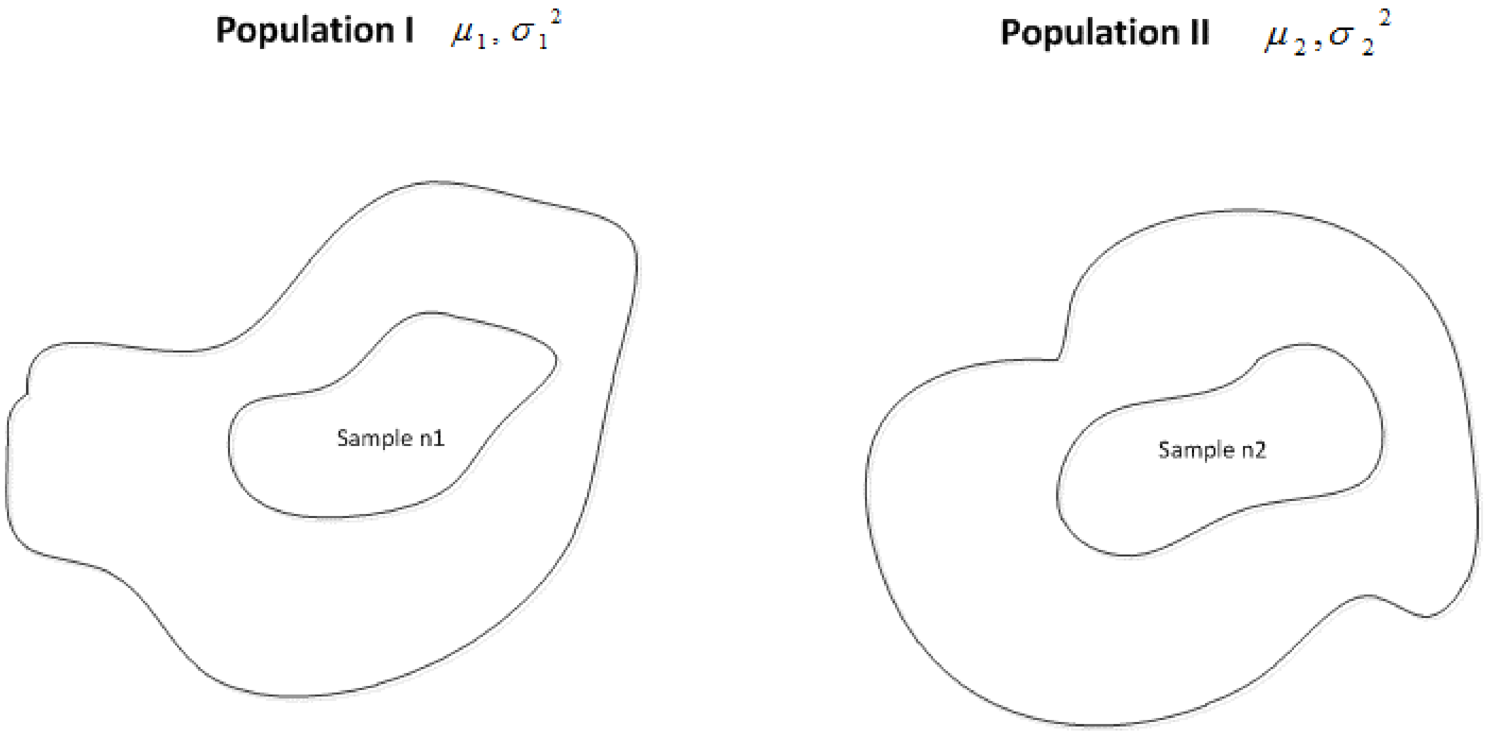
Comparing two means and two variances

There are two populations of interest each with unknown mean and variance. One random sample is drawn from the first population and one from the second population. Samples selected from two populations are independent of one another.



Point estimator of $\mu_1 - \mu_2$: $\widehat{\mu_1 - \mu_2} = \bar{X}_1 - \bar{X}_2$. Is this an unbiased estimator?

Comparing two means and two variances

Distribution of $\bar{X}_1 - \bar{X}_2$: let $\bar{X}_1$ and $\bar{X}_2$ be the sample means based on independent samples of size n_1 and n_2 drawn from normal distributions with means μ_1 and μ_2 and variances ${\sigma_1}^2$ and ${\sigma_2}^2$ respectively. Find the distribution of $\bar{X}_1 - \bar{X}_2$

It is known that $\bar{X}_1 \sim N\left(\mu_1, \frac{\sigma_1^2}{n_1}\right)$ and $\bar{X}_2 \sim N\left(\mu_2, \frac{\sigma_2^2}{n_2}\right)$, and $m_{\bar{X}_1}(t) = e^{(\mu_1 t + \frac{1\sigma_1^2}{2n_1}t^2)}$; $m_{\bar{X}_2}(t) = e^{(\mu_2 t + \frac{1\sigma_2^2}{2n_2}t^2)}$

We can find out the moment generating function of $\bar{X}_1 - \bar{X}_2$ as:

$$m_{\bar{X}_1 - \bar{X}_2}(t) = m_{\bar{X}_1}(t) m_{\bar{X}_2}(-t) = e^{(\mu_1 t + \frac{1\sigma_1^2}{2n_1}t^2)} e^{(-\mu_2 t + \frac{1\sigma_2^2}{2n_2}t^2)} = e^{(\mu_1 - \mu_2)t + \frac{1}{2}\left(\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right)t^2}$$

