

1 Objectives

The objective for this lab was to build a quantizer for PCM and creating a Delta modulator in MATLAB. The secondary objective is to understand how to quantify the quantization noise of a PCM signal and the granulation noise of a delta modulated signal.

2 Procedure

The MATLAB code that was used in the lab is found in Appendix A. Supplementary functions are found in Appendix B.

2.1 PCM

We began defining a signal that will be Pulse Code Modulated. The signal that was used was defined as:

$$y(t) = \sin(2\pi t) + \sin(4\pi t) + \sin(5\pi t) + \sin(9\pi t) + \sin(10\pi t) + \sin(24\pi t)$$

The time interval was from 0 to 1 with a time step of 0.001.

The signal was quantized using the custom function `quantized sample`. The variable `quantized data` holds integer values that represent the level of the signal. These values would normally be encoded into a binary stream and then transmitted but for the purpose of this lab that wouldn't be necessary. The resulting data was reconstructed using the custom function `reconstructed data`.

2.2 Delta Modulation

The signal that was used for this portion of the lab was defined as:

$$e^{-2t^2} \cos(\pi t)$$

with a time interval from -3 to 3 with a time step of 0.01.

The optimal step size was calculated using:

$$E = \frac{\dot{m}(t)\omega}{f_s}$$

3 Results

3.1 PCM

The following figures are the results of the MATLAB code.

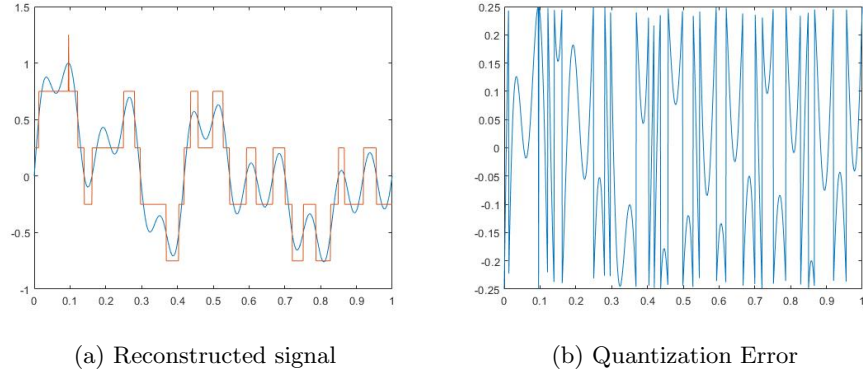


Figure 1: Signal Quantized with 4 Levels

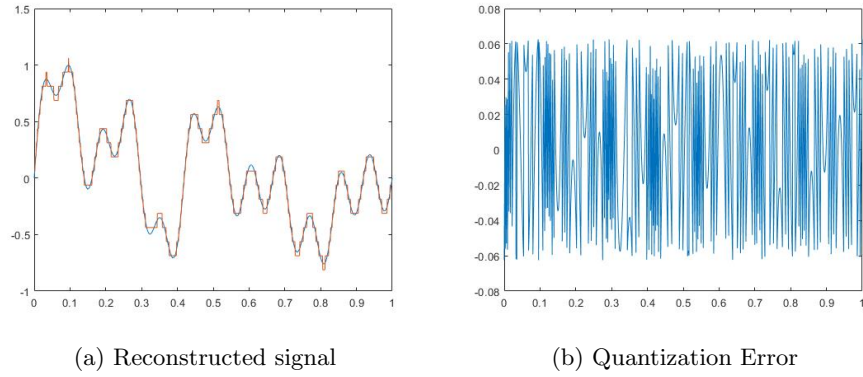


Figure 2: Signal Quantized with 16 Levels

The equation used to calculate the theoretical error is:

$$\epsilon = \frac{\hat{m}(t)^2}{3L}$$

The theoretical error that was calculated for a normalized, quantized signal with 4 levels was 0.0208.

