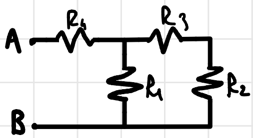
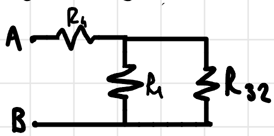
ES. 1 RESISTORI



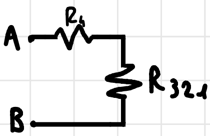
$R_1 = 1\,\Omega = R_2$
$R_3 = 3\,\Omega$
$R_4 = 2\,\Omega$

$R_{AB} = ?$

$R_{32} = R_3 + R_2 = 4\,\Omega$

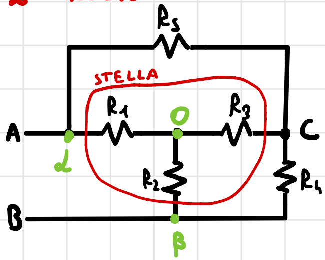


$$R_{321} = \left(\frac{1}{R_{32}} + \frac{1}{R_1}\right)^{-1} = \left(\frac{5}{4}\right)^{-1} = \frac{4}{5}\,\Omega$$



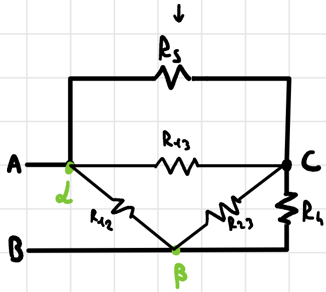
$$R_{AB} = R_4 + R_{321} = 2 + \frac{4}{5} = \frac{14}{5}\,\Omega$$

ES. 2 RESISTORI



$R_1 = 3\,\Omega = R_2 = R_3$

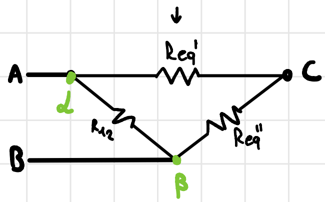
$R_4 = 2\,\Omega = R_5$



$R_{13} = R_{12} = R_{23} = 3R_1 = 9\,\Omega$ $(Y \rightarrow \Delta)$

$Req' = \left(\frac{1}{R_5} + \frac{1}{R_{13}}\right)^{-1} = \frac{R_{13}R_5}{R_{13}+R_5} = \frac{18}{11}$

$Req'' = \frac{R_{23}R_4}{R_{23}+R_4} = \frac{18}{11}$

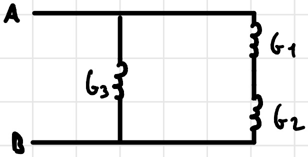


$R_{\alpha C\beta} = Req' + Req'' = \frac{36}{11}$

$R_{AB} = \left(\frac{1}{R_{12}} + \frac{1}{R_{\alpha C\beta}}\right)^{-1} = \frac{R_{\alpha C\beta}\cdot R_{12}}{R_{\alpha C\beta}+R_{12}} = \frac{\frac{36}{11}\cdot 9}{\frac{36+99}{11}} = \frac{36\cdot 9}{11}\cdot\frac{11}{135} = \frac{324}{135} =$

$= 2{,}4\ \Omega$

ES. 3 RESISTORI



$G_1 = 0,3\ S$

$G_2 = 0,4\ S$

$G_3 = 1,3\ S$

$G_{AB} = ?$

$R_1 = 1/G_1$, $R_2 = 1/G_2$, $R_3 = 1/G_3$

$$R_{12} = \frac{1}{G_1} + \frac{1}{G_2} = \frac{1}{0,3} + \frac{1}{0,4} = \frac{10}{3} + \frac{10}{4} = \frac{70}{12}$$

$$R_{123} = \left(G_3 + \frac{12}{70}\right)^{-1} = \left(\frac{13}{10} + \frac{12}{70}\right)^{-1} = \left(\frac{7 \cdot 13 + 12}{70}\right)^{-1} = \left(\frac{103}{70}\right)^{-1} = \frac{70}{103}\ \Omega$$

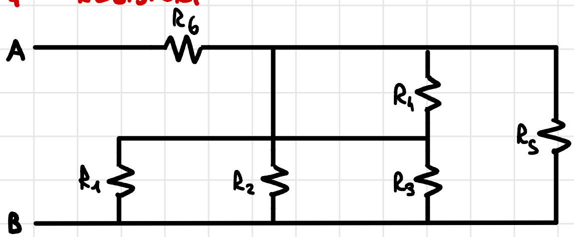
$$\parallel$$

$$R_{AB} = \frac{1}{G_{AB}} \Rightarrow G_{AB} = \frac{103}{70}$$

SI POTEVA ANCHE FARE DIRETTAMENTE CON CONDUTTANZE !!!

PS: SERIE E PARALLELO FUNZ. AL CONTR. RISPETTO A RESISTENZE

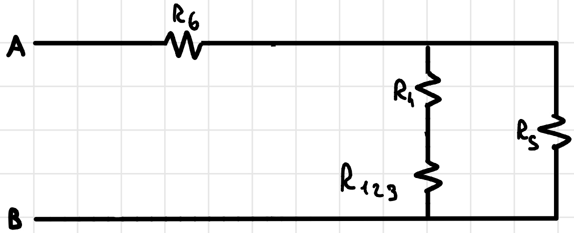
ES. 4 RESISTORI



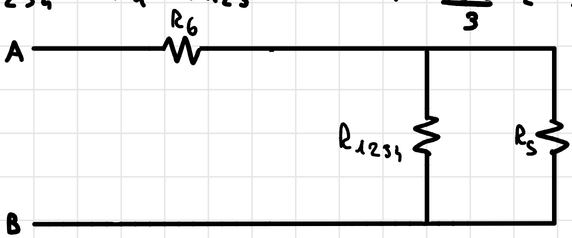
$R_1 = \dots = R_6 = 100\,\Omega$

$R_{AB} = ?$

$$R_{123} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1} = \frac{100}{3}$$



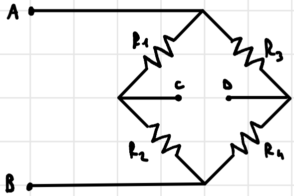
$$R_{1234} = R_4 + R_{123} = 100 + \frac{100}{3} = \frac{400}{3}$$



$$R_{12345} = \left(\frac{1}{R_{1234}} + \frac{1}{R_5}\right)^{-1} = \left(\frac{3}{400} + \frac{1}{100}\right)^{-1} = \frac{400}{7}$$

$$R_{AB} = R_{12345} + R_6 = \frac{400}{7} + 100 = \frac{1100}{7}$$

ES. 5 RESISTORI



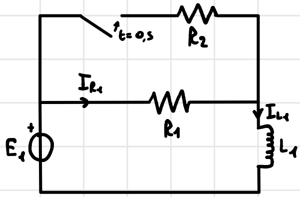
?

$R_{12} = R_1 + R_2 =$

$R_{34} = R_3 + R_4 =$

$$R_{AB} = \frac{R_{34} \cdot R_{12}}{R_{34} + R_{12}} =$$

ES. 1 TRANSISTORI

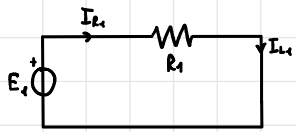


$R_1 = 1\,\Omega = R_2$ $\quad$ $t = 0^-$ $\quad$ $I_{L_1} = ?$

$L_1 = 1\,mH$ $\quad$ $t > 0$ $\quad$ $I_{L_1}(t) = ?$

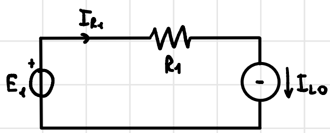
$E_1 = 10V$ $\quad$ $t > 0$ $\quad$ $I_{R_1}(t) = ?$

$t = 0^-$)



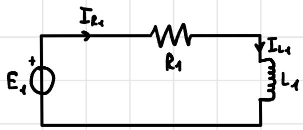
$$I_{L_1} = I_{R_1} = \frac{E_1}{R_1} = 10\,A$$

$t = 0^+$)



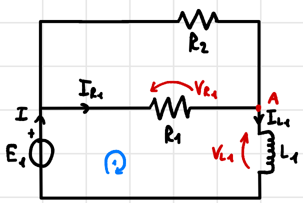
$$I_{L10} = I_{L_1} = 10\,A$$

$0^+ \leq t \leq 0{,}5$)



$$I_{L_1}(t) = (10 - 10)e^{-t/\tau} + 10 = 10\,A = I_{R_1}(t)$$

$t > 0{,}5$)

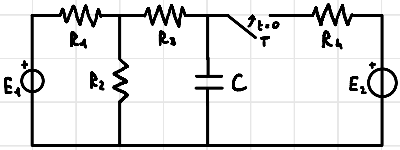


$$I_{L_1}(t) = \left(I_{L_1}(0) - I_{L_1}(+\infty)\right)e^{-t/\tau} + I_{L_1}(+\infty) =$$
$$= \left(10 - E_1 \cdot \frac{(R_1+R_2)}{R_1 R_2}\right) e^{-\frac{t R_p}{L}} + E_1 \cdot \frac{(R_1+R_2)}{R_1 R_2} =$$
$$= (10 - 20)\, e^{-t\frac{R_p}{L}} + 20 = 10\left(2 - e^{-\frac{t}{2}\cdot 10^3}\right)$$

× LKT 1 : $E_1 - V_{R_1} - V_{L_1} = 0 \longrightarrow V_{R_1} = E_1 - V_{L_1} = -L_1 \cdot \frac{d I_{L_1}(t)}{dt} + E_1 = -L_1 \cdot 10\left(-e^{-\frac{t}{2}\cdot 10^3}\right) \cdot -\frac{1}{2}10^3 + E_1 =$

$$= -\frac{L_1}{2} e^{-\frac{t}{2}10^3} \cdot 10^4 + 10 = 10\left(1 - 10^3 \cdot \frac{L_1}{2} e^{-\frac{t}{2}\cdot 10^3}\right) = 10\left(1 - \frac{1}{2} e^{-\frac{t}{2}\cdot 10^3}\right)$$

ES. 2 TRANSITORI



$R_1 = 60\ k\Omega$ $\quad C = 2\ \mu F$

$R_2 = 60\ k\Omega$ $\quad E_1 = 8V$

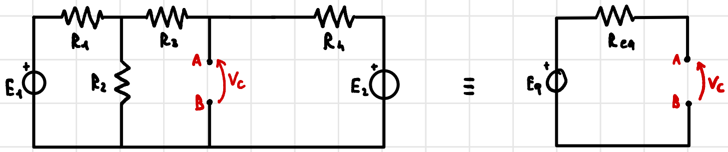
$R_3 = 30\ k\Omega$ $\quad E_2 = 2V$

$R_4 = 20\ k\Omega$

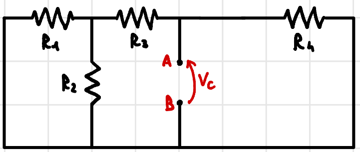
1) $V_C(t) = ?$

2) $t^* = ?$ $\quad : \quad W_C = \frac{1}{2} W_{C0}$

× NORTON



Passivo generatori → R_{eq}?

