Problem 1

An LTI system is described by the following difference equation:

$y[n] + 2y[n-1] - 3y[y-2] = y[n]$. The initial conditions: $y[-1] = 1$; $y[-2] = -1$.

i. [10 points] Represent the difference equation using a block diagram.

ii. [10 points] Determine the step response (i.e., $y[n] = u[n]$) of the LTI system in a recursive manner. Write a MATLAB code that will use the initial conditions, the input, and the previous values in y[n] sequence to generate y[n] for n=[1,10]. Plot y[n].

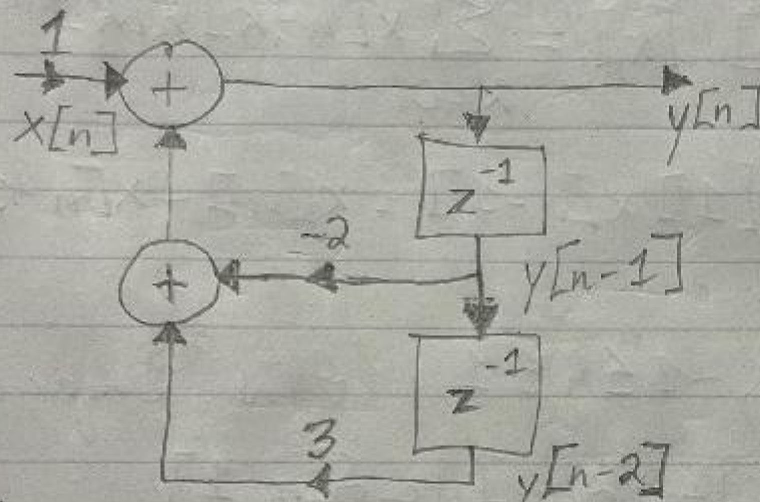
iii. [10 points] Determine the solution y[n], where y[n] is the complete solution of the system when excited by a unit step function. (use analytical approach to find homogeneous and particular solution)

Homework 3

1. $y[n] + 2y[n-1] - 3y[n-2] = x[n]$

$y[-1] = 1 \qquad y[-2] = -1$

i. $y[n] = x[n] - 2y[n-1] + 3y[n-2]$



ii. $y[0] = x[0] - 2y[-1] + 3y[-2] = 1 - 2 - 3 = -4$

$y[1] = x[1] - 2y[0] + 3y[-1] = 1 + 8 + 3 = 12$

iii.

Homogeneous:

$x[n] = 0$ and $y[n] = \lambda^n$

$\lambda^n + 2\lambda^{n-1} - 3\lambda^{n-2} = \lambda^{n-2}(\lambda^2 + 2\lambda - 3)$

$\lambda^{n-2}(\lambda_1 - 1)(\lambda_2 + 3) \rightarrow \lambda_1 = 1 \quad \lambda_2 = -3$

$y_h[n] = \alpha_1 \lambda_1^n + \alpha_2 \lambda_2^n = \alpha_1 (1)^n + \alpha_2 (-3)^n$

2

$$k \cdot n \cdot u[n] + 2\left(k \cdot (n-1) \cdot u[n-1]\right) - 3\left(k \cdot (n-2) \cdot u[n-2]\right) =$$

