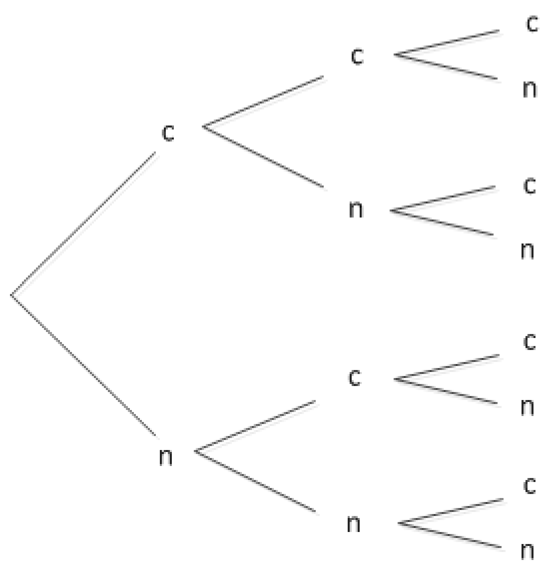
Solutions of HW1:

Chapter 1:

5.(a)



(b) $\{ccc, ccn, cnc, ncc, cnn, ncn, nnc, nnn\}$

(c)

A_1: $\{ccc, ccn, cnc, ncc, cnn, ncn, nnc\}$, "*at least one neutron is captured*

A_2: $\{ccc\}$, "*all three neutrons are captured*"

A_3: $\{nnn\}$, "*none of the three nuetrons are captured*"

(d) $A_1 \cap A_2 = \{ccc\} \neq \varnothing$, A_1 and A_2 are not mutually exclusive

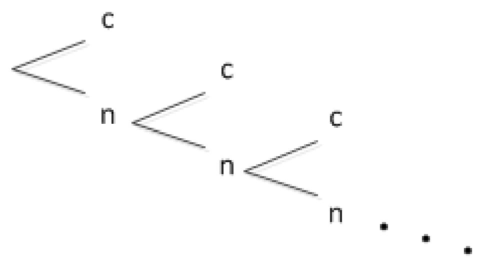
$A_1 \cap A_3 = \varnothing$, A_1 and A_3 are mutually exclusive

$A_2 \cap A_3 = \varnothing$, A_3 and A_2 are mutually exclusive

A_1 , A_2 and A_3 are not mutually exclusive

(e) No. All eight sample points are not equally likely because the probability for each neutron to be captured is different.

7. (a)



(b) No.

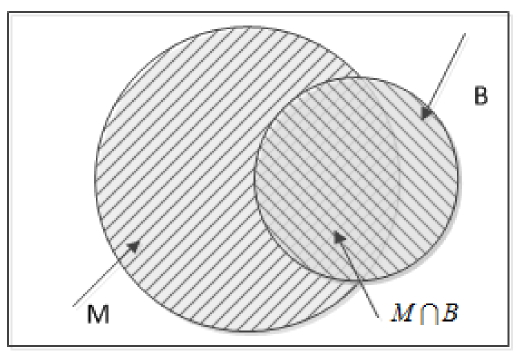
(c) {c, nc, nnc, nnnc,…}, the list will last forever.

(d) $A:\{c, nc, nnc, nnnc\}$

(e) $A_1:\{c\}, A_2:\{nc\}$ are mutually exclusive

Chapter 2

4. Let M represent the event that the main engine is operable, and B represent the event that the backup engine is operable; These events are shown in the following Vann diagram:



It is given that $P[M]=0.95; P[B]=0.80; P[M \cup B]=0.99$.

