Image Processing and Computer Vision

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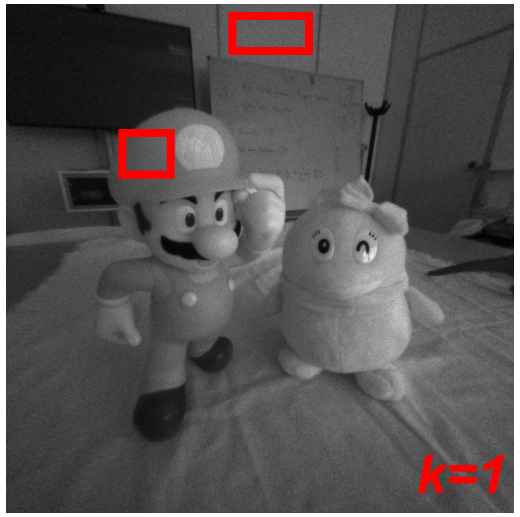
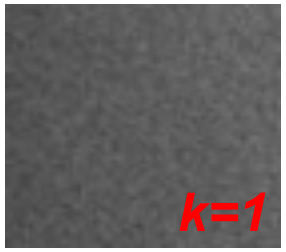
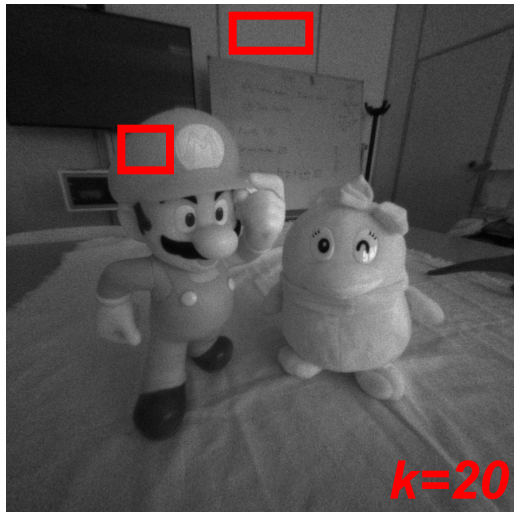
Visualizing and Understanding Noise

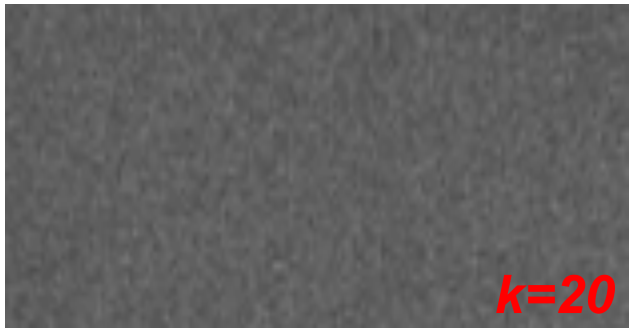
THERE WILL ALWAYS BE NOISE → YOU'LL NEVER HAVE 2 IDENTICAL PHOTOS EVEN IF YOU DON'T MOVE

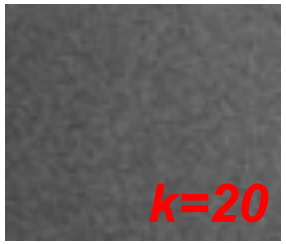
t_k











$$I_k(p) = \tilde{I}(p) + n_k(p) \quad \text{with} \quad n_k(p)\ i.i.d. \quad \text{and} \quad n_k(p) \sim N(0, \sigma)$$

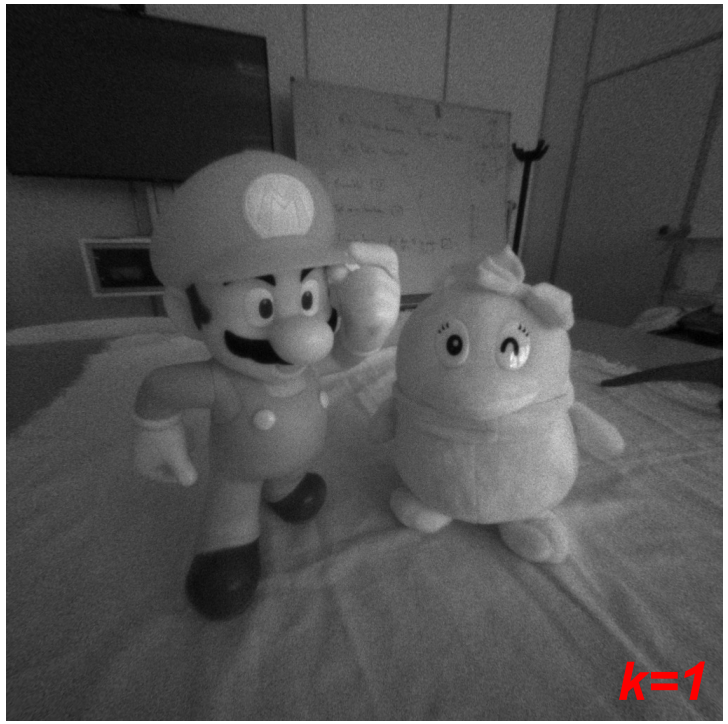
Denoising by taking a mean across time

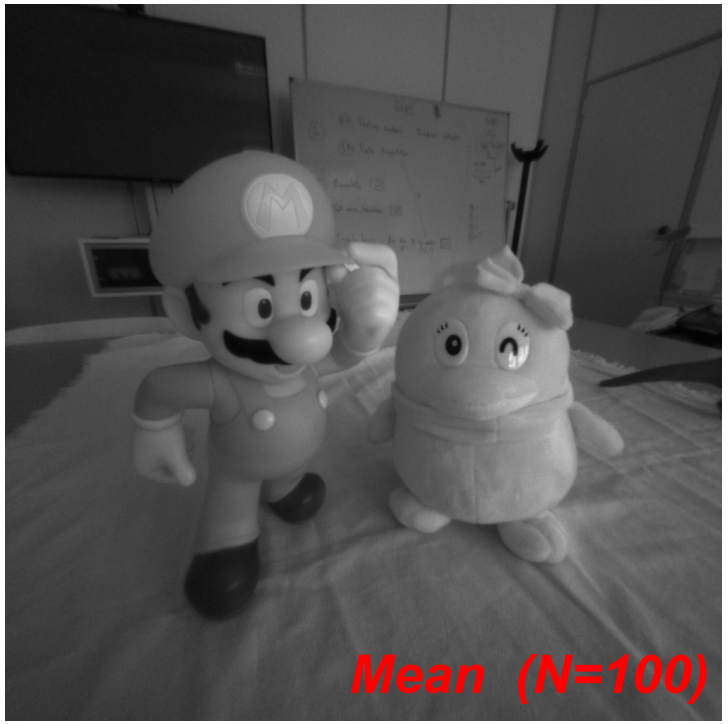
IN ORDER TO OVERCOME NOISE, WE COMPUTE A MEAN OVER TIME TO TRY TO ESTIMATE THE REAL IMAGE