$$k\left(n \cdot u[n] + 2(n-1) \cdot u[n-1] - 3(n-2) \cdot u[n-2]\right) = u$$

$$k(2 \cdot 1 + 2(1) \cdot 1 + 0) = 1$$

$$K(2 + 2 + 0) = 1 \rightarrow K = \frac{1}{4}$$

$$y_p[n] = \frac{1}{4} n \cdot u[n]$$

$$y[n] = \alpha_1 (1)^n + \alpha_2 (-3)^n + \frac{1}{4} n \cdot u[n]$$

$$y[0] = \alpha_1 + \alpha_2 + 0 = -4 \qquad \alpha_1 + \alpha_2 + 0 = -$$

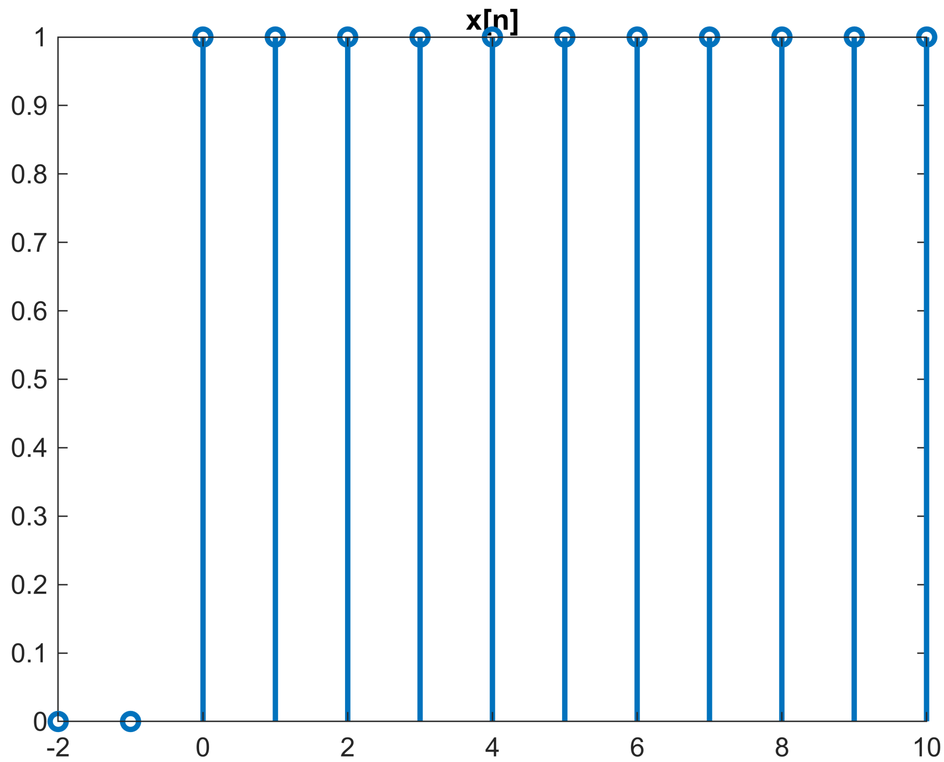
$$y[1] = \alpha_1 - 3\alpha_2 + \frac{1}{4} = 12 \rightarrow -\alpha_1 + 3\alpha_2 - \frac{1}{4} =$$

$$4\alpha_2 - \frac{1}{4} = -16 \rightarrow 4\alpha_2 = -\frac{63}{4} = \frac{-63}{16}$$

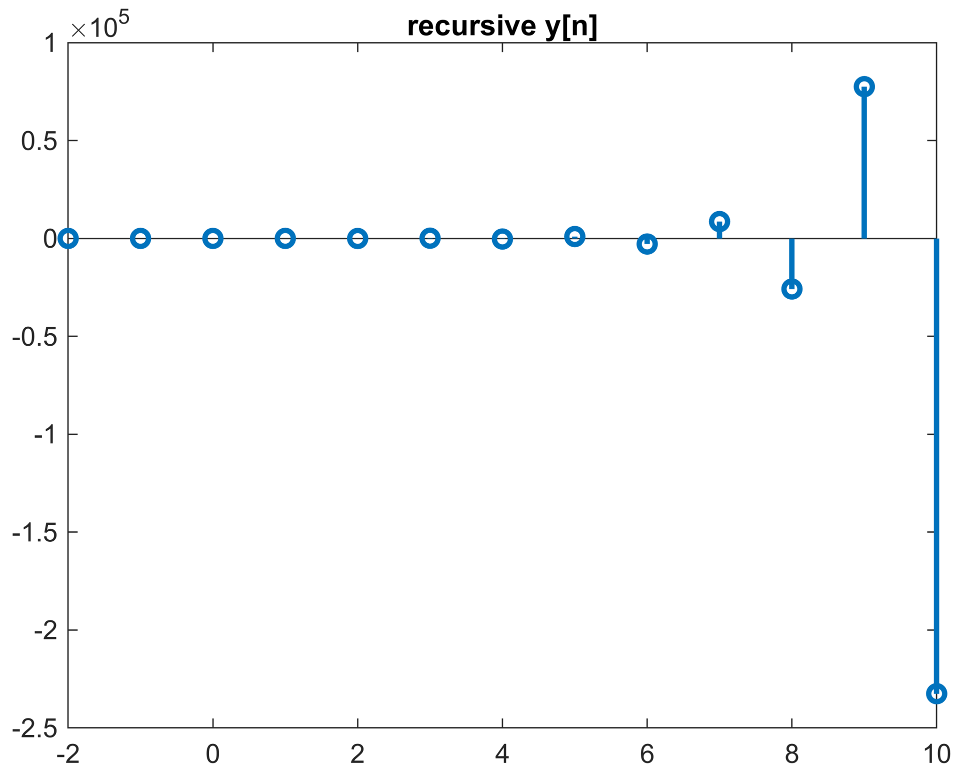
$$\alpha_1 - \frac{63}{16} = -4 \rightarrow \alpha_1 = -\frac{1}{16}$$