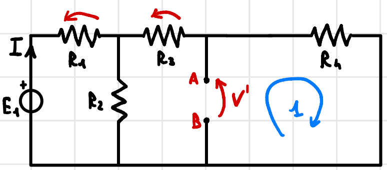


$$R_{eq} = \left((R_1 // R_2) -- R_3\right) // R_4 \;(\;$$

$$= \left(\frac{R_1 R_2}{R_1+R_2} + R_3\right) // R_4 = \frac{60 \cdot 2\cancel{0}}{6\cancel{0}+2\cancel{0}} \cdot \frac{10^{\cancel{6}}}{\cancel{10^3}} = \frac{120}{8} \cdot 10^3 =$$

$$= 15\ k\Omega$$

Passivo E_2 → $V' = ?$



× LKT1 : $V' = V_{R_4} = I_4 \cdot R_4 = I \cdot \frac{R_2}{R_2 + R_3 + R_4} \cdot R_4 =$

$$= \frac{E_1}{R_{eq}} \cdot \frac{R_2 R_4}{R_2 + R_3 + R_4} = \text{(*)}$$

$R_1 -- (R_2 // (R_3 -- R_4))$

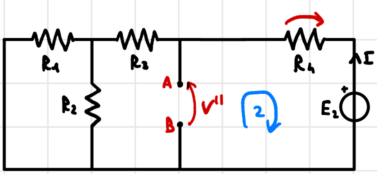
$$\frac{60 \cdot 2\cancel{0}}{11\cancel{0}} = \frac{120}{11} \cdot 10^3$$

$$R_1 + \frac{R_2(R_3+R_4)}{R_2+R_3+R_4} = 60 + \frac{60 \cdot 5\cancel{0}}{11\cancel{0}} =$$

$$= 60 + \frac{300}{11} = \frac{60 \cdot 11 + 300}{11} \cdot 10^3$$

$$\text{(*)} = E_1 \cdot \frac{\cancel{11}}{96\cancel{0}} \cdot \frac{12\cancel{0}}{\cancel{11}} \cdot \frac{\cancel{10^3}}{\cancel{10^3}} = \frac{E_1}{8} = 1\ V$$

Passivo E_1 → $V'' = ?$



LKT2: $V'' + V_{R_4} - E_2 = 0 \qquad V'' = E_2 - I \cdot R_4$

$$R_{eq} = (R_4 -- R_3) -- (R_1 // R_2) = R_4 + R_3 + \frac{R_1 R_2}{R_1 + R_2} = 80 \cdot 10^3$$

$$I = \frac{E_2}{R_{eq}} = \frac{2}{80 \cdot 10^3} = 0{,}25 \cdot 10^{-4} = 2{,}5 \cdot 10^{-5}\ A$$

$\Rightarrow \quad V'' = 1{,}5\ V$

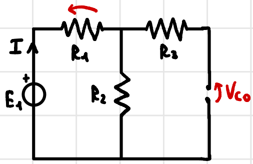
$$V_{eq} = V' + V'' = 1{,}5 + 1 = 2{,}5\,V$$

× RC : $V_C(t) = (V_{C0} - E)\, e^{-t/RC} + E$

$\downarrow$

$V_C(t) = (V_{C0} - E_{eq})\, e^{-t/R_{eq}C} + E_{eq}$

$\boxed{t = 0^+}$



$V_{C0} = V_{R_2} = I \cdot R_2 = \frac{E_1}{R_1+R_2} \cdot R_2 = \frac{E_1}{2} = 4V$

$\Rightarrow \quad V_C(t) = (4 - 2,5)\, e^{-t/15\cdot 2\cdot 10^{-3}} + 2,5 =$

$= 1,5 \cdot e^{-\frac{t}{3}\cdot 10^2} + 2,5$

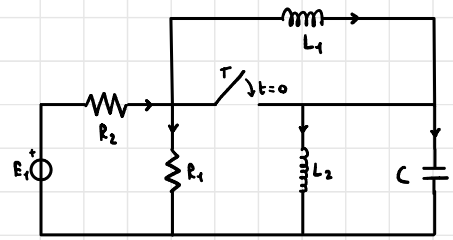
$\mathcal{E}_C(t) = \frac{1}{2} C\, V_C(t)^2$, se $\cancel{\frac{1}{2}}\cancel{C}\, V_C(t^*)^2 = \frac{1}{2\cancel{\cdot 2}}\cancel{C}\, V_{C0}^2 \quad t^* = ?$

$V_C(t^*) = \frac{V_{C0}}{\sqrt{2}} \rightarrow 1,5\, e^{-\frac{t^*}{3}\cdot 10^2} + 2,5 = \frac{4}{\sqrt{2}} \rightarrow e^{-\frac{t^*}{3}\cdot 10^2} = \frac{2\sqrt{2} - 2,5}{1,5} = \frac{4\sqrt{2} - 5}{3}$

$\rightarrow -\frac{t^*}{3}\cdot 10^2 = \ln\left(\frac{4\sqrt{2}-5}{3}\right) \rightarrow t^* = -3\ln\left(\frac{4\sqrt{2}-5}{3}\right)\cdot 10^{-2} = 0,0456\ s$

$\Rightarrow t^* = 4,56 \cdot 10^{-2}\ s$

ES. 3 TRANSITORI



$R_1 = 10\,\Omega = R_2$

$L_1 = 0{,}1\,H = L_2$

$C = 200\,\mu F$

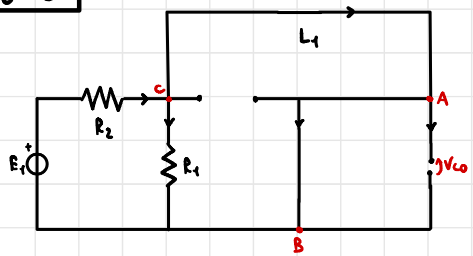
$E_1 = 20\,V$

$t=0^-)$ $\mathcal{E}_{L_1}, \mathcal{E}_{L_2}, \mathcal{E}_C$

$t=0^+)$ $\frac{dI_{L_1}}{dt}, \frac{dI_{L_2}}{dt}$

$t=+\infty)$ $\mathcal{E}_{L_1}, \mathcal{E}_{L_2}, \mathcal{E}_C$

$\boxed{t=0^-}$



$A \equiv B \rightarrow V_{C0} = 0 \rightarrow \mathcal{E}_C = \frac{1}{2} C V_{C0}^2 = 0\,J$

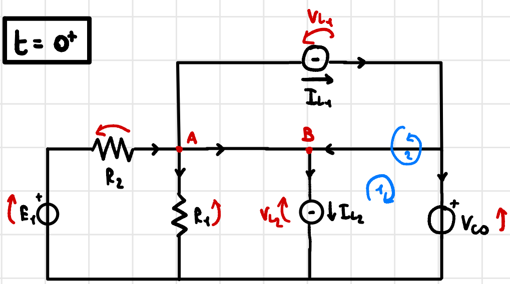
$\mathcal{E}_L = \frac{1}{2} I^2 L \rightarrow \mathcal{E}_{L_1} = \frac{1}{2} I_{L_1}^2 L_1$

$\mathcal{E}_{L_2} = \frac{1}{2} I_{L_2}^2 L_2$

corto $\rightarrow I_{L_1} = I_{L_2} = \frac{E_1}{R_2} = 2\,A$ corrente non passa da R_1 x colpa di cortocircuito

$\rightarrow \mathcal{E}_{L_1} = \mathcal{E}_{L_2} = \frac{1}{2} 4 \cdot 0{,}1 = 0{,}2\,J$

$\boxed{t=0^+}$



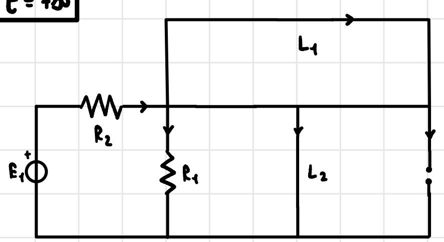
$V_{L_1} = L_1 \frac{dI_{L_1}}{dt} \rightarrow \frac{dI_{L_1}}{dt} = \frac{V_{L_1}}{L_1} \rightarrow V_{L_1}, V_{L_2} = ?$

LKT1 : $V_{L_2} - V_{C0} = 0$ $\quad V_{L_2} - 0 = 0$ $\quad V_{L_2} = 0$

LKT2 : $V_{L_1} + V_{C0} - V_{L_2} = 0$ $\quad V_{L_1} = 0$

$$\rightarrow \left.\frac{dI_{L_1}}{dt}\right|_{t=0^+} = \left.\frac{dI_{L_2}}{dt}\right|_{t=0^+} = 0$$

$\boxed{t=+\infty}$

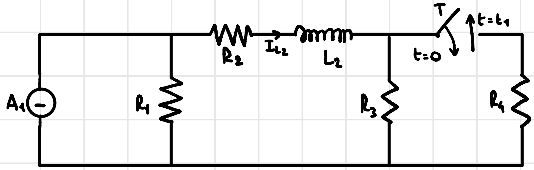


$V_C = 0\,V$ $\quad \rightarrow \mathcal{E}_C = \frac{1}{2} C V_C^2 = 0\,J$

$I_{L_1} = \frac{I_{L_2}}{2} = 1A$ $\quad \rightarrow \mathcal{E}_{L_1} = \frac{1}{2} I_{L_1}^2 L_1 = \frac{1}{2} \cdot 1 \cdot 0{,}1 = 0{,}05\,J$

$\rightarrow \mathcal{E}_{L_2} = \frac{1}{2}\, 4 \cdot 0{,}1 = 0{,}2\,J$

ES. 4 TRANSITORI



$R_1 = 25\ k\Omega$ $\quad L_1 = 150\ mH$

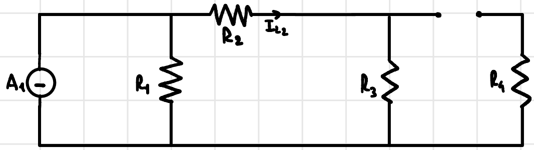
$R_2 = 10\ k\Omega$ $\quad A_1 = 4\ mA$

$R_3 = 20\ k\Omega$ $\quad t_1 = 5 \cdot 10^{-6} s$

$R_4 = 30\ k\Omega$

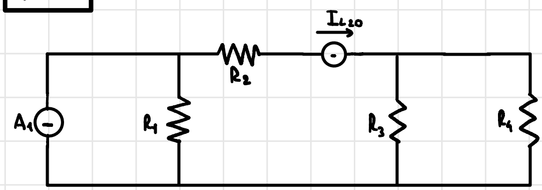
I_{L_2} per $t > 0$?

$\boxed{t = 0^-}$



$$I_{L_2} = A_1 \cdot \frac{R_1}{R_1 + R_2 + R_3} = 1,82\ mA$$

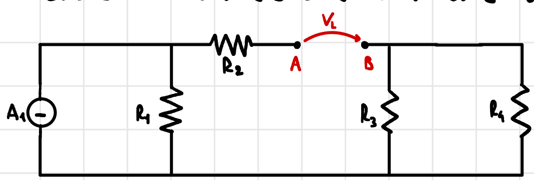
$\boxed{t = 0^+}$



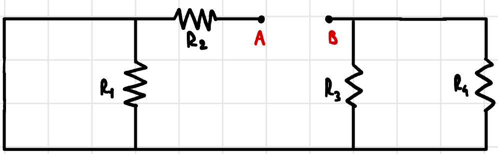
$$I_{L20} = I_{L2}(t = 0^-) = 1,82\ mA$$

$\boxed{t = t_1^-}$

Ricavo il circuito RL equivalente e così ottengo la $I_L(t)$ da cui posso ricavare la situazione del circuito in $t = t_1$.

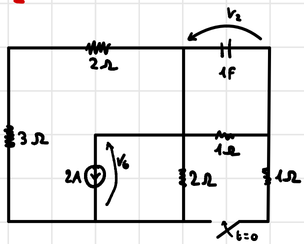


× V_{eq}

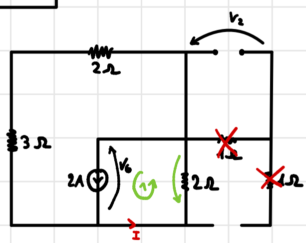


× R_{eq}

ES. 1



$\boxed{t=0^-}$

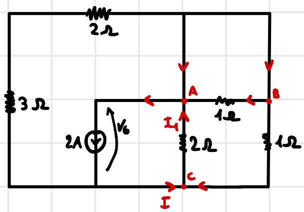


$V_2 = 0 \quad \times LKT$

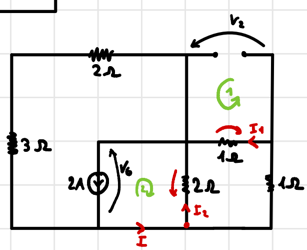
$V_6 = -2 \cdot I = -\frac{20}{7}$

$I = 2 \cdot \frac{3+2}{3+2+2} = 2 \cdot \frac{5}{7} = \frac{10}{7}$

$\boxed{t=0^+}$



$\boxed{t=+\infty}$

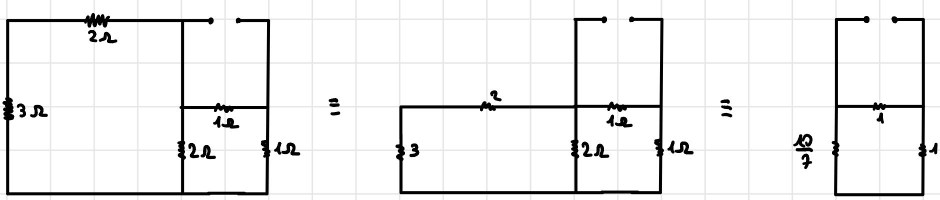


$\times LKT1 : V_2 = -I_1 \cdot 1 = -5/6$

$\times LKT2 : V_6 = -I_2 \cdot 2 = -5/3$

$I_2 = \underbrace{2 \cdot \frac{3+2}{3+2+1}}_{I} \cdot \frac{2}{2+2} = \frac{5}{6} = I_1$