Therefore,

$$\bar{X}_1 - \bar{X}_2 \sim N(\mu_1 - \mu_2, \frac{{\sigma_1}^2}{n_1} + \frac{{\sigma_2}^2}{n_2})$$

Comparing two variances

Comparing the variances of two populations can take the following two forms of hypothesis test:

I.	$H_0: {\sigma_1}^2 = {\sigma_2}^2, \quad H_1: {\sigma_1}^2 > {\sigma_2}^2$	right-tailed test
II.	$H_0: {\sigma_1}^2 = {\sigma_2}^2, \quad H_1: {\sigma_1}^2 < {\sigma_2}^2$	left-tailed test
III.	$H_0: {\sigma_1}^2 = {\sigma_2}^2, \quad H_1: {\sigma_1}^2 \neq {\sigma_2}^2$	two-tailed test

The test statistic we choose is: $\frac{{S_1}^2}{{S_2}^2}$

Then, when the observed value of $\frac{{S_1}^2}{{S_2}^2}$ is close to 1, the null hypothesis tend to be true;

when the observed value of $\frac{{S_1}^2}{{S_2}^2}$ is much larger or less than 1, the two variances tend to be unequal.

$\frac{{S_1}^2}{{S_2}^2}$ follows a **F distribution**

F distribution

***F distribution**:* Let $\mathrm{X}^2{}_{\gamma_1}$ *and* $\mathrm{X}^2{}_{\gamma_2}$ be independent chi-squared random variables with γ_1 and γ_2 degree of freedom, respectively. The random variable:

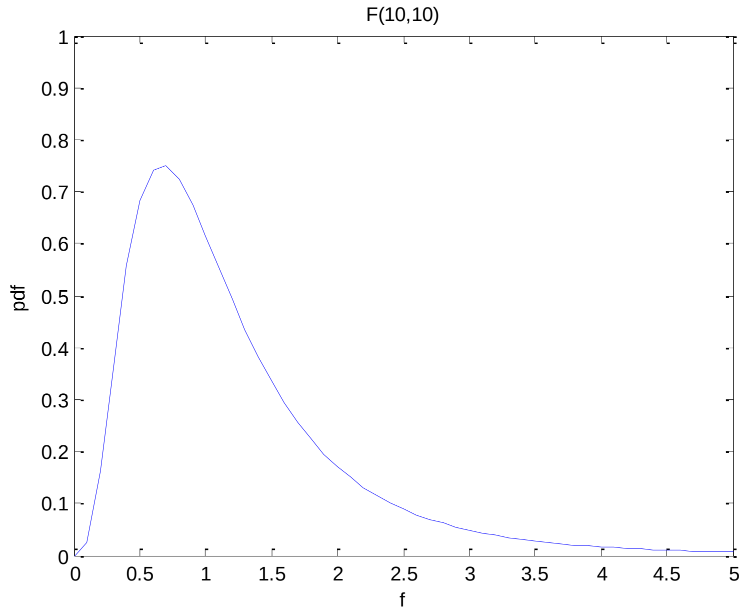
$$\frac{\mathrm{X}^2{}_{\gamma_1}/\gamma_1}{\mathrm{X}^2{}_{\gamma_2}/\gamma_2}$$

Follows the ***F distribution*** with γ_1 and γ_2 degree of freedom.

We use F_{γ_1,γ_2} to denote an F random variable with γ_1 and γ_2 degree of freedom.

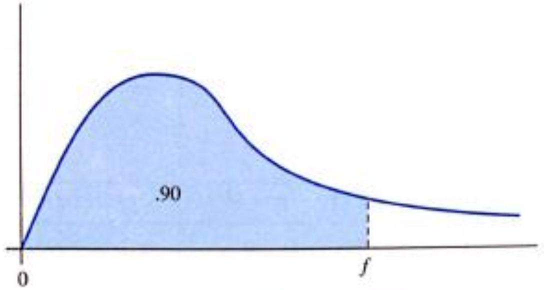
F_{γ_1,γ_2} is continuous and $F_{\gamma_1,\gamma_2} > 0$

The density of F_{γ_1,γ_2} is asymmetric.



