Solution of HW3

Chapter 3

9. Let X denotes the number of computer systems operable at the launch time. The values of X can be from $\{0,1,2,3\}$.

(a) The probability density table of X is:

x	0	1	2	3
f(x)	$(1-0.9)^3$	$3(0.9)(1-0.9)^2$	$3(0.9)^2(1-0.9)$	$(0.9)^3$

(b) When $x = 0,\ k(0) = \binom{3}{0} = 1$.

When $x = 1,\ k(1) = \binom{3}{1} = 3$

When $x = 2,\ k(2) = \binom{3}{2} = 3$

When $x = 3,\ k(3) = \binom{3}{3} = 1$

(c) The cumulative distribution table of X ($F(x) = P[X \le x]$) is:

x	0	1	2	3
F(x)	0.001	0.028	0.271	1.00

(d) The probability that at least one system is operable at launch time is:
$P[X \ge 1] = 1 - P[X \le 0] = 1 - 0.001 = 0.999$

(e) The probability that at most one system is operable at launch time is:
$P[X \le 1] = 0.028$

11. Let X be the number of times per day that a specific machine is stopped. The density function of X is:

$$f(x) = \left(\frac{16}{31}\right)\left(\frac{1}{2}\right)^x, \; x = 0,1,2,3,4$$

(a) The density table is:

x	0	1	2	3	4
f(x)	16/31	8/31	4/31	2/31	1/31

It is easy to verify that all $f(x)$ in the above table add up to 1

(b) If $x < 0, \; f(x) = 0.$ Hence, $F(x) = 0.$
(c) If $x > 4, \; f(x) = 0.$ Hence, $F(x) = F(4) = 1.$

12. Let X represent the number of loose rivets found per 10 feet beam on bridge over 20 years old.

(a) since, $F(x_0) = \sum_{x \le x_0} f(x)$. The following density table is calculated from the cumulative distribution table:

x	0	1	2	3	4	5	6
f(x)	.05	.10	.20	.30	.20	.10	.05

(b) $x = 0, \; f(0) = \dfrac{4 - |0-3|}{20} = 0.05$

$$x = 1, \; f(1) = \frac{6 - 2 \times |1-3|}{20} = 0.10$$

$$x = 2, \; f(2) = \frac{6 - 2 \times |2-3|}{20} = 0.20$$

$$x = 3, \; f(3) = \frac{6 - 2 \times |3-3|}{20} = 0.30$$

$$x = 4, \; f(4) = \frac{6 - 2 \times |4-3|}{20} = 0.20$$

$$x = 5,\ f(5) = \frac{6 - 2\times|5-3|}{20} = 0.10$$

$$x = 6,\ f(6) = \frac{4 - |6-3|}{20} = 0.05$$

This verified that $f(x) = \frac{4 - |x-3|}{20}, x = 0,6$ and $f(x) = \frac{6 - 2\times|x-3|}{20}, x = 1,2,3,4,5$

14. Let X represent the number of grafts that fail.

(a) $E[X] = \sum_x xf(x)$

$E\left[X\right]$ = 0(.7) + 1(.2) + 2(.05) + 3(.03) + 4(.01) + 5(.01) = .48

(b) μ_X = $E\left[X\right]$ = .48

(c) $E[X^2] = \sum_x x^2 f(x)$

$E\left[X^2\right]$ = $0^2(.7) + 1^2(.2) + 2^2(.05) + 3^2(.03) + 4^2(.01) + .5^2(.01)$ = 1.08

(d) $VarX$ = $E\left[X^2\right] - \left(E\left[X\right]\right)^2$ = .8496

(e) σ_X^2 = $VarX$ = .8496

(f) $\sigma_X = \sqrt{.8496}$ = .9217

(g) grafts that fail

16. Let X denotes the number of computer systems operable at the launch time. The density table of X is:

x	0	1	2	3
f(x)	.001	.027	.243	.729

$E\left[X\right]$ = (0(.001) + 1(.027) + 2(.243) + 3(.729) = 2.7

$VarX$ = $E\left[X^2\right] - \left(E\left[X\right]\right)^2$ = 7.56 – $(2.7)^2$ = .27

$E\left[X\right]$ = (n)(p) = (3)(.9) and $VarX$ = (n)(p)(1-p) = (3)(.9)(.1)

21. (a) $E[3X+Y-8]=3E[X]+E[Y]-8=3\times 3+10-8=11$

(b) $E[2X-3Y+7]=2E[X]-3E[Y]+7=2\times 3-3\times 10+7=-17$

(c) $VarX=E[X^2]-(E[X])^2=25-9=16$

(d) $\sigma_X=\sqrt{VarX}=4$

(e) $VarY=E[Y^2]-(E[Y])^2=164-100=64$

(f) $\sigma_Y=\sqrt{VarY}=8$

(g) $Var[3X+Y-8]=9Var[X]+Var[Y]=9\times 16+64=208$

(h) $Var[2X-3Y+7]=4Var[X]+9Var[Y]=4\times 16+9\times 64=640$

(i) $E[(X-3)/4]=E[\frac{1}{4}X]-\frac{3}{4}=\frac{1}{4}\times 3-\frac{3}{4}=0$

$Var[(X-3)/4]=Var[\frac{1}{4}X]+Var[-\frac{3}{4}]=\frac{1}{16}\times 16=1$

(j) $E[(Y-10)/8]=E[\frac{1}{8}Y]-\frac{10}{8}=\frac{1}{8}\times 10-\frac{5}{4}=0$

$Var[(Y-10)/8]=Var[\frac{1}{8}Y]+Var[-\frac{5}{4}]=\frac{1}{64}\times 64=1$

(k) In general, we have

$$E\left[\frac{X-\mu_X}{\sigma_X}\right]=0, Var\left[\frac{X-\mu_X}{\sigma_X}\right]=1$$