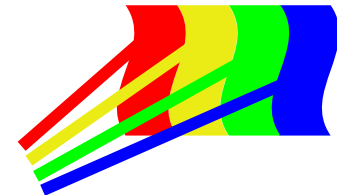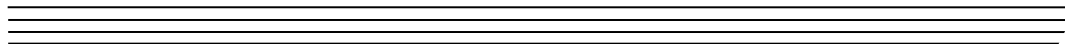# Digital Electronics
Principles & Applications
Seventh Edition

Roger L. Tokheim

Chapter 4
Using Logic Gates

INTRODUCTION

- **Logic Circuit from a Boolean expression**

- **Minterm and maxterm Boolean expressions**

- **Boolean expression from a truth table**

- **Truth table from a Boolean expression**

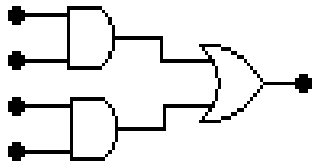- **Simplifying Boolean expressions**

- **Karnaugh mapping**

INTRODUCTION (continued)

- **Variations in Boolean expressions**

- **DeMorgan's theorem**

- **PLDs (programmable logic devices)**

"Tools of the Trade" for Solving Logic Problems

- **Gate symbols**
- **Truth tables**
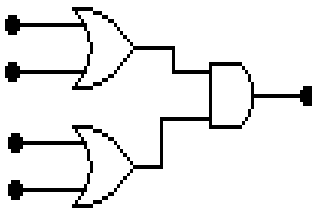- **Boolean expressions**

Combinational logic circuits:

AND-OR pattern of gates

from

Sum-of-products Boolean expression such as:

AB + CD = Y

OR-AND pattern of gates

from

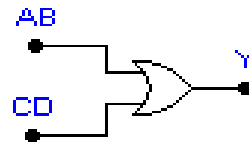Product-of-sums Boolean expression such as:

(A+B) (C+D) = Y

Logic Circuit From Boolean Expression

Example:

Draw the AND-OR logic diagram for the Boolean expression: AB + CD = Y

Step 1: OR AB with CD

Step 2: Add top AND gate
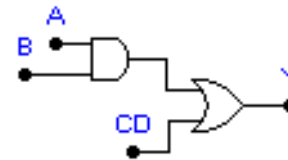
Step 3: Add bottom AND gate
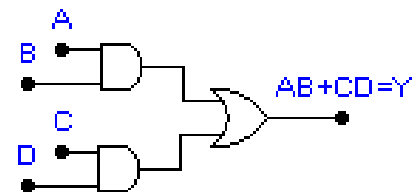
QUIZ

Draw the OR-AND logic diagram for the Boolean expression: (A+B) • (C+D) = Y

Step 1:
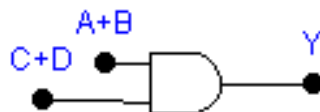
Step 2:
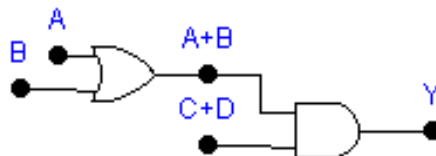
Step 3:
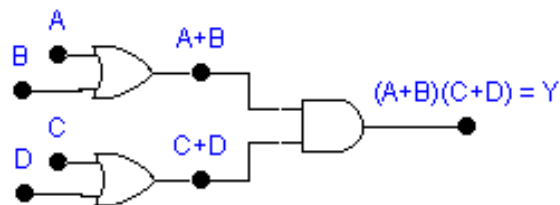
Boolean Expressions

Sum-of-products form:

$$\overline{A} \cdot B + C \cdot \overline{D} = Y$$

Also called the minterm form

Product-of-sums form:

$$(A + \overline{B}) \cdot (\overline{C} + D) = Y$$

Also called the maxterm form

QUIZ

1. The Boolean expression AB + BC = Y is in _____ (product-of-sums, sum-of-products) form.

   sum-of-products

2. The Boolean expression (A+B) (B+C) + Y is in _____ (product-of-sums, sum-of-products) form.

   product-of-sums

3. The Boolean expression (A+B) (B+C) = Y is in _____ (maxterm, minterm) form.

   maxterm

4. The Boolean expression AB + BC = Y is in _____ (maxterm, minterm) form.

   minterm

Boolean Expression from Truth Table

Write the Boolean expression that describes the logic in this truth table.