Critical value for the left-tailed probability

TABLE IX
***F* distribution**



Column heading = numerator degrees of freedom

Row heading = denominator degrees of freedom

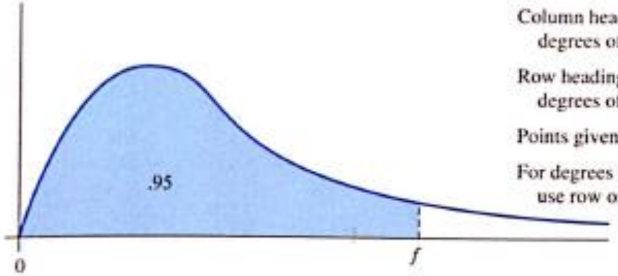
Points given are $f_{.10}$ points

For degrees of freedom > 120, use row or column 120

$P[F_{\gamma_1,\gamma_2} \le f] = .90$

γ_2 \ γ_1	1	2	3	4	5	6	7	8	9	10
1	39.862	49.500	53.593	55.833	57.240	58.204	58.906	59.439	59.857	60.195
2	8.526	9.000	9.162	9.243	9.293	9.326	9.349	9.367	9.381	9.392
3	5.538	5.462	5.391	5.343	5.309	5.285	5.266	5.252	5.240	5.231
4	4.545	4.325	4.191	4.107	4.051	4.010	3.979	3.955	3.936	3.920
5	4.060	3.780	3.619	3.520	3.453	3.405	3.368	3.339	3.316	3.297
6	3.776	3.463	3.289	3.181	3.108	3.055	3.014	2.983	2.958	2.937
7	3.589	3.257	3.074	2.961	2.883	2.827	2.785	2.752	2.725	2.703
8	3.458	3.113	2.924	2.806	2.726	2.668	2.624	2.589	2.561	2.538
9	3.360	3.006	2.813	2.693	2.611	2.551	2.505	2.469	2.440	2.416
10	3.285	2.924	2.728	2.605	2.522	2.461	2.414	2.377	2.347	2.323
11	3.225	2.860	2.660	2.536	2.451	2.389	2.342	2.304	2.274	2.248
12	3.177	2.807	2.606	2.480	2.394	2.331	2.283	2.245	2.214	2.188
13	3.136	2.763	2.560	2.434	2.347	2.283	2.234	2.195	2.164	2.138
14	3.102	2.726	2.522	2.395	2.307	2.243	2.193	2.154	2.122	2.095
15	3.073	2.695	2.490	2.361	2.273	2.208	2.158	2.119	2.086	2.059
16	3.048	2.668	2.462	2.333	2.244	2.178	2.128	2.088	2.055	2.028
17	3.026	2.645	2.437	2.308	2.218	2.152	2.102	2.061	2.028	2.001
18	3.007	2.624	2.416	2.286	2.196	2.130	2.079	2.038	2.005	1.977
19	2.990	2.606	2.397	2.266	2.176	2.109	2.058	2.017	1.984	1.956
20	2.975	2.589	2.380	2.249	2.158	2.091	2.040	1.999	1.965	1.937
21	2.961	2.575	2.365	2.233	2.142	2.075	2.023	1.982	1.948	1.920
22	2.949	2.561	2.351	2.219	2.128	2.061	2.008	1.967	1.933	1.904
23	2.937	2.549	2.339	2.207	2.115	2.047	1.995	1.953	1.919	1.890
24	2.927	2.538	2.327	2.195	2.103	2.035	1.983	1.941	1.906	1.877
25	2.918	2.528	2.317	2.184	2.092	2.024	1.971	1.929	1.895	1.866
26	2.909	2.519	2.307	2.174	2.082	2.014	1.961	1.919	1.884	1.855
27	2.901	2.511	2.299	2.165	2.073	2.005	1.952	1.909	1.874	1.845
28	2.894	2.503	2.291	2.157	2.064	1.996	1.943	1.900	1.865	1.836
29	2.887	2.495	2.283	2.149	2.057	1.988	1.935	1.892	1.857	1.827
30	2.881	2.489	2.276	2.142	2.049	1.980	1.927	1.884	1.849	1.819
31	2.875	2.482	2.270	2.136	2.042	1.973	1.920	1.877	1.842	1.812
32	2.869	2.477	2.263	2.129	2.036	1.967	1.913	1.870	1.835	1.805
33	2.864	2.471	2.258	2.123	2.030	1.961	1.907	1.864	1.828	1.799
34	2.859	2.466	2.252	2.118	2.024	1.955	1.901	1.858	1.822	1.793
35	2.855	2.461	2.247	2.113	2.019	1.950	1.896	1.852	1.817	1.787
36	2.850	2.456	2.243	2.108	2.014	1.945	1.891	1.847	1.811	1.781
37	2.846	2.452	2.238	2.103	2.009	1.940	1.886	1.842	1.806	1.776
38	2.842	2.448	2.234	2.099	2.005	1.935	1.881	1.838	1.802	1.772
39	2.839	2.444	2.230	2.095	2.001	1.931	1.877	1.833	1.797	1.767
40	2.835	2.440	2.226	2.091	1.997	1.927	1.873	1.829	1.793	1.763
[illegible]	[illegible]	2.412	2.197	2.061	1.966	1.895	1.840	1.796	1.760	1.729

TABLE IX
F* distribution *(continued)



