Solutions of HW2:

Chapter 1:

9. (a) $9! = 362880$
   (b) $6! = 720$
   (c) ${}_7P_3 = \frac{7!}{4!} = 7 \times 6 \times 5 = 210$
   (d) ${}_6P_2 = \frac{6!}{4!} = 6 \times 5 = 30$
   (f) ${}_6P_6 = \frac{6!}{0!} = 6! = 720$

10. (a) (4)(3)=12

    (b) (4)(3)(5) = 60

    (c) (4)(3)(5)(6) = 360

14. (a) $2^4 = 16$

    (b) $2^x = 32 \Rightarrow x = 5$

15. (a) 5! = 120
    (b) Out of 5 test, there are total 8 arrangements that two coatings from the same manufacturer be tested back to back. For each of these arrangements, there can be 6 different testing orders. Therefore, there are total (6)(8) = 48 test orders that the two coatings are tested back to back. The test orders without any constraints are 5! = 120.

    Hence, the probability is 48/120=0.4

17. (a) $_9C_4 = \frac{9!}{4!\times 5!} = \frac{9\times 8\times 7\times 6}{4!} = 126$

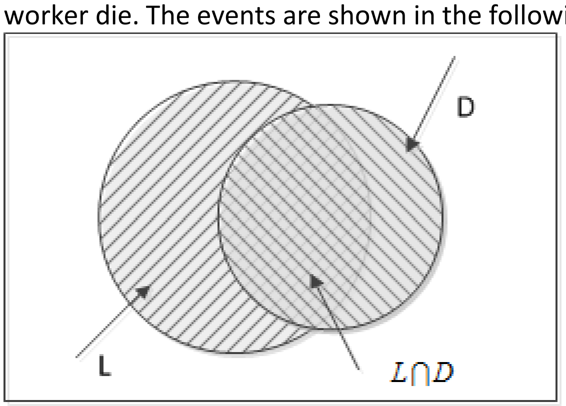
(b) $_8C_3 = \frac{8!}{3!\times 5!} = \frac{8\times 7\times 6}{3!} = 56$

(c) $\binom{8}{5} = \frac{8!}{3!\times 5!} = \frac{8\times 7\times 6}{3!} = 56$

(d) $\binom{8}{0} = \frac{8!}{0!\times 8!} = 1$

Chapter 2

13. Let L represent the event that a worker is exposed to LD_{50}, and D represent the event that a worker die. The events are shown in the following Vann diagram:



The following probabilities are given:

$P[L \cap D] = 0.30; P[D] = 0.40; P[L \cup D] = 0.68$

From the Vann diagram, it is easy to find that $P[D \cap L'] = P[D] - P[D \cup L] = 0.4 - 0.3 = 0.1$

From the general addition rule, we have

$$P[L \cup D] = P[L] + P[D] - P[L \cap D]$$
$$\Rightarrow P[L] = P[L \cup D] + P[L \cap D] - P[D] = 0.68 + 0.3 - 0.4 = 0.58$$

(a) $P[D \mid L] = \frac{P[L \cap D]}{P[L]} = \frac{0.30}{0.58} = \frac{30}{58}$

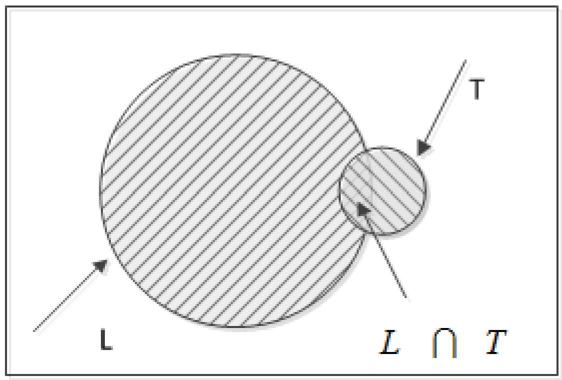
(b) $P[D' \mid L] = 1 - P[D \mid L] = \frac{28}{58}$

(c) Theorem 2.1.2

(d) $P[D \mid L'] = \dfrac{P(D \cap L']}{P[L']} = \dfrac{0.10}{1-0.58} = \dfrac{10}{42}$

(e) No. Exposure to lethal dose of radiation increase the probability that a worker dies.

17. Let T represent the event that a power failure is caused by transformer damage; and L represent the event that a power failure is caused by line damage;
Then the relations of these events can be shown in the following Vann Diagram:



The following probabilities are given:

$P[T] = 0.05; P[L] = 0.80; P[L \cap T] = 0.01$

(a) $P[L \mid T] = \dfrac{P[L \cap T]}{P[T]} = \dfrac{0.01}{0.05} = 0.20$

(b) $P[T \mid L] = \dfrac{P[L \cap T]}{P[L]} = \dfrac{0.01}{0.80} = 0.0125$

(c) $P[T \cap L'] = P[T] - P[L \cap T] = 0.05 - 0.01 = 0.04$

(d) $P[T \mid L'] = \dfrac{P[L' \cap T]}{P[L']} = \dfrac{0.04}{0.20} = 0.20$

(e) $P[T \cup L] = P[T] + P[L] - P[L \cap T] = 0.05 + 0.80 - 0.01 = 0.84$

19. According to the general addition rule, $P[A_1 \cup A_2] = P[A_1] + P[A_2] - P[A_1 \cap A_2]$. Then,

We have, $P[A_1 \cap A_2] = P[A_1] + P[A_2] - P[A_1 \cup A_2] = 0.6 + 0.4 - 0.8 = 0.2$

Since $P[A_1 \cap A_2] = 0.2$ and $P[A_1]P[A_2] = 0.6 \times 0.4 = 0.24$

i.e., $P[A_1 \cap A_2] \neq P[A_1]P[A_2]$

Therefore, A_1 and A_2 are not $independent$

27. Let PD represents the event that a blood unit was donated by a paid donor; and H represent the event that a blood unit contracts hepatitis;

The following probabilities are given:

$$P[PD]=0.67; P[E\mid PD]=0.0144; P[E\mid PD']=0.0012;$$

$$\begin{aligned}
&P[E]=P[E\cap PD]+P[E\mid PD']\\
&=P[E\mid PD]P[PD]+P[E\mid PD']P[PD']\\
&=0.0144\times 0.67+0.0012\times 0.33\\
&=0.01
\end{aligned}$$

32. According to the definition of mutually exclusive event, we have

$$A_1\cap A_2=\varnothing \qquad \Rightarrow \quad P[A_1\cap A_2]=P[\varnothing]=0\neq P[A_1]\cdot P[A_2]>0$$

Hence, they are not independent.

35. Let PT represent the event that the test result is positive; and D represent the event that a person from that group has the disease;

The following probabilities are given:

$$P[D]=0.10; P[PT\mid D]=0.85; P[PT\mid D']=0.04$$

From the Bayes' Theorem. We have,

$$P[D\mid PT]=\frac{P[PT\mid D]P[D]}{P[PT]}=\frac{P[PT\mid D]P[D]}{P[PT\mid D]P[D]+P[PT\mid D']P[D']}$$

$$=\frac{0.85\times 0.10}{0.85\times 0.10+0.04\times 0.90}=0.7025$$