

## 4. SMT Technology

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### **Combinatorial Decision Making and Optimization**

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# SMT Solvers

- Given a theory  $\mathcal{T}$ , a  $\mathcal{T}$ -solver is a procedure for deciding whether a conjunction of  $\mathcal{T}$ -literals is **satisfiable**
  - $\mathcal{T} = \text{EUF, arithmetic, arrays, bit-vectors, ...}$
- We can define a **SMT solver** as a collection of  $\mathcal{T}_i$ -solvers for different theories  $\mathcal{T}_i$ 
  - Maybe combinations of these theories
- SMT solvers can handle formulas involving variables of different **sort**
  - $\text{sort} \equiv \text{type (Integer, Real, Array, String, ...)}$
- The user **interacts** with SMT solver through **queries**
  - e.g., to check the satisfiability of a formula or add new formulas

- Nowadays plenty of SMT solvers available
  - Especially used for **software analysis** applications
- Some of them are “**special-purpose**”
  - E.g., only for solving bit-vectors or non-linear arithmetic formulas
- Some others more “**general-purpose**”
  - They can handle disparate theories
  - We shall see 2 of them: **Z3** and **CVC5**

- Z3 is a well-known SMT solver with specialized algorithms for efficiently tackling **several theories**
  - First paper describing Z3: De Moura, L., and N. Bjørner. “Z3: *An efficient SMT solver*” **TACAS 2008**
- Z3 is **open source**: <https://github.com/z3prover/z3>
- It provides **APIs** for common programming languages like **Python, Java, C++, .Net**
  - We'll see some examples of **Z3Py**
  - <https://ericpony.github.io/z3py-tutorial/guide-examples.htm>
- Let's see some of the Z3 main features
  - From <https://theory.stanford.edu/~nikolaj/programmingz3.html>

# Z3Py Example

```
from z3 import *
Tie, Shirt = Bools('Tie Shirt')
s = Solver()
s.add(
    Or(Tie, Shirt),
    Or(Not(Tie), Shirt),
    Or(Not(Tie), Not(Shirt))
)
print(s.check())
print(s.model())
```

- The solver check if  $(Tie \vee Shirt) \wedge (\neg Tie \vee Shirt) \wedge (\neg Tie \vee \neg Shirt)$  is satisfiable, and in case prints a model
  - Is this formula satisfiable?

tie\_shirt.py

# Z3Py Example

```
from z3 import *
Tie, Shirt = Bools('Tie Shirt')
s = Solver()
s.add(
    Or(Tie, Shirt),
    Or(Not(Tie), Shirt),
    Or(Not(Tie), Not(Shirt))
)
print(s.check())
print(s.model())
```

- The solver check if  $(Tie \vee Shirt) \wedge (\neg Tie \vee Shirt) \wedge (\neg Tie \vee \neg Shirt)$  is satisfiable, and in case prints a model
  - Is this formula satisfiable?

```
sat
[Tie = False, Shirt = True]
```

# Z3 Sorts

- Z3 handles different **sorts** apart from built-in **Bool**, e.g.
  - **Int**
  - **Real**
  - **BitVec**
  - **Array**
  - **String**
- **Formulas** are terms of **Bool** sort. They may include (un-)interpreted functions and constants. E.g., in:

```
B = BoolSort() ; Z = IntSort()
f = Function('f', B, Z) ; g = Function('g', Z, B)
a = Bool('a')
solve(g(1+f(a)))
```

we have  $f : \mathbb{B} \rightarrow \mathbb{Z}, g : \mathbb{Z} \rightarrow \mathbb{B}, a \in \mathbb{B}$ , with  $\mathbb{B} = \{true, false\}$  and we ask if  $g(1 + f(a))$  is satisfiable

# Example

```
Z = IntSort()
f = Function('f', Z, Z)
x, y, z = Ints('x y z')
A = Array('A', Z, Z)
fml = Implies(x + 2 == y,
              f(Store(A, x, 3)[y-2]) == f(y-x+1))
solve(Not(fml))
```

Here fml corresponds to formula:

$$x + 2 = y \implies f(\text{write}(A, x, 3)[y - 2]) = f(y - x + 1)$$

so Not(fml) is:

$$x + 2 = y \wedge f(\text{write}(A, x, 3)[y - 2]) \neq f(y - x + 1)$$

Is Not(fml) satisfiable?

