SCHEMI MATEMATICA APPLICATA

CALCOLO COMBINATORIO

FORMULE $D_{n,k} = \frac{n!}{(n-k)!}$ $\quad D^{R}_{n,k} = n^{k}$ $\quad P_n = n!$ $\quad P^{R}_{k_1+\dots+k_p} = \frac{n!}{k_1! \cdot k_p!}$ $\quad C_{n,k} = \binom{n}{k} = \frac{n!}{(n-k)!\,k!}$ $\quad C^{R}_{n,k} = \binom{n+k-1}{k}$

∩/∪ $(E \cup F)^c = E^c \cap F^c$ $\quad (E \cap F)^c = E^c \cup F^c$ $\quad P(A \cup B) = P(A) + P(B) - P(A \cap B)$

PROB COND $P(A|B) = P(A \cap B)/P(B)$

PROB TOT $P(E) = \sum P(E|H_J) \cdot P(H_J)$

BAYES $P(H_J|E) = \dfrac{P(E|H_J) \cdot P(H_J)}{\sum P(E|H_K) \cdot P(H_K)}$

VARIABILI CASUALI

FUNZIONI →
- MASSA: $P(a) = P(X = a)$ $\quad$ DISTRIB: $F(a) = P(X \leq a)$ $\quad$ X discreta
- DENSITÁ: $P(X \in B) = \int_B f(s)\,ds$ $\quad$ DISTRIB: $F(a) = \int_{-\infty}^{a} f(s)\,ds$ | $\frac{dF(a)}{da} = f(a)$ $\quad$ X continua

INDIPENDENZA $\iff$ $F(a,b) = F_X(a) \cdot F_Y(b)$ $\quad \cup \quad$ $p(a,b) = p_X(a) \cdot p_Y(b)$ $\quad \cup \quad$ $f(a,b) = f_X(a) \cdot f_Y(b)$

E[x]
- def. → $E[X] = \begin{cases} \sum_{k=1}^{m} x_k p(x_k) & X \text{ disc.} \\ \int_{-\infty}^{+\infty} x f(x) & X \text{ cont.} \end{cases}$
- prop → $E[g(X)] = \begin{cases} \sum_{k=1}^{m} g(x_k) p(x_k) & X \text{ disc.} \\ \int_{-\infty}^{+\infty} g(x) f(x) & X \text{ cont.} \end{cases}$ | $E[h(X,Y)] = \begin{cases} \sum_{k=1}^{m} \sum_{j=1}^{n} h(x_k, y_j)\, p(x_k, y_j) & X \text{ disc} \\ \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} h(x,y) f(x,y)\,dx\,dy & X \text{ cont.} \end{cases}$ | $E[\alpha X + \beta Y] = \alpha E[X] + \beta E[Y]$

Var(x)
- def. → $Var(x) = \begin{cases} \sum_{k=1}^{m} (x_k - \mu)^2 p(x_k) & X \text{ disc} \\ \int_{-\infty}^{+\infty} (x-\mu)^2 f(x)\,ds & X \text{ cont.} \end{cases}$
- prop. → $Var(x) = E[x^2] - E[x]^2$ | $Var(\alpha x + \beta) = \alpha^2 Var(x)$

Cov(X,Y)
- def → $Cov(X,Y) = E[(x - E[x]) \cdot (y - E[y])]$
- prop. → $Cov(X,Y) = Cov(Y,X)$ | $Cov(X,Y) = E[X \cdot Y] - E[X]E[Y]$ | $Cov(X,X) = Var(X)$ | $Var(X+Y) = Var(X) + Var(Y) + 2\,Cov(X,Y)$

MEDIA CAMPIONARIA def. → $\bar{X} = \sum_{k=1}^{N} \frac{X_K}{N}$ e se $\begin{matrix} E[X_K] = \mu \\ Var(X_K) = \sigma^2 \end{matrix}$ $\Rightarrow$ $E[\bar{X}] = \mu$ e $Var(\bar{X}) = \frac{\sigma^2}{N}$

MARKOV $P(X \geq a) \leq \frac{\mu}{a}$

ČEBYČËV $P(|X - E[X]| \geq r) \leq \frac{Var(x)}{r^2}$

MODELLI V.C.