$$y[n] = -\frac{1}{16}(1)^n - \frac{63}{16}(-3)^n + \frac{1}{4} n \cdot u[n]$$

```
clear all, close all, clc

index = -2:10;
x=zeros(1,length(index));
y=zeros(1,length(index));

x(index > -1) = 1;
y(index == -2) = -1;
y(index == -1) = 1;
y(index == 0) = -4;

for i=1:index(end)
    y(index==i) = -2.*y(index==i-1)+3.*y(index==i-2)+x(index==i);
end

figure, stem(index, x, 'LineWidth', 2), title('x[n]')
```




```
figure, stem(index, y, 'LineWidth', 2), title('recursive y[n]')
```



Problem 2

An LTI system is described by the following difference equation:

$2y[n] + y[n-1] - 2y[n-2] - y[n-3] = 2x[n] - x[n-1]$.

Assume that the system is initially at rest (i.e., $y[n] = 0$, when $n < 0$) and $x[n] = 4nu[n]$

i. [10 points] Represent the difference equation using a block diagram.

ii. [10 points] Determine the response (i.e., $x[n] = u[n]$) of the LTI system in a recursive manner. Write a MATLAB code that will use the initial conditions, the input and the previous values in y[n] sequence to generate y[n] for n=[1,10]. Plot y[n].

iii. [15 points] Determine the solution y[n], where y[n] is the complete solution of the system. (use analytical approach to find homogeneous and particular solution)

3

2. $2y[n] + y[n-1] - 2y[n-2] - y[n-3] = 2x[n] - x[$

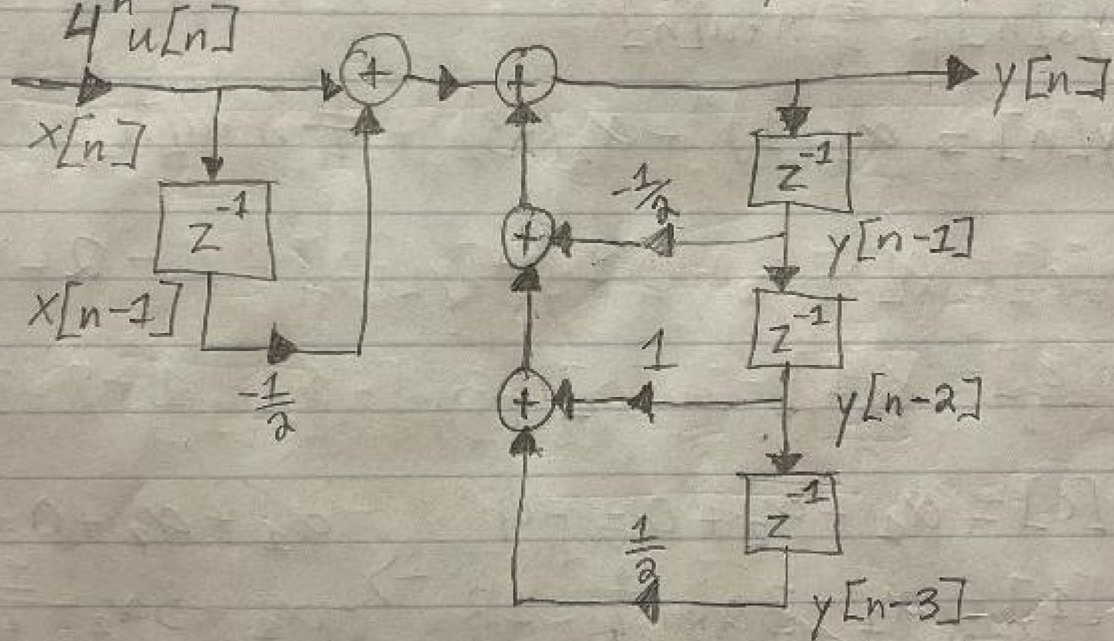
$y[-1] = 0 \quad x[n] = 4^n u[n]$

$y[-2] = 0 \quad y[-3] = 0$

i.