not\_fml.smt2



- Z3 supports several well-known theories. The “baseline” is the **EU**F theory. E.g., we can write something like:

```
S = DeclareSort('S')
a, b, c, d, e, s, t = Consts('a b c d e s t', S)
f = Function('f', S, S, S) ; g = Function('g', S, S)
solve([a == b, b == c, d == e, b == s, d == t,
      f(a, g(d)) != f(g(e), b)])
```

Is this formula satisfiable?

euf.py

# EUF theory

- Z3 supports several well-known theories. The “baseline” is the EUF theory. E.g., we can write something like:

```
S = DeclareSort('S')
a, b, c, d, e, s, t = Consts('a b c d e s t', S)
f = Function('f', S, S, S) ; g = Function('g', S, S)
solve([a == b, b == c, d == e, b == s, d == t,
      f(a, g(d)) != f(g(e), b)])
```

Is this formula satisfiable? Yes. The returned model is:

```
s = S!val!0, b = S!val!0, a = S!val!0, c = S!val!0,
d = S!val!1, e = S!val!1, t = S!val!1,
f = [(S!val!2, S!val!0) -> S!val!4,
      else -> S!val!3],
g = [else -> S!val!2]
```

This is computed with a standard congruence closure procedure

# Arithmetic theory

- Arithmetical constraints are clearly fundamental. Z3 has different procedures according to which **fragment** of arithmetic is used:

Logic	Description	Solver	Example
LRA	Linear Real Arithmetic	Dual Simplex [28]	$x + \frac{1}{2}y \leq 3$
LIA	Linear Integer Arithmetic	Cuts + Branch	$a + 3b \leq 3$
LIRA	Mixed Real/Integer	[7, 12, 14, 26, 28]	$x + a \geq 4$
IDL	Integer Difference Logic	Floyd-Warshall	$a - b \leq 4$
RDL	Real Difference Logic	Bellman-Ford	$x - y \leq 4$
UTVPI	Unit two-variable / inequality	Bellman-Ford	$x + y \leq 4$
NRA	Polynomial Real Arithmetic	Model based CAD [42]	$x^2 + y^2 < 1$
NIA	Non-linear Integer Arithmetic	CAD + Branch [41] Linearization [15]	$a^2 = 2$

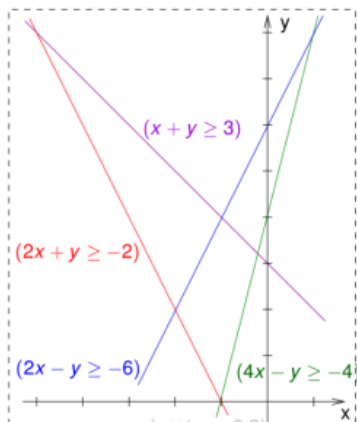
CAD = Cylindrical Algebraic Decomposition

- If we need to precisely model **finite precision** arithmetic, then using fixed-width **bit-vectors** is probably a better choice
  - Z3 handles bit-vectors with eager SAT encoding (**bit-blasting**)

# Example

$$\begin{aligned}\varphi \stackrel{\text{def}}{=} & (\neg A_1 \vee (2x + y \geq -2)) \\ & \wedge (A_1 \vee (x + y \geq 3)) \\ & \wedge (\neg A_2 \vee (4x - y \geq -4)) \\ & \wedge (A_2 \vee (2x - y \geq -6))\end{aligned}$$

```
from z3 import *
x, y = Ints('x y')
A1, A2 = Bools('A1 A2')
s = Solver()
s.add(Or(Not(A1), 2*x + y >= -2))
s.add(Or(A1, x + y >= 3))
s.add(Or(Not(A2), 4*x - y >= -4))
s.add(Or(A2, 2*x - y >= -6))
print(s.check())
print(s.model())
```