NOISE DEPENDS ON TIME WHILE IMAGE NOT

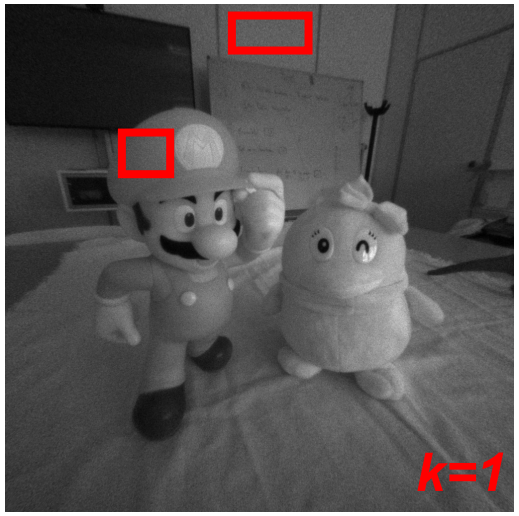
$$O(p) = \frac{1}{N}\sum_{k=1}^{N} I_k(p) = \frac{1}{N}\sum_{k=1}^{N}\left(\tilde{I}(p) + n_k(p)\right) = \frac{1}{N}\sum_{k=1}^{N}\tilde{I}(p) + \frac{1}{N}\sum_{k=1}^{N} n_k(p) \cong \tilde{I}(p)$$

IT WORKS ONLY IF THE IMAGE IS STATIC !!!

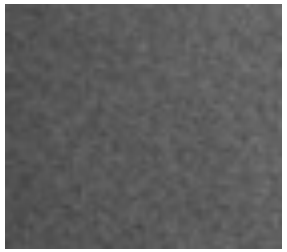




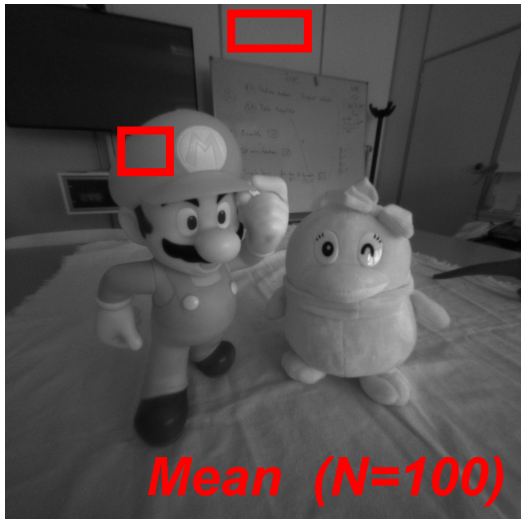
The mean image is far less noisy



k=1



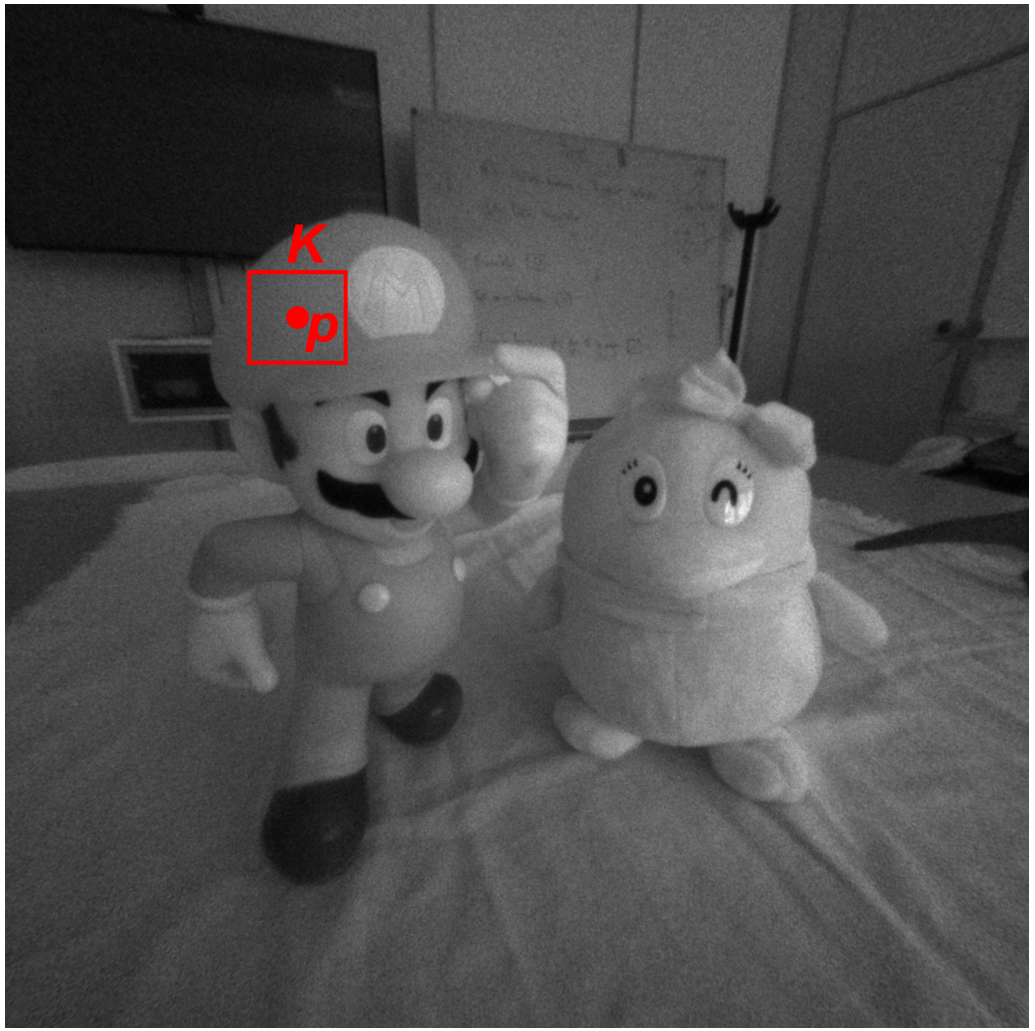
k=1



Mean (N=100)

Mean (N=100)

What if we are given a single image ?



- We may compute a mean across neighbouring pixels, i.e. a spatial rather than temporal mean

$$O(p) = \frac{1}{|K|}\sum_{q \in K} I(q) = \frac{1}{|K|}\sum_{q \in K}\left(\tilde{I}(q) + n(q)\right) =$$

SIZE OF THE NEIGHBORHOOD

$$\frac{1}{|K|}\sum_{q \in K}\tilde{I}(q) + \frac{1}{|K|}\sum_{q \in K} n(q) \cong \tilde{I}(q)$$

NOT SO SENSIBLE TO THE REAL IMAGE

TO MANY PIXELS TOGETHER

WORKS ONLY IN UNIFORM REGIONS

IF THERE ARE MANY DETAILS IT CAN'T COMPUTE THEM WELL

Denoising Filters

TOO BIG

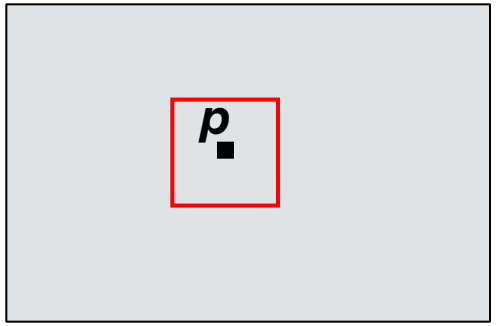
TOO SMALL

TOO SENSIBLE TO NOISE

- How large should K be?
  - It is a trade-off!