Truth Table

Input	Output
ABC	Y
0 0 0	0
0 0 1	0
0 1 0	1
0 1 1	0
1 0 0	0
1 0 1	0
1 1 0	0
1 1 1	1

Step 1: Focus only on the truth table lines with outputs of 1.

Step 2: AND the inputs for these two lines and logically OR the ANDed groups.

$$\overline{A} \cdot B \cdot \overline{C} \ + \ A \cdot B \cdot C = Y$$

Minterm Boolean expression: $\overline{A} \, B \, \overline{C} \ + \ A \, B \, C = Y$

QUIZ

Truth Table

Write the Boolean expression that describes the logic in this truth table.

Input ABC	Output Y
0 0 0	**1**
0 0 1	0
0 1 0	0
0 1 1	0
1 0 0	0
1 0 1	0
1 1 0	0
1 1 1	**1**

$$\overline{A} \cdot \overline{B} \cdot \overline{C} + A \cdot B \cdot C = Y$$

Minterm Boolean expression: $\overline{A}\,\overline{B}\,\overline{C} + A\,B\,C = Y$

QUIZ

Truth Table

Input ABC	Output Y
0 0 0	0
0 0 1	0
0 1 0	0
0 1 1	1
1 0 0	0
1 0 1	0
1 1 0	0
1 1 1	1

1. **Write the sum-of-products Boolean expression for this truth table.**

   $$A'BC + ABC = Y$$

2. **Write the minterm Boolean expression for this truth table.**

   $$A'BC + ABC = Y$$

Truth Table From Boolean Expressions

Fill in a truth table from a minterm Boolean Expression.

Minterm Boolean expression: $\overline{A} \cdot \overline{B} \cdot \overline{C} + \overline{A} \cdot B \cdot C + A \cdot B \cdot \overline{C} = Y$

Truth Table

Input ABC	Output Y
0 0 0	1
0 0 1	0
0 1 0	0
0 1 1	1
1 0 0	0
1 0 1	0
1 1 0	1
1 1 1	0

Step 1: Place three 1s in output column.

Step 2: Place five 0s in blanks in output column of truth table.

QUIZ

1. **Fill in the output column of the truth table for the Boolean expression A'B'C' + ABC' = Y.**

Truth Table

Input ABC	Output Y
0 0 0	1
0 0 1	0
0 1 0	0
0 1 1	0
1 0 0	0
1 0 1	0
1 1 0	1
1 1 1	0

Simplifying Boolean Expressions

Truth Table

Unsimplified Boolean Expression

$$\overline{A}\cdot\overline{B}\cdot\overline{C} + \overline{A}\cdot\overline{B}\cdot C + A\cdot\overline{B}\cdot\overline{C} + A\cdot B\cdot\overline{C} = Y$$

Input ABC	Output Y
0 0 0	1
0 0 1	1
0 1 0	0
0 1 1	0
1 0 0	1
1 0 1	0
1 1 0	1
1 1 1	0

This line is not to be considered in the loop.

Simplified Expression: $A\cdot\overline{C} + \overline{A}\cdot\overline{B} = Y$

RULE:

Eliminate term within loop that contains a term and its complement.

Simplify Boolean Expression
(Karnaugh map method)
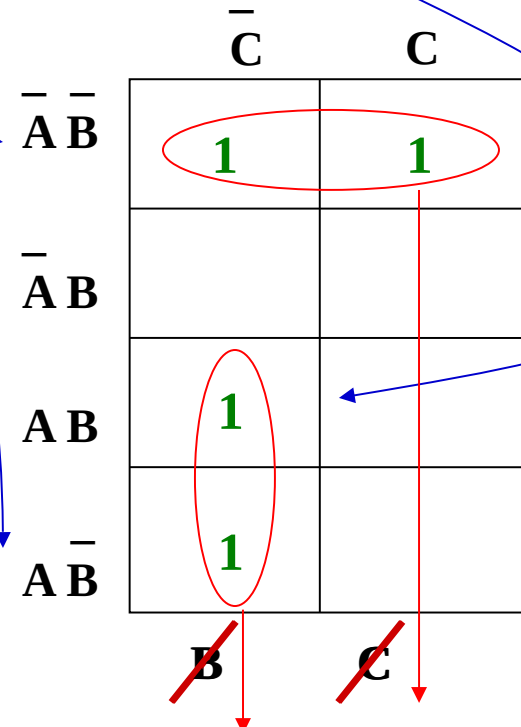
Unsimplified Boolean expression (3 variables):

