} N\geq 0 \rightarrow x\geq 0 \\ D>0 \rightarrow x<-1 \vee x>1 \end{matrix}$$

	-1		0		1		
$-$	|	$-$	$\bullet$	$+$	|	$+$	N
$+$	$\circ$	$-$	|	$-$	$\circ$	$+$	D
$-$	$\circ$	$+$	$\bullet$	$-$	$\circ$	$+$	$f(x)$

$$\begin{cases} f(x)\geq 0 \rightarrow -1<x\leq 0 \vee x>1 \\ f(x)<0 \rightarrow x<-1 \vee 0\leq x<1 \end{cases}$$

5) LIMITI:

$$\lim_{x \to \pm\infty} f(x) = 0 \to \underline{y=0} \text{ ASINTOTO ORIZZONTALE}$$

$$\lim_{x \to -1^{\pm}} f(x) = \pm\infty \to \underline{x=-1} \text{ ASINTOTO VERTICALE}$$

$$\lim_{x \to 1^{\pm}} f(x) = \pm\infty \to \underline{x=1} \text{ ASINTOTO VERTICALE}$$

6) DERIVATE :

$$f'(x) = \frac{2(x^2-1) - 2x \cdot 2x}{(x^2-1)^2} = \frac{-2x^2-2}{(x^2-1)^2}$$

$$f'(x) \geq 0 \to -2(x^2+1) \geq 0 \to \nexists x \in D$$

$$\underline{f'(x) < 0 \quad \forall x \in D} \to f(x) \searrow \to \text{NO MAX} \ \text{NO MIN}$$

$$f''(x) = \frac{-4x(x^4+1-2x^2)+(2x^2+2)(2x^2-2)\cdot 2x}{(x^2-1)^4}$$

$$f''(x) \geq 0 \to -4x^5 - 4x + 8x^3 + 8x^5 - 8x \geq 0$$

$$4x^5 + 8x^3 - 12x \geq 0 \to x(x^4+2x^2-3) \geq 0 \to$$

$$\to x(x^2+3)(x^2-1) \geq 0$$

$x \geq 0$

$x^2+3 \geq 0 \quad \forall x \in D$

$x^2-1 \geq 0 \to x \leq -1 \vee x \geq 1$

		-1		0		1	
	$-$		$-$		$+$		$+$
	$+$		$+$		$+$		$+$
	$+$		$-$		$-$		$+$
$f''(x)$	$-$		$+$		$-$		$+$
$f(x)$	$\cap$		$\cup$	F	$\cap$		$\cup$

$\underline{F(0,0) \text{ FLESSO}}$

$$f(x)=\frac{1}{\sqrt{4-x^2}}$$

1) DOMINIO: $\begin{cases} 4-x^2 \geq 0 \\ \sqrt{4-x^2} \neq 0 \end{cases} \rightarrow 4-x^2>0$

$-2<x<2 \rightarrow D=(-2;2)$

2) SIMMETRIE: $f(-x)=f(x) \rightarrow$ $f(x)$ PARI

3) INT. CON GLI ASSI:

$\begin{cases} x=0 \\ f(x)=\frac{1}{2} \end{cases} \rightarrow \left(0;\frac{1}{2}\right) \in f(x)$ $\quad \begin{cases} f(x)=0 \\ 1=0 \end{cases} \rightarrow \emptyset$

4) SEGNO:

$\left.\begin{array}{l} N>0 \rightarrow 1>0 \ \forall x \in D \\ D>0 \rightarrow \sqrt{4-x^2}>0 \ \forall x \in D \end{array}\right\} f(x)>0 \ \forall x \in D$

5) LIMITI:

$$\lim_{x\to -2^+} f(x) = \lim_{x \to 2^-} f(x) = \frac{1}{0^+} = +\infty$$

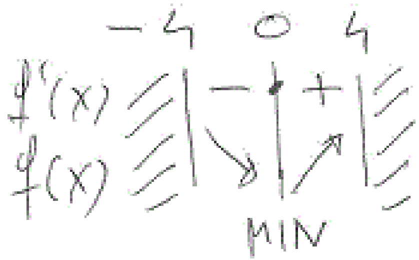
$x=\pm 2$ ASINTOTI VERTICALI

6) DERIVATE:

$$f'(x) = -\frac{1}{2\sqrt{4-x^2}} \cdot (-2x) \cdot \frac{1}{|4-x^2|}$$

$$= \frac{x}{|4-x^2|\sqrt{4-x^2}}$$

$$f'(x) \geq 0 \quad \begin{cases} N \geq 0 \rightarrow x \geq 0 \\ D > 0 \rightarrow -4 < x < 4 \end{cases}$$

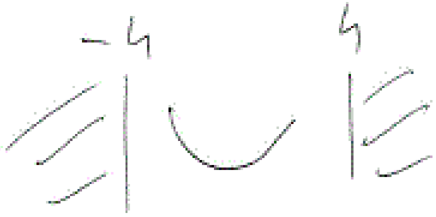


$f(0) = \frac{1}{2} \rightarrow \underline{\left(0; \frac{1}{2}\right) \text{ MIN}}$

$$f''(x) = \frac{1}{\sqrt{(4-x^2)^3}} + x\left(-\frac{3}{2}\right) \cdot \frac{-2x}{\sqrt{(4-x^2)^5}}$$

$$= \frac{1}{\sqrt{(4-x^2)^3}} + \frac{3x^2}{\sqrt{(4-x^2)^5}}$$

$\underline{f''(x) > 0 \quad \forall x \in D}$



<u>NO FLESSI</u>