Solution of HW8

Chapter 6:

21. $\bar{x} = 7.94; s^2 = 1.22; s = 1.11$

22. $\bar{x} = \dfrac{115.7}{50} = 2.31$

$$s^2 = \frac{50(314.49)-(115.7)^2}{50(49)} = .954$$

$$s = \sqrt{.954} = .98$$

24. (b) $\bar{x} = \dfrac{247}{24} = 10.29$ or 10,229 hours

(c) The median location is $\dfrac{n+1}{2} = \dfrac{25}{2} = 12.5$. Sorting the numbers from small to large allows us to easily find the 12th and 13th numbers, 9.6 and 10.0, respectively. Therefore, $\tilde{x} = \dfrac{9.6+10.0}{2} = 9.8$ or 9,800 hours, which implies that 50% of these new bulbs have a lifespan of at least 9,800 hours

(d) $s^2 = \dfrac{24(2585.6)-(247)^2}{24(23)} = 1.894$

$$s = \sqrt{1.894} = 1.38 = 1{,}380 \text{ hours}$$

(e) The normal probability rule is not applicable here

(f) At least $1-\dfrac{1}{2^2} = 75\%$ of the bulbs will have life spans between 7,530 and 13,050 hours ($\bar{x} \pm 2s$)

Chapter 7

1. It is known that $\mu = 8, \sigma^2 = 5,\ \ n = 20.$

   We already find out that $E[\bar{X}] = \mu, Var\bar{X} = \frac{\sigma^2}{n}$

   Hence, $E[\bar{X}] = 8, Var\bar{X} = \frac{\sigma^2}{n} = \frac{1}{4}$

2. We choose the estimator as $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$

   $E[\bar{X}] = E\left[\frac{1}{n}\sum_{i=1}^{n} X_i\right] = \frac{1}{n}\sum_{i=1}^{n} E[X_i] = \frac{1}{n} n\lambda s = \lambda s$

   Hence, $\bar{X}$ is an unbiased estimator of λs

5. X_1, X_2, X_3, X_4, X_5 are random sample from a binomial distribution with parameter

   $n = 10\ and\ p\ unknown$. Then, $E[X_i] = np = 10p, i = 1,2,3,4,5$

   (a) $E[\bar{X}/10] = \frac{1}{10}E[\bar{X}] = \frac{1}{10}np = p$. Hence, $\bar{X}/10$ is an unbiased estimator for p.

   (b) $\bar{x} = 4.4,\ \hat{p} = \frac{4.4}{10} = 0.44$

9. Since $\bar{X}_i, i = 1,2, \dots, k$ are unbiased estimators of μ, we have,

   $E[\bar{X}_i] = \mu, i = 1,2, \dots, k.$

   (a) $E\left[\frac{1}{k}\sum_{i=1}^{k} \bar{X}_i\right] = \frac{1}{k}\sum_{i=1}^{k} E[\bar{X}_i] = \mu$. Hence, this is an unbiased estimator

   (b) $\frac{\bar{x}_1+\bar{x}_2+\bar{x}_3}{3} = \frac{2.45}{3} \approx 0.817$. This is not a reasonable estimate of the mean.

   Because the sample size of $\bar{x}_3$ (200) is much larger than the other two.

   (c) $E[\hat{\mu}_W] = E\left[\frac{n_1\bar{X}_1+n_2\bar{X}_2+\cdots+n_k\bar{X}_k}{n_1+n_2+\cdots+n_k}\right] = \frac{n_1E[\bar{X}_1]+n_2E[\bar{X}_2]+\cdots+n_kE[\bar{X}_n]}{n_1+n_2+\cdots+n_k}$

   $= \frac{n_1\mu+n_2\mu+\cdots+n_k\mu}{n_1+n_2+\cdots+n_k} = \mu$

17. For a Poisson random variable X with parameter λs, we have $E[X] = \lambda s$

Replace both sides by its estimators, we have,

$$M_1 = \widehat{\lambda s}$$

i.e.,

$$\widehat{\lambda s} = M_1 = \bar{X}$$

Also,

$$\hat{\lambda} = \bar{X}/s$$

18. $2\hat{\lambda} = \bar{x} = 3.1 \quad \Rightarrow \quad \hat{\lambda} = \dfrac{\bar{x}}{2} = \dfrac{3.1}{2} = 1.55$

20. Since $M_1 = \dfrac{\sum_{i=1}^{n} X_i}{n} = \dfrac{28.8}{15} = 1.92$ and $M_2 = \dfrac{\sum_{i=1}^{n} X_i^2}{n} = \dfrac{58.98}{15} = 3.932$,

$$\hat{\beta} = \frac{M_2 - M_1^2}{M_1} = \frac{3.932 - (1.92)^2}{1.92} = .1279$$

$$\hat{\alpha} = \frac{M_1}{\hat{\beta}} = \frac{1.92}{.1279} = 15.0117$$

$$\hat{\mu} = \hat{\alpha}\hat{\beta} = (15.0117)(.1279) = 1.92$$

$$\hat{\sigma}^2 = M_2 - M_1^2 = 3.932 - (1.92)^2 = .2456$$