Image Filters

- *Image Filters* are image processing operators that compute the new intensity (colour) of a pixel, ***p***, based on the intensities (colours) of those belonging to a neighbourhood (aka *support*) of ***p***.



$$o(p) = f(S(p))$$

- They accomplish a variety of useful image processing functions, such as e.g. *denoising* and *sharpening* (edge enhancement).

- An important sub-class of filters is given by *Linear* and *Translation-Equivariant (LTE)* operators

- *Signal theory:* their application in image processing consist in a *2D convolution* between the input image and the *impulse response function* (*point spread function* or *kernel*) of the LTE operator.

- LTE operators are used as *feature extractors* in **CNNs** (Convolutional Neural Networks).

LTE Operators and Convolution

- Given an input 2D signal $i(x,y)$, a 2D operator, $\mathrm{T}\{\cdot\}$: $o(x,y) = T\{\, i(x,y)\,\}$, is said to be linear *iff*:

$$T\{\alpha\, i_1(x,y) + \beta\, i_2(x,y)\} = \alpha\, o_1(x,y) + \beta\, o_2(x,y) \quad \text{with} \quad o_1 = T\{i_1\} \quad \text{and} \quad o_2 = T\{i_2\}$$

LINEAR COMBINATION

  and α, β are two constants.

- The operator is said to be translation-equivariant *iff*:

$$T\{\, i(x - x_0, y - y_0)\} = o(x - x_0, y - y_0)$$

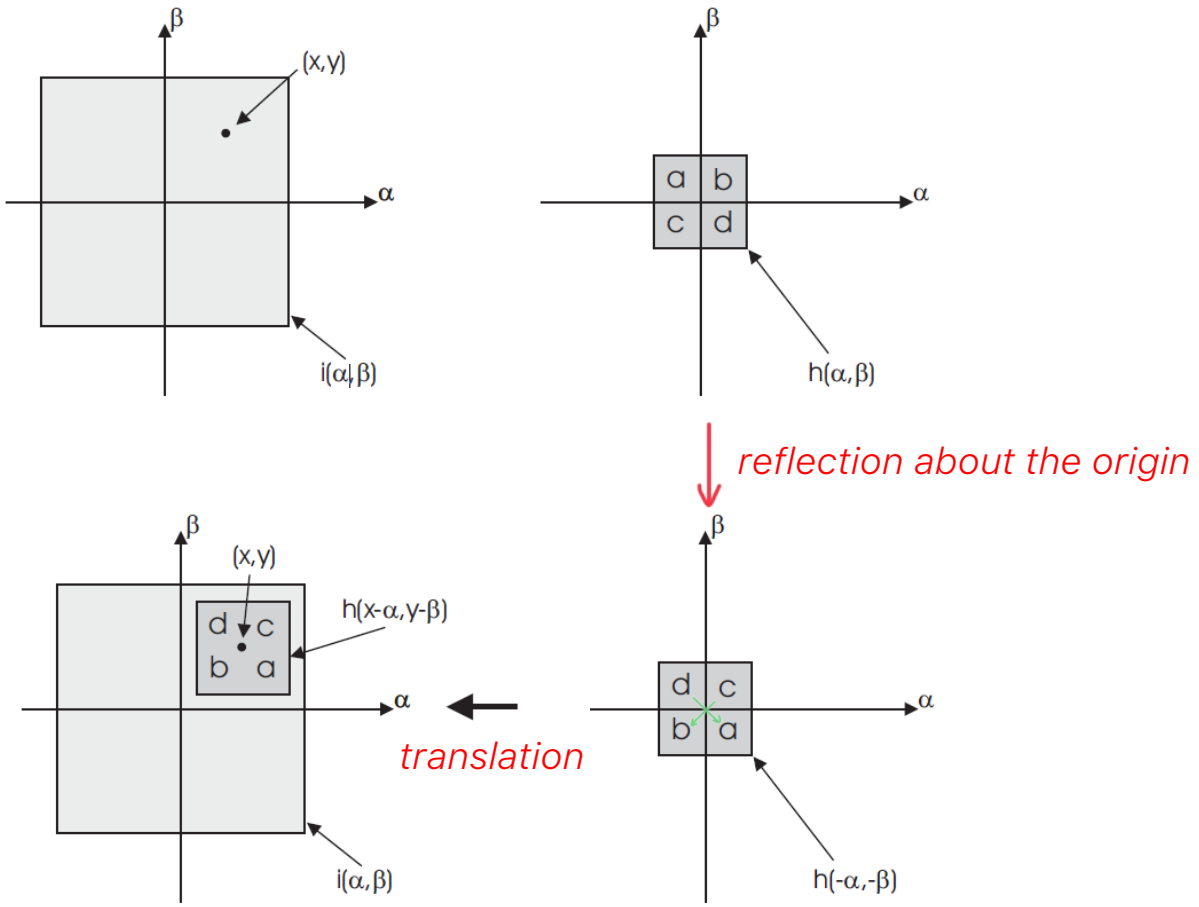
- If the operator is LTE, the output signal is given by the convolution between the input signal and the *impulse response (point spread function)*, $h(x,y) = T\{\delta(x,y)\}$, of the operator:

Unit impulse (Dirac delta function)

$$o(x,y) = T\{i(x,y)\} = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} i(\alpha,\beta) h(x-\alpha, y-\beta)\, d\alpha\, d\beta$$

A Graphical View of Convolution

$$o(x,y) = T\{i(x,y)\} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} i(\alpha,\beta)h(x-\alpha,y-\beta)\,d\alpha\,d\beta$$



Properties of Convolution

- The convolution operation is often denoted using the symbol "$*$", e.g.

$$o(x, y) = i(x, y) * h(x, y)$$

- Some useful properties of convolution are as follows:

WE CAN SPLIT UP CONVOLUTIONS → SPEED UP COMPUTATION

  - Associative Property $f * (g * h) = (f * g) * h$
  - Commutative Property $f * g = g * f$
  - Distributive Property wrt the Sum $f * (g + h) = f * g + f * h$
  - Convolution Commutes with Differentiation $(f * g)' = f' * g = f * g'$

IMPORTANT FOR EDGE DETECTION

Correlation

- The correlation of signal *i(x,y)* wrt signal *h(x,y)* is defined as:

$$i(x,y) \circ h(x,y) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} i(\alpha,\beta) h(x+\alpha, y+\beta)\, d\alpha\, d\beta$$

- Accordingly, the correlation of *h(x,y)* wrt *i(x,y)* is given by:

$$h(x,y) \circ i(x,y) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} h(\alpha,\beta) i(x+\alpha, y+\beta)\, d\alpha\, d\beta$$