$2y[n] = 2x[n] - x[n-1] - y[n-1] + 2y[n-2] + y[$

$y[n] = x[n] - \frac{1}{2}x[n-1] - \frac{1}{2}y[n-1] + y[n-2] + \frac{1}{2}$



ii.

$y[0] = x[0] - \frac{1}{2}x[-1] - \frac{1}{2}y[-1] + y[-2] + \frac{1}{2}y[-3] = 1$

$y[1] = x[1] - \frac{1}{2}x[0] - \frac{1}{2}y[0] + y[-1] + \frac{1}{2}y[-2]$

$= 4 - \frac{1}{2} - \frac{1}{2} = 3$

$y[2] = x[2] - \frac{1}{2}x[1] - \frac{1}{2}y[1] + y[0] + \frac{1}{2}y[-1]$

$= 16 - \frac{1}{2} \cdot 4 - \frac{1}{2} \cdot 3 + 1 = \frac{27}{2}$

4

iii. Homogeneous:

$2x[n] - x[n-1] = 0$ and $y[n] = \lambda^n$

$2\lambda^n + \lambda^{n-1} - 2\lambda^{n-2} - \lambda^{n-3} = \lambda^{n-3}(2\lambda^3 + \lambda^2 - 2\lambda - 1$

$\lambda^2(2\lambda+1) - (2\lambda+1) \rightarrow (\lambda^2-1)(2\lambda+1) \rightarrow (\lambda-1)($

$\lambda_1 = 1 \quad \lambda_2 = -1 \quad \lambda_3 = -\frac{1}{2}$

$y_h[n] = \alpha_1 \lambda_1^n + \alpha_2 \lambda_2^n + \alpha_3 \lambda_3^n = \alpha_1(1)^n + \alpha_2(-1)$

Particular:

Guess $y_p[n]$ based on $x[n] = 4^n u[n]$

$y_p[n] = K \cdot 4^n \cdot u[n]$

$2 \cdot 4^n \cdot u[n] - 4^{n-1} \cdot u[n-1] = 2 \cdot K \cdot 4^n \cdot u[n] + K \cdot 4^{n-1} \cdot u[n-1] - 2 \cdot K$
$- K \cdot 4^{n-3} \cdot u[n-3]$

$2 \cdot 4^n \cdot u[n] - 4^{n-1} \cdot u[n-1] = K(2 \cdot 4^n u[n] + 4^{n-1} \cdot u[n-1] - 2 \cdot 4^{n-2} \cdot$
$- 4^{n-3} \cdot u[n-3]), n=3$