$$\overline{A}\cdot\overline{B}\cdot C + \overline{A}\cdot\overline{B}\cdot\overline{C} + \overline{A}\cdot\overline{B}\cdot C + A\cdot B\cdot\overline{C} = Y$$

Step 1: Plot 1s

Step 2: Loop groups

Step 3: Eliminate variables

Step 4: Form simplified minterm expression

	$\overline{C}$	C
$\overline{A}\ \overline{B}$	1	1
$\overline{A}\ B$		
A B	1	
A $\overline{B}$	1	
	B	C

Simplified Expression: $A\,\overline{C} + \overline{A}\,\overline{B} = Y$

QUIZ

Simplify Boolean Expression (Karnaugh map method)

Unsimplified Boolean expression (4 variables):

$$\overline{A}\overline{B}C\overline{D} + \overline{A}BC\overline{D} + \overline{A}\overline{B}CD + \overline{A}\overline{B}C\overline{D} = Y$$

Step 1: Plot 1s

Step 2: Loop groups

Step 3: Eliminate variables

Step 4: Form simplified minterm expression

	$\overline{C}\,\overline{D}$	$\overline{C}\,D$	$C\,D$	$C\,\overline{D}$
$\overline{A}\,\overline{B}$			**1**	**1**
$\overline{A}\,B$				
$A\,B$	**1**			
$A\,\overline{B}$	**1**			

Simplified Expression: $A\overline{C}\overline{D} + \overline{A}\overline{B}C = Y$
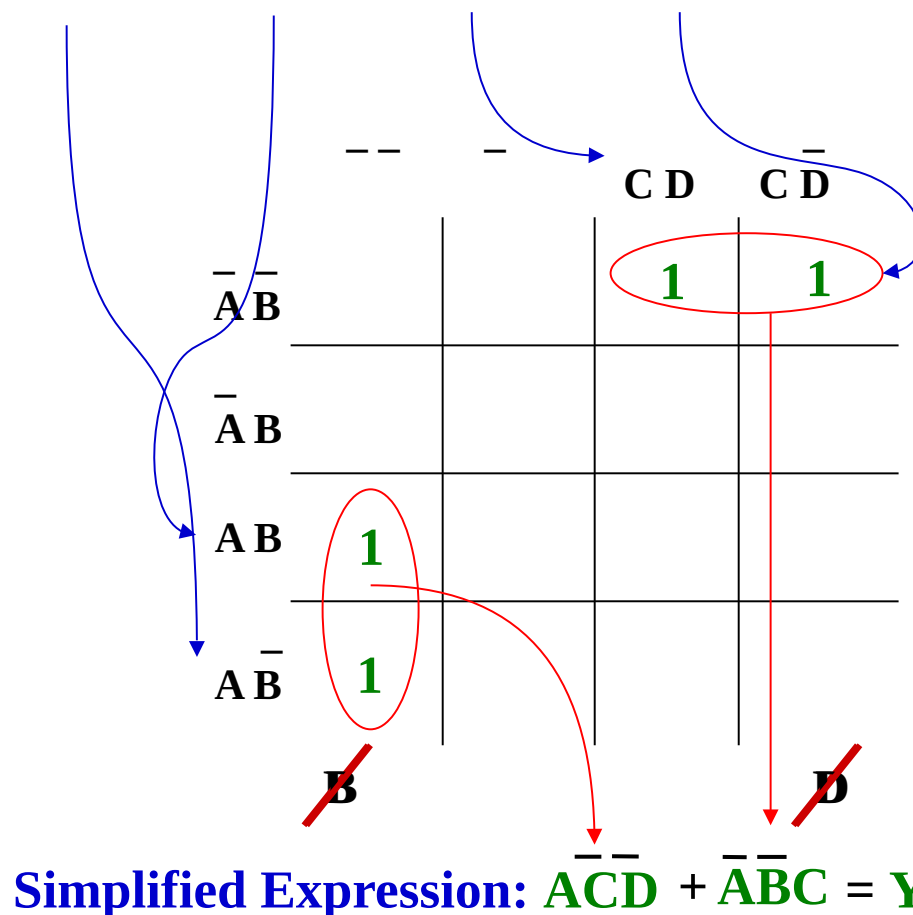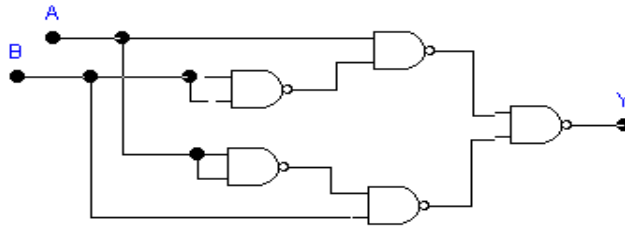
Developing a NAND Logic Diagram