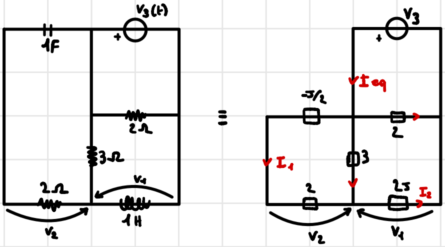
τ:



$R_{eq} = \left(1 + \frac{1}{\frac{10}{7}+1}\right)^{-1} = \frac{17}{24}$

$\tau = R_{eq} \cdot C = \frac{17}{24}$

Es. 17 AC



$V_3(t) = 2\sin(2t + 165°) = 2\cos(2t + 75)$

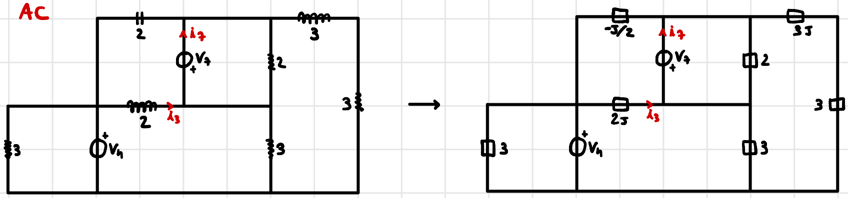
$V_3 = \sqrt{2}\, e^{165j} = 0{,}366 + 1{,}366j$

$Z_{eq} = 2 /\!/ \underbrace{(2j + \underbrace{3 /\!/ (2 - j/2)}_{1{,}218 - 0{,}178j})}_{1{,}218 + 1{,}822}= 1{,}039 + 0{,}333j \rightarrow I_{eq} = 0{,}794 + 0{,}891j$

$V_2 = -2 I_1 = -2 \cdot I_{eq} \cdot \dfrac{2}{2 + 2j + 3/\!/(2 - j/2)} \cdot \dfrac{3}{3 + 2 - j/2} = -0{,}701 - 0{,}319j = 0{,}771 \angle -156 \rightarrow 1{,}09 \cos(2t - 156°)$

$V_1 = 2j I_2 = 2j \cdot I_{eq} \cdot \dfrac{2}{2 + 2j + 3/\!/(2 - j/2)} = -0{,}416 + 1{,}222 = 1{,}29 \angle 109° \rightarrow 1{,}823 \cos(2t + 109°)$

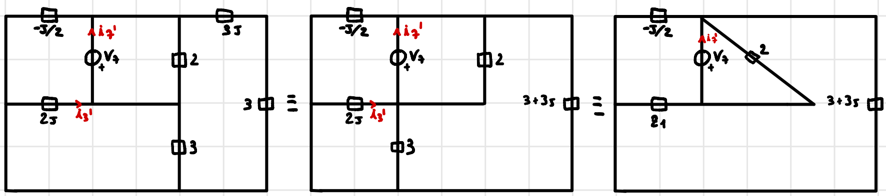
Es. 18 AC

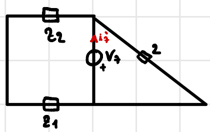


$V_7(t) = \cos(t - 75°) \longrightarrow V_7 = 0,18301 - 0,68301\,j$

$V_4(t) = 2\cos(t - 135°) \longrightarrow V_4 = -1 - j$

$V_7)$





$Z_1 = -\frac{3}{2}j \,//\, (3+3j) = 0,01918 - 0,51093\,j$

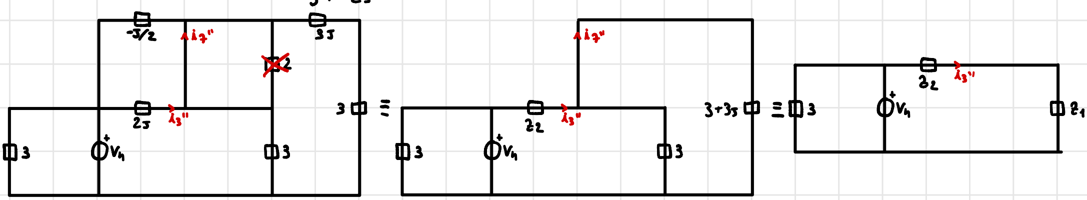
$Z_2 = 2j \,//\, 3 = 0,92308 + 1,38462\,j$

$\rightarrow Z_{eq} = 2 \,//\, (Z_1 + Z_2) = 1,08675 - 0,50913\,j$

$-I_{eq} = +i_7' = -V_7 / Z_{eq} = -0,37954 + 0,45068\,j$

$$i_3' = i_7' \cdot \frac{2}{2 + Z_2 + Z_1} \cdot \frac{3}{3 + 2j} = i_7' \cdot (0,34934 - 0,40978\,j) = 0,05201 + 0,31306\,j$$

$V_4)$



$Z_1 = (3+3j) \,//\, 3 = 1,8 + 0,6\,j$

$Z_2 = -\frac{3}{2}j \,//\, 2j = -0,66666\,j$

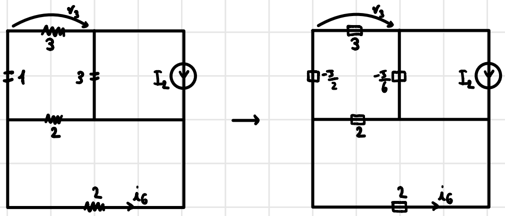
$Z_{eq} = 3 \,//\, (Z_2 + Z_1) = 1,12536 - 0,02603\,j \rightarrow I_{eq} = -0,86758 - 0,90867\,j$

$$i_3'' = I_{eq} \cdot \frac{3}{3 + Z_1 + Z_2} = -0,53425 - 0,57534\,j \longrightarrow i_7'' = i_3'' \cdot \frac{3}{3 + 3 + 3j} = -0,32877 - 0,12323\,j$$

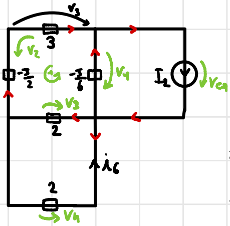
$\Rightarrow i_3 =$

$\Rightarrow i_7 =$

Es. 19 AC



$I_2 = \frac{1}{\sqrt{2}} \angle 105 = -0{,}18301 + 0{,}68301\,j$



$Z_{eq} = -\frac{j}{6} /\!/ \left(1 + 3 - \frac{j}{2}\right) = 0{,}00676 - 0{,}16334\,j$

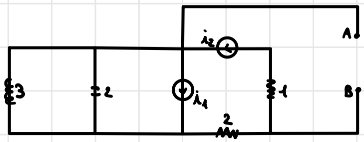
$V_{eq} = Z_{eq} \cdot I_2 = 0{,}11183 + 0{,}03491\,j$

x LKT1 : $V_2 + V_3 - V_1 = 0$ $\quad V_3 = V_1 - V_2 = V_{eq} - V_2$

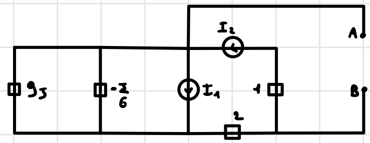
$i_6 = -V_4/2 = -V_3/2 = -\frac{(V_{eq} - V_2)}{2} = -0{,}013225 - 0{,}00602\,j = 0{,}02055 \angle -156$

$V_2 = -V_{eq} \cdot \frac{3 - \frac{j}{2}}{3 - \frac{j}{2} + 1} = 0{,}08538 + 0{,}02288\,j \rightarrow V_3 = -V_2 \cdot \frac{3}{3 - \frac{j}{2}} = \dots = 0{,}123 \angle -156$

ES. 1 N/T AC



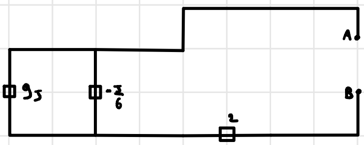
$\rightarrow$



$i_1 = 2\cos(3t - 75)$ $\qquad$ $I_1 = \sqrt{2} \angle -75 = 0,366 - 1,366\,j$

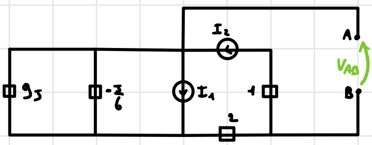
$i_2 = 3\sin(3t - 75) = 3\cos(3t - 165)$ $\qquad$ $I_2 = \frac{3}{\sqrt{2}} \angle -165 = -2,049 - 0,549\,j$

Z_{eq})

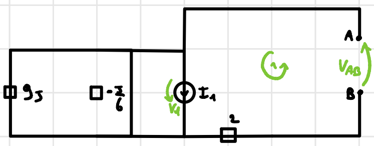


$Z_{eq} = 9j // -\frac{j}{6} + 2 = 2 - 0,17\,j$

V_{eq})



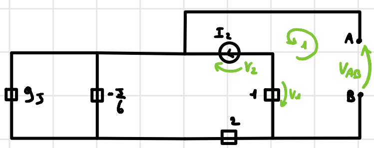
I_1)



x LKT1: $V'_{AB} = -V_1 = -V_{eq} = 0,23196 + 0,06215\,j$

$V_{eq} = I_1 \cdot \left(9j // -\frac{j}{6}\right) = -0,23196 - 0,06215\,j$

I_2)



x LKT1: $V''_{AB} = V_2 - V_1$

$V_2 = I_2 \cdot \left(3 + 9j // -\frac{j}{6}\right) = -6,24 - 1,299\,j$

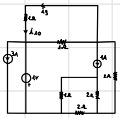
$V_1 = I_2 \cdot 1 = -2,049 - 0,549\,j$

$V''_{AB} = -4,191 - 0,75\,j$

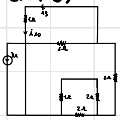
$\Rightarrow V_{AB} = -3,95904 - 0,68785\,j$?

ES. 8 1° ORDINE

$t = 0^-$

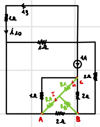


3A ≠ 0)



$i_3' = i_{10}' = 0$

1A ≠ 0)

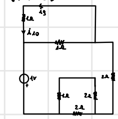


$i_3'' = i_{10}'' = -I/2 = -14/37$

$R_A = 2/5 \quad R_B = 4/5 \quad R_C = 2/5$

$I = 1 \cdot \frac{2 + R_B}{2 + R_B + R_A + 0{,}5} = \frac{28}{37}$

1V ≠ 0)



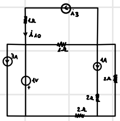
$R_{eq} = 0{,}5 + 2 + 3 // 2 = 37/10$

$I_{eq} = 1/R_{eq} = 10/37$

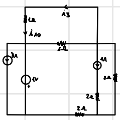
$i_3''' = i_{10}''' = I_{eq}/2 = 5/37\ A$

$i_3 = i_{10} = -14/37 + 5/37 = -9/37\ A$

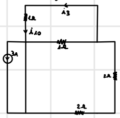
$t = 0^+$



$t = +\infty$

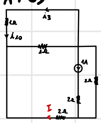


3A ≠ 0)



$i_3' = i_{10}' = 0A$

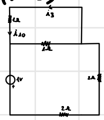
1A ≠ 0)



$i_3'' = i_{10}'' = -I/2 = -2/9$

$I = 1 \cdot \frac{2}{2 + 2 + 0{,}5} = \frac{4}{9}$

1V ≠ 0)



$i_3''' = i_{10}''' = I_{eq}/2 = 1/9$

$I_{eq} = \frac{1}{R_{eq}} = \frac{2}{9}$

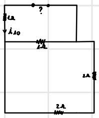
$R_{eq} = 2 + 2 + 0{,}5 = \frac{9}{2}$

$i_3 = i_{10} = -2/9 + 1/9 = -1/9\ A$

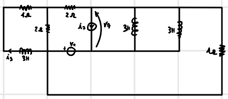
$$\Rightarrow i_3(t) = i_{10}(t) = \left(i_3(0) - i_3(+\infty)\right)e^{-t/\tau} + i_3(+\infty) = \left(-9/37 + 1/9\right)e^{-t/\tau} - \frac{1}{9} = -\frac{44}{333}e^{-t/\tau} - \frac{1}{9} =$$

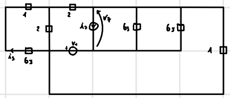
$$= -0{,}132\, e^{-t/\tau} - \frac{1}{9}$$

con $\tau = \frac{L}{R_T}$ dove $R_T = 1 + 1 // 4 = \frac{9}{5} \rightarrow \tau = \frac{15}{9} = \frac{5}{3}$



ES. 20 AC

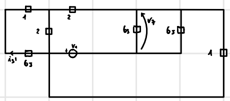




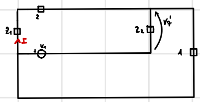
$v_1(t) = 3\sin(2t-150) = 3\cos(2t-240) \rightarrow V_1 = \frac{3}{\sqrt{2}}\angle -240 = -1{,}06066 + 1{,}83712j$