BERNOULLI $\xrightarrow{X \sim Be(P)}$ $X \in \{0,1\}$ e $\begin{matrix} P(0) = q \\ P(1) = p \end{matrix}$ $\quad E[X] = p \;;\; Var(x) = pq$

BINOMIALI $\xrightarrow{X \sim B(n,p)}$ $X =$ n° che si presenta A; $n =$ n° esperimenti; $P(A) = p$ $\quad E[X] = \sum_{k=0}^{n} k \binom{n}{k} p^k q^{n-k} \;;\; Var(x) = npq \;;\; P(K) = \binom{n}{k} p^k q^{n-k}$

GEOMETRICHE $\xrightarrow{X \sim G(p)}$ $X \in N - \{0\}$; $X =$ n° esp. fino ad A; $P(A) = p$ $\quad E[X] = \frac{1}{p} \;;\; Var(X) = \frac{q}{p^2} \;;\; P(K) = p\,q^{K-1}$

BINOMIALI NEGATIVE $\xrightarrow{X \sim NB(r,p)}$ r oss. di A; $X =$ n° esp.; $P(A) = p$ $\quad E[X] = \frac{r}{p} \;;\; Var(X) = \frac{rq}{p^2} \;;\; P(K) = \binom{K-1}{r-1} p^r q^{K-r}$

POISSON $\xrightarrow{X \sim Po(\lambda)}$ λ media $\quad E[X] = \lambda \;;\; Var(X) = \lambda \;;\; P(K) = \frac{\lambda^K}{K!} e^{-\lambda}$

UNIFORME $\xrightarrow{X \sim U(\alpha,\beta)}$ $\alpha < \beta$ intervalli in cui c'è X $\quad E[X] = \frac{\beta + \alpha}{2} \;;\; Var(X) = \frac{(\beta - \alpha)^2}{12} \;;\; P(X \in [a,b]) = \frac{b-a}{\beta - \alpha} \;;\; f(x) = \begin{cases} \frac{1}{\beta - \alpha} & \text{se } x \in [\alpha, \beta] \\ 0 & \text{altr.} \end{cases} \rightarrow F(a) = \begin{cases} 0 & \text{se } a < \alpha \\ \frac{a - \alpha}{\beta - \alpha} & \text{se } a \in [\alpha, \beta] \\ 1 & \text{se } a > \beta \end{cases}$

ESPONENZIALE $\xrightarrow{X \sim E(\lambda)}$ $\quad E[X] = \frac{1}{\lambda} \;;\; Var(X) = \frac{1}{\lambda^2} \;;\; P(X > a) = \begin{cases} 1 & \text{se } a < 0 \\ e^{-a\lambda} & \text{se } a \geq 0 \end{cases} \;;\; f(x) = \begin{cases} \lambda e^{-\lambda x} & \text{se } x \geq 0 \\ 0 & \text{se } x < 0 \end{cases} \rightarrow F(a) = \begin{cases} 0 & \text{se } a < 0 \\ 1 - e^{-a\lambda} & \text{se } a \geq 0 \end{cases}$

GAUSSIANA $\xrightarrow{X \sim N(\mu, \sigma^2)}$ $\quad E[X] = \mu \;;\; Var(X) = \sigma^2 \;;$
- normalizzare → $X \sim N(A,B)$, $P(X < K)$? $\quad P(X < K) = P\left(\frac{X - A}{\sqrt{B}} < \frac{K - A}{\sqrt{B}}\right) = P\left(Z < \frac{K-A}{\sqrt{B}}\right) = F_Z\left(\frac{K-A}{\sqrt{B}}\right)$
- proprietà → 1) $P(Z > a^*) = P(Z < -a^*)$ $\quad$ 2) $1 - P(Z \leq a^*) = F_Z(-a^*)$ $\quad$ 3) $1 - F_Z(a^*) = F_Z(-a^*)$