$2 \cdot 64 - 16 = K(2 \cdot 64 + 16 - 2 \cdot 4 - 1) = \frac{112}{135}$

$y_p[n] = \frac{112}{135} \cdot 4^n \cdot u[n]$

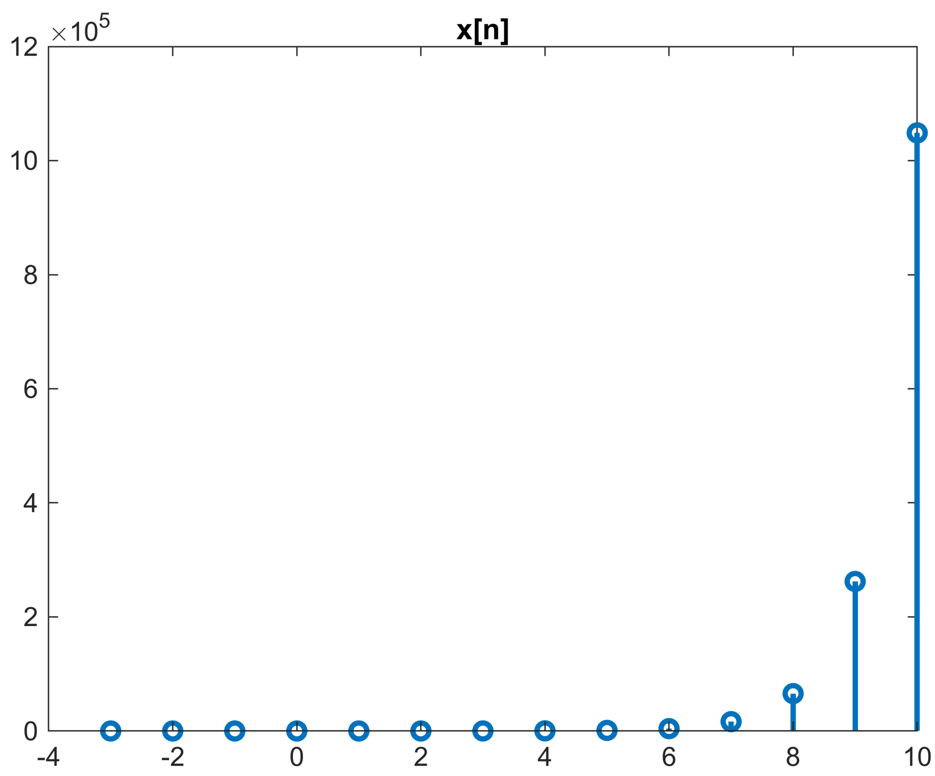
$y[n] = \alpha_1(1)^n + \alpha_2(-1)^n + \alpha_3\left(-\frac{1}{2}\right)^n + \frac{112}{135} \cdot 4^n \cdot u[n]$

$y[0] = \alpha_1 + \alpha_2 + \alpha_3 + \frac{112}{135} = 1$ $\qquad \alpha_1 =$

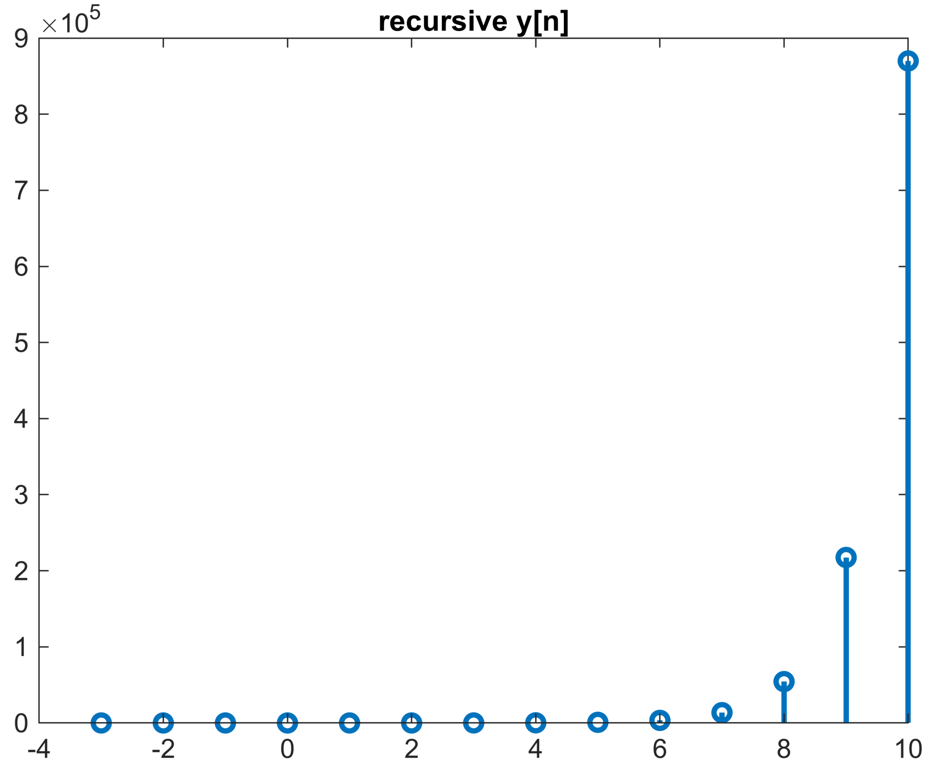
$y[1] = \alpha_1 - \alpha_2 - \frac{1}{2}\alpha_3 + \frac{448}{135} = 3 \rightarrow$ rref $\rightarrow \qquad \alpha_2 =$