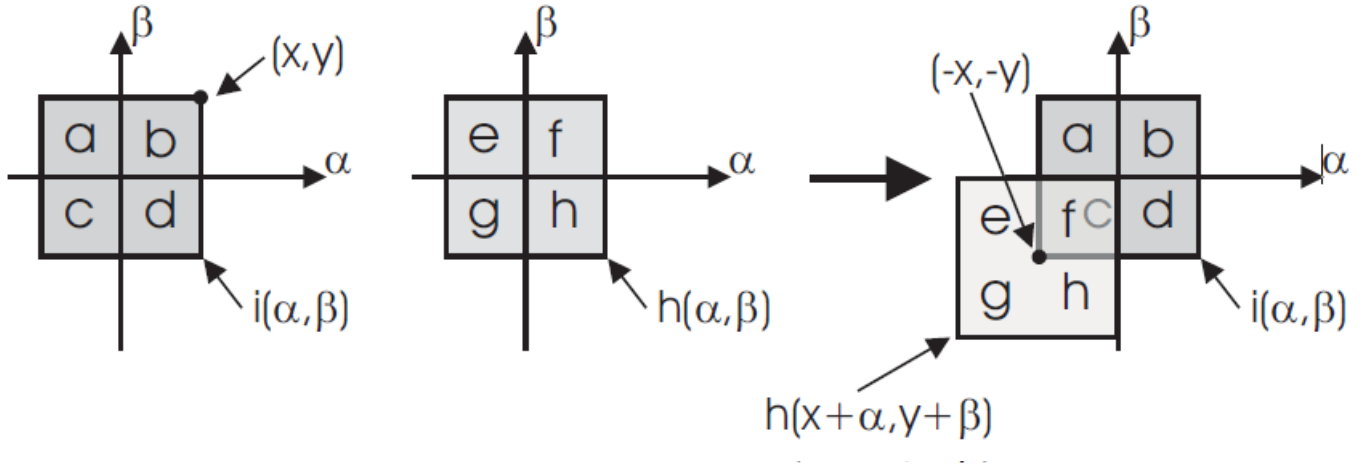
- Unlike convolution, correlation is not commutative:

$$\begin{aligned} h(x,y) \circ i(x,y) &= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} h(\alpha,\beta)\, i(x+\alpha, y+\beta)\, d\alpha\, d\beta \\ &= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} i(\xi,\eta)\, h(\xi-x, \eta-y)\, d\xi\, d\eta \\ &= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} i(\alpha,\beta)\, h(\alpha-x, \beta-y)\, d\alpha\, d\beta \\ &\neq i(x,y) \circ h(x,y) \end{aligned}$$

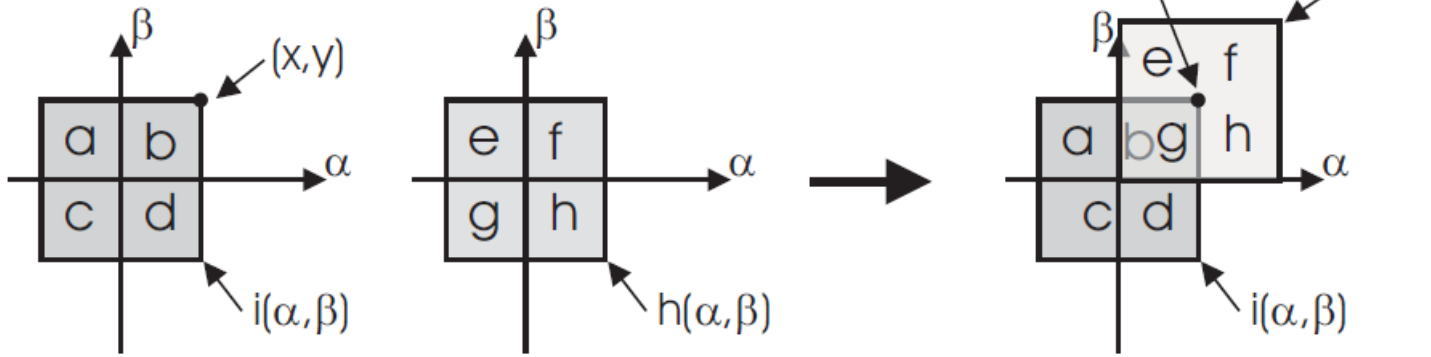
A Graphical View of Correlation

JUST A TRANSLATION, WE DON'T HAVE FLIPS LIKE IN CONVOLUTION

$i \circ h\,(x,y)$:



$h \circ i\,(x,y)$:



Convolution and Correlation

- The correlation of ***h*** wrt ***i*** is similar to convolution: the product of the two signals is integrated after translating ***h*** without reflection.
  - Hence, if ***h*** is an even function $(h(x,y) = h(-x,-y))$, the convolution between ***i*** and ***h***, $(i * h = h * i)$, is the same as the correlation of ***h*** wrt ***i***:

$$i(x,y) * h(x,y) = h(x,y) * i(x,y) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} i(\alpha,\beta)\, h(x-\alpha, y-\beta)\, d\alpha\, d\beta$$

$$= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} i(\alpha,\beta)\, h(\alpha-x, \beta-y)\, d\alpha\, d\beta$$

$$= h(x,y) \circ i(x,y)$$

we can swap

Convolution (i , h) = Correlation (h, i)

- It is worth observing that correlation is never commutative, even if ***h*** is an even function. To recap:
  - Convolution is commutative: $i * h = h * i$
  - Correlation is not commutative: $i \circ h \neq h \circ i$
  - *If h is an even function:* $i * h = h * i = h \circ i$

Discrete Convolution

WE WORK ON FINTE IMAGES WITH A DISCRETE NUMBER OF PIXELS

$$o(x,y) = T\{i(x,y)\} = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} i(\alpha,\beta)h(x-\alpha, y-\beta)\, d\alpha\, d\beta$$

$$O(i,j) = T\{I(i,j)\} = \sum_{m=-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} I(m,n)H(i-m, j-n)$$

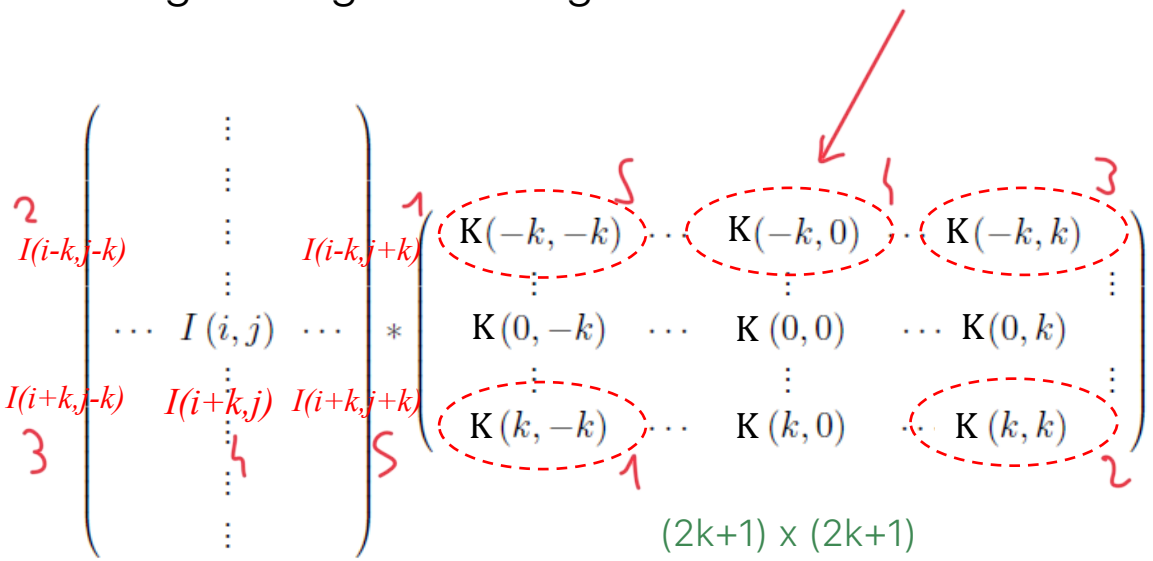
- where $I(i,j)$ and $O(i,j)$ are the discrete 2D input and output signals, respectively, and $\mathrm{H}(\mathrm{i},j) = T\{\delta(i,j)\}$ is the kernel of the discrete LTE operator, i.e. the response to the 2D discrete unit impulse (Kronecker delta function), $\delta(i,j)$.

