

The accuracy is:

**Scegli un'alternativa:**

- a. The number of correct significant digits in approximating some quantity.
- b. The number of digits with which a number is expressed.
- c. None of the above.

Given two random variables  $X$  and  $Y$ , Bayes Theorem implies that

$$p(y|x) = \frac{p(x|y)p(y)}{p(x)}$$
 where:

**Scegli un'alternativa:**

- a.  $p(x|y)$  is called prior distribution on  $x$ .
- b.  $p(x|y)$  is called posterior distribution on  $y$ .
- c.  $p(x|y)$  is called likelihood on  $y$ .

Given two random variables  $X$  and  $Y$ , Bayes Theorem implies that

$$p(y|x) = \frac{p(x|y)p(y)}{p(x)} \text{ where:}$$

**Scegli un'alternativa:**

- a.  $p(y|x)$  is called prior distribution on  $x$ .
- b.  $p(y|x)$  is called likelihood on  $y$ .
- c.  $p(y|x)$  is called posterior distribution on  $y$ .

If  $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ ,  $f(x_1, x_2) = x_1 e^{x_2}$ ,  $g : \mathbb{R} \rightarrow \mathbb{R}^2$ ,  $g(t) = (e^t, t)$ , then, if  $h(t) = f(g(t))$ :

**Scegli un'alternativa:**

- a.  $h'(t) = te^t$ .
- b.  $h'(t) = 2e^{2t}$ .
- c.  $h'(t) = e^{2t}(t + 1)$ .

If  $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ ,  $f(x_1, x_2) = x_1^2 + x_1x_2$ ,  $g : \mathbb{R} \rightarrow \mathbb{R}^2$ ,  $g(t) = (\sin(t), \cos(t))$ ,  
then, if  $h(t) = f(g(t))$ :

**Scegli un'alternativa:**

- a.  $h'(t) = \sin(2t) - \sin^2(t)$ .
- b.  $h'(t) = \sin(t) - \sin^2(2t)$ .
- c.  $h'(t) = \sin(t)\cos(t) - \sin^2(t)$ .

If  $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ ,  $f(x_1, x_2) = x_1^2 + x_2^2$ ,  $g : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ ,  $g(x_1, x_2) = (x_2, x_1)$ ,  
then, if  $h(x_1, x_2) = f(g(x_1, x_2))$ :

**Scegli un'alternativa:**

- a.  $\nabla h(x_1, x_2) = (2x_1, 2x_2)$ .
- b.  $\nabla h(x_1, x_2) = (2x_2, 2x_1)$ .
- c.  $\nabla h(x_1, x_2) = (1, 1)$ .

If  $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ ,  $f(x_1, x_2) = x_1 e^{x_2}$ ,  $g : \mathbb{R} \rightarrow \mathbb{R}^2$ ,  $g(t) = (t, \log t)$ , then, if  
 $h(t) = f(g(t))$ :

**Scegli un'alternativa:**

- a.  $h'(t) = t + 1$ .
- b.  $h'(t) = t^2 + 1$ .
- c.  $h'(t) = 2t$ .

If  $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ ,  $f(x_1, x_2) = x_1^2 + x_1x_2$ ,  $g : \mathbb{R} \rightarrow \mathbb{R}^2$ ,  $g(t) = (t^2, t)$ , then, if  $h(t) = f(g(t))$ :

Scegli un'alternativa:

- a.  $h'(t) = t(2t - 1)^2 + t.$
- b.  $h'(t) = 4t^2 + 2t + 1.$
- c.  $h'(t) = t(2t + 1)^2 - 2t^2.$

If  $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ ,  $f(x_1, x_2) = x_1^2 + x_2^2$ ,  $g : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ ,  $g(x_1, x_2) = (x_1 e^{x_2}, x_2)$ ,  
then, if  $h(x_1, x_2) = f(g(x_1, x_2))$ :

**Scegli un'alternativa:**

- a.  $\nabla h(x_1, x_2) = (2x_1 e^{x_2}(e^{x_2} + x_1), 2e^{x_2})$ .
- b.  $\nabla h(x_1, x_2) = (2x_1 e^{2x_2}(e^{x_1} + x_1), 2x_2)$ .
- c.  $\nabla h(x_1, x_2) = (2x_1 e^{x_2}(e^{x_2} + x_1), 2x_2)$ .

If  $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ ,  $f(x_1, x_2) = x_1 x_2$ ,  $g : \mathbb{R} \rightarrow \mathbb{R}^2$ ,  $g(t) = (t, t^2)$ , then, if  $h(t) = f(g(t))$ :

**Scegli un'alternativa:**

- a.  $h'(t) = 3t^2$ .
- b.  $h'(t) = 3t^3$ .
- c.  $h'(t) = t^2$ .

If

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

then:

**Scegli un'alternativa:**

- a.  $K_2(A) = 1.$
- b.  $K_2(A) = 4.$
- c.  $K_2(A) = \frac{1}{2}.$

If

$$A = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

then:

**Scegli un'alternativa:**

- a.  $K_2(A) = 4.$
- b.  $K_2(A) = 2.$
- c.  $K_2(A) = \frac{4}{3}.$

If vector  $v = (10^6, 0)^T$  is approximated by vector  $\tilde{v} = (999996, 1)^T$ , then in  $\|\cdot\|_2$  the relative error between  $v$  and  $\tilde{v}$  is:

Scegli un'alternativa:

- a.  $\sqrt{17} \cdot 10^{-6}$ .
- b. None of the above.
- c.  $4 \cdot 10^{-6}$ .

If

$$A = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

then:

Scegli un'alternativa:

- a.  $K_2(A) = \frac{1}{2}.$
- b.  $K_2(A) = 4.$
- c.  $K_2(A) = 2.$

A random variable  $X : \Omega \rightarrow \mathcal{T}$  is continuous when:

**Scegli un'alternativa:**

- a.  $\mathcal{T}$  is countable.
- b.  $\mathcal{T} = \mathbb{R}$ .
- c.  $\Omega$  is continuous.

A random variable  $X : \Omega \rightarrow \mathcal{T}$  is discrete when:

Scegli un'alternativa:

- a.  $\mathcal{T} = \mathbb{R}$ .
- b.  $\Omega$  is countable.
- c.  $\mathcal{T}$  is countable.

If

$$A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$$

Then:

**Scegli un'alternativa:**

- a.  $x = (1, 2)^T$  is an eigenvector of  $A$ .
- b.  $x = (2, 1)^T$  is an eigenvector of  $A$ .
- c.  $x = (0, 0)^T$  is an eigenvector of  $A$ .

If

$$A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$$

Then:

**Scegli un'alternativa:**

- a.  $\lambda = 5$  is the eigenvalue associated with the eigenvector  $x = (2, 1)^T$ .
- b.  $\lambda = 2$  is the eigenvalue associated with the eigenvector  $x = (2, 1)^T$ .
- c.  $\lambda = 2$  is the eigenvalue associated with the eigenvector  $x = (1, 2)^T$ .

