Solutions of Assignment #13 (Practice problems)

Chapter 10:

9. (a) $H_0: \sigma_1^2 = \sigma_2^2; H_1: \sigma_1^2 < \sigma_2^2$

(b) Assuming H_0 is true, the test statistic is $\frac{S_2^2}{S_1^2} \sim F_{9,9}$. The observed value of the test statistic is $\frac{S_2^2}{S_1^2} = \frac{11.9}{10.89} = 1.1$.

The p-value is:

$$p_{value} = P[F_{9,9} \geq 1.1]$$

(Actually, the original hypothesis is equivalent to : $H_0: \frac{\sigma_2^2}{\sigma_1^2} = 1; H_1: \frac{\sigma_2^2}{\sigma_1^2} > 1$

When the test statistic $\frac{S_2^2}{S_1^2}$ is used, the rejection region is right-tailed. Hence, the equivalent test is a right-tailed test.)

From the F-table, we have,

$P\big[F_{9,9} \geq 2.440\big] = 0.1 \; and \; P\big[F_{9,9} \geq 3.179\big] = 0.01$

Hence, we have, $p_{value} > 0.1$.

At the significance level of 0.1, H_0 can not be rejected. i.e., $\sigma_1^2 = \sigma_2^2$

10. $H_0: \sigma_{SC}^2 = \sigma_M^2$; $H_1: \sigma_{SC}^2 \neq \sigma_M^2$

The observed value of the test statistic is $f = \frac{s_M^2}{s_{SC}^2} = \frac{.011964}{.001476} = 8.11$

Assuming H_0 is true, the test statistic $\frac{S_M^2}{S_{SC}^2} \sim F_{11,9}$

The P value for this test is $2P[F_{11,9} \geq 8.11]$

From the F table, we have,

$$P\big[F_{11,9} \geq 2.396\big] = 0.1 \; and \; P\big[F_{11,9} \geq 3.102\big] = 0.05$$

Hence, we have,

$p_{value} < 2 \times 0.05 = 0.1$

Therefor, we reject H_0 at the significance level of 0.1. i.e., the variation in the prices of regular unleaded gasoline in South Carolina is different from the variation in prices in Michigan.

13. The information about the sampled data:

Journals: $n_1 = 13, \bar{x}_1 = 1.7515, s_1^2 = 0.034$

Unpublished reports: $n_2 = 13, \bar{x}_2 = 2.4215, s_2^2 = 0.0525$

(a) we first test: $H_0: \sigma_1^2 = \sigma_2^2; H_1: \sigma_1^2 \neq \sigma_2^2$

Assuming H_0 is true, the test statistic is: $\frac{S_2^2}{S_1^2} \sim F_{12,12}$. The observed value of the test statistic is $\frac{S_2^2}{S_1^2} = \frac{0.0525}{0.034} = 1.5441>1$. Since this is a two-tailed test, the p-value is:

$$p_{value} = 2P[F_{12,12} \geq 1.5441]$$

From the F-table, we find out

$$P\left[F_{12,12} \leq 2.147\right] = 0.9 \Rightarrow P\left[F_{12,12} > 2.147\right] = 0.1.$$

Hence, $P\left[F_{12,12} \geq 1.5441\right] > 0.1 \Rightarrow p_{value} > 0.2$

At the significance level of 0.2, H_0 can not be rejected. i.e., $\sigma_1^2 = \sigma_2^2$ holds.

(b) $S_p^2 = \frac{12\times0.034+12\times0.0525}{24} = 0.0432$

(c) we have, $\frac{(\bar{X}_1-\bar{X}_2)-(\mu_1-\mu_2)}{\sqrt{S_p{}^2(\frac{1}{13}+\frac{1}{13})}} \sim T_{24}$, For a T_{24} random variable, we find out

$t_{0.05} = 1.711$

The boundaries of the 90% confidence interval of $\mu_1 - \mu_2$ are :

$$1.7515 - 2.4215 \pm 1.711\sqrt{0.0432 \times \left(\frac{1}{13} + \frac{1}{13}\right)} = -0.67 \pm 0.14$$

14. (a) $H_0: \sigma_1^2 = \sigma_2^2$

$H_1: \sigma_1^2 \neq \sigma_2^2$

At $\alpha = .2$ the critical points are $f_{.9}(60,\ 60 \text{ df}) = .717$ and $f_{.1}(60,\ 60 \text{ df}) = 1.395$

The observed value of the test statistic is $f = \frac{s_1^2}{s_2^2} = \frac{24.9}{22.7} = 1.097$, which is not in the critical region. Since we cannot reject H_0, pooling is appropriate.

(b) $s_p^2 = \frac{24.9 + 22.7}{2} = 23.8$

(c) Approximating the T_{120} distribution with T_{100}, $t_{0.05} = 1.645$. The confidence interval is, therefore, $(40 - 29) \pm 1.645\sqrt{23.8\left(\frac{1}{61} + \frac{1}{61}\right)} = 11 \pm 1.45$

We can be 90% confident that $\mu_1 - \mu_2$ is between 9.55 and 12.45 bar codes per second.

(d) yes, the confidence interval on $\mu_1 - \mu_2$ contains all positive numbers.

(e) Central Limit Theorem