Solution of HW12

Chapter 8

28. (a) H_0: $p \geq .2$; H_1: $p < .2$

    (b) Type I: we will conclude that the failure rate has been reduced by new technology and stricter quality controls when, in fact, it has not.

    Type II: we do not detect an apparently reduced failure rate

    (c) Let X: the number of failures during the first 1000 hours in the 20 trials

    X is binomial with n = 20 and p = .2 when H_0 is true

    $E[X] = np = (20)(.2) = 4$

    (d) α = P[reject H_0 | H_0 is true] = $P[X \leq 1 \mid p = .2] = .0692$

    (e) β = P[fail to reject H_0 | p = .1] = $P[X > 1 \mid p = .1]$

    $= 1 - P[X \leq 1 \mid p = .1] = 1 - .3917 = .6083$

    power = $1 - \beta = 1 - .6083 = .3917$

    (f) Increase α by changing the critical region to C = {0, 1, 2}

    no, $\alpha = P[X \leq 2 \mid p = .2] = .2061$

    increase the sample size

29. $P[X \geq 14|p = 0.4] = 1 - P[X \leq 13|p = 0.4] = 1 - 0.9935 = 0.0065$

    $P[X \geq 14|p = 0.3] = 1 - P[X \leq 13|p = 0.3] = 1 - 0.9997 = 0.0003$

    $P[X \geq 14|p = 0.2] = 1 - P[X \leq 13|p = 0.2] \approx 0.0$

    $P[X \geq 14|p = 0.1] = 1 - P[X \leq 13|p = 0.1] \approx 0.0$

    All these probabilities are less than 0.0577

30. (a) $\bar{X}$ is normal with mean 20 and variance $\frac{20}{9} = 2.7778$

(b) α is the area to the right of 25 under the normal curve with mean 20

(c) $\alpha = P[\bar{X} > 25 \mid \mu = 20] = P\left[Z > \frac{25-20}{5/3}\right] = 1 - P[Z \le 3] = 1 - .9987 = .0013$

(d) $\bar{X}$ is normal with mean 28 and variance $\frac{25}{9}$

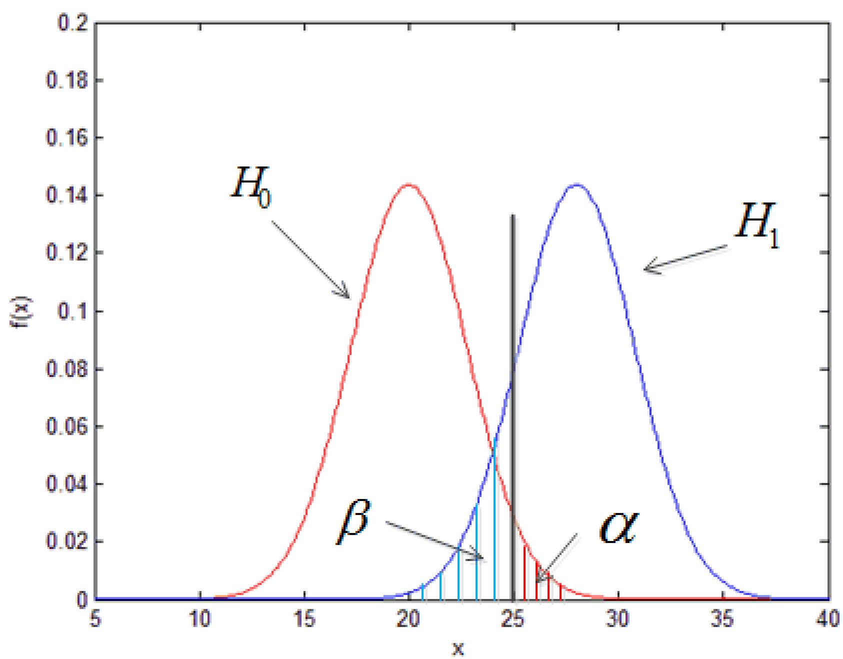
(e) β is the area to the left of 25 under the normal curve with mean 28

(f) $\beta = P\left[\bar{X} \le 25 \mid \mu = 25\right] = P\left[Z \le \frac{25-28}{5/3}\right] = P\left[Z \le -1.8\right] = .0359$

(g) power = 1 – β = 1 – .0359 = .9641

(h) The curves will become narrower and will have less overlap

(i) α and β will both decrease



32. (a) $H_0: \mu \geq .6$ g / mi; $H_1: \mu < .6$ g / mi

(b) Type I: we will conclude that the new engine has a mean emission level below .6 g / mi when, in fact, this is not true.
Type II: we will not detect the fact that the new engine has a mean emission level below the current standard of .6 g / mi.

(c) P value = $P\left[\bar{X} \leq ,5 \mid \mu = .6,\ \sigma = .4\right] = P\left[Z \leq \frac{.6-.6}{.4/\sqrt{64}}\right] = .0228$

yes, reject H_0 because the chance of being wrong if you do is .0228, which is quite small.

