Problem 1

Sampling a Periodic Signal: Write a MATLAB code to generate a sinusoid signal with an amplitude of 1 a.u. (arbitrary unit) and frequency of 0.5 Hz. Plot the analog signal as a function of time. Sample the signal with a sampling time, �� of 0.1 second. Plot the sampled signal, �[�] as a function of sample number, n. [repeat question from assignment 1]

(4 points) Generate a noise signal using ‘rand’ function in MATLAB within the range [-0.5,0.5].

Plot the noise signal.

(4 Points) Add the noise to �[�] to generate a noisy signal, ������[�].

(9 Points) Apply a moving average transformation on ������[�], with a symmetric window when

(i) window size=3 samples (M1 = 1, M2=1)

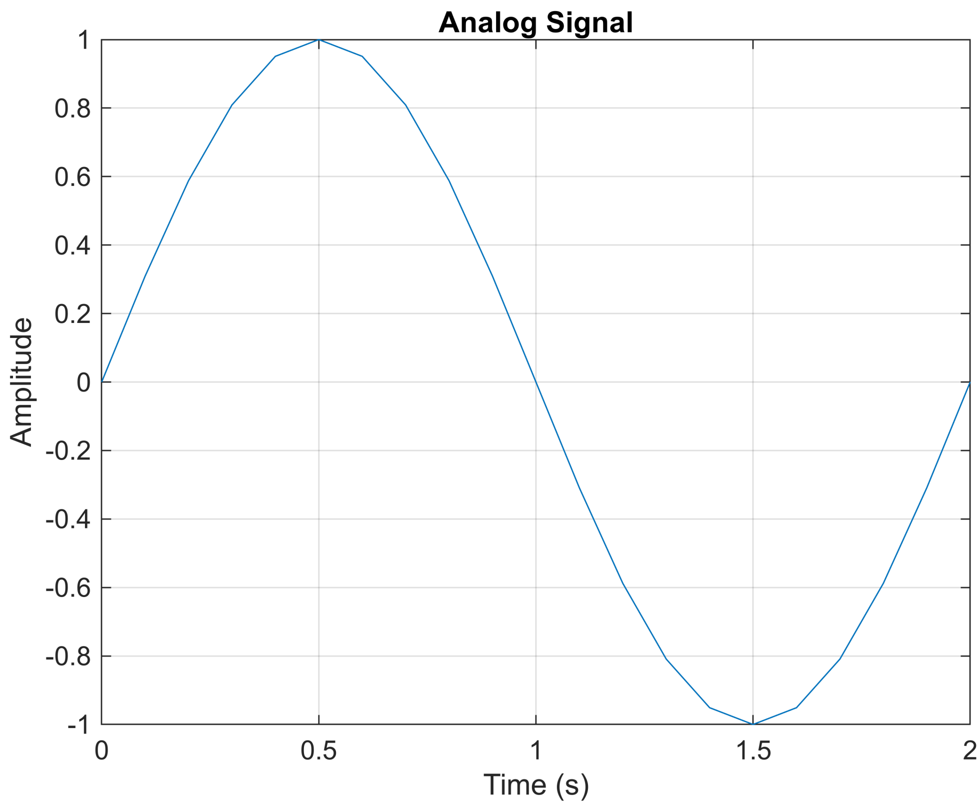
(ii) window size=5 samples (M1 = 2, M2=2)

(iii) window size=7 samples (M1 = 3, M2=3)

