Expected Value

Parameters to describe a RV

Definition: Let X be a discrete random variable (R.V.) with density $f(x)$**.**Let $H(X)$ be a function of random variable X. The expected value of $H(X)$, denoted by $E[H(X)]$, is given by:

$$E[H(X)] = \sum_{all\ x} H(x)f(x)$$

A special case of this definition, $H(X) = X,\ E[X] = \sum_{all\ x} xf(x)$

Example: Given the density of a discrete random variable X and Y as in the table. Calculate $E[X]\ and\ E[Y]$

x	1	2	3	4	5
f(x)	0.2	0.2	0.2	0.2	0.2

y	1	2	3	4	5
f(y)	0.9	0.05	0.02	0.02	0.01

Solution: $\mathrm{E}[\mathrm{X}] = 1 \times 0.2 + 2 \times 0.2 + 3 \times 0.2 + 4 \times 0.2 + 5 \times 0.2 = 3$

$E[Y] = 1 \times 0.9 + 2 \times 0.05 + 3 \times 0.02 + 4 \times 0.02 + 5 \times 0.01 = 1.19$

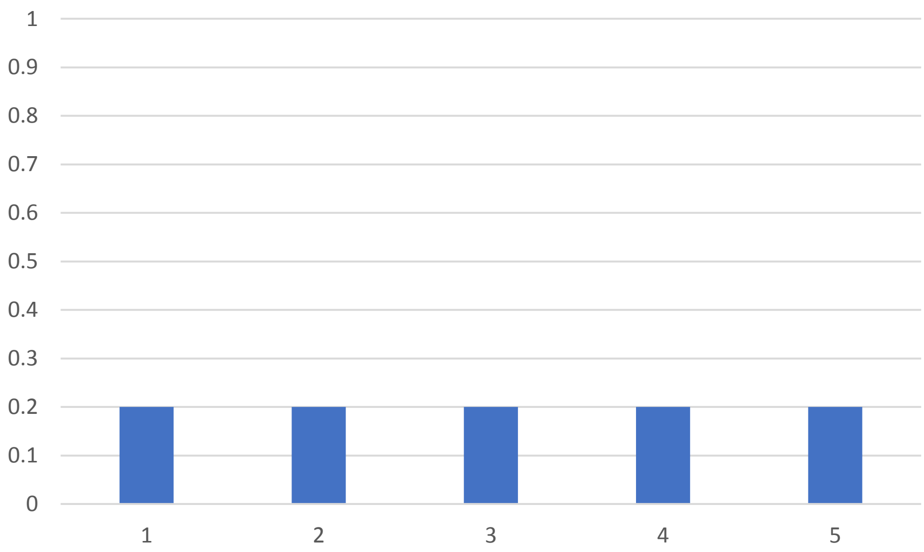
Expected Value

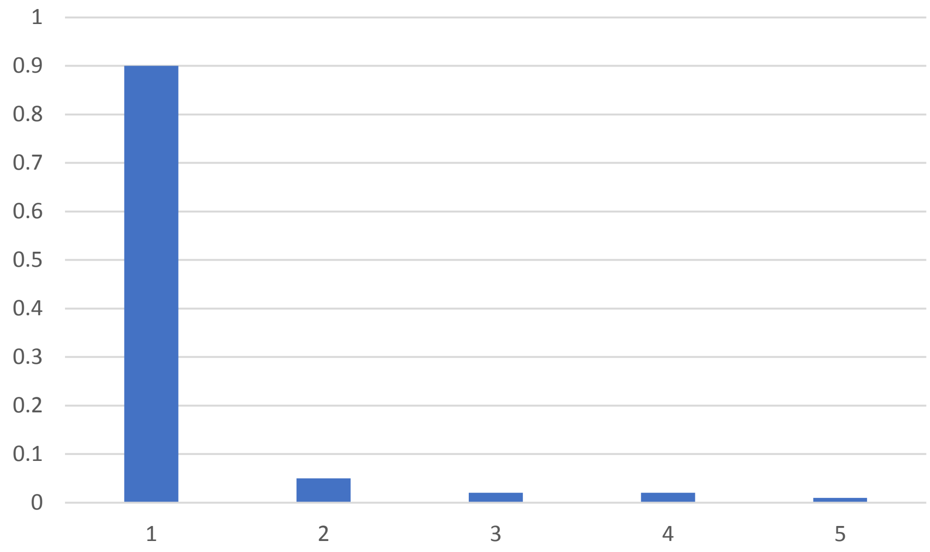
Parameters to describe a RV

Example: Given the density of a discrete random variable X and Y as in the table. Calculate $E[X]\ and\ E[Y]$