Type I error might be committed

35. For a hypothesis test on the mean with unknown variance σ^2, there are three cases of test:

The right-tailed test: $H_0: \mu = \mu_0;\ H_1: \mu > \mu_0$

The left-tailed test: $H_0: \mu = \mu_0;\ H_1: \mu < \mu_0$

The two-tailed test: $H_0: \mu = \mu_0;\ H_1: \mu \neq \mu_0$

To find the critical region, assuming H_0 is true, the test statistic $\frac{\bar{X}-\mu_0}{S/\sqrt{n}} \sim T_{n-1}$. The critical points can be found as:

For right-tailed test: $P[T_{n-1} \geq t_\alpha] = \alpha$

For left-tailed test: $P[T_{n-1} \leq t_{1-\alpha}] = \alpha$

For two-tailed test: $P\left[T_{n-1} \geq t_{\alpha/2}\right] + P\left[T_{n-1} \leq t_{1-\alpha/2}\right] = \alpha$

Hence,

(a) Left-tailed test. The critical point can be found from T_{24} table. $t_{0.95} = -1.711$

(b) Left-tailed test. The critical point can be found from T_∞ table. $t_{0.90} = -1.282$

(c) Right-tailed test. The critical point can be found from T_{19} table. $t_{0.025} = 2.093$

(d) Right-tailed test. The critical point can be found from T_{15}table. $t_{0.01} = 2.602$

(e) Two-tailed test. The critical point can be found from T_{19}table.

$t_{0.05} = 1.729, t_{0.95} = -1.729$

(f) Two-tailed test. The critical point can be found from T_{29}table.

$t_{0.025} = 2.045, t_{0.975} = -2.045$

38. (a) This is a two-tailed test. Assuming H_0 is true, the test statistic is $\frac{\bar{X}-9.5}{S/\sqrt{50}} \sim T_{49}$ The critical values can be found as: $\pm t_{0.025} = \pm 2.010$

(b) The observed value of the test statistic is $\frac{\bar{X}-9.5}{S/\sqrt{50}} = \frac{9.8-9.5}{1.2/\sqrt{50}} = 1.768$

Since $-2.010 < 1.768 < 2.010$, 1.768 is not in the critical region so we do not reject H_0 at $\alpha = .05$

Therefore, we cannot say that the mean predicted by the model is different from 9.5 million barrels per day.

In this case, we are subject to Type II error.

40. (a) $H_0 : \mu = 4.6$ mg / litre
$H_1 : \mu > 4.6$ mg / litre

(b) Assuming H_0 is true, the test statistic is $\frac{\bar{X}-4.6}{S/\sqrt{28}} \sim T_{27}$. The observed value of the test statistic is $\frac{\bar{x}-\mu_0}{s/\sqrt{n}} = \frac{5.2-4.6}{1.6/\sqrt{28}} = 1.98$

Since this is a right-tailed test, we have

$p_{value} = P[T_{27} \geq 1.98]$

From the T-table, we have, $P[T_{27} \geq 1.703] = 0.05$

Hence, $P[T_{27} \geq 1.98] < 0.05$

At the significance level of $\alpha = 0.05$, H_0 should be rejected.

(c) The mean silicon concentration in the river has increased, thus, the mineral content in the soil is being depleted.

43. (a) $H_0: \mu = 5;\ H_1: \mu < 5$

(b) Let's carry out a hypothesis test on (a). Assuming H_0 is true, the test statistic is $\frac{\bar{X}-5}{S/\sqrt{16}} \sim T_{15}$.The observed value of the test statistic is $\frac{\bar{X}-5}{S/4} = \frac{4.28-5}{0.828/4} = -3.472$

Since this is a left-tailed test, the p-value is:

$$p_{value} = P[T_{15} < -3.472]$$

From the T-table, we found out: $P[T_{15} < -2.947] = 0.005$

Hence,

$$p_{value} = P[T_{15} < -3.472] < 0.005$$

At the significance level of $\alpha = 0.01$, H_0 should be rejected. i.e., the sampled data support the contention.

(c) At the significance level of $\alpha = 0.05$, H_0 should be rejected.

(d) Probably not. Since $\bar{x} = 4.28\ inch$ is still a significant number.

48. (a) Assuming H_0 is true, the test statistic is $\frac{\bar{X}-1.3}{S/\sqrt{30}} \sim T_{29}$. The observed value of the test statistic is

$$\frac{3.97-1.3}{1.89/\sqrt{30}} = 7.738$$

Since this value is above the critical point $t_{.01}(\gamma = 29) = 2.462$, we reject H_0 at $\alpha = .01$

(b) The observed value of the test statistic is $\chi^2 = \frac{29(1.89)^2}{(.6)^2} = 287.75$

and the critical point is $\chi^2_{.01}(\gamma = 29) = 49.6$. Thus, we reject H_0 at .$\alpha = .01$

The design specifications are not met.

This can also be done using the p-value method:

$p_{value} = P[X^2_{29} \geq 287.75] = 1 - P[X^2_{29} \leq 287.75]$

From the chi-squared probability table, we find that $P[X^2_{29} \leq 52.3] = 0.995$

Hence, we have, $p_{value} < 0.005 < \alpha = 0.01$

Therefore, we reject H_0 at $\alpha = 0.01$