ECGR 5105 Homework 1: Linear Regression

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GitHub Link

Click here to view the code

Problem 1

1. Linear Models for Each Variable (Where Alpha = 0.01):

Model for X1:
The theta values found can be seen below:

$$\Theta_0 = 5.71850653 \quad \Theta_1 = -1.9568206$$

Predicted outputs can be found using this formula:

$$h(X1) = \Theta_1 * X1 + \Theta_0 = -1.9568206 * X1 + 5.71850653$$

1500 iterations were used to find these theta values. The cost was calculated and was found to have the value shown below:

$$J = 0.9905894438682062$$

Model for X2:
The theta values found can be seen below:

$$\Theta_0 = 0.71988473 \quad \Theta_1 = 0.56390334$$

Predicted outputs can be found using this formula:

$$h(X2) = \Theta_1 * X2 + \Theta_0 = 0.56390334 * X2 + 0.71988473$$

1500 iterations were used to find these theta values. The cost was calculated and was found to have the value shown below:

$$J = 3.5993993982305357$$

Model for X3:
The theta values found can be seen below:

$$\Theta_0 = 2.78048129 \quad \Theta_1 = -0.48451631$$

Predicted outputs can be found using this formula:

$$h(X2) = \Theta_1 * X2 + \Theta_0 = -0.48451631 * X2 + 2.78048129$$

1500 iterations were used to find these theta values. The cost was calculated and was found to have the value shown below:

$$J = 3.6305262475389664$$

2. Plots of Linear Regression and Gradient Descent for Each Variable:

The models were plotted with three different alpha values, so the effect of learning rate can be seen on each variable.

Figure 1: X1 Plots

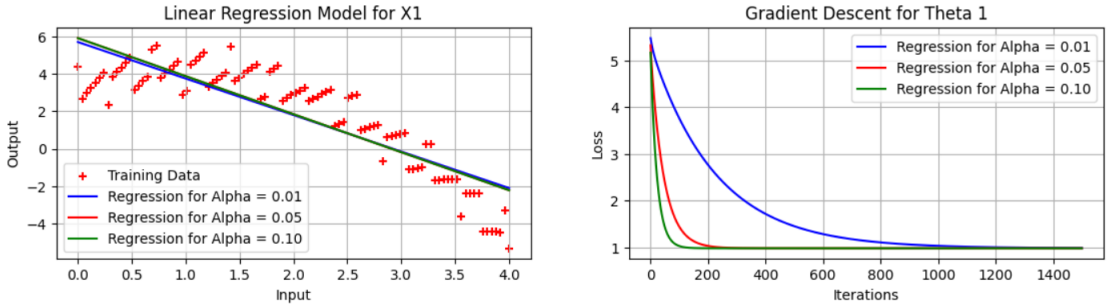


Figure 2: X2 Plots

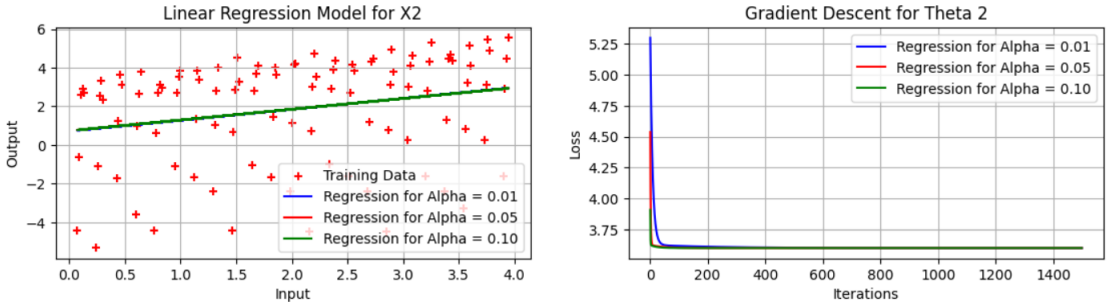
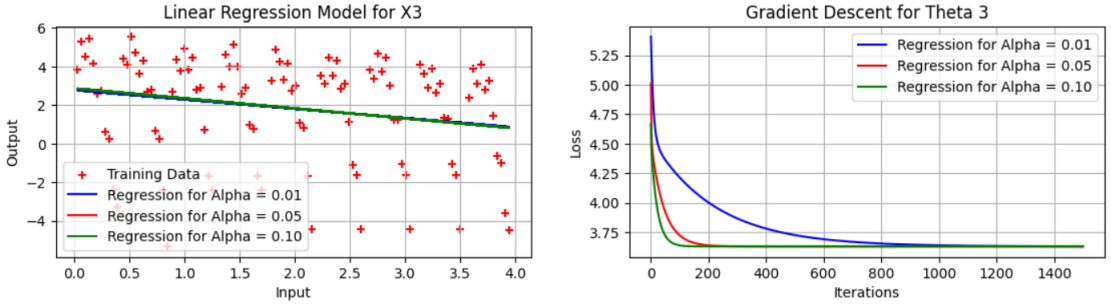


