3.1

Сложение

A: x + 0 = x

Умножение
C: x * 0 = 0
D: x * s(y) = (x * y) + x

Удвоение
E: d(0) = 0

Доказать: Число в степени единицы равно самому числу (x^s(0) = x)

1. x * s(0) = x
P(x) = (x * s(0) = x)
P(0) = (0 * s(0) = 0)

Base induction:
Left part: 0 * s(0) = [D:x=0, y=0] = (0 * 0) + 0 = [A:x=0*0] = 0*0 = [C:x=0] = 0
Right part: 0 = [E] = 0
Induction: P(x) => P(s(x))
P(s(x)) = (s(x) * s(0) = s(x))
Left part: s(x) * s(0) = [D:x=s(x), y=0] = (s(x) * 0) + s(x) = [C:x=s(x)] = 0 + s(x) = [A:x=s(x)] = s(x)
Right part: s(x)

3.2

Сложение

A: x + 0 = x
B: x + s(y) = s(x + y)

Умножение

C: x * 0 = 0
D: x * s(y) = (x * y) + x

Удвоение

E: d(0) = 0
F: d(s(x)) = s(s(d(x)))

Доказать: (x + s(y) = s(x) + y)

2. x + s(y) = s(x) + y

P(x) = (x + s(y) = s(x) + y)
(ИП) P(0) = (0 + s(y) = s(0) + y)

Base induction:
Left part: 0 + s(y) = [B:x=0, y=y] = s(0 + y) = [ИП] = s(0) + 0 + y = [A:x=y] = s(0) + y
Right part: s(0) + y

Induction: P(x) => P(s(x))

P(s(x)) = (s(x) + s(y) = s(s(x)) + y)

Left part: s(x) + s(y) = [B:x=s(x), y=y] = s(s(x) + y) = [ИП] = s(0) + s(x) + y = [B:x=s(0), y=x] = s(s(0) + x) + y = [ИП] = 0 + s(s(x)) + y = [A:x=s(s(x)) + y] = s(s(x)) + y
Right part: s(s(x)) + y