If

$$A = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

Then:

**Scegli un'alternativa:**

- a.  $x = (1, 1, 0)^T$  is an eigenvector of  $A$ .
- b.  $x = (0, 1, 0)^T$  is an eigenvector of  $A$ .
- c.  $x = (0, -1, 1)^T$  is an eigenvector of  $A$ .

If

$$A = \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$$

Then:

**Scegli un'alternativa:**

- a.  $x = (0, 0)^T$  is an eigenvector of  $A$ .
- b.  $x = (1, 0)^T$  is an eigenvector of  $A$ .
- c.  $x = (1, 1)^T$  is an eigenvector of  $A$ .

If

$$A = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$$

Then:

**Scegli un'alternativa:**

- a.  $\lambda = 2$  is the eigenvalue associated with the eigenvector  $x = (1, 0)^T$ .
- b.  $\lambda = 2$  is the eigenvalue associated with the eigenvector  $x = (0, 1)^T$ .
- c.  $\lambda = 1$  is the eigenvalue associated with the eigenvector  $x = (1, 0)^T$ .

If  $A \in \mathbb{R}^{n \times n}$ ,  $x \in \mathbb{R}^n$  and

$$Ax = \lambda x$$

For  $\lambda \in \mathbb{R}$ , then:

Scegli un'alternativa:

- a. For any  $c \in \mathbb{R}$ ,  $c \neq 0$ ,  $cx$  is an eigenvector of  $A$ .
- b.  $cx$  is an eigenvector of  $A$  if and only if  $c = 1$ .
- c. None of the above.

If

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

Then:

**Scegli un'alternativa:**

- a.  $\lambda = 2$  is the eigenvalue associated with the eigenvector  $x = (0, 1, 0)^T$ .
- b.  $\lambda = -1$  is the eigenvalue associated with the eigenvector  $x = (0, 0, 1)^T$ .
- c.  $\lambda = 1$  is the eigenvalue associated with the eigenvector  $x = (1, 0, 0)^T$ .

In  $\mathcal{F}(10, 2, -2, 2)$ , if  $x = \pi$ ,  $y = e$ , and  $z = fl(x) - fl(y)$ , then:

**Scegli un'alternativa:**

- a.  $fl(z) = 0.43 \times 10^1$ .
- b.  $fl(z) = 0.44 \times 10^1$ .
- c.  $fl(z) = 0.40 \times 10^1$ .

In  $\mathcal{F}(10, 2, -2, 2)$ , if  $x = \pi$ ,  $y = e$ , and  $z = fl(x) * fl(y)$ , then:

**Scegli un'alternativa:**

- a.  $fl(z) = 0.84 \times 10^1$ .
- b.  $fl(z) = 0.0837 \times 10^2$ .
- c.  $fl(z) = 0.837 \times 10^1$ .

In  $\mathcal{F}(10, 6, -3, 3)$ , if  $x = 192.403$ ,  $y = 0.635782$ , and  $z = fl(x) + fl(y)$ , then:

Scegli un'alternativa:

- a.  $fl(z) = 0.193039 \times 10^3$ .
- b.  $fl(z) = 0.193038 \times 10^3$ .
- c.  $fl(z) = 0.193038782 \times 10^3$ .

In  $\mathcal{F}(10, 2, -2, 2)$ , if  $x = \pi$ ,  $y = e$ , and  $z = fl(x) + fl(y)$ , then:

**Scegli un'alternativa:**

- a.  $fl(z) = 0.585 \times 10^1$ .
- b.  $fl(z) = 0.58 \times 10^1$ .
- c.  $fl(z) = 0.59 \times 10^1$ .

If  $f : \mathbb{R}^n \rightarrow \mathbb{R}$ ,  $f \in \mathcal{C}^1(\mathbb{R}^n)$ , then  $x^*$  is a minimum point if and only if:

**Scegli un'alternativa:**

- a.  $\nabla f(x^*) = 0$  and  $\nabla^2 f(x^*)$  is positive semi-definite.
- b.  $\nabla f(x^*) = 0$ .
- c.  $\nabla f(x^*) = 0$  and  $\nabla^2 f(x^*)$  is positive definite.

## Gradient descent methods:

Scegli un'alternativa:

- a. If  $\alpha$  is suitable chosen,  $f \in C^1$ , for any  $x_0$ , always converges to a stationary point of  $f(x)$ .
- b. If  $\alpha$  is suitable chosen,  $f \in C^1$ , for any  $x_0$ , always converges to a minimum of  $f(x)$ .
- c. Always converges to a minimum of  $f(x)$ .

Gradient descent methods solves the optimization problem

$$\min_x f(x)$$

By:

**Scegli un'alternativa:**

- a. Generating a sequence  $\{x_k\}_k$  such that, given  $x_0$ , computes  $x_{k+1} = x_k - \alpha \nabla f(x_k)$  for  $\alpha > 0$  step-size.
- b. Generating a sequence  $\{x_k\}_k$  such that, given  $x_0$ , computes  $x_{k+1} = x_k + \alpha \nabla f(x_k)$  for  $\alpha > 0$  step-size.
- c. Generating a sequence  $\{x_k\}_k$  such that, given  $x_0$ , computes  $x_{k+1} = x_k - \alpha \nabla f(x_k)$  for  $\alpha \neq 0$  step-size.

$$x^{(1)} = x^{(0)} - \alpha \nabla f(x^{(0)}) = (1, 1) - \alpha (2, 2) = (1 - 2\alpha, 1 - 2\alpha)$$

If  $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ ,  $f(x_1, x_2) = x_1^2 + x_2^2$ , then if the initial guess for a gradient descent iteration is  $x^{(0)} = (1, 1)^T$  and  $\alpha > 0$ , then  $|f(x^{(1)})| < |f(x^{(0)})|$  if:

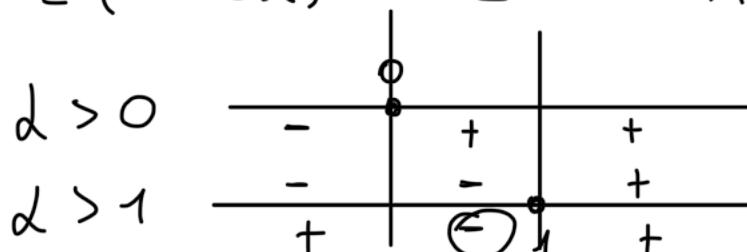
Scegli un'alternativa:

- a.  $0 < \alpha < 1$ .
- b.  $\alpha > 0$ .
- c.  $\alpha > \frac{1}{2}$ .



$$|(1 - 2\alpha)^2 + (1 - 2\alpha)^2| < |1^2 + 1^2|$$

$$2(1 - 2\alpha)^2 < 2 \quad 1 + 4\alpha^2 - 4\alpha < 1 \quad 4\alpha(\alpha - 1) < 0$$



$$0 < \alpha < 1$$

$$x^{(1)} = x^{(0)} - \lambda \nabla f(x^{(0)}) = (0,0) - \lambda (1,0) = (-\lambda, 0)$$