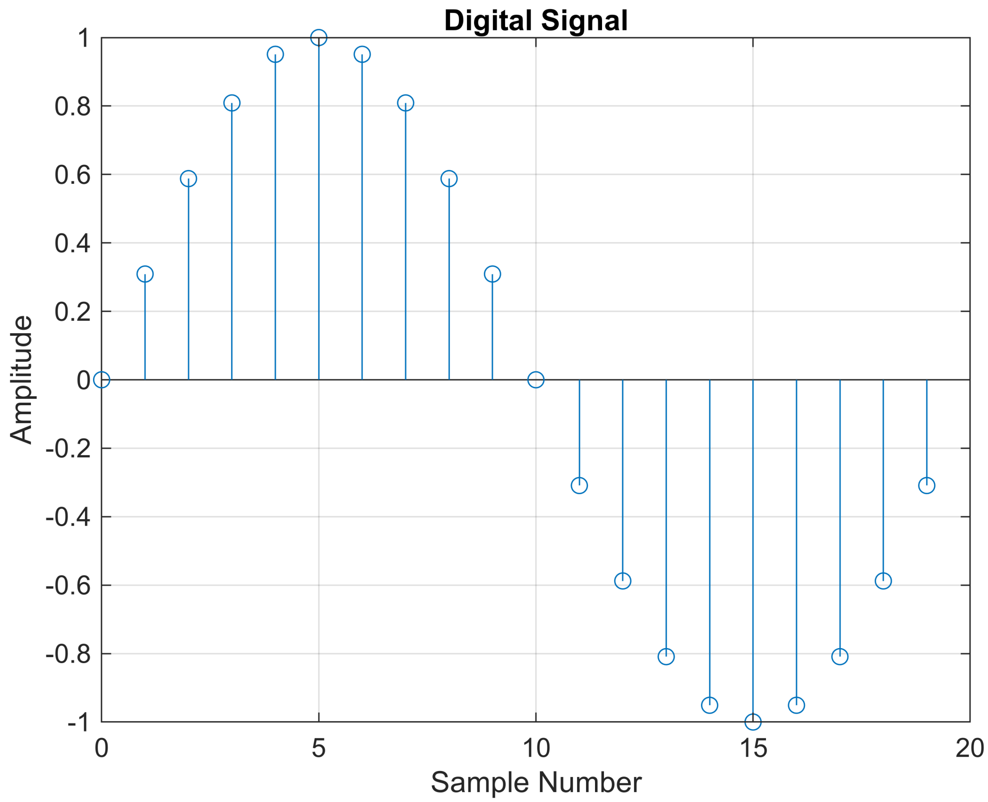
Comment on the optimal window size that recovers the trend of the original signal.

```
clear
clc
% Parameters
A = 1;  % Amplitude in arbitrary units
f = 0.5;  % Frequency in Hz
w = f*2*pi; % Angular frequency
Ts = 0.1; % Sampling Period
N = (2*pi)/(w*Ts);
T = 1/f; % Period

% Generate sinusoid signal
t = 0:0.1:T;
signal = A*sin(w*t);

% Generate sinusoid signal
n = 0:N-1;
t_sampled = n*Ts;
sampled_signal = A*sin(w*t_sampled);

% Generate noise signal
noise_signal = (rand(1, length(n)) - 0.5);

% Add the original signal to the noise signal
x_noise = sampled_signal + noise_signal;

% Moving Average for a window size of 3, 5, and 7
window_3_output = movmean(x_noise, 3);
window_5_output = movmean(x_noise, 5);
window_7_output = movmean(x_noise, 7);
```

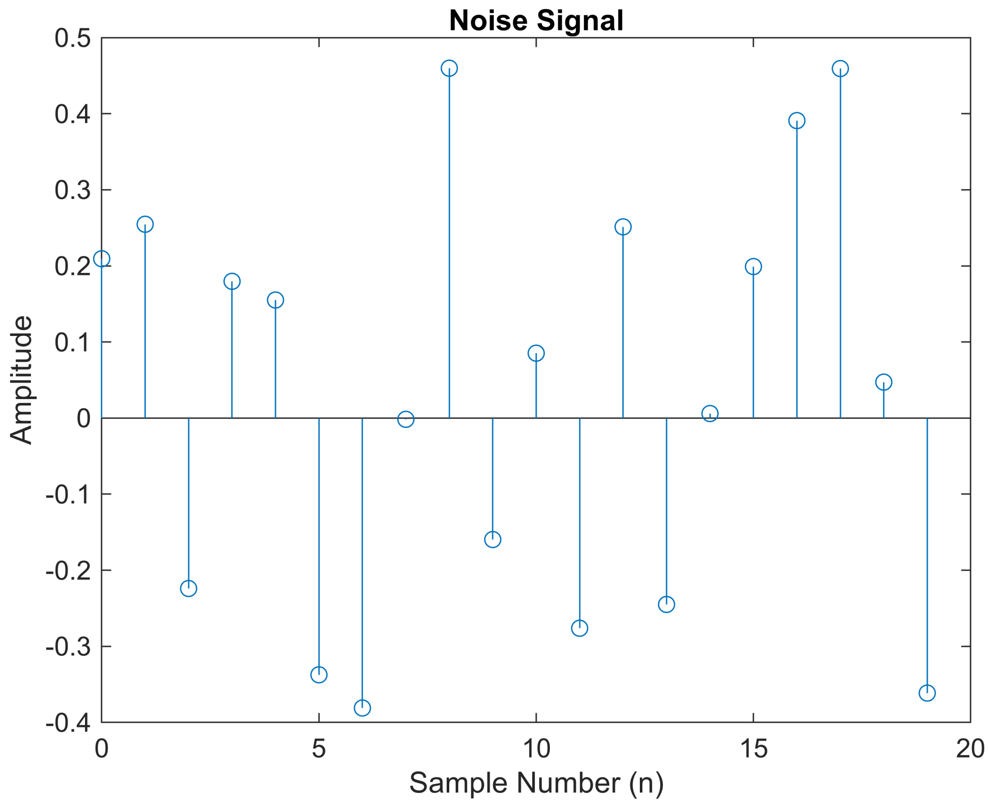
```
% Plot the original analog signal
figure(1);
plot(t, signal);
xlabel('Time (s)');
ylabel('Amplitude');
title('Analog Signal');
grid on;
```