- As for continuous signals, discrete convolution consists in summing the product of the two signals where one has been reflected about the origin and translated.

- The previously highlighted four major convolution properties hold for discrete convolution too.

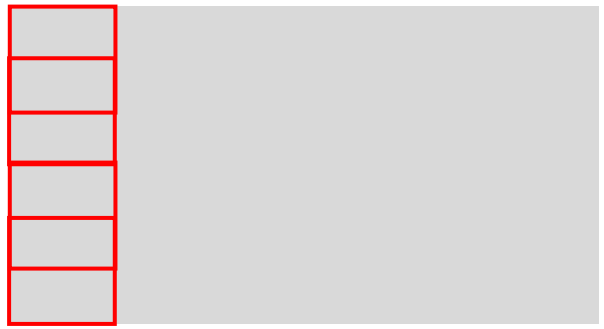
Practical Implementation

- In image processing both the input image and the kernel are stored into matrices of given finite sizes, with the image being much larger than the ***kernel***. One would cycle through the kernel:



$$O(i,j) = \sum_{m=-k}^{k} \sum_{n=-k}^{k} K(m,n) I(i-m, j-n)$$

Conceptually, to obtain the output image we need to slide the kernel across the whole input image and compute the convolution at each pixel (do not overwrite the input matrix!)



Practical Implementation

- In image processing both the input image and the kernel are stored into matrixes of given finite sizes, with the image being much larger than the ***kernel***. One would cycle through the kernel:

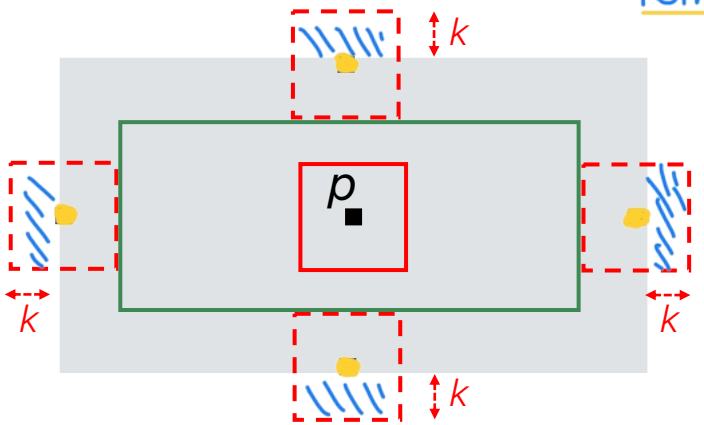
$$\begin{pmatrix} \vdots \\ \cdots\ I(i,j)\ \cdots \\ \vdots \end{pmatrix} * \begin{pmatrix} \mathrm{K}(-k,-k) & \cdots & \mathrm{K}(-k,0) & \cdots & \mathrm{K}(-k,k) \\ \vdots & & \vdots & & \\ \mathrm{K}(0,-k) & \cdots & \mathrm{K}(0,0) & \cdots & \mathrm{K}(0,k) \\ \vdots & & \vdots & & \\ \mathrm{K}(k,-k) & \cdots & \mathrm{K}(k,0) & \cdots & \mathrm{K}(k,k) \end{pmatrix}$$

(2k+1) x (2k+1)

$$O(i,j) = \sum_{m=-k}^{k} \sum_{n=-k}^{k} K(m,n) I(i-m, j-n)$$

Conceptually, to obtain the output image we need to slide the kernel across the whole input image and compute the convolution at each pixel (do not overwrite the input matrix!)

MISSING VALUES FOR CONVOLUTION OF POINTS



- Border Issue, two main options:
  - CROP (common in image processing) ⟶ drop unconvoluted values
  - PAD (preferred in CNNs) ⟶ put padding in unconvoluted values
    - How to pad: zero-padding, replicate (aaa|a.....d|ddd), reflect (cba|abc.....dfg|gfd), reflect_101 (dcb|abcd.....efgh|gfe),

OUTPUT < INPUT

Mean Filter

- Mean filtering is the simplest (and fastest) way to denoise an image. It consists in replacing each pixel intensity by the average intensity over a chosen neighbourhood (e.g. 3x3, 5x5, 7x7…).

- The Mean Filter is an LTE operator as it can be expressed as a convolution with a kernel.
  Below, the kernels for a 3x3 and 5x5 mean filter:

$$\begin{bmatrix} 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1/25 & 1/25 & 1/25 & 1/25 & 1/25 \\ 1/25 & 1/25 & 1/25 & 1/25 & 1/25 \\ 1/25 & 1/25 & 1/25 & 1/25 & 1/25 \\ 1/25 & 1/25 & 1/25 & 1/25 & 1/25 \\ 1/25 & 1/25 & 1/25 & 1/25 & 1/25 \end{bmatrix} = \frac{1}{25}\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

- According to signal processing theory, the Mean Filter carries out a *low-pass filtering* operation, which in image processing is also referred to as *image smoothing*.

- Smoothing is often aimed at image denoising, though sometimes the purpose is to cancel out small-size unwanted details that might hinder the image analysis task.

- Mean filtering is inherently fast because multiplications are not needed.

Example: Gaussian Noise

Original Image

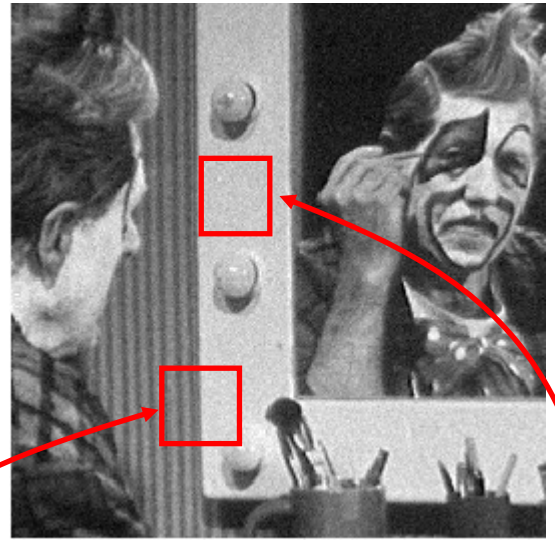


Image corrupted by Gaussian Noise ($\mu=0$, $\sigma=8$)