If  $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ ,  $f(x_1, x_2) = e^{x_1} + x_2^2$ , then if the initial guess for a gradient descent iteration is  $x^{(0)} = (0, 0)^T$  and  $\alpha > 0$ , then  $|f(x^{(1)})| < |f(x^{(0)})|$  if:

Scegli un'alternativa:

a.  $\alpha > \frac{1}{2}$ .

b.  $\alpha > 0$ .

c.  $0 < \alpha < 1$ .

$$|e^{-\lambda} + 0| < |1 + 0|$$

$$e^{-\lambda} < 1$$

$$e^{-\lambda} < e^0$$

$$-\lambda < 0 \quad \lambda > 0$$

$$\begin{aligned}
 x^{(1)} &= x^{(0)} - \alpha \nabla f(x^{(0)}) = (1,1) - \frac{1}{2} (e,e) = \\
 &= (1,1) - \frac{1}{2} (e,e) = \\
 &= \left(1 - \frac{e}{2}, 1 - \frac{e}{2}\right)
 \end{aligned}$$

If  $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ ,  $f(x_1, x_2) = x_1 e^{x_2}$ , then if the initial guess for a gradient descent iteration is  $x^{(0)} = (1, 1)^T$  and  $\alpha = \frac{1}{2}$ , then:

Scegli un'alternativa:

- a.  $x^{(1)} = (1 - \frac{e}{2}, 1 - \frac{e}{2})^T$ .
- b.  $x^{(1)} = (1 + \frac{e}{2}, 1 + \frac{e}{2})^T$ .
- c.  $x^{(1)} = (\frac{1}{2} - \frac{e}{2}, \frac{1}{2} - \frac{e}{2})^T$ .

$$\begin{aligned}x^{(1)} &= x^{(0)} - \alpha \nabla f(x^{(0)}) = (0, 0) - 1 \cdot (1, 0) = \\&= (-1, 0)\end{aligned}$$

If  $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ ,  $f(x_1, x_2) = x_1 e^{x_2}$ , then if the initial guess for a gradient descent iteration is  $x^{(0)} = (0, 0)^T$  and  $\alpha = 1$ , then:

Scegli un'alternativa:

- a.  $x^{(1)} = (1, 0)^T$ .
- b.  $x^{(1)} = (-1, 0)^T$ .
- c.  $x^{(1)} = (0, 0)^T$ .

$$x^{(1)} = x^{(0)} - \alpha \nabla f(x^{(0)}) = (1,1) - \alpha (2,2)$$

If  $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ ,  $f(x_1, x_2) = x_1^2 + x_2^2$ , then if the initial guess for a gradient descent iteration is  $x^{(0)} = (1, 1)^T$  and  $\alpha > 0$ , then:

Scegli un'alternativa:

- a.  $x^{(1)} = (1 - 2\alpha, 1 - 2\alpha)^T$ .
- b.  $x^{(1)} = (1 - \alpha, 1 - \alpha)^T$ .
- c.  $x^{(1)} = (1 + 2\alpha, 1 + 2\alpha)^T$ .

$$x^{(1)} = x^{(0)} - \alpha \nabla f(x^{(0)}) = (0,0) - \alpha (1,0) = (-\alpha, 0)$$

If  $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ ,  $f(x_1, x_2) = e^{x_1} + x_2^2$ , then if the initial guess for a gradient descent iteration is  $x^{(0)} = (0, 0)^T$  and  $\alpha > 0$ , then:

Scegli un'alternativa:

- a.  $x^{(1)} = (-\alpha, 0)^T$ .
- b.  $x^{(1)} = (0, 0)^T$ .
- c.  $x^{(1)} = (-\alpha, 2)^T$ .

For Standard IEEE, double precision representation is:

**Scegli un'alternativa:**

- a.  $\mathcal{F}(2, 64, -1024, 1023)$ .
- b. None of the above.
- c.  $\mathcal{F}(2, 53, -1024, 1023)$ .

For Standard IEEE, single precision representation is:

**Scegli un'alternativa:**

- a.  $\mathcal{F}(2, 24, -128, 127)$ .
- b. None of the above.
- c.  $\mathcal{F}(2, 32, -128, 127)$ .

Given two independent random variables  $X$  and  $Y$ , then:

**Scegli un'alternativa:**

- a.  $p(x) = p(y)$
- b.  $p(y) = p(y|x)$
- c.  $p(x|y) = p(y)$

Given two random variables  $X$  and  $Y$ , Bayes Theorem implies that:

Scegli un'alternativa:

- a.  $p(x) = p(y)p(y|x) / p(y|x).$
- b.  $p(x) = p(x|y)p(y|x) / p(y).$
- c.  $p(y) = p(y|x)p(x) / p(x|y).$

$$P(x|y) = \frac{P(y|x) \cdot P(x)}{P(y)}$$

If  $f : \mathbb{R}^n \rightarrow \mathbb{R}$ ,  $f(x) = \|Ax - b\|_2^2$  for  $A \in \mathbb{R}^{m \times n}$ ,  $b \in \mathbb{R}^m$ , then:

Scegli un'alternativa:

a.  $\nabla f(x) = 2A^T(Ax - b)$ .

b.  $\nabla f(x) = A^T(Ax - b)$ .

c.  $\nabla f(x) = A(A^T x - b)$ .

$$\nabla f(x) = 2 A^T (Ax - b)$$

$$\nabla f(x) = 0 \rightarrow A^T(Ax - b) = 0 \longrightarrow A^T A x - A^T b = 0$$

If  $f : \mathbb{R}^n \rightarrow \mathbb{R}$ ,  $f(x) = \|Ax - b\|_2^2$  for  $A \in \mathbb{R}^{m \times n}$ ,  $b \in \mathbb{R}^m$ , then the solution of  $\nabla f(x) = 0$  is:

Scegli un'alternativa:

- a.  $A^T A x = b$
- b.  $A^T A x = A^T x$
- c.  $A x = b$ .

$$A^T A x - A^T b = 0$$



The machine precision  $\epsilon$  can be defined as:

**Select one:**

- a. The smallest number  $\epsilon$  such that  $fl(1 + \epsilon) = 1$ .
- b. The smallest number  $\epsilon$  such that  $fl(1 + \epsilon) > 1$ .
- c. None of the above.

$$\begin{aligned}
 x^* &= \arg \max_x \left\{ P(x) \cdot P(y|x)^2 \right\} = \left\{ \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} \cdot ce^{-|y-ax|^2} \right\} = \\
 &= \left\{ \log \left( e^{-\frac{1}{2}x^2} \cdot e^{-|y-ax|^2} \right) \right\} = \left\{ -\frac{1}{2}x^2 - |y-ax|^2 \right\} \\
 &= \arg \min_x \left\{ \frac{1}{2}x^2 + |y-ax|^2 \right\}
 \end{aligned}$$

Given two random variables  $X$  and  $Y$  such that  $p(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$  and  $p(y|x) = ce^{-|y-ax|}$ , then the MAP reads:

Scegli un'alternativa:

- a.  $x^* = \arg \min_x |y-ax| + \frac{1}{2}x^2$ .
- b.  $x^* = \arg \min_x |y-ax|$ .
- c.  $x^* = \arg \min_x \frac{1}{2}(y-ax)^2$ .