$y[2] = \alpha_1 + \alpha_2 + \frac{1}{4}\alpha_3 + \frac{1792}{135} = \frac{27}{2} \qquad \alpha_3 =$

```matlab
clear all, close all, clc

index = -3:10;
x=zeros(1,length(index));
y=zeros(1,length(index));

x(index < 0) = 0;
y(index == -2) = 0;
y(index == -1) = 0;
y(index == -3) = 0;

for i=4:14
    x(i) = 4^(i-4);
end

for i=0:index(end)
    y(index==i) = x(index==i) -0.5.*x(index==i-1) -0.5.*y(index==i-1) +
1.*y(index==i-2) + 0.5.*y(index==i-3);
end

figure, stem(index, x, 'LineWidth', 2), title('x[n]')
```




```matlab
figure, stem(index, y, 'LineWidth', 2), title('recursive y[n]')
```



Problem 3

An LTI system is described by the following difference equation:

$2y[n] - 3y[n-1] + y[n-3] = x[n] - x[n-1]$.

Assume that the system is initially at rest (i.e., $y[n] = 0$, when $n < 0$) and $x[n] = (-1)^n u[n]$

i. [10 points] Represent the difference equation using a block diagram.

ii. [10 points] Determine the response (i.e., $x[n] = u[n]$) of the LTI system in a recursive manner. Write a MATLAB code that will use the initial conditions, the input and the previous values in y[n] sequence to generate y[n] for n=[1,10]. Plot y[n].

iii. [15 points] Determine the solution y[n], where y[n] is the complete solution of the system (use analytical approach to find homogeneous and particular solution).

5

3. $2y[n]-3y[n-1]+y[n-3]=x[n]-x[n-1]$

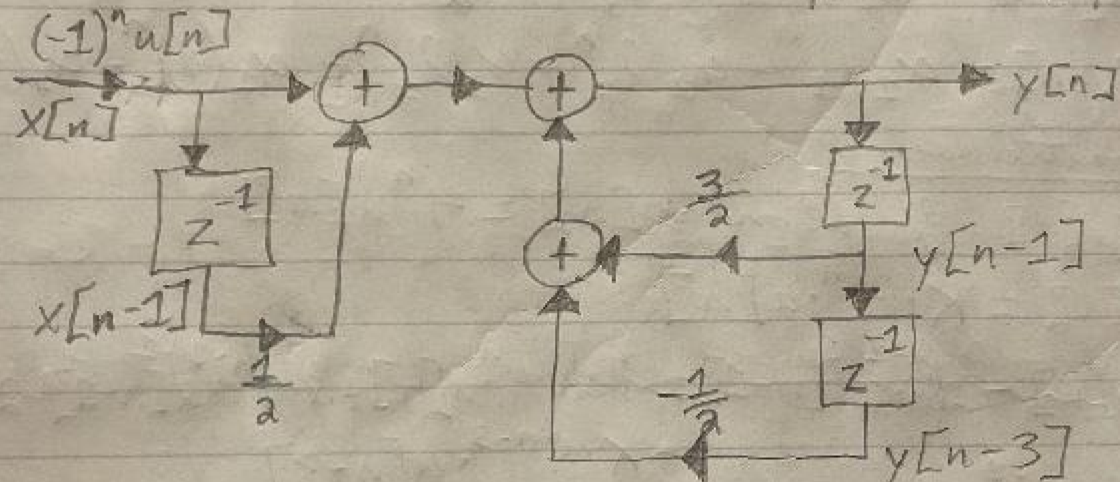
$y[-1]=0 \qquad x[n]=(-1)^n u[n]$

$y[-2]=0 \quad y[-3]=0$

i.

$2y[n]=x[n]-x[n-1]+3y[n-1]-y[n-3]$

$y[n]=\frac{1}{2}x[n]-\frac{1}{2}x[n-1]+\frac{3}{2}y[n-1]-\frac{1}{2}y[n-3]$



ii.