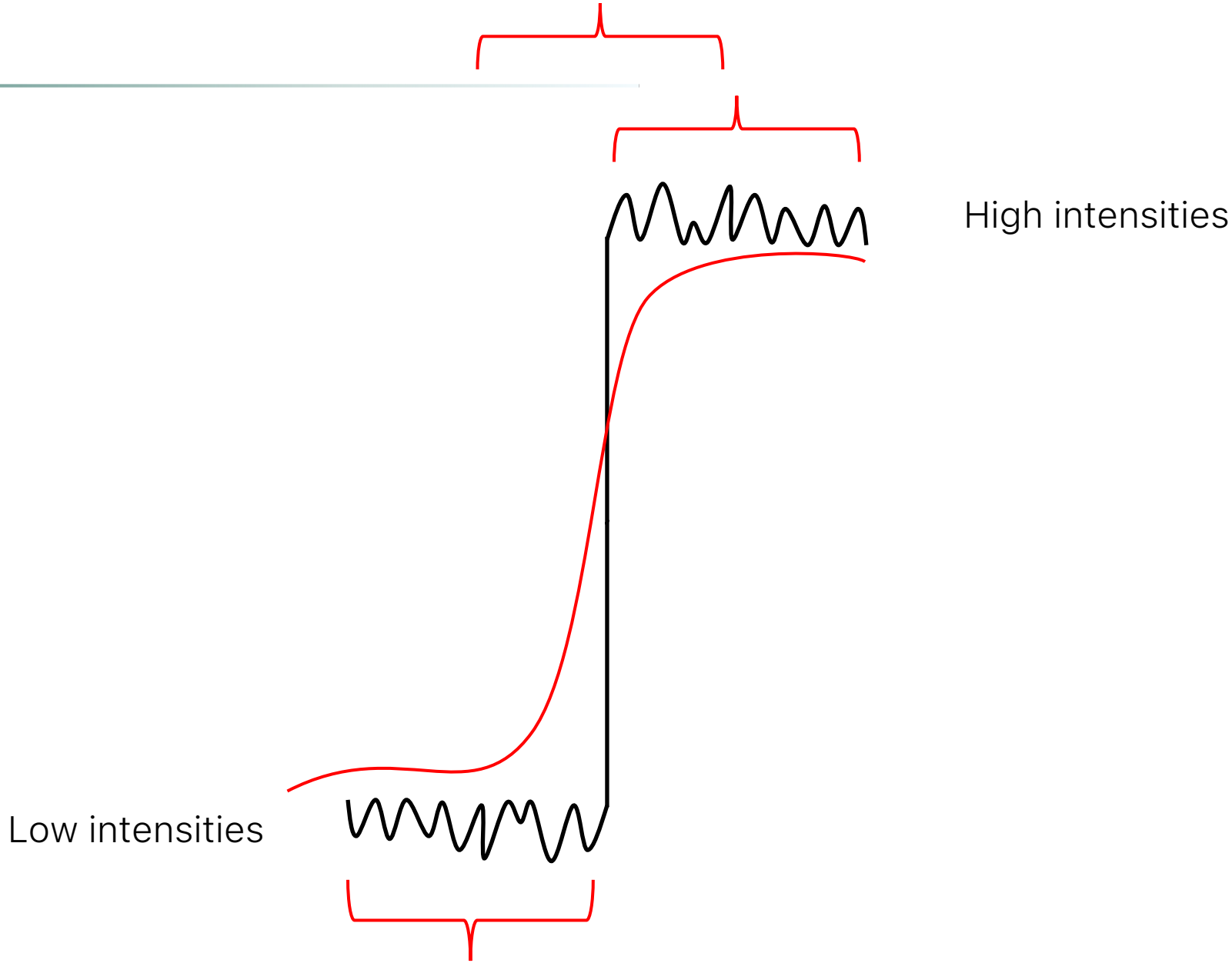
Smoothing by a 3x3 Mean

Smoothing by a 5x5 Mean

What happens here?

The ideal noiseless value of these pixels is the same?

Linear filtering does reduce noise but blurs the image

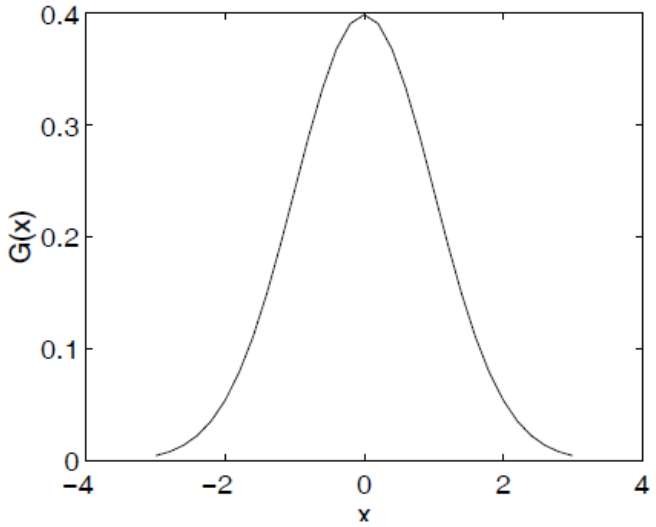

High intensities
Low intensities

Gaussian Filter

- LTE operator whose impulse response is a 2D Gaussian function (with zero mean and constant diagonal covariance matrix).

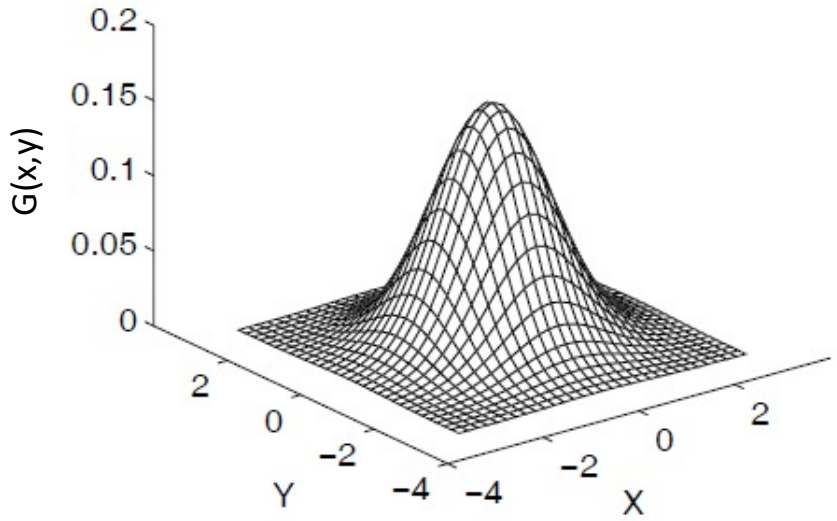
1D Gaussian

$$G(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}}$$



2D Gaussian

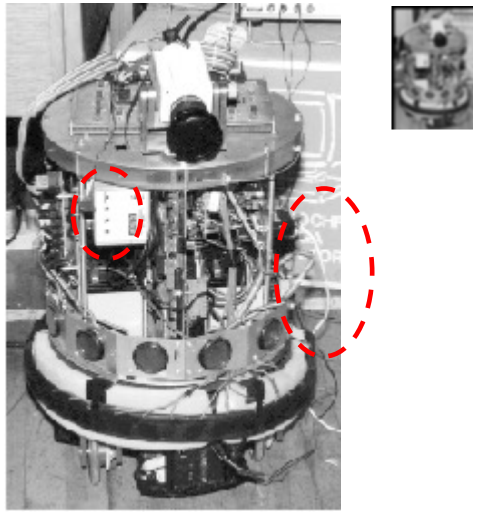
$$G(x,y) = G(x)G(y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$



Circularly Symmetric

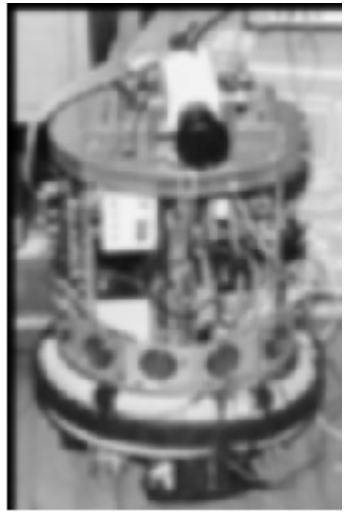
Parameter σ sets the amount of smoothing

- The higher σ, the stronger the smoothing caused by the filter. This can be understood, e.g., by observing that as σ increases, the weights of closer points get smaller while those of farther points get larger.