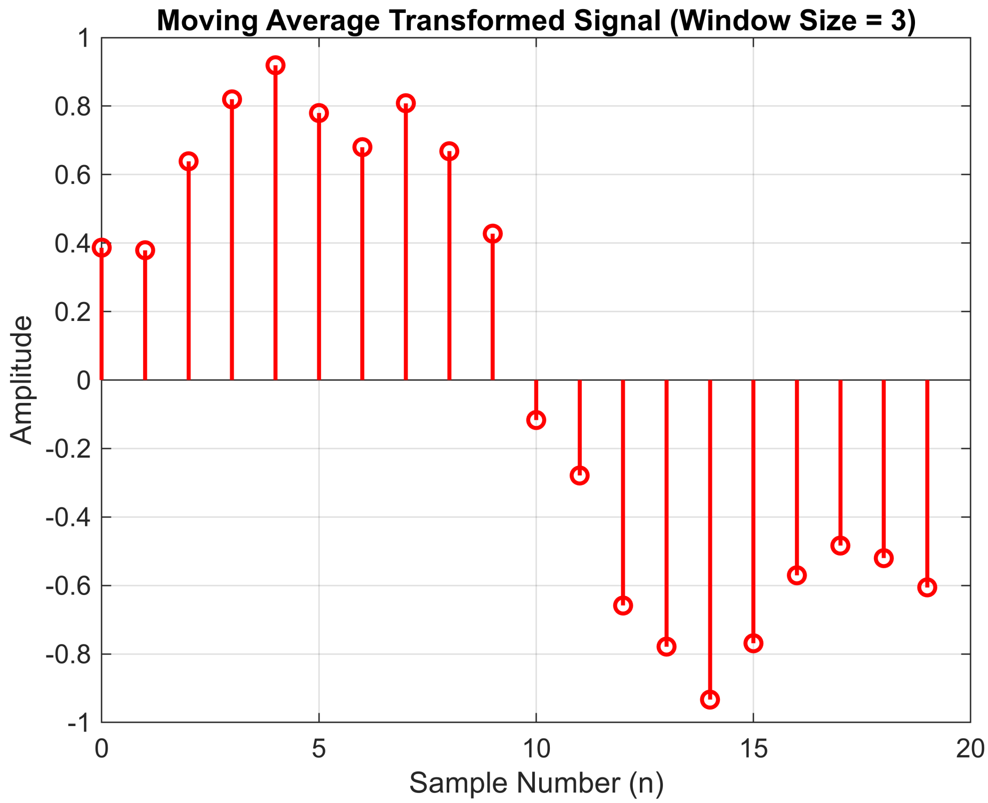

```
% Plot the original digital signal
figure(2);
stem(n, sampled_signal, 'Marker', 'o');
xlabel('Sample Number');
ylabel('Amplitude');
title('Digital Signal');
grid on;
```