x	1	2	3	4	5
f(x)	0.2	0.2	0.2	0.2	0.2

y	1	2	3	4	5
f(y)	0.9	0.05	0.02	0.02	0.01





Expected Value

Parameters to describe a RV

Notes:

- In a statistical setting, the expected value of a random variable X is also called its ***mean value***, or ***average value***. Usually denoted by symbol $\boldsymbol{\mu}$
- μ_x is a measure of the location of the ***center*** of X distribution. For this reason, μ_x is called a ***location*** parameter.

Properties:

$E[c] = c$

$E[cX] = cE[X]; E[X + b] = E[X] + b$

$E[X + Y] = E[X] + E[Y]$ (will be proved in Chapter 5)

$E[aX + bY] = aE[X] + bE[Y]$

Where, a, b, c are real numbers.

Example: Let X and Y be random variables with $E[X] = 7\ and\ E[Y] = -5$. Calculate $E[4X - 2Y + 6]$

Solution: $\mathrm{E}[4\mathrm{X} - 2\mathrm{Y} + 6]] = 4 \times 7 - 2 \times (-5) + 6 = 44$

Variance

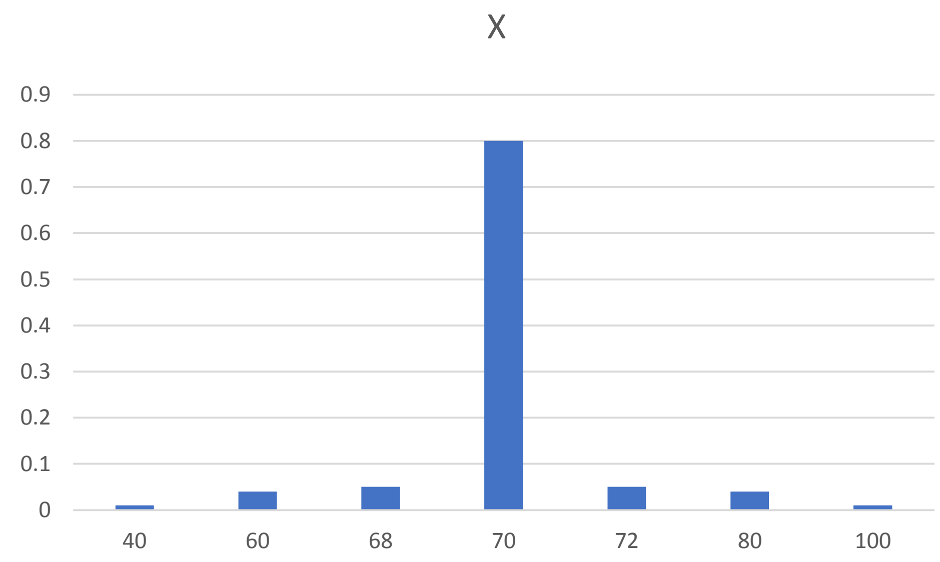
Parameters to describe a RV

Example: Consider two random variables $X \; and \; Y$ with the following distributions