Original Image

Smoothing by a Gaussian Filter with σ = 1



Smoothing by a Gaussian Filter with σ = 2

Smoothing by a Gaussian Filter with σ = 4

- As σ gets larger, small details disappear and the image content deals with larger size structures. Thus, filtering with a chosen σ can be thought of as setting the "scale" of interest to analyse image content.

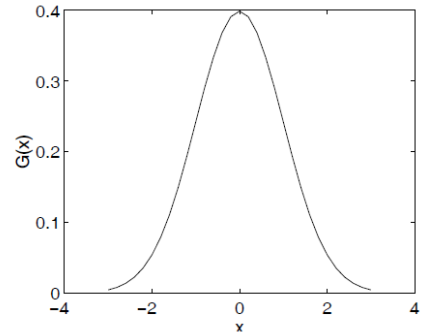
Practical Implementation

HOW SHOULD I CHOOSE σ ? → sampling the continuous gaussian at specific coordinates and depending on how much i want to smooth the image, i will choose sigma → we'll see a rule

- The discrete Gaussian kernel can be obtained by sampling the corresponding continuous function, which is however of infinite extent. A finite size must therefore be properly chosen.

- To this purpose, we can observe that:
  - The larger is the size, the more accurate turns out the discrete approximation of the ideal continuous filter.
  - The computational cost grows with filter size.
  - The Gaussian gets smaller and smaller as we move away from the origin.

$$G(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}}$$



e.g.

	-3	-2	-1	0	1	2	3
k	0.0044	0.054	0.242	0.3989	0.242	0.054	0.0044

- We should use larger sizes for filters with high σ, smaller sizes whenever σ is smaller.

- Rule-of-thumb to choose the size of the filter given σ:
  - as the interval $[-3\sigma, +3\sigma]$ captures 99% of the area ("energy") of the Gaussian function, a typical rule-of-thumb dictates taking a $(2k+1)\times(2k+1)$ kernel with $k = [3\sigma]$

$$\sigma = 1 \Rightarrow 7x7$$
$$\sigma = 1.5 \Rightarrow 11x11$$
$$\sigma = 2 \Rightarrow 13x13$$
$$\sigma = 3 \Rightarrow 19x19$$

Deploying Separability

- To further speed-up the filtering operation, one can deploy the separability property: due to the 2D Gaussian being the product of two 1D Gaussians, the original 2D convolution can be split into the chain of two 1D convolutions, i.e. either along x first and then along y, or viceversa.

$$I(x,y) * G(x,y) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} I(\alpha,\beta) G(x-\alpha, y-\beta) d\alpha\, d\beta$$

$$G(x,y) = G(x)G(y)$$

A convolution with a 7×7 → normal kernel performs 49 sums + 49 products
→ Gaussian kernel performs 7 sums + 7 products

$$I(x,y) * G(x,y) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} I(\alpha,\beta) G(x-\alpha) G(y-\beta) d\alpha\, d\beta$$

$$I(x,y) * G(x,y) = \int_{-\infty}^{+\infty} G(y-\beta) \left(\int_{-\infty}^{+\infty} I(\alpha,\beta) G(x-\alpha) d\alpha \right) d\beta$$

$$I(x,y) * G(x,y) = (I(x,y) * G(x)) * G(y) = (I(x,y) * G(y)) * G(x)$$

Accordingly, the speed-up (S) brought in by the separability property can be expressed as:

$$\begin{cases} 2D\ Filter\text{:}\ N_{OPS} = 2 \cdot (2k+1)^2 \\ 1D\ Filter\text{:}\ N_{OPS} = 2 \cdot 2 \cdot (2k+1) \end{cases}$$

SPEED UP LINEAR WITH RESPECT TO K

$$S = \frac{2 \cdot (2k+1)^2}{2 \cdot 2 \cdot (2k+1)} = k + \frac{1}{2}$$

Example: Impulse Noise

Original Image

Image corrupted by Impulse Noise (aka Salt-and-Pepper Noise)

Smoothing by a 3x3 Mean

Smoothing by a 5x5 Mean

Linear filtering is ineffective toward impulse noise (and blurs the image)

Median Filter

NO MORE CONVOLUTIONS

- *Non-linear* filter whereby each pixel intensity is replaced by the median over a given neighbourhood, the median being the value falling half-way in the sorted set of intensities.

$$median\,[A(x) + B(x)] \neq median\,[A(x)] + median\,[B(x)]$$

123	125	126	130	140
122	124	126	127	135
118	120	150	125	134
119	115	119	123	133
111	116	110	120	130

Neighbourhood values:

115, 119, 120, 123, 124, 125, 126, 127, 150

Median value: 124

- Median filtering counteracts impulse noise effectively, as outliers (i.e. noisy pixels) tend to fall at either the top or bottom end of the sorted intensities.

- Median filtering tends to keep sharper edges than linear filters such as the Mean or Gaussian:

$$\begin{aligned} \ldots\ 10\ 10\ 40\ 40\ \ldots &\Rightarrow \ldots\ 10\ 20\ 30\ 40\ \ldots \quad (Mean) \\ &\Rightarrow \ldots\ 10\ 10\ 40\ 40\ \ldots \quad (Median) \end{aligned}$$

Example

Original Image

Image Corrupted by impulse noise (+100, 5% of randomly picked pixels)

Filtering by a 3×3 Median

Filtering twice by a 3×3 Median

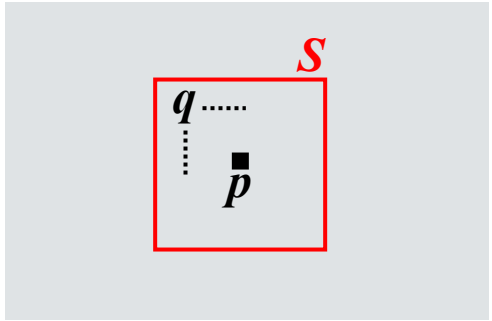
- When dealing with impulse noise, the Median Filter can effectively denoise the image without introducing significant blur.

- Yet, Gaussian-like noise, such as sensor noise, cannot be dealt with by the Median, as this would require computing new noiseless intensities.
  - Purposely, the Median may be followed by linear filtering.