Given two random variables  $X$  and  $Y$  such that  $p(x) = ce^{-|x|}$  and  $p(y|x) = \frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}(y-ax)^2}$ , then the MAP reads:

Scegli un'alternativa:

- a.  $x^* = \arg \min_x \frac{1}{2}(y - ax)^2 + |x|.$
- b.  $x^* = \arg \min_x \frac{1}{2}(y - ax)^2 + \frac{1}{2}x^2.$
- c.  $x^* = \arg \min_x \frac{1}{2}(y - ax)^2.$



Given two random variables  $X$  and  $Y$  such that  $p(x) = \frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}x^2}$  and  $p(y|x) = \frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}(y-ax)^2}$ , then the MAP reads:

Scegli un'alternativa:

a.  $x^* = \arg \min_x \frac{1}{2}(y - ax)^2 + \frac{1}{2}x^2.$



b.  $x^* = \arg \min_x \frac{1}{2}(y - ax)^2 + x^2.$

c.  $x^* = \arg \min_x \frac{1}{2}(y - ax)^2.$

$$A \cdot A^T = \begin{bmatrix} 2 & 1 & -2 \\ -1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & -1 & -1 \\ 1 & 0 & 2 \\ -2 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 4+1+4 & -2+0-2 & -2+0-2 \\ -2+0-2 & 1+0+4 & -2+2+1 \\ -2+0-2 & -2+2+1 & 4+1+4 \end{bmatrix} \neq I$$

If

$$A = \begin{bmatrix} 2 & 1 & -2 \\ -1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix}$$

Then:

Scegli un'alternativa:

- a.  $A$  is orthogonal.
- b. None of the above.
- c.  $A$  is symmetric and definite positive.



If

$$2 > 0 \quad , \quad 2 \cdot 1 + 0 \cdot 0 > 0$$

$$A = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$$



Then:

Scegli un'alternativa:

- a.  $A$  is symmetric and positive definite.
- b.  $A$  is non-symmetric and not positive definite.
- c.  $A$  is symmetric but not positive definite.

If

$$A = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} & \frac{2}{3} \\ -\frac{2}{3} & \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} & -\frac{2}{3} \end{bmatrix}$$

Then:



**Scegli un'alternativa:**

- a.  $A$  is symmetric and definite positive.
- b.  $A$  is symmetric but not definite positive.
- c.  $A$  is orthogonal.

$$2 > 0, \quad 0 \cdot 3 - 2 \cdot 2 < 0$$

If

$$A = \begin{bmatrix} 2 & 2 & -1 \\ 2 & 0 & 2 \\ -1 & 2 & 3 \end{bmatrix}$$

Then:

Scegli un'alternativa:

- a.  $A$  is symmetric but not definite positive.
- b.  $A$  is symmetric and definite positive.
- c.  $A$  is orthogonal.



If

$$A = \begin{bmatrix} -1 & 1 \\ 0 & 3 \end{bmatrix}$$

Then:

**Scegli un'alternativa:**

- a.  $A$  is symmetric but not positive definite.
- b.  $A$  is symmetric and positive definite.
- c.  $A$  is non-symmetric and not positive definite.

If

$$A = \begin{bmatrix} 9 & 6 \\ 6 & 5 \end{bmatrix}$$

Then:

**Scegli un'alternativa:**

- a.  $A$  is symmetric but not positive definite.
- b.  $A$  is non-symmetric and not positive definite.
- c.  $A$  is symmetric and positive definite.

If

$$A = \begin{bmatrix} -1 & 0 \\ 0 & 3 \end{bmatrix}$$

Then:

**Scegli un'alternativa:**

- a.  $A$  is symmetric and positive definite.
- b.  $A$  is symmetric but not positive definite.
- c.  $A$  is non-symmetric and not positive definite.

$$\begin{bmatrix} 9 & & \\ & \ddots & \\ & & 1 \end{bmatrix} \not\simeq I$$

If

$$A = \begin{bmatrix} 2 & 1 & 2 \\ -2 & 2 & 1 \\ 1 & 2 & -2 \end{bmatrix} \quad \begin{bmatrix} 2 & -2 & 1 \\ 1 & 2 & 2 \\ 2 & 1 & -2 \end{bmatrix}$$

Then:

**Scegli un'alternativa:**

- a.  $A$  is not orthogonal.
- b.  $A$  is orthogonal.
- c.  $A$  is symmetric but not definite positive.

If

$$A = \begin{bmatrix} 1 & 0 & 3 \\ 1 & 1 & 0 \\ -1 & 1 & 2 \end{bmatrix} \quad \begin{bmatrix} -1 & 1 & 2 \\ 0 & 2 & 2 \\ 0 & 0 & -8 \end{bmatrix}$$

Then:

Scegli un'alternativa:

- a.  $\text{rank}(A) = 2.$
- b.  $\text{rank}(A) = 1.$
- c.  $\text{rank}(A) = 3.$

$$\begin{bmatrix} 0 & 1 & 5 \\ 0 & 2 & 2 \\ -1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 3 \\ 0 & 6 & 8 \\ 2 & 4 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 6 & 8 \\ 0 & 4 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 10 & 0 \\ 0 & 4 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 8 \\ 0 & 10 & 0 \\ 0 & 0 & -3 \end{bmatrix}$$

If

$$A = \begin{bmatrix} 0 & 6 & 8 \\ 2 & 4 & 0 \\ 1 & 0 & 8 \end{bmatrix}$$

Then:

Scegli un'alternativa:

- a.  $\text{rank}(A) = 3.$
- b.  $\text{rank}(A) = 2.$
- c.  $\text{rank}(A) = 1.$

If

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Then:

Scegli un'alternativa:

- a.  $\text{rank}(A) = 2.$
- b.  $\text{rank}(A) = 1.$
- c.  $\text{rank}(A) = 3.$

If

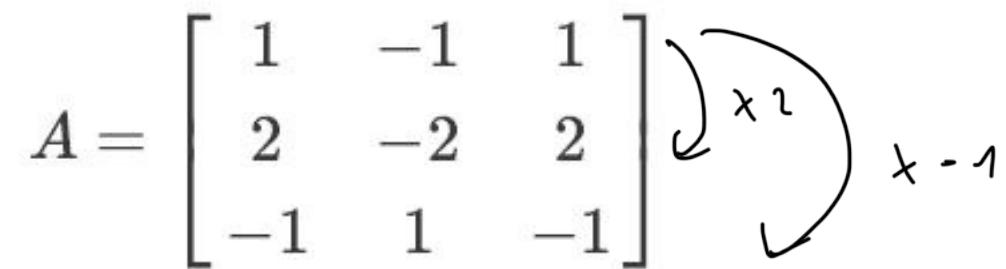
$$A = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

then:

**Scegli un'alternativa:**

- a.  $\text{rank}(A) = 3.$
- b.  $\text{rank}(A) = 4.$
- c.  $\text{rank}(A) = 2.$

If

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ -1 & 1 & -1 \end{bmatrix}$$


Then:

Scegli un'alternativa:

- a.  $\text{rank}(A) = 3.$
- b.  $\text{rank}(A) = 1.$
- c.  $\text{rank}(A) = 2.$

If

$$A = \begin{bmatrix} 0 & 2 & -1 \\ 1 & 1 & 0 \\ 1 & 3 & -1 \end{bmatrix}$$

Then:

Scegli un'alternativa:

- a.  $\text{rank}(A) = 2.$
- b.  $\text{rank}(A) = 3.$
- c.  $\text{rank}(A) = 1.$

If

$$A = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

then:

Scegli un'alternativa:

- a.  $\text{rank}(A) = 4.$
- b.  $\text{rank}(A) = 2.$
- c.  $\text{rank}(A) = 3.$

If

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 2 & 1 & 2 \end{bmatrix}$$

Then:

Scegli un'alternativa:

- a.  $\text{rank}(A) = 1.$
- b.  $\text{rank}(A) = 3.$
- c.  $\text{rank}(A) = 2.$

Given two random variables  $X$  and  $Y$  such that  $p(x) = \frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}x^2}$  and  $p(y|x) = ce^{-|y-ax|}$ , then the MLE reads:

Scegli un'alternativa:

- a.  $x^* = \arg \min_x |y - ax| + x^2$ .
- b.  $x^* = \arg \min_x \frac{1}{2}(y - ax)^2$ .
- c.  $x^* = \arg \min_x |y - ax|$ .



Given two random variables  $X$  and  $Y$  such that  $p(x) = ce^{-|x|}$  and  $p(y|x) = \frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}(y-ax)^2}$ , then the MLE reads:

Scegli un'alternativa:

- a.  $x^* = \arg \min_x \frac{1}{2}(y - ax)^2 + |x|.$
- b.  $x^* = \arg \min_x \frac{1}{2}(y - ax)^2 + \frac{1}{2}x^2.$
- c.  $x^* = \arg \min_x \frac{1}{2}(y - ax)^2.$



Given two random variables  $X$  and  $Y$  such that  $p(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x}$  and  $p(y|x) = ce^{-|y-ax|}$ , then the MLE reads:

Scegli un'alternativa:

- a.  $x^* = \arg \min_x |y - ax| + x^2$ .
- b.  $x^* = \arg \min_x |y - ax|$ .
- c.  $x^* = \arg \min_x \frac{1}{2}(y - ax)^2$ .



Given two random variables  $X$  and  $Y$  such that  $p(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$  and  $p(y|x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(y-ax)^2}$ , then the MLE reads:

Scegli un'alternativa:

- a.  $x^* = \arg \min_x \frac{1}{2}(y - ax)^2 + \frac{1}{2}x^2.$
- b.  $x^* = \arg \min_x \frac{1}{2}(y - ax)^2 + x^2.$
- c.  $x^* = \arg \min_x \frac{1}{2}(y - ax)^2.$



If

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

then:

**Scegli un'alternativa:**

- a. The 2-norm of  $A$  is  $\|A\|_2 = 1$ .
- b. The 2-norm of  $A$  is  $\|A\|_2 = 0$ .
- c. The 2-norm of  $A$  is  $\|A\|_2 = 3$ .

If

$$A = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

then:

Scegli un'alternativa:

- a. The 2-norm of  $A$  is  $\|A\|_2 = 2$ .
- b. The 2-norm of  $A$  is  $\|A\|_2 = 4$ .
- c. The 2-norm of  $A$  is  $\|A\|_2 = 2$ .

If

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

then:

Scegli un'alternativa:

- a. The 2-norm of  $A$  is  $\|A\|_2 = 3$ .
- b. The 2-norm of  $A$  is  $\|A\|_2 = 0$ .
- c. The 2-norm of  $A$  is  $\|A\|_2 = 3$ .

If  $A$  is an  $n \times n$  matrix, then

Scegli un'alternativa:

- a.  $\|A\|_1 = \sqrt{\sum_{i=1}^n \sum_{j=1}^n a_{i,j}^2}$ .
- b.  $\|A\|_1 = \rho(A^T A)$ .
- c. None of the above.

If  $A$  is an  $n \times n$  matrix, then

Scegli un'alternativa:

- a. None of the above.
- b.  $\|A\|_2 = \rho(A^T A)$ .
- c.  $\|A\|_2 = \sqrt{\sum_{i=1}^n \sum_{j=1}^n a_{i,j}^2}$ .

If  $A$  is an  $n \times n$  matrix, then

Scegli un'alternativa:

a.  $\|A\|_F = \sqrt{\sum_{i=1}^n \sum_{j=1}^n a_{i,j}^2}.$

b. None of the above.

c.  $\|A\|_F = \rho(A^T A).$

SIGNIFICA QUASIASI NORMA

If  $A \in \mathbb{R}^{m \times n}$ ,  $\|A\|_p = 0$ , then:

Scegli un'alternativa:

- a.  $A = 0$ .
- b.  $\text{rank}(A) = 0$ .
- c.  $A$  can be both equal or not equal to 0.

A matrix  $A \in \mathbb{R}^{n \times n}$  is orthogonal if:

**Scegli un'alternativa:**

- a.  $A^{-1}A = I = AA^{-1}$ .
- b.  $A^T A = I = AA^T$ .
- c.  $A = A^T$ .

If  $X : \Omega \rightarrow \mathcal{T}$  is a continuous random variable, then a function  $p : \mathcal{T} \rightarrow \mathbb{R}_+$  can be the PDF of  $X$  if:

Scegli un'alternativa:

- a.  $\int_{\mathcal{T}} p(x)dx = 1.$
- b.  $\int_{\Omega} p(x)dx = 1.$
- c.  $\int_{\mathcal{T}} p(x)dx < \infty.$

For a random variable  $X : \Omega \rightarrow \mathcal{T}$  with  $\mathbb{E}[X] = 0$ , it holds:

**Scegli un'alternativa:**

- a.  $Var(X) = \mathbb{E}[X]$ .
- b.  $Var(X) = \mathbb{E}[X^2]$ .
- c.  $Var(X) = 0$ .