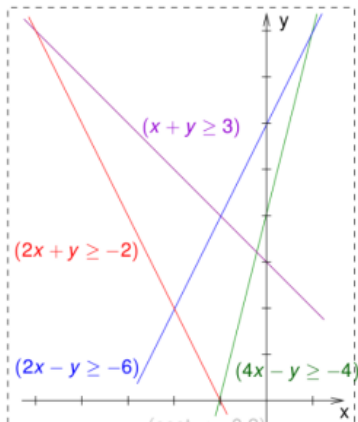


ex\_lia.py

# Example

$$\begin{aligned}\varphi \stackrel{\text{def}}{=} & (\neg A_1 \vee (2x + y \geq -2)) \\ & \wedge (A_1 \vee (x + y \geq 3)) \\ & \wedge (\neg A_2 \vee (4x - y \geq -4)) \\ & \wedge (A_2 \vee (2x - y \geq -6))\end{aligned}$$

```
from z3 import *
x, y = Ints('x y')
A1, A2 = Bools('A1 A2')
s = Solver()
s.add(Or(Not(A1), 2*x + y >= -2))
s.add(Or(A1, x + y >= 3))
s.add(Or(Not(A2), 4*x - y >= -4))
s.add(Or(A2, 2*x - y >= -6))
print(s.check())
print(s.model())
```



[A1 = True, x = -1, A2 = False, y = 1]

(note we are not optimizing here)

- Z3 offers a number of other theories:
  - Arrays
    - Via reduction to EUF
  - Floating points
    - Via reduction to Bit-vectors
  - Algebraic Datatypes
    - Captures the theory of finite trees
  - String and Sequences
    - Theory of free monoids + specific operations (length, replace, ...)

# Incremental solving

- Z3 allows **incremental** solving via **push** and **pop** operations
  - This creates **local scopes**: assertions added within a “push” are **retracted** on the matching “pop”
  - CP solvers don't have (yet) this capability!

```
p, q, r = Bools('p q r')
s = Solver()
s.add(Implies(p,q))
s.add(Not(q))
print(s.check())    # sat:    p->q /\ !q
s.push()
s.add(p)            # unsat: p->q /\ !q /\ p
print(s.check())
s.pop()             # sat:    p->q /\ !q
print(s.check())
```

# Z3 and optimization

- Z3 enables OMT via the **Optimize** module in 2 ways:
  - By specifying an **objective function**
  - Via **soft constraints**
- The objective function is either a **linear arithmetical** term or a **bit-vector** term
- Soft constraints are assertions that the solver can ignore. The goal is **maximizing** the satisfied soft constraints
  - **MaxSMT**
- Soft constraints might have an optional **weight**. In this case the goal is **minimizing** the sum of the weights of unsatisfied constraints



# Example

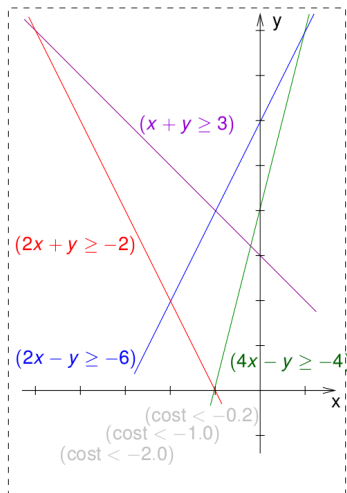
[w. pure-literal filt.  $\Rightarrow$  partial assignments]

- OMT( $\mathcal{LRA}$ ) problem:

$$\begin{aligned} \varphi &\stackrel{\text{def}}{=} (\neg A_1 \vee (2x + y \geq -2)) \\ &\wedge (A_1 \vee (x + y \geq 3)) \\ &\wedge (\neg A_2 \vee (4x - y \geq -4)) \\ &\wedge (A_2 \vee (2x - y \geq -6)) \\ &\wedge (\text{cost} < -0.2) \\ &\wedge (\text{cost} < -1.0) \\ &\wedge (\text{cost} < -2.0) \end{aligned}$$

$$\text{cost} \stackrel{\text{def}}{=} x$$

$$\mu = \left\{ \begin{array}{l} A_1, \neg A_1, A_2, \neg A_2, \\ (4x - y \geq -4), \\ (x + y \geq 3), \\ (2x + y \geq -2), \\ (2x - y \geq -6) \\ (\text{cost} < -0.2) \\ (\text{cost} < -1.0) \\ (\text{cost} < -2.0) \end{array} \right\}$$