Column heading = numerator degrees of freedom

Row heading = denominator degrees of freedom

Points given are $f_{.05}$ points

For degrees of freedom > 120, use row or column 120

$P[F_{\gamma_1,\gamma_2} \le f] = .95$

γ_2 \ γ_1	1	2	3	4	5	6	7	8
1	161.448	199.500	215.707	224.583	230.161	233.985	236.768	238.882
2	18.513	19.000	19.164	19.247	19.296	19.329	19.353	19.371
3	10.128	9.552	9.277	9.117	9.013	8.941	8.887	8.845
4	7.709	6.944	6.591	6.388	6.256	6.163	6.094	6.041
5	6.608	5.786	5.409	5.192	5.050	4.950	4.876	4.818
6	5.987	5.143	4.757	4.534	4.387	4.284	4.207	4.147
7	5.591	4.737	4.347	4.120	3.972	3.866	3.787	3.726
8	5.318	4.459	4.066	3.838	3.687	3.581	3.500	3.438
9	5.117	4.256	3.863	3.633	3.482	3.374	3.293	3.230
10	4.965	4.103	3.708	3.478	3.326	3.217	3.135	3.072
11	4.844	3.982	3.587	3.357	3.204	3.095	3.012	2.948
12	4.747	3.885	3.490	3.259	3.106	2.996	2.913	2.849
13	4.667	3.806	3.411	3.179	3.025	2.915	2.832	2.767
14	4.600	3.739	3.344	3.112	2.958	2.848	2.764	2.699
15	4.543	3.682	3.287	3.056	2.901	2.790	2.707	2.641
16	4.494	3.634	3.239	3.007	2.852	2.741	2.657	2.591
17	4.451	3.592	3.197	2.965	2.810	2.699	2.614	2.548
18	4.414	3.555	3.160	2.928	2.773	2.661	2.577	2.510
19	4.381	3.522	3.127	2.895	2.740	2.628	2.544	2.477
20	4.351	3.493	3.098	2.866	2.711	2.599	2.514	2.447
21	4.325	3.467	3.072	2.840	2.685	2.573	2.488	2.420
22	4.301	3.443	3.049	2.817	2.661	2.549	2.464	2.397
23	4.279	3.422	3.028	2.796	2.640	2.528	2.442	2.375
24	4.260	3.403	3.009	2.776	2.621	2.508	2.423	2.355
25	4.242	3.385	2.991	2.759	2.603	2.490	2.405	2.337
26	4.225	3.369	2.975	2.743	2.587	2.474	2.388	2.321
27	4.210	3.354	2.960	2.728	2.572	2.459	2.373	2.305
28	4.196	3.340	2.947	2.714	2.558	2.445	2.359	2.291
29	4.183	3.328	2.934	2.701	2.545	2.432	2.346	2.278
30	4.171	3.316	2.922	2.690	2.534	2.421	2.334	2.266
31	4.160	3.305	2.911	2.679	2.523	2.409	2.323	2.255
32	4.149	3.295	2.901	2.668	2.512	2.399	2.313	2.244
33	4.139	3.285	2.892	2.659	2.503	2.389	2.303	2.235
34	4.130	3.276	2.883	2.650	2.494	2.380	2.294	2.225
35	4.121	3.267	2.874	2.641	2.485	2.372	2.285	2.217
36	4.113	3.259	2.866	2.634	2.477	2.364	2.277	2.209
37	4.105	3.252	2.859	2.626	2.470	2.356	2.270	2.201
38	4.098	3.245	2.852	2.619	2.463	2.349	2.262	2.194
39	4.091	3.238	2.845	2.612	2.456	2.342	2.255	2.187
40	4.085	3.232	2.839	2.606	2.449	2.336	2.249	2.180
50	4.034	3.183	2.790	2.557	2.400	2.286	2.199	2.130

Comparing two variances

Distribution of $\frac{\boldsymbol{S_1}^2}{\boldsymbol{S_2}^2}$:

Let ${S_1}^2, {S_2}^2$ be sample variances based on independent samples of size $n_1 \; and \; n_2$ drawn from normal populations with means $\mu_1 \; and \; \mu_2$ and variances ${\sigma_1}^2 \; and \; {\sigma_2}^2$ respectively. If ${\sigma_1}^2 = {\sigma_2}^2$, then the statistic $\frac{{S_1}^2}{{S_2}^2}$ follows an **F distribution** with $n_1 - 1 \; and \; n_2 - 1$ degree of freedom, i.e., $\frac{{S_1}^2}{{S_2}^2} \sim F_{n_1-1, n_2-1}$

Proof:

We already know that $\frac{(n_1-1)S_1^2}{\sigma_1^2} \sim \mathrm{X}^2_{n_1-1}$ and $\frac{(n_2-1)S_2^2}{\sigma_2^2} \sim \mathrm{X}^2_{n_2-1}$. Then, from the definition of F distribution,

$$\frac{\dfrac{(n_1-1)S_1^2}{\sigma_1^2(n_1-1)}}{\dfrac{(n_2-1)S_2^2}{\sigma_2^2(n_2-1)}} = \frac{\sigma_2^2 S_1^2}{\sigma_1^2 S_2^2} \sim F_{(n_1-1),(n_2-1)}$$

When $\sigma_1^2 = \sigma_2^2$, then

$$\frac{S_1^2}{S_2^2} \sim F_{(n_1-1),(n_2-1)}$$

Comparing two variances

Example: The following are the information on two independent samples from two normal distributions with means μ_A *and* μ_B and variances ${\sigma_A}^2$ *and* ${\sigma_B}^2$. Test the following hypothesis: H_0: ${\sigma_A}^2 = {\sigma_B}^2$, H_1: ${\sigma_A}^2 \neq {\sigma_B}^2$ $(\alpha = 0.1)$

Population A	Population B
$n_A = 25$	$n_B = 16$
$\bar{x}_A = 380$	$\bar{x}_B = 370$
$S_A^2 = 100$	$S_B^2 = 400$