$i_7(t) = \sin(2t+45) = \cos(2t-45) \rightarrow I_7 = \frac{1}{\sqrt{2}}\angle -45 = 0{,}5 - 0{,}5j$

$V_1 \neq 0$)



$\equiv$



$Z_1 = (1+6j)/\!/2 = 1{,}7333 + 0{,}5333j$

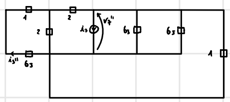
$Z_2 = 6j/\!/6j = 3j$

$Z_{eq} = 1/\!/(Z_1+2) + Z_2 = 0{,}79138 + 3{,}02351j$

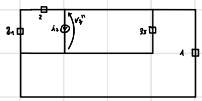
$I_{eq} = \frac{V_1}{Z_{eq}} = 0{,}48272 + 0{,}47715j \rightarrow V_7' = Z_2 \cdot I_{eq} = -1{,}43146 + 1{,}44816j$

$i_s' = I \cdot \frac{2}{3+6j} = I_{eq} \cdot \frac{1}{3+Z_1} \cdot \frac{2}{3+6j} = 0{,}03844 - 0{,}01809j$

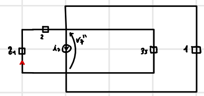
$I_7 \neq 0$)



$\equiv$



$\equiv$



$Z_1 = (1+6j)/\!/2 = 1{,}7333 + 0{,}5333j$

$Z_{eq} = 1/\!/3j/\!/(2+Z_1) = 0{,}72917 + 0{,}21418j$

$V_7'' = -Z_{eq} \cdot I_7 = -0{,}47168 + 0{,}2575j$

$i_s'' = I \cdot \frac{2}{3+6j} = I_7 \cdot \frac{3j/\!/1}{2+Z_1+\underbrace{3j/\!/1}_{0{,}9+0{,}3j}} \cdot \frac{2}{3+6j} = -0{,}00752 - 0{,}04481j$

$\Rightarrow V_7 = -1{,}90314 + 1{,}70566j = 2{,}5556\angle 138° \longrightarrow 3{,}614\cos(2t+138°)$

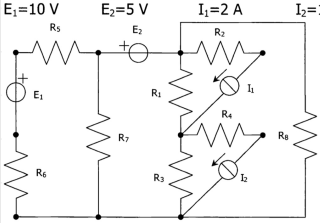
$\Rightarrow I_s = 0{,}03092 - 0{,}0599j = 0{,}104\angle -48°$?

ES. FOTO

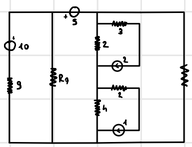
$R_1=2\ \Omega$ $R_2=3\ \Omega$ $R_3=4\ \Omega$ $R_4=2\ \Omega$ $R_5=1\ \Omega$

$R_6=8\ \Omega$ $R_7=2.25\ \Omega$ $R_8=5\ \Omega$

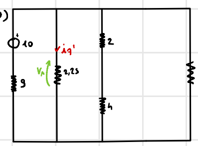
$E_1=10$ V $E_2=5$ V $I_1=2$ A $I_2=1$ A



$R_q = ?$



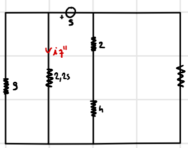
10≠0)



$$i_7' = \frac{V_A}{R_7} = \frac{400}{747}$$

$$V_A = \frac{\frac{10}{9}}{\frac{1}{9}+\frac{1}{2,25}+\frac{1}{6}+\frac{1}{3}} = \frac{100}{83} = \frac{300}{249}$$

5≠0)

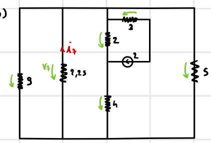


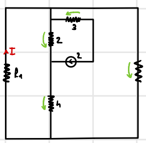
$$R_{eq} = 9/\!/2,25 + 6/\!/5 = \frac{249}{55}$$

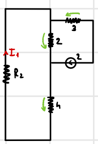
$$I_{eq} = 5/R_{eq}$$

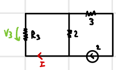
$$i_7'' = I_{eq}\cdot\frac{9}{9+2,25} = \frac{45}{R_{eq}\cdot 11,25} = \frac{220}{249}$$

2≠0)





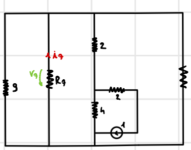


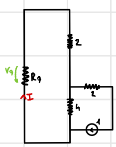


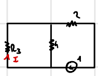
$$R_{eq} = 3 + 2/\!/R_3 = 3 + 2/\!/(R_2+4) = 3 + 2/\!/\underbrace{(4+\overbrace{5/\!/9/\!/2,25}^{45/34})}_{181/34} = \frac{1109}{249}$$

$$I = 2\cdot\frac{2}{2+\frac{181}{34}} = \frac{136}{249} \rightarrow V_3 = \frac{136}{249}\cdot\frac{45}{34} = \frac{60}{83} = V_7 \rightarrow i_7 = -\frac{V_7}{2,25} = -\frac{80}{249}$$

1≠0)





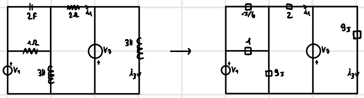


$$R_{eq} = 2 + 4/\!/\underbrace{(2+\overbrace{5/\!/9/\!/2,25}^{45/34})}_{113/34} = \frac{950}{249}$$

$$i_7 = -\frac{V_7}{2,25} = -\frac{I\cdot 45/34}{2,25} = -\frac{4}{4+\frac{113}{34}}\cdot\frac{45/34}{2,25} = -\frac{80}{249}$$

$$\Rightarrow\ i_7 = \frac{400}{747} + \frac{220}{249} - \frac{80}{249} - \frac{80}{249} = \frac{580}{747} = 0,77644\ A$$

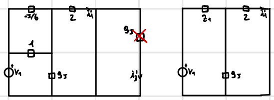
ES. 21 AC



$V_1(t) = \cos(3t - 105°)$ $\rightarrow V_1 = \frac{1}{\sqrt{2}} \angle -105° = -0,18301 - 0,68301 j$

$V_7(t) = 2\sin(3t + 135°) = 2\cos(3t + 45°)$ $\rightarrow V_7 = \sqrt{2} \angle 45° = 1 + j$

$V_1 \neq 0$)



$i_3' = 0$

$R_{eq} = Z_1 + 9j // 2 = 1 // -j/6 + 9j // 2 = 1,93291 + 0,26137 j$

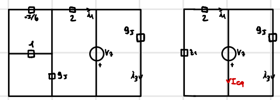
$\downarrow$

$I_{eq} = V_1 / R_{eq} = -0,1399 - 0,33444 j$

$\downarrow$

$i_1' = I_{eq} \cdot \frac{9j}{2 + 9j} = -0,06249 - 0,34833 j$

$V_7 \neq 0$)



$Z_1 = 9j // 1 // -j/6 = 0,02803 - 0,16505 j$

$Z_{eq} = 9j // (2 + Z_1) = 1,99317 + 0,29077 j$

$\downarrow$

$I_{eq} = V_7 / Z_{eq} = 0,56109 + 0,41186 j$

$\downarrow$

$i_3'' = -I_{eq} \cdot \frac{Z_1 + 2}{Z_1 + 2 + 9j} = -0,11111 + 0,11111 j$

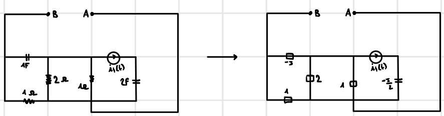
$\downarrow$

$i_1'' = I_{eq} + i_3'' = 0,44998 + 0,52971 j$

$\Rightarrow \quad i_3 = -0,11111 + 0,11111 j = 0,15713 \angle 135° \longrightarrow i_3(t) = 0,222 \cos(3t + 135°)$

$\Rightarrow \quad i_1 = 0,38219 + 0,18138 j = 0,42784 \angle 25° \longrightarrow i_1(t) = 0,605 \cos(3t + 25°)$

ES. 2 N/T AC (T)

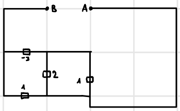


$i_1(t) = 3\cos(t - 30)$

$\downarrow$

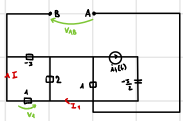
$I_1 = \frac{3}{\sqrt{2}} \angle -30 = 1,83712 - 1,06066 j$

Z_{eq})



$Z_{eq} = 1 // (-j + 2 // 1) = 0,55882 - 0,26471 j$ ✓

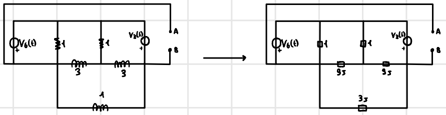
V_{eq})



X LKT 1: $V_{AB} + V_1 = 0$ $\quad V_{AB} = -V_1$

$V_1 = 1 \cdot I = I_1 \cdot \frac{2}{2 + 1 - j} = I_1 \cdot \underbrace{\frac{1}{1 + 2 // (1 - j)}}_{0,52941 + 0,11765 j} \cdot \frac{2}{3 - j} = 0,7275 + 0,01221 j$

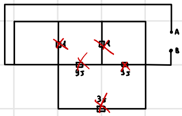
ES. 3 N/T AC (T)



$V_3(t) = 2\sin(3t - 15°) = 2\cos(3t - 105°) \longrightarrow V_3 = \sqrt{2}\ \angle -105° = -0,366 - 1,366\,j$

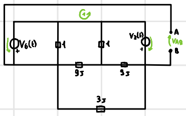
$V_6(t) = \cos(3t - 75°) \longrightarrow V_6 = \frac{1}{\sqrt{2}} \angle -75 = 0,18301 - 0,68301\,j$

Zeq)



$Z_{eq} = 0$

VAB)

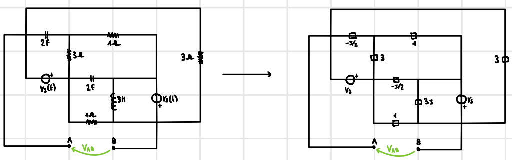


x LKT1: $V_{AB} - V_6 + V_3 = 0$

$\rightarrow V_{AB} = V_6 - V_3 = 0,54901 + 0,68299\,j$

IN QUESTI ES., SOLO SU AUTOCIRCUITS, NON USA IL VALORE EFFICACE

ES. 4 N/T AC

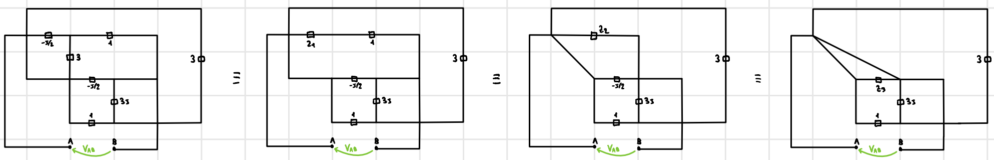


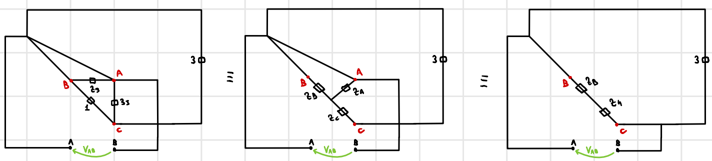
$V_2(t) = 2\sin(t - 30°) = 2\cos(t - 120°) = 2\angle -120 = -1 - \sqrt{3}\,j = -1 - 1,73205\,j$

$\sqrt{2}$ → COME ANDREBBE FATTO

$V_5(t) = \sin(t + 135°) = \cos(t + 45°) = \frac{1}{\sqrt{2}} \angle +45° = 0,70711 + 0,70711\,j$

Zeq)





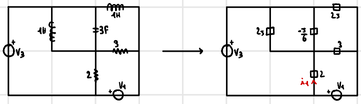
$Z_{eq} = (Z_B + Z_4) // 3 = (Z_B + Z_C // Z_A) // 3$

$Z_B = \frac{1 \cdot Z_3}{1 + Z_3 + 3j} \qquad Z_A = \frac{Z_3 \cdot 3j}{1 + Z_3 + 3j} \qquad Z_C = \frac{1 \cdot 3j}{1 + Z_3 + 3j}$

$Z_3 = -\frac{j}{2} // Z_2 = -\frac{j}{2} // (3 // -\frac{j}{2} + 1)$

ES. 1 POT. ASSORBITA

RISPOSTA : $\underbrace{2{,}277 \text{ W}}_{P} + \underbrace{0 \text{ VAR}}_{Qj}$

