3x+1 Problem

Joshua Wong

February 15, 2024

The Collatz Problem, also known as the $3x + 1$ problem, is defined as:

$$c(x) = \left\{ \begin{array}{ll} x/2, & \text{if } x = 0 \pmod 2 \\ 3x+1, & \text{if } x = 1 \pmod 2 \end{array} \right\}, x \in \mathbb{N}^+$$

Or the "shortcut" version:

$$t(x) = \left\{ \begin{array}{ll} x/2, & \text{if } x = 0 \pmod 2 \\ (3x+1)/2, & \text{if } x = 1 \pmod 2 \end{array} \right\}, x \in \mathbb{N}^+$$

$$f(x) = \left\{ \begin{array}{ll} x/4, & \text{if } x = 0 \pmod 4 \\ (3x+1)/2, & \text{if } x = 2 \pmod 4 \\ (3x+1)/2, & \text{if } x = 1 \text{ or } 3 \pmod 4 \end{array} \right\}, x \in \mathbb{N}^+$$

This process will eventually reach the number 1, regardless of which positive integer is chosen initially.

1 Proving odd inputs of Problem are even

The second part of the Collatz Conjecture (3x+1) applies to every number in the set: $\{x \in \mathbb{N}^+ \mid 2x + 1\}$.
Let c be a random number from this set. $c(c) = 3(2x + 1) + 1$
Factor out the variables: $6x + 3 + 1 = 6x + 4 = 2(3x + 2)$
Since the expression does not follow the form for an odd number, $2x + 1$, it is an even number. Therefore, all odd inputs of the Collatz Conjecture are even.

2 Proving the Collatz Problem for the set 2^n

$\{x \in \mathbb{N}^+ : 2^x\}$
In this case, $c^{(log_2 x)}(x) = 1$.
Ex. $x = 2, c(2) = \frac{2}{2} = 1$
Therefore, the conjecture for the set 2^n is true.

3 Finding functions $c^2(x)$ and beyond

Since $c(x)$ is a recursive function, we can repersent it as $c(c(x))$ or $c^2(x)$ and beyond. Let's start with the case of $\{x \in \mathbb{N}^+ : 2x + 1\}$

$c^2(x) = \frac{3x+1}{2}$
Using the proof discussed earlier, as odd inputs of $c(x)$ always return a even value, we can deduce that $c^2(x) = \frac{3x+1}{2}, x \in \mathbb{N}^+ \mid 2x+1$.

4 Finding the worst case number for $3x + 1$

Given a number $\{x \in \mathbb{N}^+ : 2x + 1\}$ after $t(x) = \frac{3x+1}{2}$ will still be part of this set, over time, the number will look like this (change r to the amount of times you should run the Collatz shortcut function).

$$w^r(x) = \frac{3x+1}{2} + \sum_{i=1}^{r-1} \frac{3^i x + 3^i}{2^{(i+1)}}$$

Or:

$$w^r(x) = \left(\frac{3}{2}\right)^r * (x+1) - 1$$

Example: 31

$$w^5(31) = \frac{3x+1}{2} + \sum_{i=1}^{5-1} \frac{3^i x + 3^i}{2^{(i+1)}} = 242$$

Since 31 leads to an even number, it will not break the Collatz Conjecture.

5 Finding the average case number for $3x + 1$

Given a number $\{x \in \mathbb{N}^+ : 2x + 1\}$

6 Finding the prime factors for $3x + 1 = 2^n$

Is there a pattern when it comes to prime factors of numbers of this format? $\{x \in N^+ : \frac{2^{2^n}-1}{3}\}$

7 Finding the zero series for $3x + 1$

Find a mix of $c(x) = x/2, x \in N^+ : 2x$ and $c(x) = 3x + 1, x \in N^+ : 2x + 1$ such that

$$o^r(x) = \sum_{i=1}^{r-1} a_i = 0$$

$$w^r(x) = \frac{3x+1}{2} + \sum_{i=1}^{r-1} \frac{3^i x + 3^i}{2^{(i+1)}}$$

8 Finding the Limit of the Worst Case Function

$$\frac{3x+1}{2} + \sum_{i=1}^{\frac{\ln \frac{2(3x+2)}{3(x+1)}}{\ln \frac{3}{2}}} \frac{3^i x + 3^i}{2^{(i+1)}}$$