If  $X : \Omega \rightarrow \mathcal{T}$  is a discrete random variable, then a function  $f_X : \mathcal{T} \rightarrow [0, 1]$  can be the PDF of  $X$  if:

**Scegli un'alternativa:**

- a.  $\sum_{i \in \Omega} f_X(i) = 1.$
- b.  $\int_{\Omega} f_X(x) dx = 1.$
- c.  $\sum_{i \in \mathcal{T}} f_X(i) = 1.$

Given a discrete random variable  $X : \Omega \rightarrow \mathcal{T}$ , with  $\mathcal{T} = \{1, 2, \dots, 6\}$ , and  $f_X = \{\frac{1}{6}, \frac{1}{6}, \dots, \frac{1}{6}\}$ , then:

Scegli un'alternativa:

a.  $\mathbb{E}[X] = 21$ .

b.  $\mathbb{E}[X] = 3.5$ .

c.  $\mathbb{E}[X] = \frac{1}{6}$ .

$$\begin{aligned}\mathbb{E}[x] &= 1 \cdot \frac{1}{6} + 2 \cdot \frac{1}{6} + 3 \cdot \frac{1}{6} + 4 \cdot \frac{1}{6} + 5 \cdot \frac{1}{6} + 6 \cdot \frac{1}{6} \\ &= \frac{21}{6} =\end{aligned}$$

Given a continuous random variable  $X : \Omega \rightarrow \mathcal{T}$ , with  $\mathcal{T} = [0, 1]$ , and  $p(x) = 3x^2$  its PDF, then:

Scegli un'alternativa:

- a.  $E[X] = 2.$
- b.  $E[X] = 3.$
- c.  $E[X] = \frac{3}{4}.$

$$\begin{aligned} E[X] &= \int_0^1 x \cdot 3x^2 \, dx = \int_0^1 3x^3 \, dx = \\ &= \left[ \frac{3}{4}x^4 \right]_0^1 = \frac{3}{4} \end{aligned}$$

Given a continuous random variable  $X : \Omega \rightarrow \mathcal{T}$ , with  $\mathcal{T} = [0, 1]$ , and  $p(x) = 2x$  its PDF, then:

Scegli un'alternativa:

- a.  $\mathbb{E}[X] = \frac{2}{3}$ .
- b.  $\mathbb{E}[X] = 2$ .
- c.  $\mathbb{E}[X] = 1$ .

$$\int_0^1 2x^2 dx = \left[ \frac{2}{3}x^3 \right]_0^1$$

If  $X : \Omega \rightarrow \mathcal{T}$  is a continuous random variable with PDG  $p : \mathcal{T} \rightarrow \mathbb{R}_+$ , then:

Scegli un'alternativa:

- a.  $\mathbb{E}[X] = \int_{\mathcal{T}} p(x)dx.$
- b.  $\mathbb{E}[X] = \int_{\Omega} xp(x)dx.$
- c.  $\mathbb{E}[X] = \int_{\mathcal{T}} xp(x)dx.$

If  $X : \Omega \rightarrow \mathcal{T}$  is a continuous random variable, its Probability Density Function (PDF)  $p_X(x)$  is defined to be:

**Scegli un'alternativa:**

- a.  $P(X = x) = p_X(x).$
- b.  $P(X = x) = \int x p_X(x) dx.$
- c.  $P(X \in A) = \int_A p_X(x) dx.$

If  $X : \Omega \rightarrow \mathcal{T}$  is a continuous random variable, its Probability Density Function (PDF)  $p_X(x)$  is:

Scegli un'alternativa:

- a. A function  $p_X : \mathcal{T} \rightarrow \mathbb{R}_+$ .
- b. A function  $p_X : \mathcal{T} \rightarrow [0, 1]$ .
- c. A function  $p_X : \Omega \rightarrow [0, 1]$ .

If  $X : \Omega \rightarrow \mathcal{T}$  is a discrete random variable, its Probability Mass Function (PMF)  $f_x$  is:

**Select one:**

- a. A function  $f_X : \mathcal{T} \rightarrow [0, 1]$ .
- b. A function  $f_X : \Omega \rightarrow [0, 1]$ .
- c. A function  $f_X : \mathcal{T} \rightarrow \mathbb{R}$ .

Given two discrete random variable  $X_1 : \Omega \rightarrow \mathcal{T}$ ,  $X_2 : \Omega \rightarrow \mathcal{T}$  with  $\mathcal{T} = \{1, 2, 3\}$ , and  $f_{X_1} = \{\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\}$ ,  $f_{X_2} = \{\frac{1}{2}, \frac{1}{6}, \frac{1}{3}\}$  their PMF, then:

Scegli un'alternativa:

- a.  $\mathbb{E}[X_1] < \mathbb{E}[X_2]$ .
- b.  $\mathbb{E}[X_1] = \mathbb{E}[X_2]$ .
- c.  $\mathbb{E}[X_1] > \mathbb{E}[X_2]$ .

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If  $X : \Omega \rightarrow \mathcal{T}$  is a discrete random variable, its Probability Mass Function (PMF)  $f_x$  is:

Scegli un'alternativa:

- a.  $f_X(x) = \int P(x)dx.$
- b.  $f_X(x) = P(X \in x).$
- c.  $f_X(x) = P(X = x).$

If  $X : \Omega \rightarrow \mathcal{T}$  is a discrete random variable with PMG  $f_X : \mathcal{T} \rightarrow [0, 1]$ , then:

Scegli un'alternativa:

- a.  $\mathbb{E}[X] = \sum_{i \in \mathcal{T}} i$ .
- b.  $\mathbb{E}[X] = \sum_{i \in \mathcal{T}} i f_X(i)$ .  
- Exam subscription November 2

Vai a...

Given a discrete random variable  $X : \Omega \rightarrow \mathcal{T}$ , with  $\mathcal{T} = \{1, 2, 3\}$ , and  $f_X = \{\frac{1}{2}, \frac{1}{6}, \frac{1}{3}\}$  its PMF, then:

Scegli un'alternativa:

- a.  $E[X] = 6$ .
- b.  $E[X] = 2$ .
- c.  $E[X] = \frac{11}{6}$ .

$$\frac{1}{2} + \frac{2}{6} + \frac{3}{3}$$

$$\frac{1}{2} + \frac{1}{3} + 1$$

$$\frac{3+2+6}{6} = \frac{11}{6}$$

Given a discrete random variable  $X : \Omega \rightarrow \mathcal{T}$ , with  $\mathcal{T} = \{1, 2, 3\}$ , and  $f_X = \{\frac{1}{6}, \frac{1}{3}, \frac{1}{2}\}$  its PMF, then:

Scegli un'alternativa:

- a.  $\mathbb{E}[X] = 6$ .
- b.  $\mathbb{E}[X] = 2$ .
- c.  $\mathbb{E}[X] = \frac{13}{6}$ .

$$\frac{1}{6} + \frac{2}{3} + \frac{3}{2}$$

$$1 + 6 + 6$$

Given two discrete random variable  $X_1 : \Omega \rightarrow \mathcal{T}$ ,  $X_2 : \Omega \rightarrow \mathcal{T}$  with  $\mathcal{T} = \{1, 2, 3\}$ , and  $f_{X_1} = \{\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\}$ ,  $f_{X_2} = \{\frac{1}{2}, \frac{1}{6}, \frac{1}{3}\}$  their PMF, then:

Scegli un'alternativa:

- a.  $E[X_1] < E[X_2]$ .
- b.  $E[X_1] = E[X_2]$ .
- c.  $E[X_1] > E[X_2]$ .

$$2 \quad \frac{1}{2} + \frac{2}{6} + \frac{3}{3}$$

$$\frac{3+2+6}{6}$$

$$\frac{1}{6} + \frac{2}{3} + \frac{3}{2} = \frac{1+4+9}{6} = \frac{14}{6} \quad \textcircled{2}$$

Given two discrete random variable  $X_1 : \Omega \rightarrow \mathcal{T}, X_2 : \Omega \rightarrow \mathcal{T}$  with  $\mathcal{T} = \{1, 2, 3\}$ , and  $f_{X_1} = \{\frac{1}{2}, \frac{1}{6}, \frac{1}{3}\}, f_{X_2} = \{\frac{1}{6}, \frac{1}{3}, \frac{1}{2}\}$  their PMF, then:

Scegli un'alternativa:

- a.  $\mathbb{E}[X_1] > \mathbb{E}[X_2]$ .
- b.  $\mathbb{E}[X_1] = \mathbb{E}[X_2]$ .
- c.  $\mathbb{E}[X_1] < \mathbb{E}[X_2]$ .

$$\frac{1}{2} + \frac{2}{6} + \frac{3}{3} = \frac{3+2+6}{6} = \frac{11}{6} \quad \textcircled{1}$$

The precision is:

**Scegli un'alternativa:**

- a. None of the above.
- b. The number of digits with which a number is expressed.
- c. The number of correct significant digits in approximating some quantity.

A random variable  $X$  is:

**Scegli un'alternativa:**

- a. A function  $X : \Omega \rightarrow \mathcal{T}$ .
- b. A variable that returns random elements with known probability.
- c. A set that contains the possible outcomes of the experiment.

If  $\Omega$  is the sample space,  $\mathcal{A}$  is the event space and  $\mathcal{T}$  is a subset of  $\mathbb{R}$ , a random variable  $X$  is:

**Scegli un'alternativa:**

- a. A function  $X : \Omega \rightarrow \mathcal{A}$ .
- b. A function  $X : \mathcal{A} \rightarrow \mathcal{T}$ .
- c. A function  $X : \Omega \rightarrow \mathcal{T}$ .

In normalized scientific notation and base  $\beta = 10$ , if  $x = 2.71$ , then:

**Scegli un'alternativa:**

- a. The mantissa of  $x$  is 0.271 and the exponential part is  $10^1$ .
- b. The mantissa of  $x$  is 2.71 and the exponential part is  $10^0$ .
- c. None of the above.

$$\underline{A} = U \Sigma V$$

$m \times n \quad m \times n \quad n \times n$

$$mn \quad \begin{matrix} u_i & v_i \\ m \times 1 & 1 \times n \end{matrix} \rightarrow mn$$

If  $A = U\Sigma V^T$  is the SVD decomposition of  $A \in \mathbb{R}^{m \times n}$ , then a dyade  $A_i = u_i v_i^T$  of  $A$  is:

Scegli un'alternativa:

- a. None of the above.
- b. A vector of length  $mn$  that express some properties of  $A$ .
- c. A rank-1 matrix of dimension  $m \times n$ .

If  $A = U\Sigma V^T$  is the SVD decomposition of an  $m \times n$  matrix  $A$ , then:

Scegli un'alternativa:

- a.  $A^T A = V^T \Sigma^2 V$ .
- b.  $A^T A = U \Sigma^2 U^T$ .
- c.  $A^T A = V \Sigma^2 V^T$ .

$$\begin{aligned}A^T A &= (V \Sigma V^T) \cdot (U \Sigma U^T) = \\&= V \underbrace{\Sigma}_{\Sigma^2} U^T U \Sigma V^T = V \Sigma^2 V^T\end{aligned}$$

If  $A = U\Sigma V^T$  is the SVD decomposition of an  $m \times n$  matrix  $A$ , then:

**Scegli un'alternativa:**

- a. The rows of  $V^T$  are eigenvectors of  $AA^T$ .
- b. The columns of  $U$  are eigenvectors of  $AA^T$ .
- c. None of the above

If  $A = U\Sigma V^T$  is the SVD decomposition of an  $m \times n$  matrix  $A$ , then:

**Scegli un'alternativa:**

- a. The rows of  $V^T$  are eigenvectors of  $A^T A$ .
- b. None of the above
- c. The columns of  $U$  are eigenvectors of  $A^T A$ .

Given a matrix  $A \in \mathbb{R}^{m \times n}$ ,  $m > n$ , with  $r = \text{rank}(A)$ , then:

**Scegli un'alternativa:**

- a. None of the above.
- b. It is always possible to write  $A$  as  $U\Sigma V^T$ , where  $\Sigma \in \mathbb{R}^{m \times n}$  is diagonal,  $U \in \mathbb{R}^{m \times m}$ ,  $V \in \mathbb{R}^{n \times n}$  are orthogonals.
- c. It is possible to write  $A$  as  $U\Sigma V^T$ , where  $\Sigma \in \mathbb{R}^{m \times n}$  is diagonal,  $U \in \mathbb{R}^{m \times m}$ ,  $V \in \mathbb{R}^{n \times n}$  are orthogonals if and only if  $\text{rank}(A) = n$ .

$$U \Sigma V^T V \Sigma U^T$$

If  $A = U\Sigma V^T$  is the SVD decomposition of an  $m \times n$  matrix  $A$ , then:

Scegli un'alternativa:

a.  $AA^T = U^T \Sigma^2 U$ .

b.  $AA^T = U\Sigma^2 U^T$ .

c.  $AA^T = U\Sigma U^T$ .

If  $A = U\Sigma V^T$  is the SVD decomposition of an  $m \times n$  matrix  $A$ , then:

**Scegli un'alternativa:**

- a. The elements of the diagonal matrix  $\Sigma$  are the singular values of  $A$ , in decreasing order.
- b. The elements of the diagonal matrix  $\Sigma$  are the eigenvalues of  $A$ , in decreasing order.
- c. None of the above.

If  $A = U\Sigma V^T$  is the SVD decomposition of an  $m \times n$  matrix  $A$ , then:

Scegli un'alternativa:

- a. The singular values  $\sigma_i$  of  $A$  are  $\sigma_i = \sqrt{\lambda_i(A^T A)}$  where  $\lambda_i(A^T A)$  are the eigenvalues of  $A^T A$ .
- b. None of the above
- c. The singular values  $\sigma_i$  of  $A$  are  $\sigma_i = \lambda_i(A^T A)$  where  $\lambda_i(A^T A)$  are the eigenvalues of  $A^T A$ .