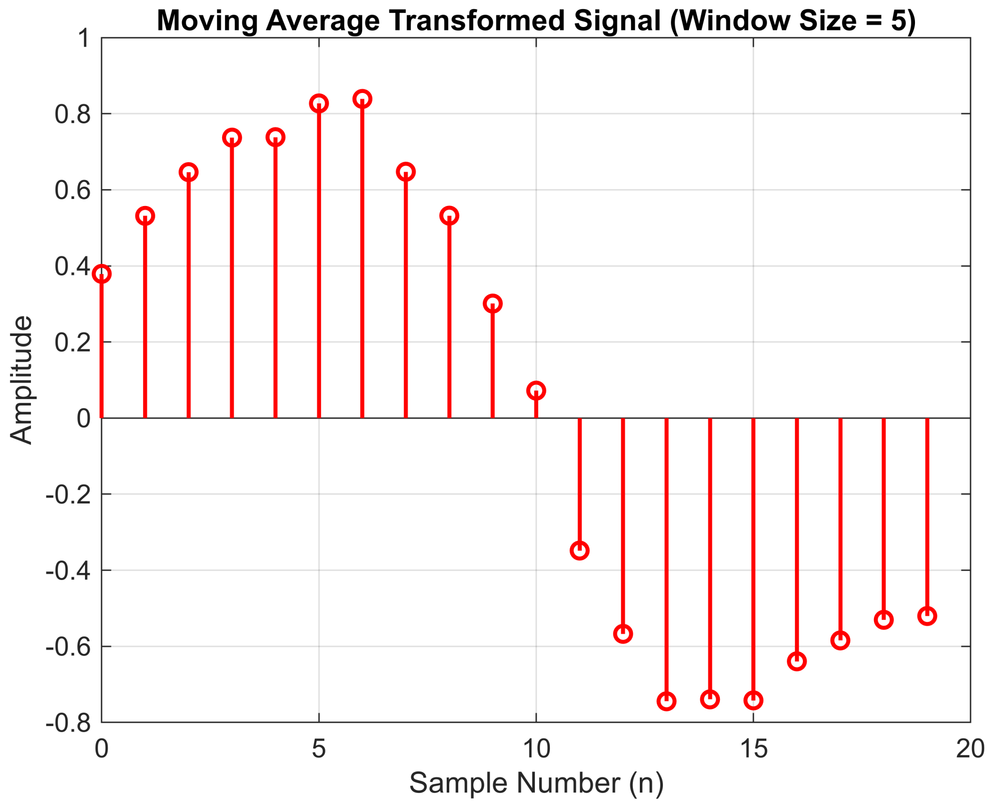

```
% Plot the noise signal
figure(3);
stem(n, noise_signal);
title('Noise Signal');
xlabel('Sample Number (n)');
ylabel('Amplitude');
```