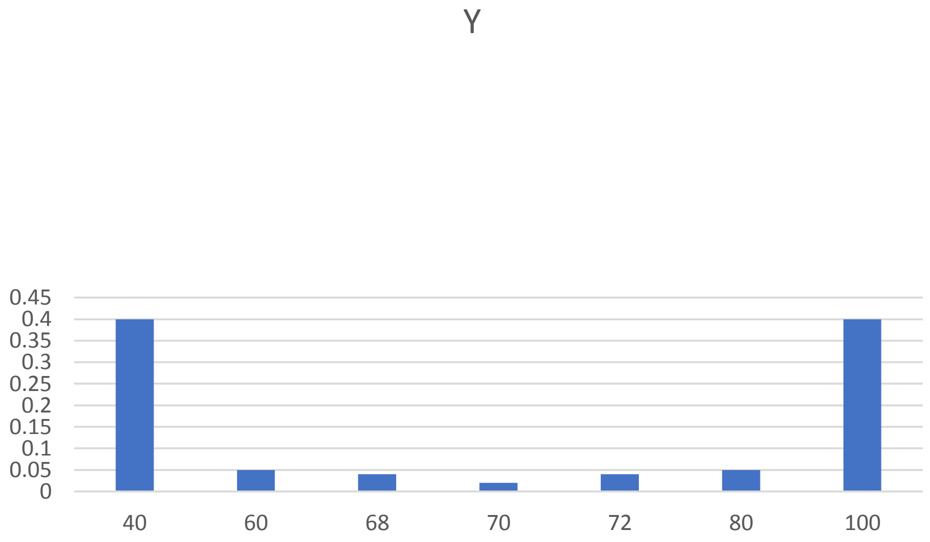
x	40	60	68	70	72	80	100
f(x)	0.01	0.04	0.05	0.80	0.05	0.04	0.01

y	40	60	68	70	72	80	100
f(y)	0.40	0.05	0.04	0.02	0.04	0.05	0.40

We need more parameters in addition to mean value to describe a distribution!



$\mu_x = 70$



$\mu_y = 70$

Variance

Parameters to describe a RV

Definition: Let X be a random variable with mean μ_X. The variance of X, denoted by $VarX \ or \ \sigma^2$, is defined as:

$$VarX = \sigma^2 = E[(X - \mu_X)^2]$$

Variance measures how far a data set spread out, or, how wide the data set scatter.

Calculate the variance of the following three data sets: (assuming uniform distribution)

(1)$\{10,10,10,10,10\}$ 0

(2)$\{10,10,10,10,12\}$ 0.64

(3)$\{10,10,10,10,10000\}$ 15968016

If you try to calculate these variances using Matlab or Excel, you might get 0, 0.80, and 19960020

Assuming uniform distribution, they are using the formula

$$\sigma^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \mu_x)^2$$

while we are using:

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu_x)^2$$

Variance

Parameters to describe a RV

Definition: Let X be a random variable with mean μ_X. The variance of X, denoted by $VarX \;\; or \; \sigma^2$, is defined as:

$$VarX = \sigma^2 = E[(X - \mu_X)^2]$$

An Important Formula to Calculate variance: $\sigma^2 = VarX = E[X^2] - (E[X])^2$

$$VarX = E[(X-\mu)^2] = E[X^2 - 2\mu X + \mu^2] = E[X^2] - 2\mu E[X] + \mu^2 = E[X^2] - 2(E[X])^2 + (E[X])^2 = E[X^2] - (E[X])^2$$

Example: Calculate variances using this new formula.

Variance

Parameters to describe a RV

Properties: Let $X \; and \;\; Y$ be random variables and c a real number. Then:

(1) $Var(c) = 0$

(2) $Var(cX) = c^2 VarX$

(3) $if \;\, X \; and \; Y \; are \; independent, then \; Var(X+Y) = VarX + VarY$

Example: Let $X \; and \; Y \; be \; independent \; with \; {\sigma_x}^2 = 9, {\sigma_y}^2 = 3$. Calculate $Var(4X + 2Y + 6)$

$$Var(4X + 2Y + 6) = 16 \times 9 + 4 \times 4 + 0 = 156$$

Standard deviation

Parameters to describe a RV

Definition: Let X be random variables with variance σ^2. Then the standard deviation of X, denoted by σ, is given by $\sigma = \sqrt{VarX}$

(1) A large standard deviation implies that the random variable X is somewhat hard to predict.
(2) Standard deviation is always in physical measurement unit that match the original data. Variance is often unit-less.