Solution of HW7

Chapter 5

16. (a) no

(b) The marginal densities for X and Y are, respectively,

x	0	1	2	3	4	5
$f_X(x)$	.525	.354	.062	.027	.022	.010

y	0	1	2	3
$f_Y(y)$	.762	.167	.053	.018

E[X] = 0(.525) + 1(.354) + ⋯ + 5(.010) = .697

E[Y] = 0(.762) + 1(.167) + 2(.053) + 3(.018) = .327

E[XY] = 0 · 0(.400) + 0 · 1(.100) + ⋯ + 5 · 2(.002) + 5 · 3(.018) = .376

Cov(X, Y) = E[XY] – E[X] · E[Y] = .376 – (.697)(.327)=0.148

Since Cov(X, Y,) > 0, a small number of syntax errors tends to associate with a small number of errors in logic and vice versa.

(c) E[X + Y] = E[X] + E[Y] = .697 + .327 = 1.024

On the average we can expect a programmer to make just over one error on the first run of a BASIC program.

20. (a) negative; as temperature increases, the time it takes for a diesel engine to get ready to start should decrease, and vice versa.

(b) From Exercise 8(c) the marginal density for X is

$$f_X(x) = \frac{1}{6640}(8x+6), \quad 0 \le x \le 40$$

Thus,

$$E[X] = \frac{1}{6640}\int_0^{40}\left(8x^2+6x\right)dx = 26.426$$

Also from Exercise 8(c) the marginal density for Y is

$$f_Y(y) = \frac{1}{6640}(80y + 3240), \quad 0 \le y \le 2$$

Thus,

E[Y] = $\frac{1}{6640}\int_0^2 (80y^2 + 3240y)\,dy = \frac{1}{6640}(6693.3333) = 1.008$

E[XY] = $\int_0^{40}\int_0^2 xy\left(\frac{1}{6640}\right)(4x + 2y + 1)\,dydx = 26.586$

Cov(X, Y) = E[XY] – E[X]E[Y] = 26.586 – (26.426)(1.008) = –.051

21. $f_{XY}(x,y) = \frac{1}{x}, 0 < y < x < 1$

$f_X(x) = \int_0^x \frac{1}{x}dy = 1,\ \ 0 < x < 1; f_Y(y) = \int_y^1 \frac{1}{x}dx = -\ln(y)\,, 0 < y < 1$

$$E[X] = \int_0^1 xf_X(x)dx = \frac{1}{2}$$

$$E[Y] = \int_0^1 yf_Y(y)dy = -\int_0^1 yln(y)dy = \frac{1}{4}$$

$$E[XY] = \int_0^1\int_0^x xyf_{XY}(x,y)dydx = \int_0^1\int_0^x ydydx = \frac{1}{6}$$

$$Cov(X,Y) = E[XY] - E[X]E[Y] = \frac{1}{24}$$

33. From Exercise 21, we have,

$$f_{XY}(x,y) = \frac{1}{x}, 0 < y < x < 1$$

$f_X(x) = \int_0^x \frac{1}{x}dy = 1,\ \ 0 < x < 1; f_Y(y) = \int_y^1 \frac{1}{x}dx = -\ln(y)\,, 0 < y < 1$

$$E[X] = \frac{1}{2}; E[Y] = \frac{1}{4}; E[XY] = \frac{1}{6}; Cov(X,Y) = \frac{1}{24}$$

We can also find that,

$$E[X^2] = \int_0^1 x^2 f_X(x)dx = \int_0^1 x^2 dx = \frac{1}{3}$$

$$E[Y^2] = \int_0^1 y^2 f_Y(y)dy = -\int_0^1 y^2 \ln(y)\,dy = \frac{1}{9}$$

$$VarX = E[X^2] - (E[X])^2 = \frac{1}{12}$$

$$VarY = E[Y^2] - (E[Y])^2 = \frac{7}{144}$$

$$\rho_{XY} = \frac{Cov(X,Y)}{\sqrt{VarXVarY}} = \frac{1/24}{\sqrt{\frac{1}{12} \times \frac{7}{144}}} = 0.655$$

35. (a) $Cov(X,Y) = E[XY] - E[X]E[Y] = E[X(\beta_0 + \beta_1 X)] - E[X]E[\beta_0 + \beta_1 X]$

$= \beta_0 E[X] + \beta_1 E[X^2] - \beta_0 E[X] - \beta_1 (E[X])^2 = \beta_1(E[X^2] - (E[X])^2)$

$= \beta_1 VarX$

(b) $VarY = Var(\beta_0 + \beta_1 X) = {\beta_1}^2 VarX$

(c) $\rho_{XY} = \frac{Cov(X,Y)}{\sqrt{(VarX)V(arY)}} = \frac{\beta_1 VarX}{\sqrt{(VarX)({\beta_1}^2 VarX)}} = \frac{\beta_1}{|\beta_1|}$

(d) $\rho_{XY} = \frac{\beta_1}{|\beta_1|} = 1 \; when \; \beta_1 > 0$

$\rho_{XY} = \frac{\beta_1}{|\beta_1|} = -1 \; when \; \beta_1 < 0$

40. (a) $f_{X|y}(x) = \frac{f_{XY}(x,y)}{f_Y(y)} = \frac{1/240}{2/240} = \frac{1}{2}, \quad 8.5 \le x \le 10.5$

$f_{X|y}(y) = f_X(x)$ because X and Y are independent; an individual's blood calcium level does not depend on the specific level of her blood cholesterol.

(b) $f_{Y|x}(y) = \frac{f_{XY}(x,y)}{f_X(x)} = \frac{1/240}{1/2} = \frac{2}{240}, \quad 120 \le y \le 240$

yes

(c) $\mu_{X|y} = \int_{8.5}^{10.5} x \cdot f_{X|y}(x)dx = \int_{8.5}^{10.5} x \cdot \frac{1}{2} dx = 9.5$

$$\mu_{Y|x} = \int_{120}^{240} y \cdot f_{Y|x}(y)dy = \int_{120}^{240} y \cdot \frac{2}{240} dy = 180$$

yes

42. (a) We first need the conditional density for X given Y=y.

$$f_{X|y}(x) = \frac{f_{XY}(x,y)}{f_Y(y)} = \frac{1/x}{-\ln y}, \quad y < x < 1$$

Then $\mu_{X|y} = \int_{y}^{1} -\frac{x}{x \ln y} dx = -\frac{x}{\ln y}\bigg|_{y}^{1} = \frac{y-1}{\ln y}$

Not linear

(b) $\mu_{X|y=.5} = \frac{.5-1}{\ln(.5)} = .721$

(c) First we need the conditional density for Y given X=x.

$$f_{Y|x}(y) = \frac{f_{XY}(x,y)}{f_X(x)} = \frac{1/x}{1} = \frac{1}{x}, \quad 0 < y < x$$

Then, $\mu_{Y|x} = \int_{0}^{x} \frac{y}{x} dy = \frac{y^2}{2x}\bigg|_{0}^{x} = \frac{x}{2}$

Yes, it is linear

(d) $\mu_{Y|x=.75} = \frac{.75}{2} = .375$