We choose $\frac{S_B^2}{S_A^2}$ as the test statistic. Then, assuming H_0 is true, $\frac{S_B^2}{S_A^2} \sim F_{15,24}$. The observed value of $\frac{S_B^2}{S_A^2}$ is 4

Since this is a two tailed test, $p - value = 2\min\{P[F_{15,24} \geq 4], P[F_{15,24} \leq 4]\}$

From the F-table, we find out that:

$$P[F_{15,24} > 2.108] = 0.05 \; and \; P[F_{15,24} \leq 2.108] = 0.95$$

Then, we have,

$$P[F_{15,24} \geq 4] < 0.05; P[F_{15,24} \leq 4] > 0.95$$

Therefore,

$$p - value < 2 \times 0.05 = 0.1$$

H_0 should be rejected at the significance level of 0.1

95% table

TABLE IX
F* distribution *(continued)

γ_2 \ γ_1	9	10	11	12	13	14	15	16
1	240.543	241.881	242.983	243.905	244.689	245.363	245.949	246.462
2	19.385	19.396	19.405	19.412	19.419	19.424	19.429	19.433
3	8.812	8.786	8.763	8.745	8.729	8.715	8.703	8.692
4	5.999	5.964	5.936	5.912	5.891	5.873	5.858	5.844
5	4.772	4.735	4.704	4.678	4.655	4.636	4.619	4.604
6	4.099	4.060	4.027	4.000	3.976	3.956	3.938	3.922
7	3.677	3.637	3.603	3.575	3.550	3.529	3.511	3.494
8	3.388	3.347	3.313	3.284	3.259	3.237	3.218	3.202
9	3.179	3.137	3.102	3.073	3.048	3.025	3.006	2.989
10	3.020	2.978	2.943	2.913	2.887	2.865	2.845	2.828
11	2.896	2.854	2.818	2.788	2.761	2.739	2.719	2.701
12	2.796	2.753	2.717	2.687	2.660	2.637	2.617	2.599
13	2.714	2.671	2.635	2.604	2.577	2.554	2.533	2.515
14	2.646	2.602	2.566	2.534	2.507	2.484	2.463	2.445
15	2.588	2.544	2.507	2.475	2.448	2.424	2.403	2.385
16	2.538	2.494	2.456	2.425	2.397	2.373	2.352	2.333
17	2.494	2.450	2.413	2.381	2.353	2.329	2.308	2.289
18	2.456	2.412	2.374	2.342	2.314	2.290	2.269	2.250
19	2.423	2.378	2.340	2.308	2.280	2.256	2.234	2.215
20	2.393	2.348	2.310	2.278	2.250	2.225	2.203	2.184
21	2.366	2.321	2.283	2.250	2.222	2.197	2.176	2.156
22	2.342	2.297	2.259	2.226	2.198	2.173	2.151	2.131
23	2.320	2.275	2.236	2.204	2.175	2.150	2.128	2.109
24	2.300	2.255	2.216	2.183	2.155	2.130	2.108	2.088
25	2.282	2.236	2.198	2.165	2.136	2.111	2.089	2.069
26	2.265	2.220	2.181	2.148	2.119	2.094	2.072	2.052
27	2.250	2.204	2.166	2.132	2.103	2.078	2.056	2.036
28	2.236	2.190	2.151	2.118	2.089	2.064	2.041	2.021
29	2.223	2.177	2.138	2.105	2.075	2.050	2.027	2.007
30	2.211	2.165	2.126	2.092	2.063	2.037	2.015	1.995
31	2.199	2.153	2.114	2.080	2.051	2.026	2.003	1.983
32	2.189	2.142	2.103	2.070	2.040	2.015	1.992	1.972
33	2.179	2.133	2.093	2.060	2.030	2.004	1.982	1.961
34	2.170	2.123	2.084	2.050	2.021	1.995	1.972	1.952
35	2.161	2.114	2.075	2.041	2.012	1.986	1.963	1.942
36	2.153	2.106	2.067	2.033	2.003	1.977	1.954	1.934
37	2.145	2.098	2.059	2.025	1.995	1.969	1.946	1.926
38	2.138	2.091	2.051	2.017	1.988	1.962	1.939	1.918
39	2.131	2.084	2.044	2.010	1.981	1.954	1.931	1.911
40	2.124	2.077	2.038	2.003	1.974	1.948	1.924	1.904
50	2.073	2.026	1.986	1.952	1.921	1.895	1.871	1.850
60	2.040	1.993	1.952	1.917	1.887	1.860	1.836	1.815
120	1.959	1.910	1.869	1.834	1.803	1.775	1.750	1.728

90% table

TABLE IX
F* distribution *(continued)