From the general addition rule, we have:

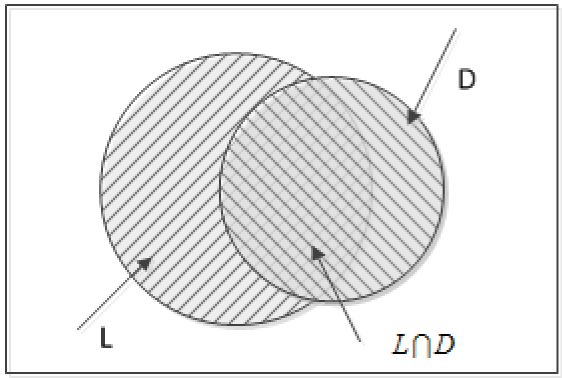
$$P[B \cap M] = .95 + .80 - .99 = .76$$

$$P[M' \cap B] = .80 - .76 = .04$$

$$P[B' \cap M] = .95 - .76 = .19$$

$$P[(M \cup B)'] = 1 - P[M \cup B] = 1 - 0.99 = 0.01$$

5. Let L represent the event that a worker is exposed to LD_{50}, and D represent the event that a worker dies. The events are shown in the following Vann diagram:



The following probabilities are given:

$$P[L \cap D] = 0.30; P[D] = 0.40; P[L \cup D] = 0.68$$

Then, using the Vann diagram, it is easy to find that:

The probability that a randomly selected worker is exposed to LD_{50} but does not die:

$$P[L \cap D^{'}] = P[L \cup D] - P[D] = 0.68 - 0.40 = 0.28$$

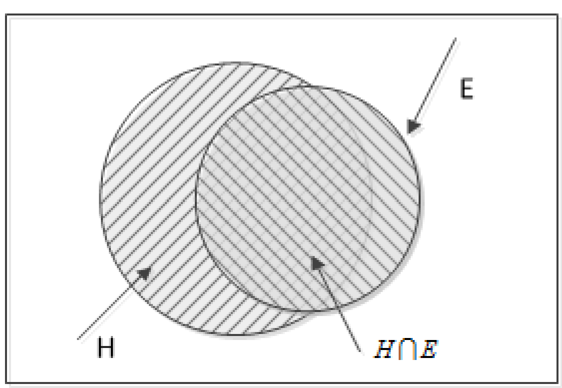
The probability that a randomly selected worker is exposed to LD_{50}:

$$P[L] = P[L \cap D'] + P[L \cap D] = 0.28 + 0.30 = 0.58$$

The probability that a randomly selected worker is not exposed to LD_{50} but dies:

$$P[L' \cap D] = P[D] - P[L \cap D] = 0.40 - 0.30 = 0.10$$

10. Let H represent the event that an accident involves human error, and E represents the event that an accident involves equipment failure. These events are shown in the following Vann diagram:



It is given that $P[H]=0.80; P[E]=0.40; P[H \cap E]=0.35$.

From the diagram, it is easy to see that the probability that an accident involves human error only is:

$$P[H \cap E']=P[H]-P[H \cap E]=0.80-0.35=0.45$$

12. (a) We know that $P[S]=1$. Also, $S=S \cup \varnothing$, and S and $\varnothing$ are mutually exclusive. Then, apply axiom (3), we have:

$$P[S]=P[S]+P[\varnothing]=1+P[\varnothing]=1$$

Hence, $P[\varnothing]=0$

(b) We know that $S=A+A'$ and A and A' are mutually exclusive. Then, according to axiom (3),

$$P[S]=P[A]+P[A']=1 \Rightarrow P[A']=1-P[A]$$

(c) We have, $B=A \cup (A' \cap B)$, also, A and $(A' \cap B)$ are mutually exclusive.

According to axiom (3),

$P[B]=P[A]+P[A' \cap B]$ and $P[A' \cap B] \geq 0$

Hence, $P[A] \leq P[B]$

(d) $A \subset S$. According to the results of (c), we have, $P[A] \leq P[S]=1$

i.e., the probability of event A is at most 1.

(e) According to the general addition rule, $P[A_1 \cup A_2]=P[A_1]+P[A_2]-P[A_1 \cap A_2]$

When A_1 and A_2 are mutually exclusive, i.e., $A_1 \cap A_2 = \varnothing$, we have,

$$P[A_1 \cup A_2]=P[A_1]+P[A_2]$$

This is the same as axiom (3)