3x+1 Problem

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The Collatz Problem, also known as the $3x+1$ problem, is defined as:

$$c(x) = \begin{Bmatrix} x/2, & \text{if } x = 0 \pmod 2 \\ 3x+1, & \text{if } x = 1 \pmod 2 \end{Bmatrix}, x \in \mathbb{N}^+$$

Or the "shortcut" version:

$$t(x) = \begin{Bmatrix} x/2, & \text{if } x = 0 \pmod 2 \\ (3x+1)/2, & \text{if } x = 1 \pmod 2 \end{Bmatrix}, x \in \mathbb{N}^+$$

This process will eventually reach the number 1, regardless of which positive integer is chosen initially.

1 Proving odd inputs of Problem are even

The second part of the Collatz Conjecture (3x+1) applies to every number in the set: $\{x \in \mathbb{N}^+ \mid 2x+1\}$.
Let c be a random number from this set. $c(c) = 3(2x+1)+1$
Factor out the variables: $6x+3+1 = 6x+4 = 2(3x+2)$
Since the expression does not follow the form for an odd number, $2x+1$, it is an even number. Therefore, all odd inputs of the Collatz Conjecture are even.

2 Proving the Collatz Problem for the set 2^n

$\{x \in \mathbb{N}^+ : 2^x\}$
In this case, $c^{(log_2 x)}(x) = 1$.
Ex. $x = 2, c(2) = \frac{2}{2} = 1$
Therefore, the conjecture for the set 2^n is true.

3 Finding functions $c^2(x)$ and beyond

Since $c(x)$ is a recursive function, we can repersent it as $c(c(x))$ or $c^2(x)$ and beyond. Let's start with the case of $\{x \in \mathbb{N}^+ \mid 2x+1\}$
$c^2(x) = \frac{3x+1}{2}$
Using the proof discussed earlier, as odd inputs of $c(x)$ always return a even value, we can deduce that $c^2(x) = \frac{3x+1}{2}, x \in \mathbb{N}^+ \mid 2x+1$.

4 Finding the worst case number for $3x + 1$

Given a number $\{x \in \mathbb{N}^+ : 2x + 1\}$ after $t(x) = \frac{3x+1}{2}$ will still be part of this set, over time, the number will look like this (change infinity to one less times you should run the Collatz shortcut function).

$$\frac{3x+1}{2} + \sum_{i=1}^{\infty} \frac{3^i x + 3^i}{2^{(i+1)}}$$

Example: 31

$$\frac{3x+1}{2} + \sum_{i=1}^{5-1} \frac{3^i x + 3^i}{2^{(i+1)}} = 242$$

Since this leads to an even number, this does not break the Collatz Conjecture.