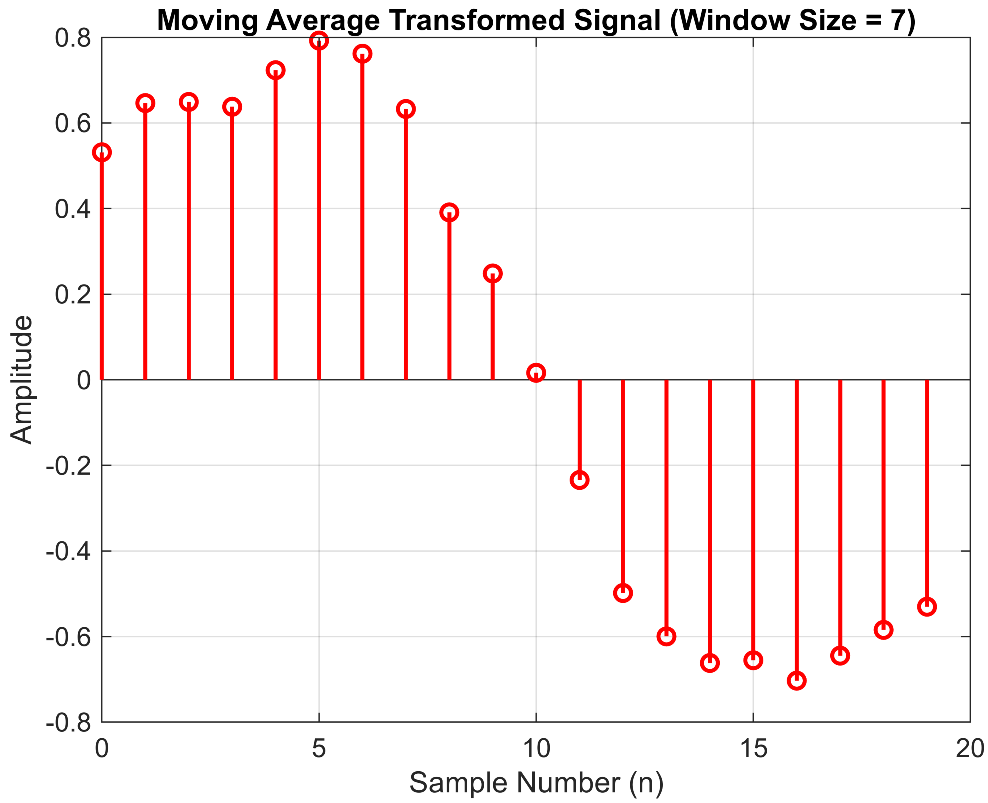

```
% Plot the moving avg for a window of 3
figure(4);
stem(n, window_3_output, 'r', 'LineWidth', 1.5);
title('Moving Average Transformed Signal (Window Size = 3)');
xlabel('Sample Number (n)');
ylabel('Amplitude');
grid on;
```




```
% Plot the moving avg for a window of 5
figure(5);
stem(n, window_5_output, 'r', 'LineWidth', 1.5);
title('Moving Average Transformed Signal (Window Size = 5)');
xlabel('Sample Number (n)');
ylabel('Amplitude');
grid on;
```




```
% Plot the moving avg for a window of 7
figure(6);
stem(n, window_7_output, 'r', 'LineWidth', 1.5);
title('Moving Average Transformed Signal (Window Size = 7)');
xlabel('Sample Number (n)');
ylabel('Amplitude');
grid on;
```


Moving Average Transformed Signal (Window Size = 7)
Amplitude
Sample Number (n)
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
0
5
10
15
20

1

2\. a. $T(x[n]) = \sum_{k=n-n_0}^{n+n_0} x[k]$

1\. assume $|x[n]| \leq M_x < \infty \ \forall n$

since the input is bounded from $n-n_0$ to $n+n_0$, the system is BIBO stable

2\. non-causal, since $x[n+n_0]$ is a part of the system and this would rely on a future input if n_0 is positive

3\. $$y[n] = \sum_{k=n-n_0}^{n+n_0} x[k]$$

$$\alpha x_1[n] + \beta x_2[n] \mapsto \sum_{k=n-n_0}^{n+n_0} \alpha x[k] + \beta x[k]$$

$$= \alpha \sum_{k=n-n_0}^{n+n_0} x[k] + \beta \sum_{k=n-n_0}^{n+n_0} x[k] \rightarrow \text{linear}$$

4\. shifted output: $y_0[n] = y[n-m] = \sum_{k=n-m-n_0}^{n-m+n_0} x[k]$

output due to a shifted input: $x_1[n] = x[n-m]$

$$y_1[n] = \sum_{k=n-n_0}^{n+n_0} x[k-m] \rightarrow \sum_{l=n-n_0-m}^{n+n_0-m} x[l] = y_0[n]$$

change of variables: so time-invariant

$l = k-m \rightarrow k = l+m$

5\. has memory since $x[k]$ depends on past or future inputs of $n+n_0$ and $n-n_0$

2

b. $T(x[n]) = x[n-n_0]$

1. assume $|x[n]| \leq M_x \leq \infty \ \forall n$

$|y[n]| = |x[n-n_0]| = M_x \leq \infty$ so it's stable

2. causal if $n_0 \geq 0$, but non-causal if $n<0$

3. $\alpha x_1[n] + \beta x_2[n] \longmapsto \alpha \underbrace{x[n-n_0]}_{y_1[n]} + \beta \underbrace{x[n-n_0]}_{y_2[n]}$

linear

4. shifted output: $y_0[n] = y[n-m] = x[n-m-n_0]$

output of shifted input:

$y_1[n] = x_1[n-n_0] = x[n-n_0-m]$

$y_0[n] = y_1[n]$ so time invariant

5. has memory since it relies on past or future input depending on the value of n_0

c. $T(x[n]) = e^{\{x[n]\}}$

1. assume $|x[n]| \leq M_x \leq \infty \ \forall n$

$|y[n]| = e^{x[n]} = e^{M_x} \leq \infty$ so $y[n]$ is stable

2. causal since it only depends on present values

3. $\bar{x}[n] = \alpha x_1[n] + \beta x_2[n]$

$\longmapsto e^{\{\alpha x_1[n] + \beta x_2[n]\}} = e^{\alpha x_1[n]} e^{\beta x_2[n]} \neq e^{\alpha x_1[n]} + e^{\beta x_2[n]}$

so non-linear

4. shifted output: $y_0[n] = y[n-m] = e^{\{x[n-m]\}}$

op due to shifted input: $y_1[n] = e^{\{x[n-m]\}}$

so time invariant

3

5. output only depends on the present input so memoryless

d. $T(x[n]) = x[-n]$

1. assume $|x[n]| \le M_x \le \infty \; \forall n$
$|y[n]| = |x[-n]| = M_x < \infty$ so it's stable

2. not causal $\rightarrow$ when n is positive, the output depends on a future input
ex. $n = 2 \rightarrow x[-2]$

3. $\bar{x}[n] = \alpha x_1[n] + \beta x_2[n] \mapsto \alpha \underbrace{x_1[-n]}_{y_1[n]} + \beta \underbrace{x_2[-n]}_{y_2[n]}$
so it's linear

4. shifted output: $y_0[n] = y[n-m] = x[-n+m]$
o/p of a shifted input: $y_1[n] = x[-n-m]$
$x_1[n] = x[n-m]$
since $y_0[n] \ne y_1[n]$, it's time variant

5. has memory since it depends on past values as n increases

e. $T(x[n]) = x[n] + 2u[n+1]$

1. assume $|x[n]| \le M_x < \infty \; \forall n$
$|y[n]| = |x[n] + 2u[n+1]| = |x[n]| + |\underbrace{2u[n+1]}_{1}|$
$|x[n]| + 2 = M_x + 2 < \infty$ so it's stable

2. causal since x[n] only depends on present inputs

3. $\alpha x_1[n] + \beta x_2[n] \mapsto \underbrace{\alpha x[n]}_{\text{only part of } y_1} + \underbrace{\beta x[n] + 2u[n+1]}_{y_2}$
not linear

4

4. shifted output: $y_0[n] = y[n-m] = x[n-m] - 2u[n-m+1]$
op of a shifted input:

$y_1[n] = x_1[n] + 2u[n+1] = x_1[n-m] + 2u[n+1] \neq y_0[n]$

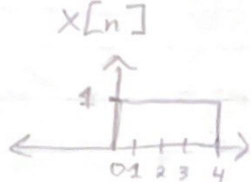
so time variant

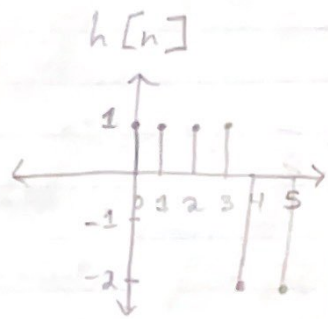
5. memoryless since $x[n]$ only depends on present inputs

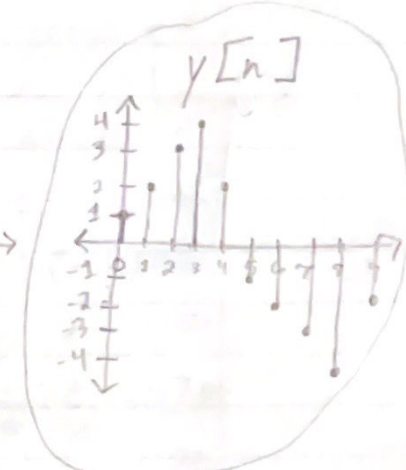
5

3. $$h[n] = \begin{cases} 0, & n<0 \\ 1, & n=0,1,2,3 \\ -2, & n=4,5 \\ 0, & n>5 \end{cases}$$