Bilateral Filter

BECAUSE

- Advanced *non-linear* filter to accomplish denoising of Gaussian-like noise without blurring the image (aka edge preserving smoothing).



$$O(p) = \sum_{q \in S} H(p,q) \cdot I_q$$

$$H(p,q) = \frac{1}{W(p)} G_{\sigma_s}\big(d_s(p,q)\big)\, G_{\sigma_r}\big(d_r(p,q)\big)$$

THE LARGER THE DISTANCES AND THE SMALLER THE GAUSSIANS.

HIGH SPACIAL DISTANCE

WHEN I COME ACROSS AN EDGE I HAVE THAT THE GAUSSIANS BALANCE THEM

I PRESERVE EDGES

(IF THEY SHOULD BE COUNTED BECAUSE HAVE SIMILAR INTENSITIES EVEN THO THE ARE FAR AWAY FROM CENTER)

GAUSSIAN ON SPACIAL DISTANCE

GAUSSIAN ON PIXEL INTENSITY DISTANCE

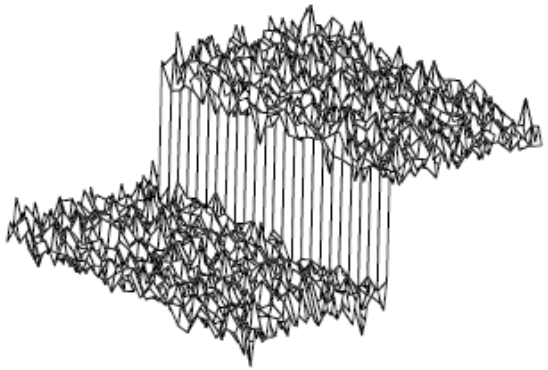
$$d_s(p,q) = \|p - q\|_2 = \sqrt{(u_p - u_q)^2 + (v_p - v_q)^2}$$ → *Spatial Distance*

$$d_r(I_p, I_q) = |I_p - I_q|$$ → *Range (Intensity) Distance*

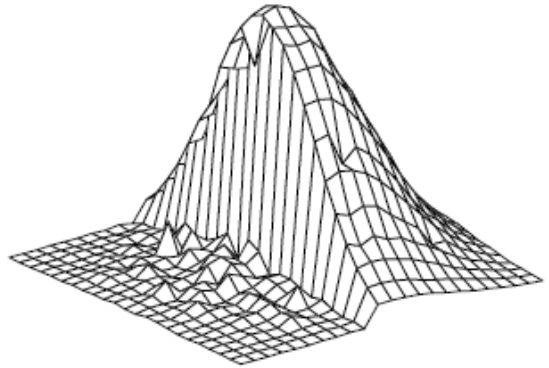
$$W(p) = \sum_{q \in S} G_{\sigma_s}\big(d_s(p,q)\big)\, G_{\sigma_r}\big(d_r(p,q)\big)$$ → *Normalization Factor (Unity Gain)*

Bilateral Filter

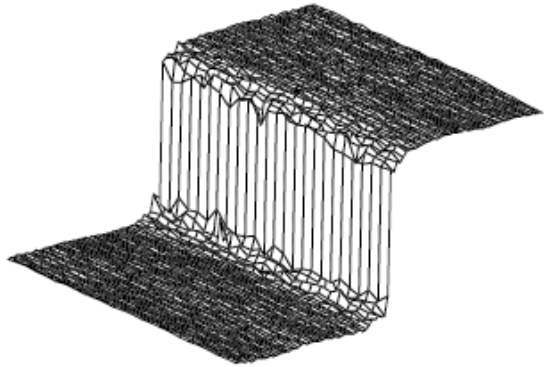
Step-edge as wide as 100 gray-levels



H(p,q) at a pixel just across the edge in the brighter region



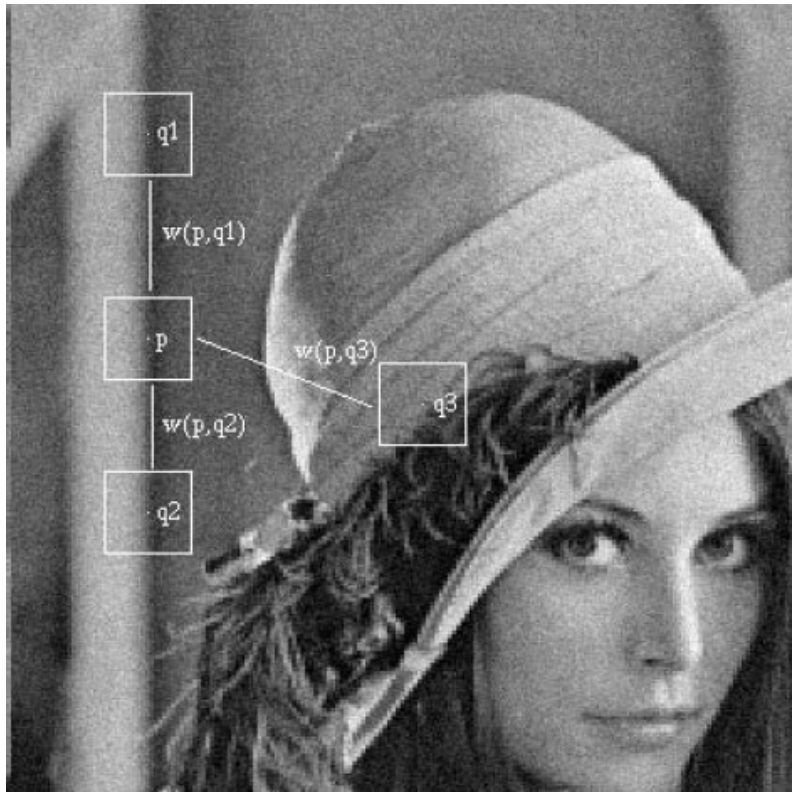
Output provided by the filter $\sigma_s = 5\,, \sigma_r = 50$

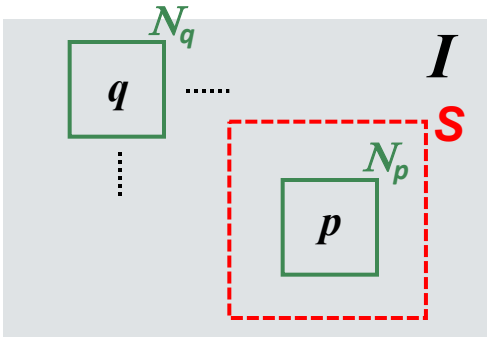


- Given the supporting neighbourhood, neighbouring pixels take a larger weight as they are both closer and more similar to the central pixel.
- At a pixel nearby an edge, the neighbours falling on the other side of the edge look quite different and thus cannot contribute significantly to the output value due to their weights being small.

Non-local Means Filter

- Another well-known *non-linear* edge preserving smoothing filter. The key idea is that the similarity among patches spread over the image can be deployed to achieve denosising.





$$O(p)=\sum_{q\in S} w(p,q)I(q)$$

$$w(p,q)=\frac{1}{Z(p)}e^{-\frac{\|N_p-N_q\|_2^2}{h^2}}$$

$$Z(p)=\sum_{q\in I} e^{\frac{\|N_p-N_q\|_2^2}{h^2}}$$

Non-local Means Filter

Image Corrupted by Gaussion Noise (σ =20)

Gaussian Filter

Non-local Means Filter

$N = 7\times7,$

$S = 21\times21,$

$h = 10 \cdot \sigma$

? h FILTERING PARAMETER

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