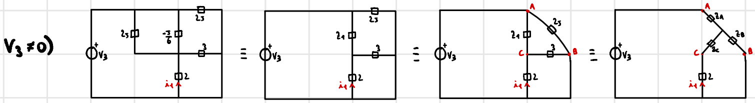


$V_1(t) = 3\cos(2t+120) = \frac{3}{\sqrt{2}} \angle 120 = -1{,}06066 + 1{,}83712\,j$

$V_3(t) = 3\cos(2t-75°) = \frac{3}{\sqrt{2}} \angle -75 = 0{,}54904 - 2{,}04904\,j$

$P_{DISS} = 2 \cdot (i_1^*)^2 = ?$

$V_3 \neq 0)$



$Z_1 = 2j // -\frac{j}{6} = -0{,}18182\,j$

con $Z_A = \frac{2j \cdot Z_1}{2j + Z_1 + 3} = 0{,}08865 - 0{,}05373\,j$

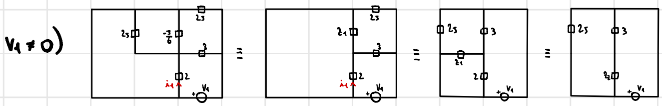
$3 + 1{,}818181\,j$

$Z_B = \frac{3 \cdot 2j}{2j + Z_1 + 3} = 0{,}9863 + 1{,}46273\,j$

$Z_C = \frac{Z_1 \cdot 3}{2j + Z_1 + 3} = -0{,}08059 - 0{,}13298\,j$

$Z_{eq} = Z_A + Z_B // (Z_C + 2) = 1{,}01143 + 0{,}46354\,j \longrightarrow I_{eq} = V_3 / Z_{eq} = -0{,}32434 - 1{,}87596\,j \longrightarrow i_1' = -I_{eq} \cdot \frac{Z_B}{Z_B + Z_C + 2} = (0{,}42011 - 0{,}36086\,j) \cdot (-1) = (-0{,}42011 + 0{,}96086\,j)$

$V_1 \neq 0)$



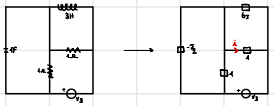
$Z_{eq} = 2j // (3 + Z_2) = 2j // (3 + Z_1 // 2) = 2j // (3 + \underbrace{2 // 2j // -\tfrac{j}{6}}_{0{,}01639 - 0{,}18033\,j}) = 0{,}97226 + 1{,}41347\,j$

$I_{eq} = V_1 / Z_{eq} = 0{,}5319 + 1{,}11626\,j$

$i_1'' = I_{eq} \cdot \frac{1/2}{\frac{1}{2} + \frac{1}{2j} + \frac{j}{-3}} = I_{eq} \cdot (0{,}00819 - 0{,}09016\,j) = 0{,}103 - 0{,}03891\,j$

$\Rightarrow i_1 = -0{,}31310 + 0{,}92205\,j \longrightarrow i_1^* = -0{,}31310 - 0{,}92205\,j \longrightarrow P_{diss} = -1{,}50178 + 1{,}16215\,j$

ES. 2 POT ASSORBITA



$V_3(t) = 2\sin(2t-150) = \cos(2t-240) \longrightarrow V_3 = \frac{1}{\sqrt{2}} \angle -240 = -0{,}35355 + 0{,}61237\,j$

$P_{ASS} = V \cdot i_2^* = 1 \cdot i_3 \cdot i_3^* =$

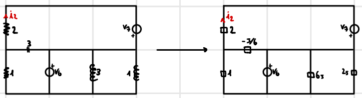
$Z_{eq} = 6j \,//\, 1 + -\frac{j}{2} \,//\, 1 = 1{,}17297 - 0{,}23784\,j \longrightarrow I_{eq} = V_3 / Z_{eq} = -0{,}39119 + 0{,}44275\,j$

$i_1 = I_{eq} \cdot \frac{6j}{1+6j} = -0{,}45244 + 0{,}36735\,j$

$i_1^* = -0{,}45244 - 0{,}36735\,j$

$P_{ass} = 0{,}33962 + 0j$

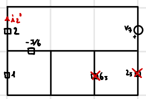
ES. 3 POT ASSORBITA



$V_6(t) = \cos(2t-30) \longrightarrow V_6 = \frac{1}{\sqrt{2}} \angle -30 = 0{,}61237 - 0{,}35355\,j$

$V_9(t) = 3\sin(2t+60) = 3\cos(2t-30) \longrightarrow V_9 = \frac{3}{\sqrt{2}} \angle -30 = 1{,}83712 - 1{,}06066\,j$

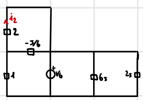
$V_9 \neq 0$)



$Z_{eq} = 2 + 1 \,//\, -\frac{3j}{6} = 2{,}02703 - 0{,}16216\,j$

$I_{eq} = 0{,}94214 - 0{,}44789\,j = i_2'$

$V_6 \neq 0$)



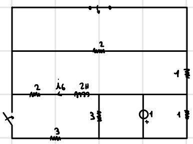
$Z_{eq} = 2j \,//\, 6j \,//\, \underbrace{(1 + 2 \,//\, -3j/6)}_{1{,}04379 - 0{,}16332\,j} = 0{,}81215 + 0{,}43094\,j$

$I_{eq} = 0{,}40811 - 0{,}65188\,j$

$i_2'' = -I \cdot \frac{-3j/6}{2-3j/6} = -I_{eq} \cdot \frac{\frac{1}{1+2//-3j/6}}{\frac{1}{1+2//-3j/6} + \frac{1}{2j} + \frac{1}{6j}} \cdot \frac{-3j/6}{2-3j/6} = 0{,}01372 + 0{,}05196\,j$

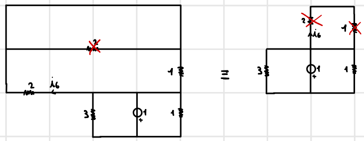
$i_2 = 0{,}95786 - 0{,}39293\,j \longrightarrow P_{diss} = 2 \cdot i_2 \cdot i_2^* = 2 \cdot 1{,}07189 = 2{,}14378 = 2{,}144 + 0j$

ES. 9 1° ORDINE



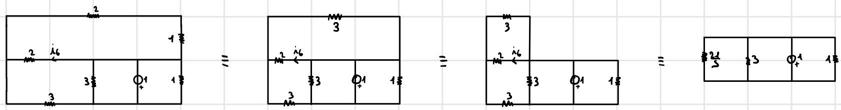
$i_6(t) = ? = (i_6(0) - i_6(+\infty))e^{-t/\tau} + i_6(+\infty)$

$\boxed{t = 0^-}$



$i_6 = 0$

$\boxed{t = +\infty}$



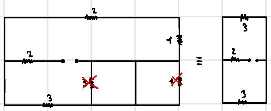
$Z_{eq} = \frac{21}{5} // 3 // 1 = \frac{7}{11} \longrightarrow I_{eq} = \frac{11}{7}\,A$

$i_6 = \dfrac{\frac{5}{21}}{\frac{5}{21} + \frac{1}{3} + 1} \cdot \dfrac{3}{2+3} \cdot (-I_{eq}) = \dfrac{5}{\cancel{33}_{11}} \cdot \dfrac{8}{5} \cdot \left(-\dfrac{\cancel{11}}{7}\right) = -\dfrac{1}{7}$

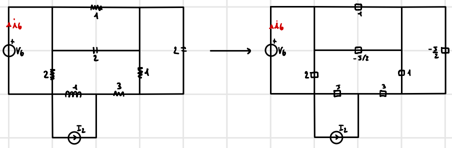
$\Rightarrow \quad i_6(t) = \left(0 - \left(-\frac{1}{7}\right)\right) e^{-t/\tau} - \frac{1}{7} = \frac{1}{7} e^{-t/\tau} - \frac{1}{7}$

$\Rightarrow \quad \tau = L/R_T = 2 \cdot \frac{2}{7} = 4/7$

$R_T = 2 + 3//3 = 2 + \frac{3}{2} = \frac{7}{2}$



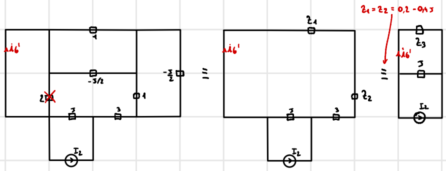
ES. 4 POT. ASSORBITA



$I_2(t) = 3\sin(t-90) = 3\cos(t-180) \longrightarrow I_2 = \frac{3}{\sqrt{2}}\angle{-180} = -2{,}12132$

$V_6(t) = 3\sin(t+60°) = 3\cos(t-30°) \longrightarrow V_6 = \frac{3}{\sqrt{2}}\angle{-30} = 1{,}83712 - 1{,}06066j$

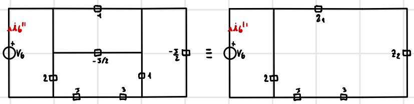
$I_2 \neq 0)$



$Z_3 = Z_1 + Z_2 + 3 = 3{,}4 - 0{,}8j$

$i_6' = -I_2 \cdot \frac{j}{j + Z_3} = +0{,}03657 + 0{,}62177j$

$V_6 \neq 0)$



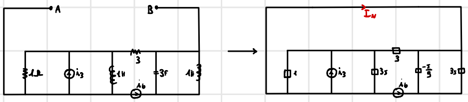
$i_6'' = I_{eq} = V_6/Z_{eq} = 1{,}43874 - 0{,}87289j$

$Z_{eq} = 2 \,//\, (Z_1 + Z_2 + 3 + j) = 1{,}26027 + 0{,}02739j$

$\Rightarrow i_6 = 1{,}47531 - 0{,}25112j$

$\Rightarrow P_{diss} = (1{,}83712 - 1{,}06066j)(1{,}47531 + 0{,}25112j) = 2{,}97667 - 1{,}10346j$

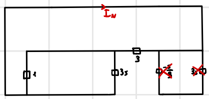
ES. N/T AC (N)



$i_3(t) = 2\cos(3t - 45°) \longrightarrow I_3 = 2\angle -45 = 1,41421 - 1,41421\,j$

$i_6(t) = 2\sin(3t-30) = 2\cos(3t-120) \longrightarrow I_6 = 2\angle -120 = -1 - 1,73205\,j$

Zeq)

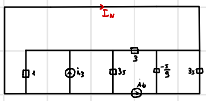


$Z_{eq} = 3 + 1 // 3j = 3,9 + 0,3\,j$

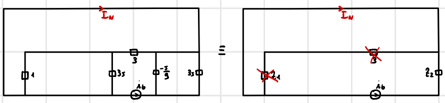
$\downarrow$

$G_{eq} = \frac{1}{Z_{eq}} = 0,2549 - 0,01961\,j$

Ieq)

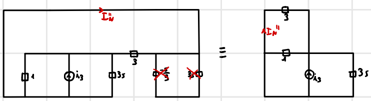


$I_6 \neq 0$)



$I_N' = -i_6 = +1 + 1,73205\,j$

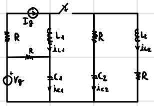
$I_3 \neq 0$)



$I_N'' = -i_3 \cdot \frac{1/3}{\frac{1}{3} + 1 + \frac{1}{3j}} = (-1,41421 + 1,41421\,j) \cdot (0,23529 + 0,05882\,j) = -0,41594 + 0,24957\,j$

$\Rightarrow I_N = 0,58406 + 1,98162\,j$

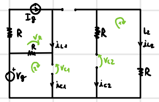
ES. 1 SIMULAZIONE ESAME



$R = 10$
$L_1 = L_2 = 0{,}2\,H$
$C_1 = C_2 = 200\,\mu F$
$I_g = 3A$
$V_g = 20V$

$t = 0^- :\ W_{L1}, W_{L2}, W_{C1}, W_{C2}$
$t = 0^+ :\ \frac{di_{L1}}{dt}, \frac{di_{L2}}{dt}$
$t = \infty :\ Q_{C1}, Q_{C2}$

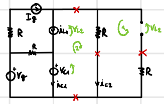
$t = 0^-$



x LKT1: $V_{C2} = 0\,V$
x LKT2: $V_g + V_R - V_{C1} = 0 \rightarrow V_{C1} = V_g + V_R = V_g + I_g \cdot R = 50V$
$i_{L1} = I_g = 3\,A$
$i_{L2} = 0\,A$

$W_{C2} = 0\,W$
$W_{C1} = \frac{1}{\cancel{2}} \cdot \cancel{2} \cdot 10^{-4} \cdot 50^2 = 0{,}25\,W$
$W_{L1} = \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{10} \cdot 3^2 = 0{,}9\,W$
$W_{L2} = 0\,W$