$x[n] = u[n] - u[n-4]$

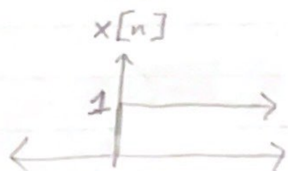


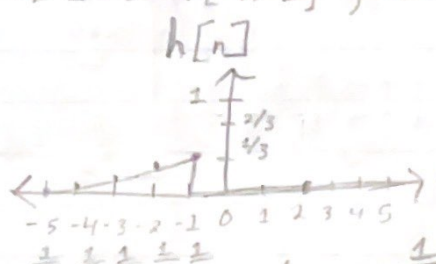


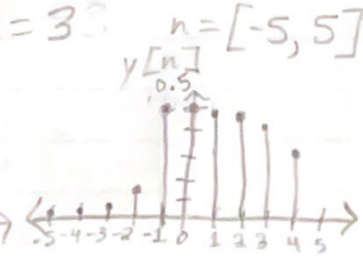


shift 0: 1
shift 1: 1+1 = 2
shift 2: 1+1+1 = 3
shift 3: 1+1+1+1 = 4
shift 4: 1+1+1+1-2 = 2
shift 5: 1+1+1-2-2 = -1
shift 6: 1+1-2-2 = -2
shift 7: 1-2-2 = -3
shift 8: -2-2 = -4
shift 9: -2

4. $x[n] = u[n]$ $h[n] = a^n u[-n-1]$, $a = 3$ $n = [-5, 5]$







$\frac{1}{243}\ \frac{1}{81}\ \frac{1}{27}\ \frac{1}{9}\ \frac{1}{3}$

at $n=-5$: $\frac{1}{243} = 0.0041$

at $n=-4$: $\frac{1}{243} + \frac{1}{81} = 0.0164$

at $n=-3$: $\frac{1}{243} + \frac{1}{81} + \frac{1}{27} = 0.0534$

at $n=-2$: $\frac{1}{243} + \frac{1}{81} + \frac{1}{27} + \frac{1}{9} = 0.165$

at $n=-1$: $\frac{1}{243} + \frac{1}{81} + \frac{1}{27} + \frac{1}{9} + \frac{1}{3} = 0.5$

at $n=0$: $\frac{1}{243} + \frac{1}{81} + \frac{1}{27} + \frac{1}{9} + \frac{1}{3} = 0.5$

at $n=1$: $\frac{1}{81} + \frac{1}{27} + \frac{1}{9} + \frac{1}{3} = 0.4938$

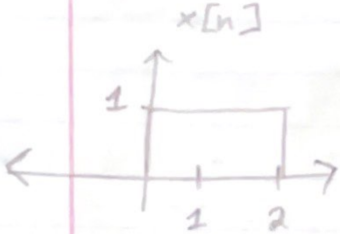
at $n=2$: $\frac{1}{27} + \frac{1}{9} + \frac{1}{3} = 0.4815$

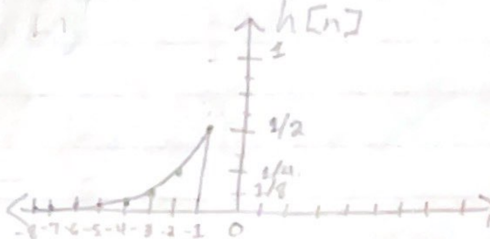
at $n=3$: $\frac{1}{9} + \frac{1}{3} = 0.444$

at $n=4$: $\frac{1}{3} = 0.333$

6

5. $x[n] = u[n] - u[n-2]$ $h[n] = 2^n u[-n-1]$ $n = [-8, 8]$





at $n = -8$: $\frac{1}{256} = 0.0039$

at $n = -7$: $\frac{1}{256} + \frac{1}{128} = 0.0117$

at $n = -6$: $\frac{1}{128} + \frac{1}{64} = 0.0234$

at $n = -5$: $\frac{1}{64} + \frac{1}{32} = 0.0469$

at $n = -4$: $\frac{1}{32} + \frac{1}{16} = 0.0938$

at $n = -3$: $\frac{1}{16} + \frac{1}{8} = 0.1875$

at $n = -2$: $\frac{1}{8} + \frac{1}{4} = 0.375$

at $n = -1$: $\frac{1}{4} + \frac{1}{2} = 0.75$

at $n = 0$: $\frac{1}{2} = 0.5$