γ_2 \ γ_1	11	12	13	14	15	16	17	18	19	20
1	60.473	60.705	60.903	61.072	61.220	61.350	61.464	61.566	61.658	61.740
2	9.401	9.408	9.414	9.420	9.425	9.429	9.432	9.435	9.438	9.441
3	5.223	5.216	5.210	5.205	5.200	5.196	5.193	5.190	5.187	5.185
4	3.907	3.896	3.886	3.878	3.870	3.864	3.858	3.853	3.849	3.844
5	3.282	3.268	3.257	3.247	3.238	3.230	3.223	3.217	3.212	3.207
6	2.920	2.905	2.892	2.881	2.871	2.863	2.855	2.848	2.842	2.836
7	2.684	2.668	2.654	2.643	2.632	2.623	2.615	2.607	2.601	2.595
8	2.519	2.502	2.488	2.475	2.464	2.455	2.446	2.438	2.431	2.425
9	2.396	2.379	2.364	2.351	2.340	2.329	2.320	2.312	2.305	2.298
10	2.302	2.284	2.269	2.255	2.244	2.233	2.224	2.215	2.208	2.201
11	2.227	2.209	2.193	2.179	2.167	2.156	2.147	2.138	2.130	2.123
12	2.166	2.147	2.131	2.117	2.105	2.094	2.084	2.075	2.067	2.060
13	2.116	2.097	2.080	2.066	2.053	2.042	2.032	2.023	2.014	2.007
14	2.073	2.054	2.037	2.022	2.010	1.998	1.988	1.979	1.970	1.962
15	2.037	2.017	2.000	1.985	1.972	1.961	1.950	1.941	1.932	1.924
16	2.005	1.985	1.968	1.953	1.940	1.928	1.917	1.908	1.899	1.891
17	1.978	1.958	1.940	1.925	1.912	1.900	1.889	1.879	1.870	1.862
18	1.954	1.933	1.916	1.900	1.887	1.875	1.864	1.854	1.845	1.837
19	1.932	1.912	1.894	1.878	1.865	1.852	1.841	1.831	1.822	1.814
20	1.913	1.892	1.875	1.859	1.845	1.833	1.821	1.811	1.802	1.794
21	1.896	1.875	1.857	1.841	1.827	1.815	1.803	1.793	1.784	1.776
22	1.880	1.859	1.841	1.825	1.811	1.798	1.787	1.777	1.768	1.759
23	1.866	1.845	1.827	1.811	1.796	1.784	1.772	1.762	1.753	1.744
24	1.853	1.832	1.814	1.797	1.783	1.770	1.759	1.748	1.739	1.730
25	1.841	1.820	1.802	1.785	1.771	1.758	1.746	1.736	1.726	1.718
26	1.830	1.809	1.790	1.774	1.760	1.747	1.735	1.724	1.715	1.706
27	1.820	1.799	1.780	1.764	1.749	1.736	1.724	1.714	1.704	1.695
28	1.811	1.790	1.771	1.754	1.740	1.726	1.715	1.704	1.694	1.685
29	1.802	1.781	1.762	1.745	1.731	1.717	1.705	1.695	1.685	1.676
30	1.794	1.773	1.754	1.737	1.722	1.709	1.697	1.686	1.676	1.667
31	1.787	1.765	1.746	7.729	1.714	1.701	1.689	1.678	1.668	1.659
32	1.780	1.758	1.739	1.722	1.707	1.694	1.682	1.671	1.661	1.652
33	1.773	1.751	1.732	1.715	1.700	1.687	1.675	1.664	1.654	1.645
34	1.767	1.745	1.726	1.709	1.694	1.680	1.668	1.657	1.647	1.638
35	1.761	1.739	1.720	1.703	1.688	1.674	1.662	1.651	1.641	1.632
36	1.756	1.734	1.715	1.697	1.682	1.669	1.656	1.645	1.635	1.626
37	1.751	1.729	1.709	1.692	1.677	1.663	1.651	1.640	1.630	1.620
38	1.746	1.724	1.704	1.687	1.672	1.658	1.646	1.635	1.624	1.615
39	1.741	1.719	1.700	1.682	1.667	1.653	1.641	1.630	1.619	1.610
40	1.737	1.715	1.695	1.678	1.662	1.649	1.636	1.625	1.615	1.605
50	1.703	1.680	1.660	1.643	1.627	1.613	1.600	1.588	1.578	1.568
60	1.680	1.657	1.637	1.619	1.603	1.589	1.576	1.564	1.553	1.543
120	1.625	1.601	1.580	1.562	1.545	1.530	1.516	1.504	1.493	1.482

Comparing two variances

Example: The following are the information on two independent samples from two normal distributions with means μ_A *and* μ_B and variances ${\sigma_A}^2$ *and* ${\sigma_B}^2$. Test the following hypothesis: H_0: ${\sigma_A}^2 = {\sigma_B}^2$, $H_1: {\sigma_A}^2 \neq {\sigma_B}^2$ ($\alpha = 0.1$)

Population A	Population B
$n_A = 25$	$n_B = 16$
$\bar{x}_A = 380$	$\bar{x}_B = 370$
$S_A^2 = 100$	$S_B^2 = 400$

We choose $\frac{S_A^2}{S_B^2}$ as the test statistic. Then, assuming H_0 is true, $\frac{S_A^2}{S_B^2} \sim F_{24,15}$. The observed value of $\frac{S_A^2}{S_B^2}$ is 1/4

Since this is a two tailed test, $p - value = 2\min\{P\left[F_{24,15} \geq \frac{1}{4}\right], P[F_{24,15} \leq \frac{1}{4}]\}$

From the F-table, we find out that:

$$P\left[F_{24,15} > 1.899\right] = 0.10 \textit{ and } P\left[F_{24,15} \leq 1.899\right] = 0.90$$

Then, we have,

$$P\left[F_{24,15} \geq 1/4\right] > 0.10; P\left[F_{24,15} \leq 1/4\right] < 0.90$$

Therefore,

$p - value$ can not be evaluated based on the F table. (actually, $P\left[F_{24,15} \leq 1/4\right] = 0.0013$ from Matlab)

No conclusion on weather H_0 should be rejected.

