Solutions of HW10

Chapter 7 47(20),49(15)

47.(a) $\bar{X} = 0.643$

(b) $L_1 = \bar{X} - \frac{1.96\sigma}{\sqrt{n}} = 0.643 - 1.96 \times \frac{0.01}{5} = 0.643 - 0.0039 = 0.6391$

$L_2 = \bar{X} + \frac{1.96\sigma}{\sqrt{n}} = 0.643 + 0.0039 = 0.6469$

(c) The 90% confidence interval is shorter than 95% confidence interval
The boundaries of $100(1-\alpha)\%$ confidence interval are: $\bar{X} \pm z_{\alpha/2}\sigma/\sqrt{n}$
When α change from 5 to 10, the value of $z_{\alpha/2}$ becomes smaller and the interval becomes shorter.
$z_{\alpha/2} = 1.64$ for 90% confidence interval.
The boundaries are: $\bar{X} \pm \frac{1.64\sigma}{\sqrt{n}} = 0.643 \pm 0.0033$

(d) The 99% confidence interval is longer than 95% confidence interval
The boundaries of $100(1-\alpha)\%$ confidence interval are: $\bar{X} \pm z_{\alpha/2}\sigma/\sqrt{n}$
When α change from 5 to 1, the value of $z_{\alpha/2}$ becomes bigger and the interval becomes longer.
$z_{\alpha/2} = 2.58$ for 99% confidence interval.
The boundaries are: $\bar{X} \pm \frac{2.58\sigma}{\sqrt{n}} = 0.643 \pm 0.0052$

49.(a) Not normal distribution. Normal distribution is for continuous variables
(b) $\bar{X} \approx 2.8000$
(c) The boundaries of the 99% confidence interval are:

$$\bar{X} \pm \frac{2.58\sigma}{\sqrt{n}} = 2.8000 \pm 0.2036$$

From the central limit theorem, we have that $\bar{X}$ is a normal random variable.
(d) $\mu = 3.0$ is inside the interval. Hence, the data does not refute the value.

Chapter 8 2(20), 4(15), 6(15),9(15)

2. (a) $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2 = \frac{1}{n-1}\sum_{i=1}^{n} x_i{}^2 - \frac{n}{n-1}\left(\frac{1}{n}\sum_{i=1}^{n} x_i\right)^2$
$= \frac{n\sum_{i=1}^{n} x_i{}^2 - \left(\sum_{i=1}^{n} x_i\right)^2}{n(n-1)} = 20.4286$

(b) $n = 30$. Hence, the table values come from a χ^2_{29} distribution.
The 90% confidence interval on $\sigma^2$is:
$L_1 = \frac{(n-1)S^2}{\chi^2_{0.05}}, L_2 = \frac{(n-1)S^2}{\chi^2_{0.95}}$

From the probability table of the χ^2_{29} distribution, we can read out:
$\chi^2_{0.05} = 42.6$; $\chi^2_{0.95} = 17.7$;

Hence the 90% confidence interval of σ^2 is:

$$\left[\frac{(n-1)s^2}{\chi^2_{.05}}, \frac{(n-1)s^2}{\chi^2_{.95}}\right] = \left[\frac{(29)(20.4286)}{42.6}, \frac{(29)(20.4286)}{17.7}\right]$$

$$= [13.9068,\ 33.4706]$$

Thus, we are 90% confident that the true variance is between 13.9068 and 33.4706.

(c) $\left[\sqrt{13.9068},\ \sqrt{33.4706}\right] = [3.7292,\ 5.7854]$
We are 90% confident that σ is between 3.7292 and 5.7854.

(d) From the sample data, we calculated $\bar{x} = \frac{1}{30}\sum_{i=1}^{30} x_i = 8.62$.
we already have $S = \sqrt{20.4286} = 4.5198$. Then, we have,
$\bar{x} \pm 2S = 8.62 \pm 2 \times 4.52 = [0, 17.66]$

yes, we are 95% confident that the surround luminance is less than 17.66, so a value of 18 would be unusual.

4. $n = 14$. Hence the degree of freedom on χ^2 is 13.
From the sample data, we calculate $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2 = 0.0105$
From the probability table of χ^2, we can read out: $\chi^2_{0.95} = 5.89$.
Then, the 95% one-sided confidence interval of $\sigma^2$is:
$L = \frac{(n-1)S^2}{\chi^2_{0.95}} = \frac{13\times 0.0105}{5.89} = 0.023$

Hence, we are 95% confident that the true variance of the length of 63 mm nails is less than .023.

6. The mean and variance of a chi-squared random variable with γ degrees of freedom are γ and 2γ, respectively. Therefore, since $\frac{(n-1)S^2}{\sigma^2}$ has a chi-squared distribution with n – 1 degrees of freedom, its mean and variance are n – 1 and 2(n – 1), respectively.

So, $\mathrm{E}\left[\frac{(n-1)S^2}{\sigma^2}\right]=n-1 \;\Rightarrow\; \frac{n-1}{\sigma^2}\mathrm{E}\left[S^2\right]=n-1 \;\Rightarrow\; \mathrm{E}\left[S^2\right]=\sigma^2$

$$Var\left[\frac{(n-1)S^2}{\sigma^2}\right] = 2(n-1) \Rightarrow \frac{(n-1)^2}{\sigma^4}Var[S^2] = 2(n-1)$$

$$\Rightarrow Var[S^2] = \frac{2\sigma^4}{n-1}$$

9. (a) 1.86
(b) -1.86
(c) -2.179
(d) 2.179
(e) 1.645
(f) 1.645
(g) 1.708
(h) 2.060
(i) 1.753
(j) 1.325
(k) 1.746
(l) 1.310