# Example

```
from z3 import *
x, y = Reals('x y')
A1, A2 = Bools('A1 A2')
s = Optimize()
s.add(Or(Not(A1), 2*x + y >= -2))
s.add(Or(A1, x + y >= 3))
s.add(Or(Not(A2), 4*x - y >= -4))
s.add(Or(A2, 2*x - y >= -6))
z = s.minimize(x)
print(s.check())
print(s.model())
print(z.value())
```

omt.py

# Example

```
from z3 import *
x, y = Reals('x y')
A1, A2 = Bools('A1 A2')
s = Optimize()
s.add(Or(Not(A1), 2*x + y >= -2))
s.add(Or(A1, x + y >= 3))
s.add(Or(Not(A2), 4*x - y >= -4))
s.add(Or(A2, 2*x - y >= -6))
z = s.minimize(x)
print(s.check())
print(s.model())
print(z.value())

sat
[A1 = True, y = 2, x = -2, A2 = False]
-2
```

- Many other SMT solvers exist apart from Z3
  - <https://smt-comp.github.io/2023/>
- A well-known family of SMT solvers is **CVC\*** (*Cooperating Validity Checker*), a saga originated at **Stanford University**
  - CVC (2002)
  - CVC Lite (2004)
  - CVC3 (2007)
  - **CVC4 (2011)**
  - **CVC5 (2022)**: Barbosa, H., et al. “*cvc5: A Versatile and Industrial-Strength SMT Solver*.” International Conference on Tools and Algorithms for the Construction and Analysis of Systems 2022.

- **CVC5** is a major improvement of CVC4 1.8 (final CVC4 version)
  - New APIs, theories, solvers and procedures
- It provides strong performance on **industrial** use cases
- It is **open-source**: <https://github.com/cvc5/cvc5>
  - Releases: <https://github.com/cvc5/cvc5/releases>
- It can be programmed via **APIs** (C++,Java,Python) or executed in **interactive mode**
  - Documentation: <https://cvc5.github.io/docs/cvc5-0.0.11>
  - **Optimization not supported**, but they're working on that:  
<https://arxiv.org/abs/2404.16122>

- What solver should we use? Z3? CVC5? Both? None of them?
  - Yices2, Vampire, Bitwuzla, (Opti)MathSAT, ...
- Clearly each SOTA solvers has strengths and weaknesses, selecting the best of them for an **unforeseen** SMT formula is not trivial
  - **Algorithm Selection** problem
- Surely we shouldn't write  $n > 1$  **programs** for solving the **same** formula with  **$n$  solvers**
  - *Model once, solve everywhere*
- Need for **standardization**

- SMT-LIB initiative started in 2003 to:
  - Provide rigorous descriptions of SMT theories
  - Develop and promote common languages for SMT solvers.
  - Connect developers, researchers and users of the SMT community
  - Establish and make available benchmarks for SMT solvers.
  - Collect and promote software tools useful to the SMT community
- SMT-LIB website: <https://smtlib.cs.uiowa.edu/>
- SMT-LIB 2.6 specifications: <https://link.springer.com/content/pdf/bbm%3A978-3-662-50497-0%2F1.pdf>

# SMT-LIB language

- SMT-LIB uses a **parenthesized prefix** notation (similar to **LISP**)
  - Designed to be **machine-readable** rather than human-readable
  - E.g.  $(= (+ a b) c)$  or  $(< (f x) (g y z))$
- 3 main components: **theory** declarations, **logic** declarations, **scripts**
- SMT-LIB **theories** are defined by **sorts** and **functions**
  - Predicates  $\equiv$  Bool-valued functions
  - E.g., theory of **Ints**, **Reals**, ...
- SMT-LIB **logic** = Theory declarations + **restrictions** on formulas
  - E.g., **QF\_IDL** logic is based on theory of **Ints** and restricts (in)equalities to be of the form  $x - y \bowtie k$  with  $\bowtie \in \{=, \neq, <, \leq, \geq, >\}$



# Theory of integers

```
(theory Ints

  :smt-lib-version 2.6
  :smt-lib-release "2017-11-24"
  :written-by "Cesare Tinelli"
  :date "2010-04-17"
  :last-updated "2015-04-25"
  :update-history
  "Note: history only accounts for content changes, not release changes.
   2015-04-25 Updated to Version 2.5.
  "

  :sorts ((Int 0))

  :funs ((NUMERAL Int)
    (- Int Int) ; negation
    (- Int Int Int :left-assoc) ; subtraction
    (+ Int Int Int :left-assoc)
    (* Int Int Int :left-assoc)
    (div Int Int Int :left-assoc)
    (mod Int Int Int)
    (abs Int Int)
    (<= Int Int Bool :chainable)
    (< Int Int Bool :chainable)
    (>= Int Int Bool :chainable)
    (> Int Int Bool :chainable)
  )
)
```