90% table

TABLE IX
***F* distribution (*continued*)**

γ_2 \ γ_1	21	22	23	24	25	26	27	28	29	30
1	61.815	61.882	61.945	62.002	62.054	62.103	62.148	62.189	62.228	62.265
2	9.444	9.446	9.448	9.450	9.451	9.453	9.454	9.456	9.457	9.458
3	5.182	5.180	5.178	5.176	5.175	5.173	5.172	5.170	5.169	1.168
4	3.841	3.837	3.834	3.831	3.828	3.826	3.824	3.821	3.819	3.817
5	3.202	3.198	3.194	3.191	3.187	3.184	3.181	3.179	3.176	3.174
6	2.831	2.827	2.822	2.818	2.815	2.811	2.808	2.805	2.803	2.800
7	2.589	2.584	2.580	2.575	2.571	2.568	2.564	2.561	2.558	2.555
8	2.419	2.414	2.409	2.404	2.400	2.396	2.392	2.389	2.386	2.383
9	2.292	2.287	2.282	2.277	2.272	2.268	2.265	2.261	2.258	2.255
10	2.194	2.189	2.183	2.178	2.174	2.170	2.166	2.162	2.159	2.155
11	2.117	2.111	2.105	2.100	2.095	2.091	2.087	2.083	2.080	2.076
12	2.053	2.047	2.041	2.036	2.031	2.027	2.022	2.019	2.015	2.011
13	2.000	1.994	1.988	1.983	1.978	1.973	1.969	1.965	1.961	1.958
14	1.955	1.949	1.943	1.938	1.933	1.928	1.923	1.919	1.916	1.912
15	1.917	1.911	1.905	1.899	1.894	1.889	1.885	1.880	1.876	1.873
16	1.884	1.877	1.871	1.866	1.860	1.855	1.851	1.847	1.843	1.839
17	1.855	1.848	1.842	1.836	1.831	1.826	1.821	1.817	1.813	1.809
18	1.829	1.823	1.816	1.810	1.805	1.800	1.795	1.791	1.787	1.783
19	1.807	1.800	1.793	1.787	1.782	1.777	1.772	1.767	1.763	1.759
20	1.786	1.779	1.773	1.767	1.761	1.756	1.751	1.746	1.742	1.738
21	1.768	1.761	1.754	1.748	1.742	1.737	1.732	1.728	1.723	1.719
22	1.751	1.744	1.737	1.731	1.726	1.720	1.715	1.711	1.706	1.702
23	1.736	1.729	1.722	1.716	1.710	1.705	1.700	1.695	1.691	1.686
24	1.722	1.715	1.708	1.702	1.696	1.691	1.686	1.681	1.676	1.672
25	1.710	1.702	1.695	1.689	1.683	1.678	1.672	1.668	1.663	1.659
26	1.698	1.690	1.684	1.677	1.671	1.666	1.660	1.656	1.651	1.647
27	1.687	1.680	1.673	1.666	1.660	1.655	1.649	1.645	1.640	1.636
28	1.677	1.669	1.662	1.656	1.650	1.644	1.639	1.634	1.630	1.625
29	1.668	1.660	1.653	1.647	1.640	1.635	1.630	1.625	1.620	1.616
30	1.659	1.651	1.644	1.638	1.632	1.626	1.621	1.616	1.611	1.606
31	1.651	1.643	1.636	1.630	1.623	1.618	1.612	1.607	1.602	1.598
32	1.643	1.636	1.628	1.622	1.616	1.610	1.604	1.599	1.595	1.590
33	1.636	1.628	1.621	1.615	1.608	1.603	1.597	1.592	1.587	1.583
34	1.630	1.622	1.614	1.608	1.601	1.596	1.590	1.585	1.580	1.576
35	1.623	1.615	1.608	1.601	1.595	1.589	1.584	1.579	1.574	1.569
36	1.617	1.609	1.602	1.595	1.589	1.583	1.578	1.572	1.567	1.563
37	1.612	1.604	1.596	1.590	1.583	1.577	1.572	1.567	1.562	1.557
38	1.606	1.598	1.591	1.584	1.578	1.572	1.566	1.561	1.556	1.551
39	1.601	1.593	1.586	1.579	1.573	1.567	1.561	1.556	1.551	1.546
40	1.596	1.588	1.581	1.574	1.568	1.562	1.556	1.551	1.546	1.541
50	1.559	1.551	1.543	1.536	1.529	1.523	1.517	1.512	1.507	1.502
60	1.534	1.526	1.518	1.511	1.504	1.498	1.492	1.486	1.481	1.476
120	1.472	1.463	1.455	1.447	1.440	1.433	1.427	1.421	1.415	1.409