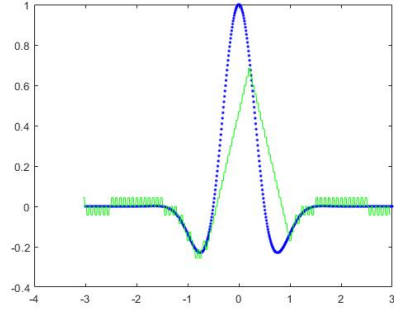
The measured error was found to be 0.0205

The theoretical error that was calculated for a normalized, quantized signal with 16 levels was 0.0013

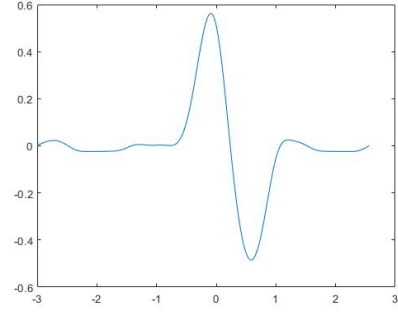
The measured error was found to be 0.0013

3.2 Delta Modulation

The following figures are the results from the Delta Modulation portion of the label

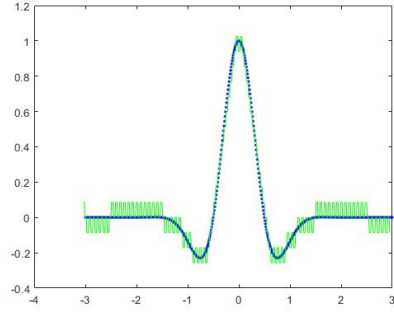


(a) Reconstructed signal

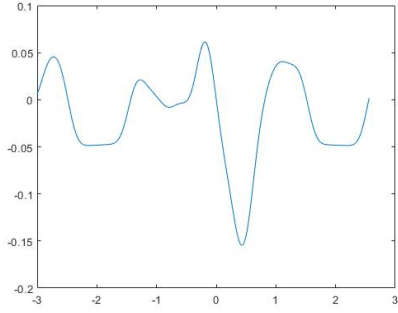


(b) Granulation Error

Figure 3: Signal Sampled with $E = 0.5E_0$ Levels

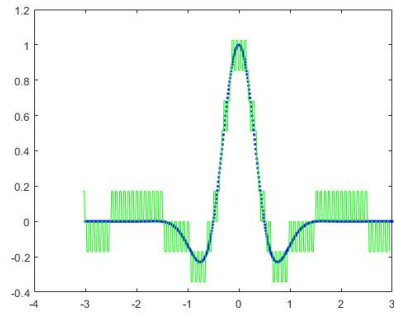


(a) Reconstructed signal

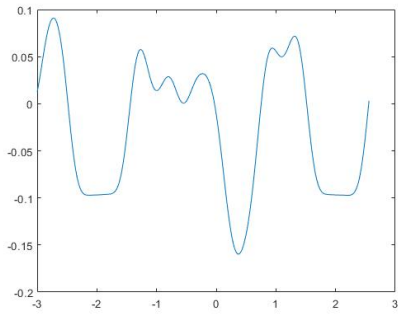


(b) Granulation Error

Figure 4: Signal Sampled with $E = E_0$ Levels



(a) Reconstructed signal



(b) Granulation Error

Figure 5: Signal Sampled with $E = 2E_0$ Levels

A MATLAB Code

```
t = 0:0.001:1;
fs = 1000;
L = 4;
signal = sin(t* 2 * pi) + sin(t* 4 * pi) + sin(t* 5 * pi)...
        + sin(t* 9 * pi) + sin(t* 10 * pi) + sin(t* 24 * pi);
signal = signal/max(abs(signal));
quantizedData = quantizedSample(signal, L);

reconstructedData = reconstructQuantized(max(abs(signal)),...
                                         L, quantizedData);

plot(t, signal, t, reconstructedData);

q = signal - reconstructedData;
theoreticalError = max(abs(signal)).^2/(3*L^2)
measuredError = mean(q.^2)
var(q)
plot(t, q);

fs= 100*10;
t = -3:1/fs:3;
signal = exp(-2*t.^2).*cos(pi*t);

sampleRatio = 4;

optimalStepSize = pi * max(myDerivative(signal,1/fs))*max(signal)/fs;

[delSignal,estime] = delQuantization(optimalStepSize, signal, fs,...
                                     fs/sampleRatio);
recon1 = cumulativeSum(delSignal);
resampled = upsample(delSignal, sampleRatio);

resampled = cumulativeSum(resampled * optimalStepSize);

test1 = cumulativeSum(delSignal * optimalStepSize);
test2 = cumulativeSum(delSignal) * optimalStepSize;
resampled = resampled(1:length(t));
plot(t, signal, 'b.', t - 5/fs, resampled, 'g');

filtered = filter(DelFilter(fs),[1],resampled);
offset = 44;
plot(t, signal, 'b.', t(1:end-offset), filtered(1+offset:end), 'g');

shift = 44;
granError = signal(1:end-shift) - filtered(1 + shift:end);