# Integer difference logic

```
(Logic QF_IDL

  :smt-lib-version 2.6
  :smt-lib-release "2017-11-24"
  :written-by "Cesare Tinelli"
  :date "2010-04-30"
  :last-updated "2015-04-25"
  :update-history
    "Note: history only accounts for content changes, not release changes.
    2015-04-25 Updated to Version 2.5.
    "

  :theories ( Ints )

  :language
    "Closed quantifier-free formulas with atoms of the form:
    - q
    - (op (- x y) n),
    - (op (- x y) (- n)), or
    - (op x y)
    where
      - q is a variable or free constant symbol of sort Bool,
      - op is <, <=, >, >=, =, or distinct,
      - x, y are free constant symbols of sort Int,
      - n is a numeral.
    "
)
```

# SMT-LIB syntax

- `set-logic` specifies the logic
  - E.g. `(set-logic QF_IDL)` or `(set-logic QF_LRA)`
- `declare-fun` introduces a new `function` symbol, so it can be used to declare `variables` too (variables  $\equiv$  uninterpreted constants)
  - E.g., function `(declare-fun f (Int Int) Bool)` or variable `(declare-fun x () Real)`
  - Command `(declare-const x () Real)` is also available
- `assert` specifies formulas, and `check-sat` checks the satisfiability of all the specified formulas
  - E.g., `(assert (= (+ a b) c))`
- Others: `(set-option :produce-models true)`, `(get-model)`, `(get-unsat-core (x))`, ...

# SMT-LIB script

- This is an example of SMT-LIB **script**, i.e., a sequence of **commands**

```
(set-logic QF_LRA)
(set-option :produce-models true)
(declare-fun x () Real)
(declare-fun y () Real)
(declare-fun A1 () Bool)
(declare-fun A2 () Bool)
(assert (or (not A1) (>= (+ (* 2 x) y) (- 2))))
(assert (or A1 (>= (+ x y) 3)))
(assert (or (not A2) (>= (- (* 4 x) y) (- 4))))
(assert (or A2 (>= (- (* 2 x) y) (- 6))))
(check-sat)
(get-model)
```

- It encodes  $A1 \Rightarrow 2x + y \geq -2 \wedge \neg A1 \Rightarrow x + y \geq 3 \wedge A2 \Rightarrow 4x - y \geq -4 \wedge \neg A2 \Rightarrow 2x - y \geq -6$

ex\_lra.smt2

# SMT-LIB script

```
$ z3 ex_lra.smt2
```

```
sat
```

```
(  
  (define-fun A1 () Bool true)  
  (define-fun y () Real (/ 13.0 5.0))  
  (define-fun x () Real (- (/ 3.0 5.0)))  
  (define-fun A2 () Bool false)  
)
```

```
$ cvc5 ex_lra.smt2
```

```
sat
```

```
(  
  (define-fun x () Real (/ 4 3))  
  (define-fun y () Real (/ 28 3))  
  (define-fun A1 () Bool false)  
  (define-fun A2 () Bool true)  
)
```

# Assertion stack

- SMT solvers react to commands by modifying an **assertion stack**
- Each stack element is called **level** and consists of a **set of assertions**
  - formulas + declarations/definitions of sorts and functions
- By default a **new assertion** always belongs to the **current level**
  - In the example above, only 1 level
- Levels can be **added** and **removed** with **push** and **pop** commands
  - pop removes **all level assertions**, including declarations/definitions

# Incremental solving

```
(declare-fun x () Real)
(declare-fun y () Real)

(push 1)
(declare-fun A () Bool)
(assert (or (not A) (>= (+ (* 2 x) y) (- 2))))
(assert (or A (>= (- (* 4 x) y) (- 4))))
(check-sat)
(get-model)
(pop 1)

(declare-fun A () Int)
(assert (or (< A 0) (>= (- (* 4 x) y) (- 4))))
(assert (or (>= A 0) (< (- (* 2 x) y) (- 6))))
(check-sat)
(get-model)
```

ex\_lra2.smt2

# Incremental solving

- In the above example we have 2 levels
- After (push 1), one decision level including declarations for Boolean variable  $A$  and  $\{A \Rightarrow 2x + y \geq -2, A \Rightarrow 4x - y \geq -4\}$  is pushed on assertion stack
  - Then we check for satisfiability and ask for a model
- After (pop 1), the last decision level is removed from the stack, so  $A$  can be declared again, this time with a different sort ( $\text{Int}$ )
  - Then we check again for satisfiability and ask for a model