$t = 0^+$

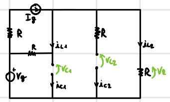


x LKT1: $V_{L2} = 0$
x LKT2: $V_{L1} = -V_{C1} = -50$

$\frac{di_{L1}}{dt} = \frac{V_{L1}}{L_1} = -250$

$\frac{di_{L2}}{dt} = \frac{V_{L2}}{L_2} = 0$

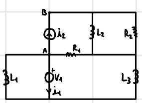
$t = +\infty$



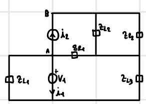
$V_{C1} = V_{C2} = V_R = 70V$
$I_g \neq 0)\ V_R' = \frac{I_g}{2} \cdot R = 3 \cdot 5 = 15\,V$
$V_g \neq 0)\ V_R'' = I_{eq} \cdot R = \frac{V_g}{2\cancel{R}} \cdot \cancel{R} = \frac{V_g}{2} = 10$
$V_R = 25\,V$

$Q_{C1} = C_1 \cdot V_{C1} = 2 \cdot 10^{-4} \cdot 25 = 0{,}005\,C$
$Q_{C2} = C_2 \cdot V_{C2} = 2 \cdot 10^{-4} \cdot 25 = 0{,}005\,C$

ES. 2 SIMULAZIONE D'ESAME



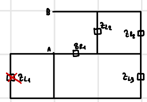
$\longrightarrow$



$R_1 = 1\Omega \quad R_2 = 3\Omega$
$L_1 = 3H \quad L_2 = 2H \quad L_3 = 3H$
$\rightarrow\ Z_{R1} = 1\Omega \quad Z_{R2} = 3\Omega$
$Z_{L1} = 6j \quad Z_{L2} = 4j \quad Z_{L3} = 6j$

$V_1 = \cos(2t - 105°) = 1/\sqrt{2} \angle -105 = -0{,}18301 - 0{,}68301j$
$I_2 = 2\sin(2t + 90) = 2\cos(2t) = \sqrt{2} \angle 0 = 1{,}41421$

Z_{AB})

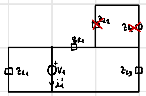


$Z_{AB} = Z_{L3} // Z_{R1} + Z_{L2} // Z_{R2} = 6j // 1 + 4j // 3 = 2{,}89297 + 1{,}60216j$

P_{diss})

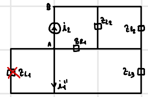
$P_{diss} = V_1 \cdot i_1^* = (-0{,}18301 - 0{,}68301j) \cdot (0{,}19132 - 0{,}18761j) = -0{,}16315 - 0{,}09634j$

$V_1 \neq 0$)



$i_1' = -I_{eq} = -\frac{V_1}{Z_{L1} // (Z_{R1} + Z_{L3})} = 0{,}22954 - 0{,}04172j$

$i_2 \neq 0$)



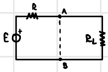
$i_1'' = -i_2 \cdot \frac{Z_{R1}}{Z_{R1} + Z_{L3}} = -0{,}03822 + 0{,}22933j$

$\Rightarrow i_1 = 0{,}19132 + 0{,}18761j = 0{,}26796 \angle 44 \rightarrow i_1(t) = 0{,}37895\cos(2t + 44)$

RIPASSO TEORIA

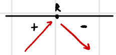
1) TABLEAU $\rightarrow$ L eq. cost ; N-1 LKC ; L-N+1 LKT

2) MAX TRASFERIMENTO DI POTENZA

Dato  allora quale valore di R_L : P_L max ? $R_L = R$

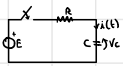
DIM: $P_L = R_L \cdot I^2 = R_L \cdot \left(\frac{E}{R+R_L}\right)^2 \longrightarrow P_L(R_L) = E^2 \cdot \frac{R_L}{(R+R_L)^2} \longrightarrow P_L(R_L)' = E^2 \cdot \left[\frac{(R+R_L)^2 - R_L \cdot 2(R+R_L)}{(R+R_L)^4}\right]$

$P_L(R_L)' \geq 0 \qquad \cancel{E^2} \cdot \left[\frac{(R+R_L)^2 - R_L \cdot 2(R+R_L)}{\cancel{(R+R_L)^4}}\right] \geq 0 \qquad R^2 + R_L^2 + 2\cancel{RR_L} - 2\cancel{RR_L} - 2R_L^2 \qquad R^2 - R_L^2 \geq 0 \qquad R_L^2 - R^2 \leq 0 \qquad R_L \leq R$

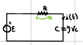


$\rightarrow R_L = R$

3) CIRCUITO RC

Dato  allora $V_C(t) = ?$ $\qquad V_C(t) = \underbrace{(V_{C0} - E)\, e^{-\frac{t}{RC}}}_{\text{RISPOSTA TRANSITORIA}} + \underbrace{E}_{\text{RISPOSTA A REGIME}} = \underbrace{V_{C0}\, e^{-\frac{t}{RC}}}_{\text{EVOLUZIONE LIBERA}} + \underbrace{E\left(1 - e^{-\frac{t}{RC}}\right)}_{\text{RISPOSTA FORZATA}}$

DIM: $\boxed{t=0^-}$ $V_C = V_{C0}$

$\boxed{t=0^+}$ 

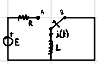
x LKT: $E - V_R - V_C = 0$
$V_R = R \cdot i$
$i = C\,\frac{dV_C}{dt}$

$\Rightarrow E - R \cdot \left(C\,\frac{dV_C}{dt}\right) - V_C = 0 \Rightarrow \frac{E}{RC} - \frac{dV_C}{dt} - \frac{V_C}{RC} = 0 \Rightarrow \frac{dV_C}{dt} + \frac{V_C}{RC} = \frac{E}{RC}$ $\qquad V_C' + \frac{V_C}{RC} = 0 \qquad \lambda + \frac{1}{RC} = 0 \qquad \lambda = -\frac{1}{RC} \longrightarrow o(t) = A\, e^{-\frac{t}{RC}}$

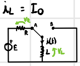
cost. $\qquad P(t) = K \qquad \frac{dP(t)}{dt}^{\,0} + \frac{P(t)}{RC} = \frac{E}{RC} \qquad \frac{K}{\cancel{RC}} = \frac{E}{\cancel{RC}} \qquad K = E$ $\qquad$ *

* $V_C(t) = A\, e^{-\frac{t}{RC}} + E \qquad$ ma $\qquad V_C(0) = V_{C0} = A\, e^{-\frac{0}{RC}} + E = A + E \rightarrow A = V_{C0} - E$

4) CIRCUITO RL

Dato  allora $i_L(t) = ?$ $\qquad i_L(t) = \underbrace{\left(I_0 - \frac{E}{R}\right) e^{-\frac{t}{L/R}}}_{\text{RISPOSTA TRANSITORIA}} + \underbrace{\frac{E}{R}}_{\text{RISPOSTA A REGIME}} = \underbrace{I_0\, e^{-\frac{t}{L/R}}}_{\text{EVOLUZIONE LIBERA}} + \underbrace{\frac{E}{R}\left(1 - e^{-\frac{t}{L/R}}\right)}_{\text{RISPOSTA FORZATA}}$

DIM: $\boxed{t=0^-}$ $i_L = I_0$

$\boxed{t=0^+}$ 

x LKT: $E - V_R - V_L = 0$
$V_R = R \cdot i$
$V_L = L\,\frac{di}{dt}$

$\Rightarrow E - Ri - L\,\frac{di}{dt} = 0 \qquad Li' + Ri = E \qquad i' + i\,\frac{R}{L} = \frac{E}{L}$ $\qquad i' + i\,\frac{R}{L} = 0 \qquad \lambda + \frac{R}{L} = 0 \qquad \lambda = -\frac{R}{L} \qquad o(t) = A\, e^{-\frac{t}{L/R}}$

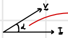
cost. $\qquad P(t) = K \qquad P'(\cancel{t}) + P(t) \cdot \frac{R}{L} = \frac{E}{L} \qquad K = \frac{E}{R}$ $\qquad$ *

* $i_L(t) = A\, e^{-\frac{t}{L/R}} + E/R \qquad$ ma $\qquad i_L(0) = I_0 = A + E/R \rightarrow A = I_0 - E/R$

5) TRASFORMATA DI STEINETZ $\longrightarrow$ $S[a(t)] = \frac{\sqrt{2}}{T}\int_0^T a(t)\, e^{-j\omega t}\, dt$

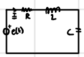
6) COMPONENTI FASORIALI

- RESISTORE: $\alpha = 0$
- INDUTTORE: $\alpha = -\pi/2$
- CONDENSATORE: $\alpha = \pi/2$



SFASAMENTO DI I RISPETTO A V $\rightarrow$ IPOTIZZATO A $\emptyset$

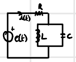
7) RISONANZA

Dato  $e(t) = \hat{V}\cos(\omega t + \alpha_V)$ $\qquad$ Facciamo studio in frequenza di $i(t)$

DIM: $Z_{eq} = R + j\overbrace{(\underbrace{\omega L}_{X_L} - \underbrace{\tfrac{1}{\omega C}}_{X_C})}^{X}$ $\qquad \exists\, \omega_0 : X = 0 \rightarrow \omega_0 L - \frac{1}{\omega_0 C} = 0 \qquad \omega_0 = \frac{1}{\sqrt{LC}}$ PULSAZIONE DI RISONANZA

1) $\omega < \omega_0 \qquad X_C > X_L$
2) $\omega = \omega_0 \qquad X_C = X_L$
3) $\omega > \omega_0 \qquad X_C < X_L$

8) ANTIRISONANZA

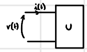
Dato  $e(t) = \hat{V}\cos(\omega t + \alpha_V)$

DIM: $Z_{eq} = j\underbrace{\left(\frac{\omega L}{1 - \omega^2 LC}\right)}_{X}$ $\qquad \exists\, \omega_0 : X = +\infty \longrightarrow 1 - \omega_0^2 LC = 0 \qquad \omega_0 = \frac{1}{\sqrt{LC}}$

1) $\omega = 0 \qquad (X_L, X_C) = (0, \infty)$
2) $\omega \to +\infty \qquad (X_L, X_C) = (\infty, 0)$
3) $\omega = \omega_0 \qquad (X_L, X_C) = (\infty, \infty) \xrightarrow{\text{APERTO}} I = \emptyset$

9) POTENZA IN REGIME SINUSOIDALE

Dati $v(t) = V\cos(\omega t)$ e $i(t) = I\cos(\omega t - \varphi)$



ENERGIA IMMAGAZZINATA DA U
$\int_0^T p(t)\,dt = VI\cos\varphi \cdot T$

DIM: $i(t) = I\cos(\omega t - \varphi) = I[\cos(\omega t)\cos(\varphi) + \sin(\omega t)\sin(\varphi)] = I\cos(\omega t)\cos(\varphi) + I\sin(\omega t)\sin(\varphi) = \underbrace{i_a(t)}_{\text{CORRENTE ATTIVA}} + \underbrace{i_r(t)}_{\text{CORRENTE REATTIVA}}$

POTENZA COMPLESSA $p(t) = v(t)\cdot i(t) = \underbrace{v(t)\cdot i_a(t)}_{\text{POT. ATTIVA} > 0} + \underbrace{v(t)\cdot i_r(t)}_{\text{POT. REATTIVA}}$

POT. REATTIVA ⟶ < 0 CARICO CAPACITIVO, > 0 CARICO INDUTTIVO

10) RIFASAMENTO IN MONOFASE ⟶ per evitare perdite lungo la linea per effetto Joule → ridurre I

$P = VI_L\cos\varphi \rightarrow I_L = \frac{P}{V\cos\varphi}$ $\quad I_L \ll$ se $\cos\varphi \gg$ ovvero $\varphi \ll$ ⟶ $\underline{V}$ e $\underline{I}$ più in fase

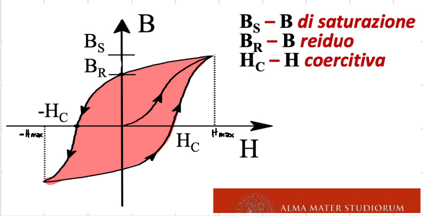
Se U e Ω-induttivo sfasa in ritardo ⟶ serve condensatore per sfasare in anticipo ⟶ C? : $\varphi \rightarrow \varphi'$ $\quad C = \frac{P(\tan\varphi - \tan\varphi')}{\omega V^2}$

11) SISTEMI TRIFASE

$\cos\varphi = \cos\left(\text{arctg}\left(\frac{Q}{P}\right)\right)$ (Q: POT REATTIVA, P: POT ATTIVA) → SIST NON BILANCIATO → $\cos\varphi = \frac{P}{\sqrt{P^2+Q^2}}$

