Solutions of HW11

11(15), 17(10), 19(15), 20(10), 21(15), 26(15), 27(20)

11. (a) $\bar{x} = 1.2896$; $S^2 = 0.0000123; S = 0.0035$

    (b) The 95% confidence interval of μ is: $\bar{X} \pm t_{0.025}\frac{S}{\sqrt{n}}$
    $n = 20$, The degree of freedom of the T random variable is 19. From the probability table, we read out:$t_{0.025} = 2.093$. Then the confidence interval is calculated as: 1.2896 ± 0.0016

    (c) 1.29 is inside the confidence interval. It is not unusual.

17. $L = \bar{X} - t_{0.05}\frac{S}{\sqrt{n}}$
    When $n = 19$, the T random variable is T_{18}. From the probability table, we read out: $t_{0.05} = 1.734$.
    From the sample data, we calculate $\bar{X} = 41.0526$; $S^2 = 98.6082; S = 9.9302$
    Then, $L = 37.1$. The one side confidence interval is $[37.1, \infty)$

19. (a) $d = z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \Rightarrow \sqrt{n} = \frac{z_{\alpha/2}\sigma}{d} \Rightarrow n = \left(\frac{z_{\alpha/2}\sigma}{d}\right)^2$. When σ is unknown, σ can be replaced by its estimator $\hat{\sigma}$, i.e., $n = \left(\frac{z_{\alpha/2}\hat{\sigma}}{d}\right)^2$
    (b) $n = \left(\frac{-1.96\times500}{50}\right)^2 \approx 385$
    (c) $n = \left(\frac{-1.65\times0.75}{0.1}\right)^2 \approx 153$

20. (a) $s = \sqrt{\frac{10(49.08)-(21.4)^2}{10(9)}} = \sqrt{.365} = .60$
    (b) $n = \left(\frac{z_{0.005}s}{d}\right)^2 = \left(\frac{2.575\times0.6}{0.2}\right)^2 = 60$

21. (a) $H_0: p \geq 0.08$; $H_1: p < 0.08$
    (b) The experiment concludes that the percentage of metal in household waste reduced when actually it has not.
    (c) The experiment does not conclude that the percentage of metal in household waste reduces when it actually reduces.
    (d) It means the probability of making type I error is 5%

26. (a) From a binomial distribution with n = 10 and p = .7,

$P[X \leq 4] = 0.0474 \approx 0.05$

Thus, the rejection (critical) region is $C = \{0,1,2,3,4\}$

(b) since $x = 5$ does not fall in the critical region, H_0 will not be rejected.

In this case, Type II error might be made.

27. (a) $H_0: p \leq 0.5;\ H_1: p > 0.5$

(b) X follows a binomial distribution with $n = 15, p = 0.5$.

Hence $E[X] = np = 7.5$.

(c) $\alpha = P[Type\ I\ error] = P[X \geq 11|p = 0.5] = 1 - P[X \leq 10|p = 0.5]$

$= 1 - 0.9408 = 0.0592$

(d) $\beta = P[Type\ II\ error] = P[X < 11|p > 0.5]$

When $p = 0.6$, $\beta = P[X < 11|p = 0.6] = 0.7827$

When $p = 0.7$, $\beta = P[X < 11|p = 0.7] = 0.4845$

When $p = 0.8$, $\beta = P[X < 11|p = 0.8] = 0.1642$

When $p = 0.9$, $\beta = P[X < 11|p = 0.9] = 0.0127$

(e) $power = 1 - \beta$. Hence,

When $p = 0.6$, power = 0.2173

When $p = 0.7$, power = 0.5155

When $p = 0.8$, power = 0.8358

When $p = 0.9$, power = 0.9873

(f) Yes, H_0 will be rejected. Type I error might be committed.

(g) No, H_0 will not be rejected. Type II error might be committed.