# Incremental solving

```
$ cvc5 -i --produce-models ex_lra2.smt2
```

```
sat
```

```
(  
  (define-fun x () Real (/ (- 9) 8))  
  (define-fun y () Real (/ (- 1) 2))  
  (define-fun A () Bool false)  
)
```

```
sat
```

```
(  
  (define-fun x () Real 0.0)  
  (define-fun y () Real 8.0)  
  (define-fun A () Int (- 1))  
)
```

# SMT-LIB and optimization

- Standard SMT-LIB does not have an explicit support to **optimization**
- One possible workaround is to implement an **offline OMT approach**
  - SMT solvers as **black-boxes**
- SMT solver should be in **incremental** mode avoiding to restart each time a new bound is found
- In **binary search** mode, one should use **push/pop** primitives

# Offline OMT( $\mathcal{LRA}$ )

**Algorithm 1** Offline OMT( $\mathcal{LA}(\mathbb{Q})$ ) Procedure based on Mixed Linear/Binary Search.

**Require:**  $\langle \varphi, \text{cost}, \text{lb}, \text{ub} \rangle$  {ub can be  $+\infty$ , lb can be  $-\infty$ }

```
1:  $l \leftarrow \text{lb}; u \leftarrow \text{ub}; \text{PIV} \leftarrow \top; \mathcal{M} \leftarrow \emptyset$ 
2:  $\varphi \leftarrow \varphi \cup \{ \neg(\text{cost} < l), (\text{cost} < u) \}$ 
3: while ( $l < u$ ) do
4:   if (BinSearchMode()) then {Binary-search Mode}
5:      $\text{pivot} \leftarrow \text{ComputePivot}(l, u)$ 
6:      $\text{PIV} \leftarrow (\text{cost} < \text{pivot})$ 
7:      $\varphi \leftarrow \varphi \cup \{ \text{PIV} \}$ 
8:      $\langle \text{res}, \mu \rangle \leftarrow \text{SMT.IncrementalSolve}(\varphi)$ 
9:      $\eta \leftarrow \text{SMT.ExtractUnsatCore}(\varphi)$ 
10:   else {Linear-search Mode}
11:      $\langle \text{res}, \mu \rangle \leftarrow \text{SMT.IncrementalSolve}(\varphi)$ 
12:      $\eta \leftarrow \emptyset$ 
13:   end if
14:   if ( $\text{res} = \text{SAT}$ ) then
15:      $\langle \mathcal{M}, u \rangle \leftarrow \text{Minimize}(\text{cost}, \mu)$ 
16:      $\varphi \leftarrow \varphi \cup \{ (\text{cost} < u) \}$ 
17:   else { $\text{res} = \text{UNSAT}$ }
18:     if ( $\text{PIV} \notin \eta$ ) then
19:        $l \leftarrow u$ 
20:     else
21:        $l \leftarrow \text{pivot}$ 
22:        $\varphi \leftarrow \varphi \setminus \{ \text{PIV} \}$ 
23:        $\varphi \leftarrow \varphi \cup \{ \neg \text{PIV} \}$ 
24:     end if
25:   end if
26: end while
27: return  $\langle \mathcal{M}, u \rangle$ 
```

$u = \text{current best bound}$

Linear search completed

Updating binary search pivot

From R. Sebastiani, S. Tomasi: *Optimization Modulo Theories with Linear Rational Costs*. ACM Trans. Comput. Log. 16(2): 12:1-12:43 (2015)

# SMT-LIB $\iff$ MiniZinc

- One may think to SMT-LIB as the SMT equivalent of **MiniZinc** language for CP problems
- SMT-LIB is actually “*lower-level*”, more similar to **FlatZinc** language
  - MiniZinc models, together with optional **data** and **solver-specific** redefinitions, are **compiled** into FlatZinc instances
- Translating **SMT-LIB**  $\rightarrow$  **MiniZinc** is quite straightforward, except that MiniZinc does not support all the standard SMT-LIB theories
  - E.g., theory of **arrays** and **strings** not officially supported by MiniZinc
  - **Global constraints** likely lost
- One can translate **SMT-LIB**  $\rightarrow$  **FlatZinc**, if target solver is known or simply ignored