- UTILIZZO ⟶ alternatori
  ⟶ potenza istantanea tot. é costante
  ⟶ più economico di singola fase × stesso trasporto di energia
- SISTEMA IMPEDENZE BILANCIATE ⟶ impedenze uguali in stella/triangolo
- SISTEMA TRIFASE BILANCIATO ⟶ sist. trifase con tensioni simmetriche e correnti equilibrate
- NEUTRO ⟶ quarta linea collegata alla stella dei generatori e alla stella del carico ⟶ ogni fase delle tensioni a stella alimenta una fase delle impedenze a stella
  ↳ quando carico é squilibrato

12) CICLO DI ISTERESI



0) Momento iniziale, campi nulli → magnetizzazione nulla

1) É stata percorsa la CURVA DI PRIMA MAGNETIZZAZIONE, B aumentato con H, raggiunto il livello di saturazione B_s

2) La corrente é diminuita, quindi anche H. Ma in $H=0$ abbiamo della MAGNETIZZAZIONE RESIDUA, quindi il campo magnetico non torna nullo. ⟶ MANTIENE PROPRIETÁ MAGNETICA ANCHE SENZA CAMPO

3) La corrente é invertita e il campo si annulla in $-H_c$ → CAMPO DI COERCIZIONE

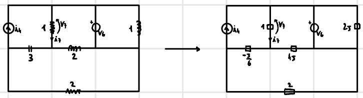
4) H é diminuito ulteriormente, facendo diventare B negativo fino a $-H_{max}$

5-7) H riaumenta fino a chiudere il ciclo

● – DENSITÁ ENERGIA DISSIPATA (W_{ist})

- MATERIALI DOLCI : $W_{ist}\downarrow\downarrow$ $\quad \Gamma_c \uparrow\uparrow$ $\quad B_m \uparrow$ $\quad H_c \downarrow\downarrow$ ⟶ × MACCHINE ELETTRICHE
- MATERIALI FORTI : $W_{ist}\uparrow\uparrow$ $\quad$? $\quad B_m \uparrow\uparrow$ $\quad H_c\uparrow\uparrow$ ⟶ × MAGNETI PERMANENTI

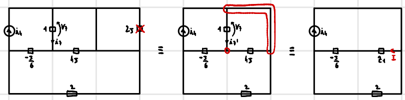
ES. AC - EDO



$V_6(t) = 2\sin(2t+105) = 2\cos(2t+15) \rightarrow V_6 = \sqrt{2}\angle 15 = 1,36603 + 0,36603\,j$

$i_4(t) = 3\sin(2t+90) = 3\cos(2t) \rightarrow I_4 = \frac{3}{\sqrt{2}}\angle 0 = 2,12132$

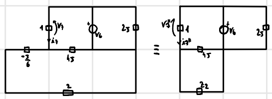
$I_4 \neq 0)$



$i_7' = I\cdot\frac{4j}{1+4j} = i_4\cdot\frac{2}{2+Z_1-j/6}\cdot\frac{4j}{1+4j} = 1,36476 + 0,30755\,j = V_7'$

$Z_1 = 1//4j = 0,94118 + 0,23529\,j$

$V_6 \neq 0)$



$Z_{eq} = 2j//\overbrace{(1+4j//Z_2)}^{2,71174+0,71947\,j} = 0,73552 + 1,26247\,j$

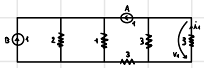
$Z_2 = 2-j/6$

$I_{eq} = 0,6871 - 0,68172\,j$

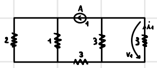
$i_7'' = I_{eq}\cdot\frac{2j}{2j+1+4j//Z_2} = 0,50409 + 0,00129\,j = V_7''$

$\Rightarrow i_7 = V_7 = 1,86885 + 0,30884\,j = 1,8942\angle 9° \longrightarrow V_7(t) = I_7(t) = 2,6788\cos(2t+9)$

ES. 4 DC

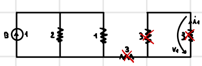


$A \neq 0)$



$i_1' = \frac{1}{2}$

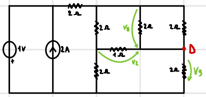
$B \neq 0)$



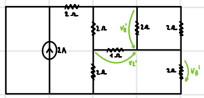
$i_1'' = 0$

$i_1 = \frac{1}{2} \rightarrow V_1 = \frac{3}{2}$

ES. 5 DC

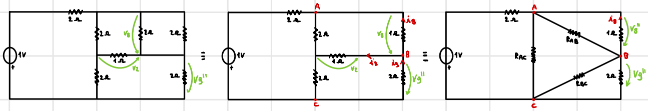


2A ≠ 0)



$V_8' = 0$
$V_2' = 0$
$V_g' = 0$

1V ≠ 0)



$$R_{AC} = \frac{R_AR_B + R_AR_C + R_BR_C}{R_B} = \frac{8}{1} = 8$$

$$R_{AB} = \frac{R_AR_B + R_AR_C + R_BR_C}{R_C} = \frac{8}{2} = 4$$

$$R_{BC} = \frac{R_AR_B + R_AR_C + R_BR_C}{R_A} = \frac{8}{2} = 4$$

$$R_{eq} = 2 + R_{AC} // (1 // R_{AB} + 2 // R_{BC}) = 2 + R_{AC} // (4/5 + 4/3) = \frac{70}{19}$$

$$I_{eq} = V / R_{eq} = \frac{19}{70}$$

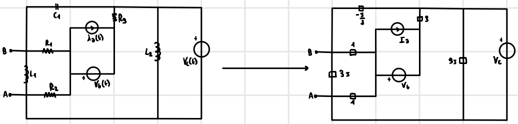
$$V_8'' = 1 \cdot i_8 = I_{eq} \cdot \frac{R_{AC}}{R_{AC} + \underbrace{1 // R_{AB}}_{4/5} + \underbrace{2 // R_{BC}}_{4/3}} \cdot \frac{R_{AB}}{R_{AB} + 1} = \frac{19}{70} \cdot \frac{15}{19} \cdot \frac{4}{5} = \frac{6}{35} \longrightarrow V_8 = \frac{6}{35}$$

$$V_g'' = 2 \cdot i_g = 2 I_{eq} \cdot \frac{R_{AC}}{R_{AC} + \underbrace{1 // R_{AB}}_{4/5} + \underbrace{2 // R_{BC}}_{4/3}} \cdot \frac{R_{BC}}{R_{BC} + 2} = 2 \cdot \frac{19}{70} \cdot \frac{15}{19} \cdot \frac{2}{3} = \frac{2}{7} \longrightarrow V_g = \frac{2}{7}$$

× LKC D: $i_g = i_2 + i_8$

$$\frac{V_g}{2} = \frac{V_2}{1} + \frac{V_8}{1} \rightarrow V_2 = \frac{V_g}{2} - V_8 = -\frac{1}{35}$$

ES. 5 REGIME SINUSOIDALE

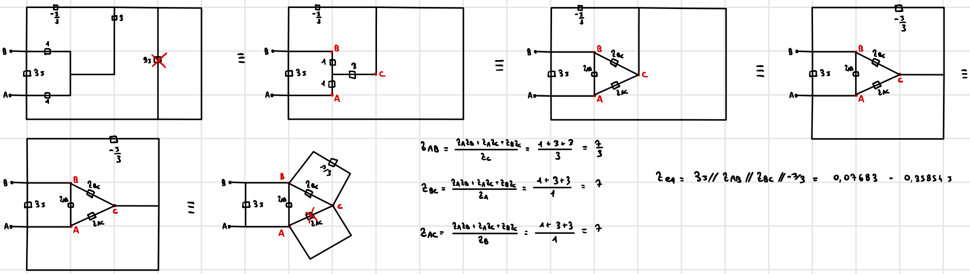


$i_a(t) = 3\cos(3t - 75)$ $\longrightarrow$ $I_a = \frac{3}{\sqrt{2}} \angle -75 = 0{,}54904 - 2{,}04904\,j$

$V_b(t) = 3\sin(3t + 105) = 3\cos(3t + 15)$ $\longrightarrow$ $V_b = \frac{3}{\sqrt{2}} \angle 15 = 2{,}04904 + 0{,}54904\,j$

$V_c(t) = 2\cos(3t - 165)$ $\longrightarrow$ $V_c = \sqrt{2} \angle -165 = -1{,}36603 - 0{,}36603\,j$

Z_{eq})



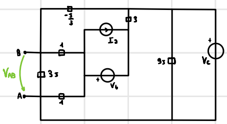
$$Z_{AB} = \frac{Z_A Z_B + Z_A Z_C + Z_B Z_C}{Z_C} = \frac{1+3+3}{3} = \frac{7}{3}$$

$$Z_{BC} = \frac{Z_A Z_B + Z_A Z_C + Z_B Z_C}{Z_A} = \frac{1+3+3}{1} = 7$$

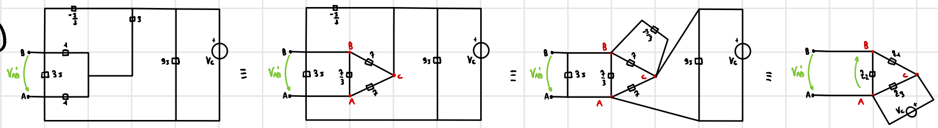
$$Z_{AC} = \frac{Z_A Z_B + Z_A Z_C + Z_B Z_C}{Z_B} = \frac{1+3+3}{1} = 7$$

$$Z_{eq} = 3j \,//\, Z_{AB} \,//\, Z_{BC} \,//\, -3/3 = 0{,}07683 - 0{,}35854\,j$$

V_{AB})



$V_C \neq 0$)



$Z_1 = 7 \,//\, -\frac{3}{3} = 0{,}01584 - 0{,}93258\,j$

$Z_2 = 3j \,//\, \frac{7}{3} = 1{,}45385 + 1{,}13077\,j$

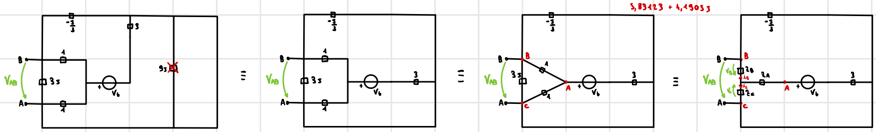
$Z_3 = 7 \,//\, 9j = 4{,}36134 + 3{,}39231\,j$

$Z_{eq} = Z_3 \,//\, (Z_2 + Z_1) = 1{,}1068 + 0{,}65663\,j$

$I_{eq} = V_c / Z_{eq} = -1{,}03803 + 0{,}29698\,j$

$$V_{AB}' = -Z_2 \cdot I_{eq} \cdot \frac{Z_3}{\underbrace{Z_3 + Z_2 + Z_1}_{5{,}83123 + 4{,}1905\,j}} = -I_{eq} \cdot \frac{Z_2 Z_3}{Z_1 + Z_2 + Z_3} = -I_{eq} \cdot (1{,}08191 + 0{,}91189\,j) = +1{,}11868 + 0{,}64261\,j$$

$V_B \neq 0$)



$Z_B = \frac{Z_{AB} \cdot Z_{BC}}{Z_{AB} + Z_{BC} + Z_{AC}} = \frac{3j}{2+3j} = 0{,}69231 + 0{,}46154\,j$

$Z_A = \frac{Z_{AB} \cdot Z_{AC}}{Z_{AB} + Z_{BC} + Z_{AC}} = \frac{1}{2+3j} = 0{,}15385 - 0{,}23077\,j$

$Z_C = \frac{Z_{AC} \cdot Z_{BC}}{Z_{AB} + Z_{BC} + Z_{AC}} = \frac{3j}{2+3j} = 0{,}69231 + 0{,}46154\,j$

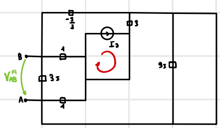
$Z_{eq} = Z_A + 3 + Z_C \,//\, (Z_B - 3/3) = 3{,}51699 - 0{,}09057\,j$

$I_{eq} = V_B / Z_{eq} = 0{,}57821 + 0{,}171\,j$

$V_{AB}'' + V_C - V_B = 0 \rightarrow V_{AB}'' = V_B - V_C = i_B \cdot Z_B - i_C \cdot Z_C = i_B Z_B - (I_{eq} - i_B) Z_C =$
$= i_B Z_B - I_{eq} Z_C + i_B Z_C = i_B (Z_B + Z_C) - I_{eq} Z_C = i_B \cdot 2 \cdot Z_C - I_{eq} Z_C =$
$= Z_C (2 i_B - I_{eq}) = -0{,}03061 + 0{,}09892\,j$

$i_B = I_{eq} \cdot \frac{Z_C}{Z_C + Z_B - 3/3} = I_{eq} \cdot (0{,}543396 + 0{,}10189\,j) = 0{,}29677 + 0{,}15183\,j$