Minterm expression: $\overline{A}B + A\overline{B} = Y$

Step 1:   Draw AND-OR logic diagram from minterm expression.
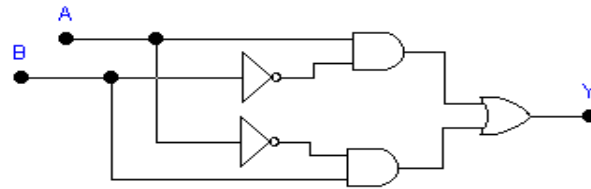
Step 2:   Substitute NAND gates for each inverter, AND, and OR gate.

NOTE: Both logic diagrams will generate the same truth table.

Variations in Boolean Expressions

Boolean expression (textbook style):

$$\overline{A} \ \overline{B} = Y$$

Boolean expression (keyboard style):

$$A' \ B' = Y$$

QUIZ

1. The Boolean expression $A\overline{B} + \overline{C}D = Y$ is written as _____ in keyboard style. $A\,B' + C'D = Y$

2. The Boolean expression $\overline{A+B+C} = Y$ is written as _____ in keyboard style. $(A+B+C)' = Y$

3. The Boolean expression $\overline{A}\;\overline{B}\;\overline{C} = Y$ is written as _____ in keyboard style. $A'\,B'\,C' = Y$

DeMorgan's Theorems

First theorem:

$$\overline{A + B} = \overline{A}\ \overline{B}$$

**Both used
to eliminate
long overbars** nd **theorem:**

$$\overline{A\ B} = \overline{A} + \overline{B}$$

DeMorgan's Theorems
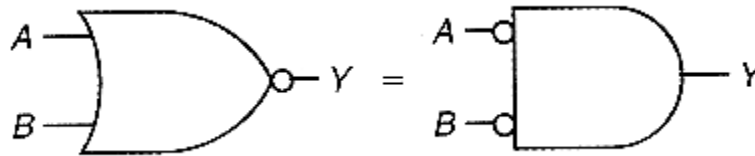
First theorem:

$$\overline{A + B} = \overline{A} \ \overline{B}$$

Second theorem:

$$\overline{A \ B} = \overline{A} + \overline{B}$$
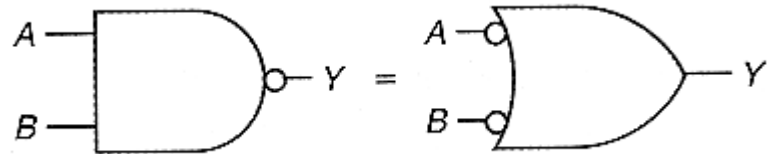
QUIZ

1. The statement $\overline{A \cdot B} = \overline{A} + \overline{B}$ is DeMorgan's
   _____ (first, second) theorem and
   suggests converting from a NAND to an OR
   situation.

   second

2. The statement $\overline{A + B} = \overline{A} \cdot \overline{B}$ is DeMorgan's
   _____ (first, second) theorem and
   suggests converting from a NOR to an AND
   situation.

   first