$y[0]=\frac{1}{2}x[0]-\frac{1}{2}x[-1]+\frac{3}{2}y[-1]-\frac{1}{2}y[-3]=\frac{1}{2}$

$y[1]=\frac{1}{2}x[1]-\frac{1}{2}x[0]+\frac{3}{2}y[0]-\frac{1}{2}y[-2]=-\frac{1}{2}-\frac{1}{2}+\frac{3}{4}=$

$y[2]=\frac{1}{2}x[2]-\frac{1}{2}x[1]+\frac{3}{2}y[1]-\frac{1}{2}y[-1]=\frac{1}{2}+\frac{1}{2}-\frac{3}{8}=$

iii. <u>Homogeneous</u>:

$x[n]-x[n-1]=0$ and $y[n]=\lambda^n$

$2\lambda^n-3\lambda^{n-1}+\lambda^{n-3}=\lambda^{n-3}(2\lambda^3-3\lambda^2+1)$

6

Particular:

Guess $y_p[n]$ based on $x[n] = (-1)^n u[n]$

$$y_p[n] = k \cdot (-1)^n \cdot u[n]$$

$$(-1)^n \cdot u[n] - (-1)^{n-1} \cdot u[n-1] = 2 \cdot k(-1)^n \cdot u[n] - 3 \cdot k(-1)^{n-1} \cdot u[n-1]$$
$$+ k(-1)^{n-3} \cdot u[n-3], n = 3$$

$$-1-1 = k(-2-3+1) \to -2 = k(-4) = \frac{1}{2}$$

$$y_p[n] = \frac{1}{2}(-1)^n \cdot u[n]$$

$$y[n] = \alpha_1 (1)^n + n\alpha_2 (1)^n + \alpha_3 \left(-\frac{1}{2}\right)^n + \frac{1}{2}(-1)^n \cdot u$$

$$y[0] = \alpha_1 \quad 0 \quad \alpha_3 \quad \frac{1}{2} = \frac{1}{2} \qquad \alpha_1 =$$

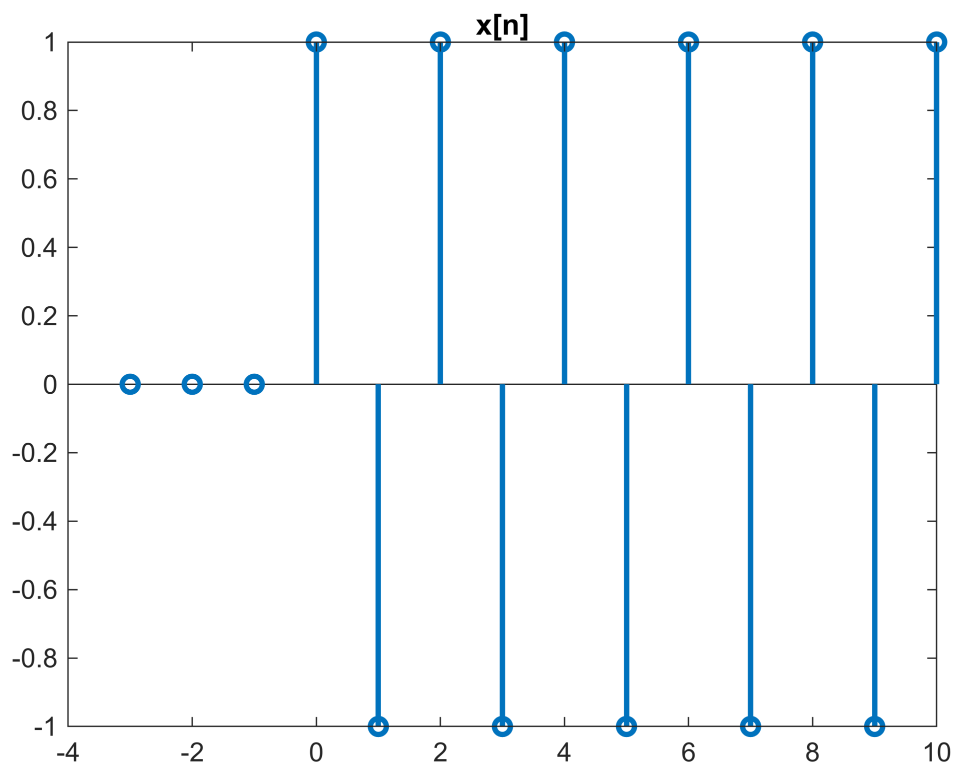
$$y[1] = \alpha_1 \quad \alpha_2 \quad -\frac{1}{2}\alpha_3 \quad -\frac{1}{2} = -\frac{1}{4} \to \alpha_2 =$$