95% table

TABLE IX
***F* distribution (*continued*)**

γ_2 \ γ_1	17	18	19	20	21	22	23	24
1	246.917	247.322	247.685	248.012	248.308	248.577	248.824	249.051
2	19.437	19.440	19.443	19.446	19.448	19.450	19.452	19.454
3	8.683	8.675	8.667	8.660	8.654	8.648	8.643	8.639
4	5.832	5.821	5.811	5.803	5.795	5.787	5.781	5.774
5	4.590	4.579	4.568	4.558	4.549	4.541	4.534	4.527
6	3.908	3.896	3.884	3.874	3.865	3.856	3.849	3.841
7	3.480	3.467	3.455	3.445	3.435	3.426	3.418	3.411
8	3.187	3.173	3.161	3.150	3.140	3.131	3.123	3.115
9	2.974	2.960	2.948	2.936	2.926	2.917	2.908	2.900
10	2.812	2.798	2.785	2.774	2.764	2.754	2.745	2.737
11	2.685	2.671	2.658	2.646	2.636	2.626	2.617	2.609
12	2.583	2.568	2.555	2.544	2.533	2.523	2.514	2.505
13	2.499	2.484	2.471	2.459	2.448	2.438	2.429	2.420
14	2.428	2.413	2.400	2.388	2.377	2.367	2.357	2.349
15	2.368	2.353	2.340	2.328	2.316	2.306	2.297	2.288
16	2.317	2.302	2.288	2.276	2.264	2.254	2.244	2.235
17	2.272	2.257	2.243	2.230	2.219	2.208	2.199	2.190
18	2.233	2.217	2.203	2.191	2.179	2.168	2.159	2.150
19	2.198	2.182	2.168	2.156	2.144	2.133	2.123	2.114
20	2.167	2.151	2.137	2.124	2.112	2.102	2.092	2.082
21	2.139	2.123	2.109	2.096	2.084	2.073	2.063	2.054
22	2.114	2.098	2.084	2.071	2.059	2.048	2.038	2.028
23	2.091	2.075	2.061	2.048	2.036	2.025	2.014	2.005
24	2.070	2.054	2.040	2.027	2.015	2.003	1.993	1.984
25	2.051	2.035	2.021	2.007	1.995	1.984	1.974	1.964
26	2.034	2.018	2.003	1.990	1.978	1.966	1.956	1.946
27	2.018	2.002	1.987	1.974	1.961	1.950	1.940	1.930
28	2.003	1.987	1.972	1.959	1.946	1.935	1.924	1.915
29	1.989	1.973	1.958	1.945	1.932	1.921	1.910	1.901
30	1.976	1.960	1.945	1.932	1.919	1.908	1.897	1.887
31	1.965	1.948	1.933	1.920	1.907	1.896	1.885	1.875
32	1.953	1.937	1.922	1.908	1.896	1.884	1.873	1.864
33	1.943	1.926	1.911	1.898	1.885	1.873	1.863	1.853
34	1.933	1.917	1.902	1.888	1.875	1.863	1.853	1.843
35	1.924	1.907	1.892	1.878	1.866	1.854	1.843	1.833
36	1.915	1.899	1.883	1.870	1.857	1.845	1.834	1.824
37	1.907	1.890	1.875	1.861	1.848	1.837	1.826	1.816
38	1.899	1.883	1.867	1.853	1.841	1.829	1.818	1.808
39	1.892	1.875	1.860	1.846	1.833	1.821	1.810	1.800
40	1.885	1.868	1.853	1.839	1.826	1.814	1.803	1.793
50	1.831	1.814	1.798	1.784	1.771	1.759	1.748	1.737
60	1.796	1.778	1.763	1.748	1.735	1.722	1.711	1.700
120	1.709	1.690	1.674	1.659	1.645	1.632	1.620	1.608

Example: The following are the information on two independent samples from two normal distributions with means μ_A *and* μ_B and variances ${\sigma_A}^2$ *and* ${\sigma_B}^2$. Test the following hypothesis: H_0: ${\sigma_A}^2 = {\sigma_B}^2$, H_1: ${\sigma_A}^2 \neq {\sigma_B}^2$ $(\alpha = 0.1)$

Population A	Population B
$n_A = 25$	$n_B = 16$
$\bar{x}_A = 380$	$\bar{x}_B = 370$
$S_A^2 = 100$	$S_B^2 = 400$

When we compare two variances, we always choose $\frac{S_B^2}{S_A^2}$ as the test statistic when the observed value of this ratio is larger then 1. In this example, assuming H_0 is true, $\frac{S_B^2}{S_A^2} \sim F_{15,24}$. The observed value of $\frac{S_B^2}{S_A^2}$ is 4.

Then, $\boldsymbol{p_{value}}$ ***can be calculated as:***

$$p_{value} = 2P\left[F_{\gamma_1,\gamma_2} \geq S_{obs}\right], where, S_{obs} > 1 \text{ is the observed value of the test statistic}$$

i.e.,

$$p_{value} = 2P\left[F_{15,24} \geq 4\right]$$

From the F-table, we find out that:

$$P\left[F_{15,24} > 2.108\right] = 0.05 \text{ and } P\left[F_{15,24} \leq 2.108\right] = 0.95$$

Then, we have, $P\left[F_{15,24} \geq 4\right] < 0.05$

Therefore, $p - value < 2 \times 0.05 = 0.1$

H_0 should be rejected at the significance level of 0.1