Figure 3: X3 Plots



3. Explanatory Variable with the Lowest Loss:

Of the explanatory variables, X1 was found to have the lowest loss for a learning rate of 0.01. The comparison of these variables can be seen in the table below:

Exp. Variable	Loss
X1	0.9905894438682062
X2	3.5993993982305357
X3	3.6305262475389664

4. Impact of Different Learning Rates on Final Loss:

As can be seen from the plots above, when a larger learning rate was used, the gradient descent reached a lower loss in less iterations. When comparing the learning rates below, it appears that a learning rate of 0.10 reported the lowest loss for all explanatory values. The impact of different learning rates on final loss for each explanatory variable can be seen in the table below:

Learning Rate	Exp. Variable	Loss
0.01	X1	0.9905894438682062
0.05	X1	0.9849930825406077
0.10	X1	0.9849930825405946
0.01	X2	3.5993993982305357
0.05	X2	3.5993660181680425
0.10	X2	3.599366018168041
0.01	X3	3.6305262475389664
0.05	X3	3.629451124607917
0.10	X3	3.6294511246079155

Problem 2

1. Best Linear Model:

The best linear model had a learning rate of 0.10. This resulted in the lowest loss.
The theta values found can be seen below:

$$\Theta_0 = 5.31416563 \quad \Theta_1 = -2.00371905 \quad \Theta_2 = 0.53256359 \quad \Theta_3 = -0.26560164$$

Predicted outputs can be found using this formula:

$$h(x) = \Theta_3 * X3 + \Theta_2 * X2 + \Theta_1 * X1 + \Theta_0$$

Which equals:

$$h(x) = -0.26560164 * X3 + 0.53256359 * X2 - 2.00371905 * X1 + 5.31416563$$

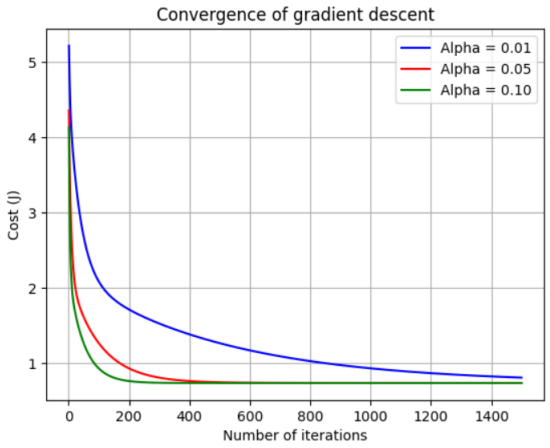
1500 iterations were used to find these theta values. The cost was calculated and was found to have the value shown below:

$$J = 0.7384642415684215$$

2. Plots of Loss over Iterations:

The model's loss was plotted with three different learning rates, so the effect of various learning rates can be seen.

Figure 4: Loss When Using All Explanatory Variables



3. Impact of Learning Rates:

Much like problem 1, when a larger learning rate was used, the gradient descent reached a lower loss in less iterations. When comparing the learning rates below, it appears that a learning rate of 0.10 reported the lowest loss. The impact of different learning rates on final loss can be seen in the table below:

Learning Rate	Loss
0.01	0.8105104429083866
0.05	0.7384646851794184
0.10	0.7384642415684215

4. Predicting Future Values:

The predicted outputs were found using the h(x) formula found above in part 1 of problem 2. The following x values were substituted into this formula to find the respective predicted values. These results can be seen in the table below:

(X1,X2,X3)	Predicted Value
(1,1,1)	3.577408529345462
(2,0,4)	0.2443209702176523
(3,2,1)	0.1025340197359193