If  $A = U\Sigma V^T$  is the SVD decomposition of  $A \in \mathbb{R}^{m \times n}$ , then the rank  $k$  approximation of  $A$  is:

**Scegli un'alternativa:**

- a.  $\hat{A}(k) = \sum_{i=1}^n \sigma_i A_i$ , where  $A_i = u_i v_i^T$  is a dyade.
- b.  $\hat{A}(k) = \sum_{i=1}^k A_i$ , where  $A_i = u_i v_i^T$  is a dyade.
- c.  $\hat{A}(k) = \sum_{i=1}^k \sigma_i A_i$ , where  $A_i = u_i v_i^T$  is a dyade.

If  $A = U\Sigma V^T$  is the SVD decomposition of an  $m \times n$  matrix  $A$ , then:

**Scegli un'alternativa:**

- a. The elements on the diagonal of  $\Sigma$  are strictly positive.
- b. None of the above.
- c. The elements of the diagonal matrix  $\Sigma$  are non-negative.

If  $A = U\Sigma V^T$  is the SVD decomposition of  $A \in \mathbb{R}^{m \times n}$ , then its rank  $k$  approximation of  $\hat{A}(k)$  satisfies:

Select one:

- a.  $\hat{A}(k) = \arg \min_{rk(B)=k} \|A - B\|_2.$
- b.  $\hat{A}(k) = \arg \min_{rk(B)=k} \|A - B\|_F.$
- c.  $\hat{A}(k) = \sigma_{k+1}.$

If  $A \in \mathbb{R}^{m \times n}$ , then:

**Scegli un'alternativa:**

- a.  $A^T A$  is symmetric but not necessarily positive definite.
- b.  $A^T A$  is always symmetric and positive definite.
- c. It depends on  $A$ .

For a random variable  $X : \Omega \rightarrow \mathcal{T}$ , its variance is defined as:

Scegli un'alternativa:

- a.  $\text{Var}(X) = \mathbb{E}[X - \mathbb{E}[X]].$
- b.  $\text{Var}(X) = \mathbb{E}[(X - \mathbb{E}[X])^2].$
- c.  $\text{Var}(X) = \mathbb{E}[X^2 - \mathbb{E}[X]^2].$

For a random variable  $X : \Omega \rightarrow \mathcal{T}$ , it holds:

**Scegli un'alternativa:**

- a.  $Var(X) = \mathbb{E}[X^2] - \mathbb{E}[X]^2$ .
- b.  $Var(X) = \mathbb{E}[X^2] + \mathbb{E}[X]^2$ .
- c.  $Var(X) = \mathbb{E}[X]^2 - \mathbb{E}[X^2]$ .

For a random variable  $X : \Omega \rightarrow \mathcal{T}$  with  $\mathbb{E}[X] = 0$ , it holds:

**Scegli un'alternativa:**

a.  $Var(X) = \mathbb{E}[X^2]$ .

b.  $Var(X) = 0$ .

c.  $Var(X) = \mathbb{E}[X]$ .

If vector  $v = (10^6, 0)^T$  is approximated by vector  $\tilde{v} = (999996, 1)^T$ , then in  $\|\cdot\|_1$  the relative error between  $v$  and  $\tilde{v}$  is:

Scegli un'alternativa:

- a.  $4 \cdot 10^{-6}$ .
- b. None of the above.
- c.  $5 \cdot 10^{-6}$ .

$$d = |(10^6 - 999996)| + |0 - 1| = 5$$

If vector  $v = (10^6, 0)^T$  is approximated by vector  $\tilde{v} = (999996, 1)^T$ , then in  $\|\cdot\|_2$  the relative error between  $v$  and  $\tilde{v}$  is:

**Scegli un'alternativa:**

- a.  $\sqrt{17} \cdot 10^{-6}$ .
- b.  $4 \cdot 10^{-6}$ .
- c. None of the above.

If vector  $v = (1, 0)^T$  is approximated by vector  $\tilde{v} = (1.1, 0.1)^T$ , then in  $\|\cdot\|_1$  the relative error between  $v$  and  $\tilde{v}$  is:

$$|1 - 1,1| + |0 - 0,1|$$

Scegli un'alternativa:

- a. 0.1.
- b. 0.2.
- c. None of the above.

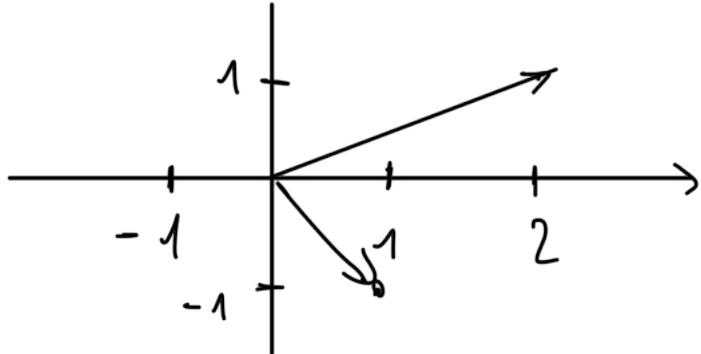
If vector  $v = (10^6, 1)^T$  is approximated by vector  $\tilde{v} = (999996, 1)^T$ , then in  $\|\cdot\|_\infty$  the relative error between  $v$  and  $\tilde{v}$  is:

Scegli un'alternativa:

- a. 4.
- b.  $4 \cdot 10^{-6}$ .
- c. None of the above.

$$d = \max \left\{ \left| \frac{10^6 - 999996}{10^6} \right|, \left| \frac{1 - 1}{1} \right| \right\} =$$

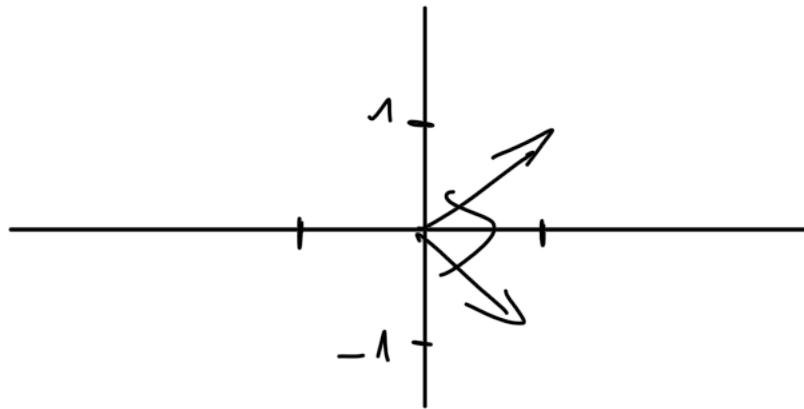
4



Given  $x = (1, -1)^T$  and  $y = (2, 1)^T$ , then:

Scegli un'alternativa:

- a. None of the above.
- b.  $x \perp y$ .
- c.  $x \parallel y$ .



Given  $x = (1, -1)^T$  and  $y = (1, 1)^T$ , then:

Scegli un'alternativa:

- a.  $x \perp y$ .
- b. None of the above.
- c.  $x \parallel y$ .