plot(t(1:end - shift), granError);
```

B Supplementary Code

B.1 quantizedSample

```
function data = quantizedSample(data, numSamples)
maxValue = max(data)*2;
data = data * numSamples / maxValue;
data = floor(data);
end
```

B.2 reconstructQuantized

```
function data = reconstructQuantized(maxValue, numLevels, data)
    levelValue = maxValue * 2 / numLevels;
    data = data*levelValue + levelValue / 2;
end
```

B.3 myDerivative

```
function dataout = myDerivative(dataIn, samplePeriod);
dataout = (dataIn(2:end) - dataIn(1:end-1))/samplePeriod;
end
```

B.4 delQuantization

```
function [data,value] = delQuantization(stepSize, signal,...
    signalSampleRate, newSampleRate)

    signal = downsample(signal, ceil(signalSampleRate../
        newSampleRate));
    data = zeros(1,length(signal));
    error = zeros(1,length(signal));
    value = zeros(1,length(signal));
    data(1) = 1;
    value(1) = stepSize;
    for i = 2:length(signal)
        error(i) = signal(i-1) - value(i-1);
        if(error(i) > 0)
            data(i) = 1;
        else
            data(i) = -1;
        end
        value(i) = value(i-1) + data(i) * stepSize;
    end
end
```

B.5 cumulativeSum

```
function dataOut = cumulativeSum(dataIn)
    dataOut = zeros(1,length(dataIn));
    dataOut(1) = dataIn(1);
    for i = 2:length(dataIn)
```

```

        dataOut(i) = dataOut(i-1) + dataIn(i);
    end
end

```

B.6 DelFilter

```

function H = DelFilter(Fs)
%DELFILTER Returns a discrete-time filter object.

% MATLAB Code
% Generated by MATLAB(R) 9.4 and Signal Processing Toolbox 8.0.
% Generated on: 06-Mar-2019 20:53:25

% Equiripple Lowpass filter designed using the FIRPM function.

% All frequency values are in Hz.
%Fs = 100; % Sampling Frequency

Fpass = 1;           % Passband Frequency
Fstop = 4;           % Stopband Frequency
Dpass = 0.057501127785; % Passband Ripple
Dstop = 0.0001;      % Stopband Attenuation
dens = 20;           % Density Factor

% Calculate the order from the parameters using FIRPMORD.
[N, Fo, Ao, W] = firpmord([Fpass, Fstop]/(Fs/2), [1 0], [Dpass, Dstop]);

% Calculate the coefficients using the FIRPM function.
b = firpm(N, Fo, Ao, W, {dens});
Hd = dfilt.dffir(b);
H = Hd.Numerator;
% [EOF]

```