$$y[2] = \alpha_1 \quad 2\alpha_2 \quad \frac{1}{4}\alpha_3 \quad \frac{1}{2} = \frac{5}{8} \qquad \alpha_3 =$$

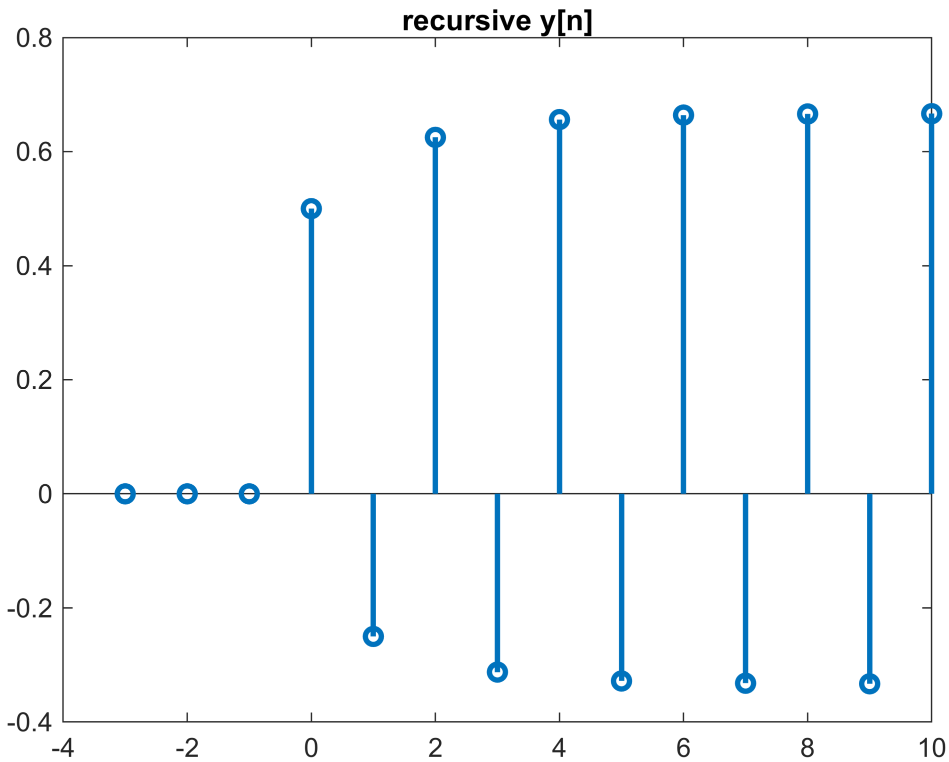
$$y[n] = \frac{1}{6}(1)^n + n \cdot 0 (1)^n - \frac{1}{6}\left(-\frac{1}{2}\right)^n + \frac{1}{2}(-1)^n \cdot u$$

$$y[n] = \frac{1}{6}(1)^n - \frac{1}{6}\left(-\frac{1}{2}\right)^n + \frac{1}{2}(-1)^n \cdot u[n]$$

```
clear all, close all, clc

index = -3:10;
x=zeros(1,length(index));
y=zeros(1,length(index));

x(index < 0) = 0;
y(index == -2) = 0;
y(index == -1) = 0;
y(index == -3) = 0;

for i=4:14
    x(i) = (-1)^(i-4);
end

for i=0:index(end)
    y(index==i) = 0.5.*x(index==i) -0.5.*x(index==i-1) +1.5.*y(index==i-1) -
0.5.*y(index==i-3);
end

figure, stem(index, x, 'LineWidth', 2), title('x[n]')
```




```
figure, stem(index, y, 'LineWidth', 2), title('recursive y[n]')
```


recursive y[n]
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-4
-2
0
2
4
6
8
10