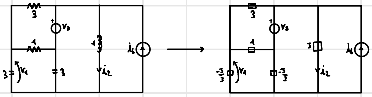
$I_a \neq 0$)



$V_{AB}''' = 0$

$$\Rightarrow \quad V_{AB} = 1{,}36807 + 0{,}74133\,j \xrightarrow{\times\sqrt{2}} 1{,}93474 + 1{,}04868\,j$$

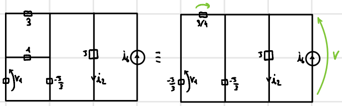
ES. 6 REGIME SINUSOIDALE



$v_a(t) = 3\sin(t+105) = 3\cos(t+15) \longrightarrow V_a = \frac{3}{\sqrt{2}}\angle 15 = 2{,}04904 + 0{,}54904\, j$

$i_b(t) = 3\sin(t-60) = 3\cos(t-150) \longrightarrow I_b = \frac{3}{\sqrt{2}}\angle -150 = -1{,}83712 - 1{,}06066\, j$

$I_b \neq 0$)

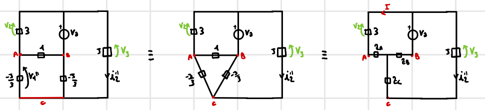


$$V = \frac{i_b}{\frac{1}{j} - \frac{3}{j} + \frac{1}{-5/3 + 3/4}} = -0{,}62838 + 0{,}45584\, j$$

$i_2' = V/j = 0{,}45584 + 0{,}62838\, j$

$V_1' = V \cdot \frac{-5/3}{-\frac{5}{3} + \frac{3}{4}} = 0{,}06549 + 0{,}30848\, j$

$V_a \neq 0$)



$Z_A = \frac{Z_{AB}\, Z_{AC}}{Z_A + Z_B + Z_C} = \frac{-5/3}{1 - \frac{2}{3}j} = \frac{2}{13} - \frac{3}{13}j$

$Z_B = \frac{Z_{AB}\, Z_{BC}}{Z_A + Z_B + Z_C} = \frac{-5/3}{1 - \frac{2}{3}j} = \frac{2}{13} - \frac{3}{13}j$

$Z_C = \frac{Z_{AC}\, Z_{BC}}{Z_A + Z_B + Z_C} = \frac{-1/9}{1 - \frac{2}{3}j} = -\frac{1}{13} - \frac{2}{39}j$

$Z_{eq} = Z_B + \overbrace{(3+Z_A)}^{\frac{41}{13} - \frac{3}{13}j} // \overbrace{(j + Z_C)}^{-\frac{1}{13} + \frac{37}{39}j} = \frac{53}{146} + \frac{51}{93}j$

$\downarrow$

$I_{eq} = V_a / Z_{eq} = 1{,}81879 - 1{,}98788\, j$

$\downarrow$

$i_2'' = I_{eq} \cdot \frac{3 + Z_A}{3 + Z_A + j + Z_C} = I_{eq} \cdot (0{,}95548 - 0{,}29793\, j) = \begin{matrix} 1{,}14554 \\ -2{,}44128\, j \end{matrix}$

$\downarrow$

$I = I_{eq} - i_2 = 0{,}67325 + 0{,}4534\, j$

$V_2 = 3I$

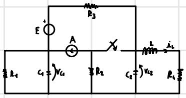
$V_3 = i_2'' \cdot j$

$V_3 - V_2 - V_1'' = 0 \qquad V_1'' = V_3 - V_2 = i_2'' j - 3I = 0{,}42153 - 0{,}21466\, j$

$\Rightarrow i_2 = 1{,}60138 - 1{,}81269\, j = 2{,}41873 \angle -49 \longrightarrow i_2(t) = 3{,}4206 \cos(t - 49°)$

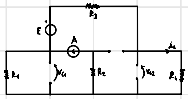
$\Rightarrow V_1 = 0{,}48702 + 0{,}09382\, j = 0{,}49597 \angle 11 \longrightarrow V_1(t) = 0{,}70141 \cos(t + 11°)$

ES. 1 22/07



$R_1 = R_2 = 2$
$R_3 = R_4 = 4$
$L = 2$
$C_1 = 1 \quad C_2 = 2$
$E = 2 \quad A = 1$

$t = 0^-$

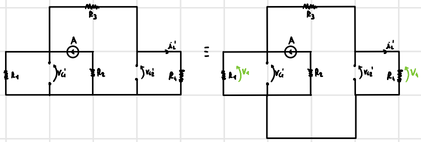


$W_L = \frac{1}{\cancel{2}} \cdot \cancel{2} \cdot i_L^2 = \frac{4}{25} = 0{,}16$

$W_{C1} = \frac{1}{2} \cdot 1 \cdot V_{C1}^2 = \frac{1}{\cancel{2}} \cdot \frac{\cancel{36}^{18}}{25} = 0{,}72$

$W_{C2} = \frac{1}{\cancel{2}} \cdot \cancel{2} \cdot V_{C2}^2 = \frac{64}{25} = 2{,}56$

A ≠ 0)



$V_{C1}' = V_1 = (A - i_L')R_1 = \frac{4}{5} \cdot 2 = \frac{8}{5}$

$V_{C2}' = V_4 = i_L' \cdot R_4 = \frac{4}{5}$

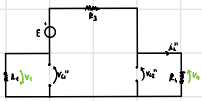
$i_L' = A \cdot \frac{R_1}{R_1 + R_3 + R_4} = \frac{2}{2 + 4 + 4} = \frac{1}{5}$

$V_{C1} = \frac{8}{5} - \frac{2}{5} = \frac{6}{5}$

$V_{C2} = \frac{4}{5} + \frac{4}{5} = \frac{8}{5}$

$i_L = \frac{1}{5} + \frac{1}{5} = \frac{2}{5}$

E ≠ 0)

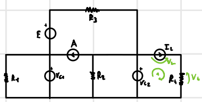


$i_L'' = \frac{E}{R_3 + R_4 + R_1} = \frac{2}{4 + 4 + 2} = \frac{1}{5}$

$V_{C2}'' = V_4 = i_L'' \cdot R_4 = \frac{4}{5}$

$V_{C1}'' = -V_1 = -i_L'' \cdot R_1 = -\frac{2}{5}$

$t = 0^+$

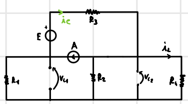


$V_{C1} = 2{,}2$
$V_{C2} = 2{,}7$
$I_L = 6{,}2$

$\frac{di_L}{dt} = \frac{V_L}{L} = \frac{22{,}1}{2} = 11{,}05$

$V_{C2} + V_L - V_4 = 0 \qquad V_L = V_4 - V_{C2} = I_L \cdot R_4 - V_{C2} = 22{,}1$

$t = +\infty$

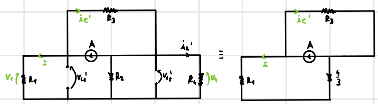


$A = 8{,}3$
$E = 2{,}1$

$R_1 = R_2 = 2$
$R_3 = R_4 = 4$
$L = 2$
$C_1 = 1 \quad C_2 = 2$

$P_E = E \cdot i_e =$
$W_L = \frac{1}{2} \cdot 2 \cdot i_L^2 =$
$Q_{C1} = C_1 V_{C1} =$
$Q_{C2} = C_2 V_{C2} =$

A ≠ 0)

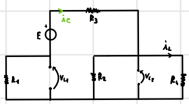


$i_e' = A \cdot \frac{R_1 + 4/3}{R_1 + 4/3 + R_3} = 8{,}3 \cdot \frac{2 + 4/3}{6 + 4/3} = \frac{83}{22} \longrightarrow I = A - i_e' = \frac{249}{55}$

$V_{C1}' = V_1 = I \cdot R_1 = \frac{249}{55} \cdot 2 = \frac{498}{55}$

$i_L' = -I \cdot \frac{R_2}{R_2 + R_3} = -\frac{249}{55} \cdot \frac{1}{3} = -\frac{83}{55} \longrightarrow V_{C2}' = V_4 = i_L' \cdot R_4 = -\frac{83}{55} \cdot 4 = -\frac{332}{55}$

E ≠ 0)



$i_e'' = \frac{E}{R_1 + R_3 + R_2 /\!/ R_4} = \frac{2{,}1}{2 + 4 + 4/3} = \frac{63}{220} \longrightarrow i_L'' = i_e'' \cdot \frac{R_2}{R_2 + R_4} = \frac{i_e''}{3} = \frac{21}{220}$

$\downarrow$

$V_{C2}'' = i_L'' \cdot R_4 = \frac{21}{220} \cdot 4 = \frac{21}{55}$

$V_{C1}'' = -i_e'' \cdot R_1 = -\frac{63}{110}$

$i_e = \frac{83}{22} + \frac{63}{220} = 4{,}05909$

$i_L = -\frac{83}{55} + \frac{21}{220} = -1{,}41364$

$V_{C1} = \frac{498}{55} - \frac{63}{110} = 8{,}48182$

$V_{C2} = -\frac{332}{55} + \frac{21}{55} = -5{,}65455$

$\Rightarrow$

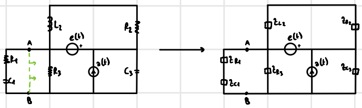
$P_E = i_e \cdot E = 8{,}524$

$W_L = \frac{1}{\cancel{2}} \cancel{2} \cdot i_L^2 = 1{,}998$

$Q_{C1} = V_{C1} \cdot C_1 = 8{,}482$

$Q_{C2} = V_{C2} \cdot C_2 = -11{,}309$

Es. 2 22/07



$R_1 = 1$ $\quad Z_{R_1} = 1$

$R_2 = R_3 = 2$ $\quad Z_{R_2} = Z_{R_3} = 2$

$L_2 = 3$ $\quad Z_{L_2} = 3j$

$C_1 = 1$ $\quad Z_{C_1} = -j$

$C_2 = 3$ $\quad Z_{C_3} = -j/3$

$e(t) = \sqrt{2}\,\sin(t-165) = \sqrt{2}\,\cos(t-255)$

$\downarrow$

$E = 1\angle 255$

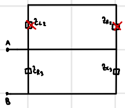
$a(t) = \cos(t+30)$

$\downarrow$

$A = \frac{1}{\sqrt{2}}\angle 30 = 0{,}70711\ \angle 30$

NUOVI DATI: $Z_{R_1} = 5{,}7$ $\quad Z_{R_2} = 8{,}7$ $\quad Z_{R_3} = 2{,}9$ $\quad Z_{L_2} = 9{,}1j$ $\quad Z_{C_1} = -0{,}3j$ $\quad Z_{C_3} = -4{,}9j$

Z_{AB})

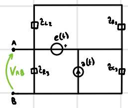


$Z_{AB} = Z_{R_3} \parallel Z_{C_3} = 2{,}14772 - 1{,}27109j$

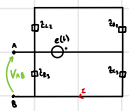
NUOVI DATI: $E = 4\angle 14$ $\quad Z_{R_1} = 5$ $\quad Z_{R_2} = 1$ $\quad Z_{R_3} = 6$ $\quad Z_{L_2} = 3j$ $\quad Z_{C_1} = -j$ $\quad Z_{C_3} = -j/6$

$A = 2\angle -87$

V_{AB})



$E \neq 0$)



$I_{eq} = \dfrac{E}{(Z_{L_2} + Z_{R_2}) \parallel (Z_{R_3} + Z_{C_3})} = 1{,}32031 - 0{,}88847j$

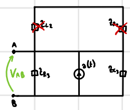
$\downarrow$

$I = I_{eq} \cdot \dfrac{Z_{L_2} + Z_{R_2}}{Z_{L_2} + Z_{R_2} + Z_{R_3} + Z_{C_3}} = I_{eq} \cdot (0{,}27179 + 0{,}31836j) = 0{,}64189 + 0{,}17911j$

$\downarrow$

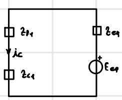
$V'_{AB} = -I \cdot Z_{R_3} = -3{,}85134 - 1{,}07466j = 3{,}998\angle -164$

$A \neq 0$)



$V''_{AB} = A \cdot \dfrac{Z_{C_3}}{Z_{C_3} + Z_{R_3}} \cdot Z_{R_3} = 0{,}33320\ \angle -175$

NUOVI DATI: $E_{eq} = 0{,}471\angle -15$ $\quad Z_{eq} = \frac{4}{3} + \frac{2}{3}j$ $\quad R_1 = 3 \rightarrow Z_{R_1} = 3$ $\quad C_1 = 4 \rightarrow Z_{C_1} = -3/4$



$i_C = I_{eq} = \dfrac{E_{eq}}{Z_{eq} + Z_{R_1} + Z_{C_1}} = 0{,}108\angle -20$