# SMT-LIB $\iff$ MiniZinc

- An early proposal to convert FlatZinc  $\rightarrow$  SMT-LIB by Bofill et al. is **fzn2smt**, used by **Yices** SMT solver in MiniZinc Challenges 2010–2013
  - Based on obsolete MiniZinc versions and no longer maintained
- A prototypical converter SMT-LIB  $\rightarrow$  MiniZinc called **smt2mzn-str** was developed by G. Gange for solving **string constraints**
  - String support in MiniZinc is still experimental
- Contaldo et al. proposed 2 compilers STM-LIB  $\leftrightarrow$  FlatZinc called **fzn2omt** and **omt2fzn**
  - Contaldo, F. et al. “*From MiniZinc to Optimization Modulo Theories, and Back*”. CPAIOR 2020.
  - They use the **default** MiniZinc  $\rightarrow$  FlatZinc decomposition for **global constraints** and SMT-LIB with optimization **extensions**

# Writing SMT-LIB

- From SMT-LIB standard: “...Preferring *ease of parsing* over human readability is reasonable in this context not only because SMT-LIB benchmarks are meant to be *read by solvers* but also because they are produced in the first place by *automated tools* like verification condition generators or translators...”
- One may write a SMT-LIB instance *manually*...
  - Tricky prefix notation, impractical for large instances
- ...or define an *ad hoc script* producing a SMT-LIB instance
  - E.g., Bash or Python script
- ...or define the instance with Z3 and then use *Z3's API* to generate a corresponding SMT-LIB instance
  - E.g., *Z3Py + to\_smt2()* method of Solver class
  - Easier writing, no need to define ad hoc script
  - Translation may introduce additional variables

# Z3Py Example

```
from z3 import *
Tie, Shirt = Bools('Tie Shirt')
s = Solver()
s.add(
    Or(Tie, Shirt),
    Or(Not(Tie), Shirt),
    Or(Not(Tie), Not(Shirt))
)
print(s.to_smt2())
print(s.dimacs())
```

- `to_smt2` returns the **SMT-LIB** instance for solver's assertions
- `dimacs` returns the **DIMACS** instance for solver's assertions

tie\_shirt.py

# SMT-LIB output

```
; benchmark generated from python API
(set-info :status unknown)
(declare-fun Shirt () Bool)
(declare-fun Tie () Bool)
(assert
  (or Tie Shirt))
(assert
  (let (($x8 (not Tie)))
    (or $x8 Shirt)))
(assert
  (let (($x8 (not Tie)))
    (or $x8 (not Shirt))))
(check-sat)
```

- Note the introduction of `$x8`, a **local** variable defined through the `let` binder



# DIMACS output

```
p cnf 2 3
1 2 0
-1 2 0
-1 -2 0
c 1 Tie
c 2 Shirt
```

- Header `p cnf 2 3` means CNF formula with 2 variables and 3 clauses
- Follows one clause per line, terminated with 0
  - `1 2 0` is the clause  $x_1 \vee x_2$
  - `-1 2 0` is the clause  $\neg x_1 \vee x_2$
  - `-1 -2 0` is the clause  $\neg x_1 \vee \neg x_2$
- Each line that begins with `c` is a **comment**

- CVC5 + IDL Theory project: write C++ code  
<https://github.com/cvc5/cvc5/blob/idl-lab/project.md>
- Define SMT-LIB specifications for well-known **NP-Hard** problems (subset-sum, knapsack, TSP, ...) and solve with different SMT solvers
- Take some MiniZinc models and manually **translate** to SMT-LIB  
<https://github.com/MiniZinc/minizinc-benchmarks>
  - compare with fzn2omt translation  
<https://github.com/PatrickTrentin88/fzn2omt>

- Z3 solver

- *Programming Z3*. N. Bjørner, L. de Moura, L. Nachmanson, and C. Wintersteiger  
<https://theory.stanford.edu/~nikolaj/programmingz3.html>
- <https://ericpony.github.io/z3py-tutorial/guide-examples.htm>

- CVC5 solver

- <https://cvc5.github.io/>
- <https://github.com/cvc5/cvc5>

- SMT-LIB initiative

- <https://smtlib.cs.uiowa.edu/>
- <https://smtlib.cs.uiowa.edu/papers/smt-lib-reference-v2.6-r2017-07-18.pdf>